Growth Risk of Nontraded Industries and Asset Pricing

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Abstract

This paper shows that output fluctuations in nontraded industries are a central risk factor driving asset prices in all countries. This is because nontraded industries entail a growth risk that is mostly non-diversifiable, and constitute the largest component of gross domestic product (GDP) of a country. In interest rate markets, movements in the growth of industries with higher nontradability feed greater risk to the economy, and therefore, stronger downward pressure on the interest rate. Empirically, the effect of an industry's growth volatility on the interest rate increases significantly with its nontradability. In currency markets, this risk factor generates carry trade profits because it induces co-movement of the investor's marginal utility and the exchange rate. Empirically, a carry trade strategy employing currency portfolios sorted on nontraded output growth volatility earns a sizable mean return and Sharpe ratio for US investors. Trade frictions do not alter these mechanisms, although incomplete markets may reverse carry trade profits.

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1 Introduction

The rational theory and practice of asset pricing center around three fundamental principles: the tradeoff between risk and return, diversification, and no arbitrage. Movements in an economy's nontraded-sector output should play a key role in the determination of domestic asset prices and their differentials across economies, because these are risks that are not easily diversified even in an arbitrage-free international market. This paper shows that the nontraded output growth risk is indeed an important determinant of international asset prices. We adopt a canonical consumption-based exchange economy setting, with multiple countries, multiple traded and nontraded goods, trade costs, and with either complete or incomplete financial markets. A new feature of our model centers on its ability to accommodate *partially traded* goods and services as they actually are in reality. This property allows us to estimate the effects of nontraded output risk that are robust to the possible classification errors in macro data employed. We verify new implications of nontraded output growth risk for the interest rates and carry trade returns using data from the Organisation for Economic Co-operation and Development (OECD) economies.

The main insight of this paper is that the nontradability of an output amplifies the impact of its growth risk on the host economy. From this insight follow all our key conceptual results, which are also verified empirically in the paper. First, at the country level, the fluctuations in gross domestic product (GDP) growth of less open-to-trade economies pose greater risk, incite higher precautionary savings motives, and thus induce relatively lower home interest rates in the cross section of economies. Second, at the industry level, the fluctuations in the output growth of less traded industries also place stronger downward pressure on interest rates. Third, in the currency market, the carry trade strategies that expose investors to larger nontraded output growth risk offer higher returns on average. Fourth, the nontraded output growth risk regulates consumption allocation, moves investors' marginal utility and exchange rates in the same direction, breaks the uncovered interest rate parity, and generates currency forward premia. In contrast, *country-specific* traded output growth risk is much less prominent, because it is subject to diversification via international trades.

The nontraded sector produces goods and services that cannot be consumed outside of the home country. It includes wholesale and retail trade, hotels and restaurants, real estate, financial intermediation, and business activities. Two stylized features of nontraded output stand out.

First, nontraded outputs feed the lion's share to the GDP and national aggregate consumption in all countries. Figure 1 shows that the ratio of real nontraded output over GDP is substantial among the OECD economies, ranging from 0.5 for Iceland to 0.7 for the United States (US). Second, the *tradabilities*, measured as the ratio of total import plus export over output, of key nontraded industries are indeed very low. In particular, Table 1 shows that the tradabilities in Financial Services, Construction Services, and Other Services rarely exceed 5% across a host of countries. These stylized facts imply that the nontraded output growth volatilities should pose a major source of risk to national economies which should be reflected in the level of domestic interest rates, stock market returns, and real exchange rates. Structurally, the primary force underlying these relationships is the precautionary savings motives of market participants.

The case of Japan illustrates the insight of nontraded output growth risk. Japan's low real interest rate and the yen's status as a favorite choice for the short currency leg in profitable carry trade strategies are well-known and perplexing issues in international finance. Interestingly, these facts fit neatly with the nontraded risk story proposed here. Among all OECD economies, Japan possesses, in relative terms (i) one of the largest nontraded sectors (figure 1), (ii) one of the most volatile nontraded sectors (figure 2), and (iii) the most "closed" economy in term of trade-to-GDP ratio (figure 3). All these empirical regularities suggest that the nontraded output growth risk is more severe in Japan than anywhere else in the OECD. As a result, Japanese risk-free bonds are highly valuable as a safe hedge against this country-specific risk, and therefore offer both a low yield and are a profitable asset to short in currency investment strategies. It is important to note that complete and fully integrated financial makets, both in Japan and in its trading-partner countries, are not able to eliminate the risk stemming from Japanese nontraded output fluctuations. This is because (i) the limited supply of nontraded physical output (ii) traded and nontraded goods are not perfect substitutes, and (iii) all financial contracts pay only in term of tradable goods. Hence, regardless of how perfect are the financial markets, they are not capable of creating additional supply of nontraded goods.[1]

[1] An analogy from one-country Lucas endowment economy helps illustrate the pertinent problem of limited ouput supply. Suppose in that economy a riskless bond and a stock contingent on country's aggregate output are available for trade. The financial market then is complete, yet any portfolio of bond and stock does not *at all* help to smooth country's consumption shock. Again, this is because bond and stock do not provide investors with any additional source of physical output, which in this case is the aggregate endowment. Similarly, in the current paper's international setting, nontraded outputs stay within each country and they suffer from similar limited supply problem as long as traded and nontraded goods are not perfect substitutes.

Beyond their dominant impacts at home, nontraded output fluctuations are an important source of risk because they also matter for *all* trade partners of the home country. In the rational framework of this paper, this inter-countries effect underlies the risk and profits of international investment strategies, including currency trades. The transmission of nontraded output shocks is facilitated by two distinctive mechanisms. The first is the *substitution effect*, in which countries can substitute their traded and nontraded consumptions to smooth their overall consumption over time. The second is the *trade effect*, in which a country's traded consumption adjustment influences the traded consumptions of its trade partners by the force of market clearing in traded goods. An example illustrates. Suppose country H receives a windfall of nontraded endowment, which makes nontraded goods relatively cheaper than traded goods. When H's elasticity of intertemporal substitution is lower than that of the traded-nontraded consumption substitution, as documented for many economies (see, e.g., Obstfeld and Rogoff (2001)), H reduces its traded consumption, and its trade partners increase traded consumptions to clear the market and accommodate this adjustment. In other words, the nontraded output risk of a country is actually priced by trade partner countries because it influences partners' consumptions and thus their marginal utilities (or pricing kernel).

We now discuss in depth the specific implications of nontraded risk on interest rates and carry trade returns. In light of the standard precautionary savings motives, volatilities of home nontraded output, trade partners' nontraded output, and global (aggregate) traded output all act to depress home interest rates because these three types of shocks are able to perturb home consumption. However, as mentioned above, although nontraded output risk is primarily internalized, the country-specific traded risk is largely internationalized and thus neutralized in the global pool of traded goods. Consequently, nontraded output volatility should influence home interest rates more strongly than does the home-specific traded output volatility. We discuss aspects of testing this intuitive result below after rigorously formulating the concept of (partial) tradability.

Nontraded output risk is an equally important factor behind carry trade profits. Why do certain currency pairs tend to generate profits, whereas others incur losses in the currency market? Let us consider a strategy of borrowing home currency and lending foreign currency. An adverse foreign nontraded shock simultaneously causes foreign currency to appreciate and home traded consumption to drop (by virtue of the substitution and trade effects mentioned above). That is, with respect to foreign nontraded risk, this strategy pays well when home investors value consumption highly, and vice versa. From the perspective of home investors, such carry trade is a good hedge against foreign nontraded output shocks, and it commands low, possibly negative, expected return to home investors with respect to this risk. By a similar argument, the same carry trade is not a good hedge against home nontraded output growth risk, and thus commands high expected returns to home investors in that regard. The overall expected profit (or loss) of the carry trade is determined by whether home (or foreign) nontraded output growth risk dominates in this process. More specifically, when home nontraded output sector is sufficiently more volatile than that of the foreign trade partner, shorting home and longing foreign currency tend to generate positive expected returns to compensate home investors[2] for bearing the dominating home nontraded risk embedded in the carry trades, and vice versa.

Nontraded output risk then presents a rational cause behind the violation of uncovered interest rate parity (UIP), i.e., the empirical regularity in which increasing-interest-rate currencies tend to appreciate. Lustig and Verdelhan (2007) document that the exchange rates (with respect to the US dollar) of high-interest-rate currencies tend to positively correlate with US consumption growth, and therefore longing high-interest-rate foreign currency and shorting US dollars pose a risk to US investors. These authors consequently attribute this positive correlation pattern to a force that breaks UIP. Movements in nontraded output sectors offer a natural way to rationalize this positive correlation. In our setting, countries having stable nontraded output sectors tend to be associated with high-interest-rate currencies. Thus, for the carry trades that pair US dollars with these currencies, US nontraded output risk dominates its foreign counterpart. As explained above, the dominating US nontraded output shocks generate both a positive correlation between endowment rates and US consumption growth, as well as positive expected profits for the respective carry trade. In contrast, US nontraded output risk does not dominate the carry trade formed between US dollars and low-interest-rate currencies, and as a result these carry trades are not profitable to US investors in the expectation.

In this paper, we devised empirical tests for the effects of nontraded growth risk on interest rates and carry trade returns for OECD economies. The first test concerned interest rates and output growth risk at the industry level. We regressed real interest rates on output growth volatilities of various industries, their tradabilities, and the interaction term, while controlling for other variables.

[2]Carry trade profits to home investors are determined after the carry trade proceeds are converted back into home currency.

Table 5 shows that across OECD economies and on average, the effect of output growth risk on real interest rates increases by 12% when the output's classification moves from traded to nontraded. Another test showed a similar result; the volatility of GDP has greater effect on home interest rates when the economy is less open to trades (i.e., having lower ratio of national trade over GDP). The next test concerned profits of investment strategies in currency markets. In particular, sorting currencies based on nontraded output risk and forming carry trade strategies accordingly yield sizable mean returns. Figure 4 shows that the long-short strategy on currency portfolios sorted on the volatility of nontraded output growth earns US investors a mean annual real return of almost 3%, and Sharpe ratio of around 20%. Though these strategies are not as profitable as the investment strategy in the US equity index,[3] this figure clearly demonstrates the consistency of the nontraded output risk rationale with the carry trade profits.

Our analysis naturally suggests two-factor pricing model for each country. The factors are nontraded and traded *consumption* growths. We note that in the current setting of exchange economies, the nontraded output is essentially the nontraded consumption and thus is largely internalized within the country. Consequently, shocks in nontraded consumption are always perceived as risk and the corresponding factor price is unambiguously positive.[4] Using carry trade portfolios as test assets and two different data sets, a two-stage GMM procedure gave a statistically significant positive estimate of 32 basis points for nontraded consumption factor price, from the US investors' perspective.

We extended our theoretical analysis to the incomplete asset market setting, where financial assets that are contingent on the nontraded outputs of certain (emerging) economies are not marketable and thus absent from markets. In this incomplete financial market, the nontraded output risk originating from developed economies can still be shared quite efficiently. However, nontraded risk from emerging countries' cannot be shared optimally because of the absence of appropriate assets contingent on these countries' nontraded outputs. In the pooling equilibrium, countries choose to spread this risk evenly within the group of developed countries, and within the group of emerging countries (although not evenly across these two groups). As a result, in the pooling equilibrium,

[3]Based on historical data, the strategy of longing S&P500 index earns real return of 7% and Sharpe ratio of 40% approximately, see e.g., Mehra and Prescott (2008).

[4]In contrast, movements in home traded *consumption* are not necessarily a risk factor to home investors because this consumption is endogenous in the model. Consequently, the factor price associated with traded consumption growth volatility is not necessarily positive.

all of the above results concerning the effects of developed economies' nontraded output risk on other developed economies remain qualitatively intact. However, the effects of nontraded output shocks from emerging economies on other economies are much weaker (because of pooling), or are even reversed, compared to those obtained in the basic setting. To illustrate, a positive shock in an emerging economy's nontraded sector may *decrease* the traded consumption at home *and* in other emerging countries. Consequently, we expect that UIP violation to be more pronounced among currency pairs of developed economies. Bansal and Dahlquist (2000) empirically observe this asymmetry in a mixed data set of developed and emerging economies. In retrospect, the mechanism of an incomplete market thus lends theoretical support to their findings.[5]

The current paper contributes to an important asset pricing literature that attempts to pin down the determinants of asset returns.[6] Different factors have been proposed and found to have statistically significant power in pricing assets in different markets. Nevertheless, many of them are ad-hoc factors that do not necessarily have clear economic intuitions. The nontraded output growth risk that this paper pursues is fully motivated from and thus backed by economic rationales. The concept and modeling of traded and nontraded goods have been widely employed in international economics and international trades. The current study instead brings this keen intuition of output nontradability to the pricing of financial assets. In this aspect our paper builds on the early leads of Stulz (1987), Stockman and Dellas (1989), Backus and Smith (1993), and Zapatero (1995). We extend these analyses by concentrating on the concept of partial nontradability and its dynamic role on prices, in particular the carry trade returns and the underlying risk. While the majority of models in international finance build on the simplified two-country two-good paradigm, the model of this paper works with multiple-country multiple-good setting with the possibility of incomplete financial markets, which is more realistic and promising as advocated by Pavlova and Rigobon (2010). In the presence of multiple economic players who face nontraded risk, we are able to derive explicit and identify the structural factors that contribute to the diversification benefits in both assets and goods markets. In previous literature concerning currency investment strategies, the international diversification benefits are studied mostly under the mean-variance efficiency and reduced-form perspectives, as in Burnside et al. (2008) and Campbell et al. (2010). Other international asset

[5] Bansal and Dahlquist (2000)'s empirical analysis also concern the differential of inflation level in these countries.

[6] This literature expands on the earlier influential Capital Asset Pricing Model (Lintner (1965), Mossin (1966), Sharpe (1964)), Intertemporal Capital Asset Pricing Model (Merton (1973)), Arbitrage Pricing Model (Ross (1976)), and more recent factor pricing model (Fama and French (1993)).

pricing puzzles concerning real exchange rate and stochastic discount factor movement, and possible solutions based on recursive utility (together with a long-run risk component), and habit formation are discussed in the work by Brandt et al. (2006), Colacito and Croce (2011), and Stathopoulos (2011) respectively. Closest to our paper is Hassan (2010)'s, who is the first to analyze the effect of economy's size on carry trade returns. The current paper instead focuses on the role of nontraded risk and makes clear that the economy's size only enter the international pricing dynamics under two premises; (i) size is always coupled with the nontraded output of the host economy, *and* (ii) size's influence is always transmitted by means of international trade. To illustrate, we consider two extreme cases in which we turn off completely one of these two premises: (i) all goods are traded (no nontraded goods), and (ii) all goods are nontraded (countries as isolated islands). In both cases, under the assumption that countries have homogeneous preferences, the sizes of economies do not contribute to the interest rate differentials across countries.

The paper is structured as follows. Section 2 presents the basic international asset pricing model with a single traded good and symmetric consumption tastes across countries. Section 3 analyzes interest rates and derives testable implications on the relationship between interest rates and nontraded output risk, both with and without trade frictions. Section 4 analyzes carry trade strategies and the associated returns, and derives their testable implications. Section 5 presents and develops a much more general international asset pricing model with multiple traded goods, arbitrary trade configuration and incomplete financial markets. Section 6 conducts empirical tests concerning the pricing of nontraded and traded risk in interest rates and carry trade strategies. Section 7 summarizes the main findings. Appendix A presents a short description of data and lists their original sources. Appendices B, C and D present derivations and proofs of technical results.

2 Basic model

The basic model of the world economy consists of K countries, engaged in trade with one another and with a single consumption good. Each country also has its country-specific nontraded consumption good, which can be consumed only in that country. We concentrate on the consumption risk in this paper and thus abstract our findings from production aspects of the economy. The countries are endowed with country-specific streams of these traded and respective nontraded goods.

Specifically, the endowments (or interchangeably, outputs) $\{\Delta_T^H, \Delta_N^H\}$ are stochastic and follow the country-specific general[7] diffusion processes

$$d\log\Delta_T^H = \mu_T^H dt + \sigma_T^H dZ_T^H; \qquad d\log\Delta_N^H = \mu_N^H dt + \sigma_N^H dZ_N^H; \qquad H = 1\ldots K,$$

where, throughout, the superscript H denotes the country and the subscripts T, N denote the traded and nontraded goods, respectively. In the above equations, Z_T^H and Z_N^H are standard (possibly multi-dimensional) Brownian motions characterizing the country-specific supply shocks of the traded and nontraded sectors. For simplicity, we also omit time index t whenever this omission does not create confusion. Let us first assume that the traded good is shipped without friction around the globe.[8] The market clearing mechanism then simply enforces that traded good outputs from all countries are bundled together, and only the *global* (aggregate) traded endowment Δ_T enters the dynamic

$$\Delta_T \equiv \sum_{H=1}^{K} \Delta_T^H; \qquad d\log\Delta_T \equiv \mu_T dt + \sigma_T dZ_T.$$

In this section, we also assume that investors can trade at least as many financial assets, i.e., contingent claims on these stochastic outputs and risk-free bonds denominated in countries' currencies, as needed to complete the world market. Incomplete markets are the topic of section 5.2. Each country features a representative agent who maximizes the expected utility weighted over traded and nontraded consumptions $C \equiv \{C_T, C_N\}$. It is important to note that in this representative-agent approach, individual investors in each country are assumed to be identical,[9] thus, these are consumptions *per capita*. The period utilities have the following standard form

$$U^H(C^H, t) = e^{-\rho t}\frac{(C^H)^{1-\gamma}}{1-\gamma} = e^{-\rho t}\frac{1}{1-\gamma}\left[\omega_T (C_T^H)^{1-\epsilon} + \omega_N (C_N^H)^{1-\epsilon}\right]^{\frac{1-\gamma}{1-\epsilon}}; \quad \omega_T + \omega_N = 1, \quad (1)$$

where ρ denotes the subjective discount factor. Utility is a power function of the consumption aggregator C^H, which in turn is a function of traded and nontraded consumptions with constant elasticity of substitution (CES). Countries may have different tastes $\{\omega_T, \omega_N\}$ for traded and non

[7] That is, the constant moments $\mu_N^H, \mu_T^H, \sigma_N^H, \sigma_T^H$ are not essential for the model's implication, although the geometric Brownian motion specification considerably eases the exposition.

[8] We reinstate the transportation cost in the next section.

[9] An alternative view is to normalize countries' populations to units.

traded goods to model the possible effect of home biases in consumption. Their normalization is purely conventional. In this setting, the intertemporal elasticity of consumption is $\frac{1}{\gamma}$, and the elasticity of substitution between traded and nontraded goods is $\frac{1}{\epsilon}$. They satisfy the conditions $\gamma > 0$, $\epsilon > 0$. The interaction between these two substitution effects drives many of the model's implications, as presented below.

Equilibrium consumption allocation

We consider the competitive equilibrium in which each country's representative takes prices as given and dynamically allocates consumption and savings (i.e., investment in financial assets) to maximize her expected utility subject to the budget constraint. Market clearing then consistently determines goods and assets prices. Because the market is complete, equilibrium consumption allocations across countries can be conveniently characterized by (i) formulating the world's representative agent (see Negishi (1960)), and (ii) constructing the *static* optimization scheme in which the world's representative agent maximizes her period utility subject to the aggregate resource constraint at *each* time and for each state (see Cox and Huang (1989)). As a result, the world's static optimization problem reads

$$\max_{\{C_T^H\}_{H=1}^K} \sum_{H=1}^K \Lambda^H \frac{e^{-\rho t}}{1-\gamma} \left[\omega_T (C_T^H)^{1-\epsilon} + \omega_N (\Delta_N^H)^{1-\epsilon}\right]^{\frac{1-\gamma}{1-\epsilon}} \quad \text{s.t.} \quad \sum_{H=1}^K C_T^H = \Delta_T.$$

Note that the intra-country market clearings allow us to explicitly replace the nontraded consumptions by the respective nontraded endowments. The $\{\Lambda^H\}_H^K$ are the countries' Pareto weights. Because individuals are identical within each country, Λ^H is proportional to the product of country H's populations and per-capita wealth. In other words, Λ^H is a measure of H's gross domestic product (GDP).

The law of one price indeed holds for the traded good because the marginal utilities of this good are necessarily equal across countries in equilibrium

$$\Lambda^H \frac{\partial U^H}{\partial C_T^H} = \Lambda^F \frac{\partial U^F}{\partial C_T^F} \equiv M_T \quad \forall H, F = 1 \ldots K. \tag{2}$$

In principle, these $K-1$ first-order equations together with the traded good's market clearing condition determine the K equilibrium consumptions $\{C_T^H\}_{H=1}^K$. In practice, because marginal utilities are highly nonlinear functions of consumption, the equilibrium allocation is not known in closed form. Instead, we log-linearize this world optimization problem to obtain an approximate but intuitive solution for the sake of analysis. Detailed derivations can be found in appendix B. Let the lower-case letters always denote the respective log quantities; $c \equiv \log C$, $\delta_T \equiv \log \Delta_T$, $\delta_N \equiv \log \Delta_N$. In equilibrium, the log per-capita consumptions are given by (see appendix B)

$$c_T^H = \delta_T + \frac{1}{\gamma\omega_T + \epsilon\omega_N} \left\{ -\rho t - (\gamma - \epsilon)\omega_N \left(\left[1 - \frac{\Lambda^H}{\Lambda} \right] \delta_N^H - \sum_{F \neq H}^{K} \frac{\Lambda^F}{\Lambda} \delta_N^F \right) \right\}, \tag{3}$$

where we recall that δ_T is the log *aggregate* traded output. $\Lambda \equiv \sum_{H=1}^K \Lambda^K$ is a measure of the global GDP, therefore $\frac{\Lambda^H}{\Lambda}$ the relative GDP size of countries. This consumption allocation was first obtained by Hassan (2010), who employs a different construction version involving initial wealth transfers among households. His interpretation centers on the relative GDP size, the hedging and the risk aversion effects. In contrast, we focus on various aspects of the nontraded output growth risk in each economy. In particular, we show that the size of economy matters only because it affects the ability of the host country to mitigate its own nontraded output growth risk through international trades.

First, it is reassuring that only the traded good aggregate endowment, but not their country-specific counterparts, explicitly enters the equilibrium consumption allocation. We note that this internationalization has more to do with the global market clearing in the traded good than with the risk sharing. A deeper and surprising result is that the traded output influences log consumptions uniformly across countries in the log-linearization approximation, regardless of the countries' nontraded endowments and sizes. This is an implication of the perfect sharing in traded output risk (i.e., equalized marginal utilities of traded good) and homogeneous preferences across countries.[10] For all countries, the traded consumption [11] necessarily increases with the global supply of the traded good in the current setting.

Second, when $\gamma > \epsilon$, country H's traded consumption c_T^H increases with its trade partners'

[10] The setting of heterogeneous tastes and other extensions are analyzed in section 5.

[11] The supply shock dZ in $\frac{d\Delta}{\Delta} = \mu dt + \sigma dZ$ is a shock to both endowment growth and endowment level, and the change in log per-capita consumption concerns the growth rate of the per-capita consumption level. For the sake of brevity, we simply refer to the changes in c (or δ) as changes in consumption (or endowment).

nontraded endowments δ_N^F and decreases with its own δ_N^H. The intuition is as follows. When the elasticity of substitution between traded and nontraded goods $\frac{1}{\epsilon}$ is higher than that of intertemporal substitution $\frac{1}{\gamma}$, investors are primarily concerned with smoothing consumption over time, and thus are always eager to adjust their traded-nontraded consumption composition to achieve this smoothing. As a result, traded consumptions c_T^H response strongly to nontraded supply shocks. All else being equal, in times of home nontraded surplus ($dZ_N^H > 0$), investors substitute traded consumption ($dc_T^H < 0$) with home nontraded good that has become relatively cheaper. Similarly, in times of foreign nontraded surplus ($dZ_N^F > 0$), foreign investors demand less, and home investors end up consuming more traded goods ($c_T^H > 0$) by force of global market clearing in the traded good. We accordingly make the following assumption throughout. Various empirical estimates reported in Obstfeld and Rogoff (2001) strongly support this assumption.

Assumption 1: *The elasticity of substitution between the traded and nontraded goods is higher than that of the intertemporal substitution,* $\frac{1}{\epsilon} > \frac{1}{\gamma}$.

The relationships discussed above are then quantified by the proportional coefficients

$$\frac{(\gamma - \epsilon)}{\gamma\omega_T + \epsilon\omega_N} = \frac{\frac{1}{\epsilon} - \frac{1}{\gamma}}{\frac{\omega_T}{\epsilon} + \frac{\omega_N}{\gamma}} = \alpha(\gamma - \epsilon); \qquad \alpha \equiv \frac{1}{\gamma\omega_T + \epsilon\omega_N}, \tag{4}$$

which indeed are measures of the relative difference between elasticities of consumption substitution and a weighted substitution elasticity respectively. Later, we will encounter these measures repeatedly in all generalized versions of the current setting.

Finally, in the above expression of equilibrium log consumption, the size of the economy is coupled only to the nontraded output because the traded output is fully internationalized. A more profound explanation is that trade-partner F's nontraded shock affects country H only through the sharing of the traded good. Because the variation in *per-capita* traded consumption of a larger country F projects a larger impact on the common marginal utility,[12] it is clear that a country's size amplifies its nontraded shock impact on the rest of the world.[13] However, it is equally interesting to see that country H's own nontraded shock has a smaller impact on H's log traded consumption when H is larger. This lessened impact arises because a larger country actually finds increasingly

[12] We recall that endowment and consumption are per-capita quantities, and thus the marginal utilities of traded good are equalized up to the size factor; $\frac{\Lambda^H}{\Lambda}\frac{\partial U^H}{\partial C_T^H} = \frac{\Lambda^F}{\Lambda}\frac{\partial U^F}{\partial C_T^F}$ $\forall H, F = 1 \ldots K$.

[13] This observation seems particularly germane in the situation in 2009-2010, when Europe and the United States are suffering significant downward shocks to their nontraded production.

less outside room to share traded consumption with its much smaller trade partners.[14] In the limit where $\frac{\Lambda^H}{\Lambda} \to 1$, the super economy H consumes nearly the entire global supply of traded output, which is exogenous and thus non-responsive to whatever happens to H's nontraded output.

Stochastic discount factors

In the current consumption-based setting, a country's currency (i.e., its numeraire) is its consumption basket, which is defined as the lowest-cost consumption bundle that delivers one unit of the respective country's utility. Consequently, the stochastic discount factor (SDF) that prices the assets in units of a country's numeraire is country-specific and equal to the country's marginal utility of its consumption aggregator (see appendix B); $M^H = e^{-\rho t}(C^H)^{-\gamma}$. We note especially that because these numeraires are different from the traded good, these country-specific SDFs M^H are not the same as the common marginal utility of the traded consumption $M_T = \Lambda^H \frac{\partial U^H}{\partial C_T^H}$.[15] Because in multiple-good settings, assets returns are *not* invariant with respect to numeraires, the country-specific SDFs M^H are the most appropriate choice to price country-specific assets (bonds and stocks).

The log SDF in the log-linearization approximation reads

$$\begin{aligned} m^H &= -\rho t - \gamma\omega_T\delta_T - \gamma\omega_N\left[\delta_N^H - \alpha(\gamma-\epsilon)\omega_T\delta_N^H + \alpha(\gamma-\epsilon)\omega_T\sum_F^K \frac{\Lambda^F}{\Lambda}\delta_N^F\right] \qquad (5)\\ &= -\rho t - \gamma\omega_T\delta_T - \gamma\omega_N\left[\delta_N^H - \alpha(\gamma-\epsilon)\omega_T\left(1-\frac{\Lambda^H}{\Lambda}\right)\delta_N^H + \alpha(\gamma-\epsilon)\omega_T\sum_{F\neq H}^K \frac{\Lambda^F}{\Lambda}\delta_N^F\right], \end{aligned}$$

where $\alpha \equiv (\gamma\omega_T + \epsilon\omega_N)^{-1}$ is a weighted elasticity of substitution, as defined earlier. First, the SDF of any country decreases with the global supply of the traded good. This effect occurs is because countries' traded consumptions increase with the aggregate endowment δ_T and higher consumptions reduce countries' marginal utilities. Reassuringly, δ_T enters countries' log SDF in a uniform manner because the traded good is globally shared without frictions.

Second, the home nontraded endowment δ_N^H impacts the country's SDF m^H through two chan-

[14]It has long been observed that small nations get more from and are more affected by international trade than are large countries, other factors equal. This observation adds an additional dimension to this dynamic.

[15]When we use the common marginal utility of traded consumption, $M_T = \Lambda^H \frac{\partial U^H}{\partial C_T^H}$, to price the assets, prices are in units of the traded good.

nels. As a direct effect (the first term within the square brackets), a surge in nontraded consumption (which equals δ_N^H) simply suppresses H's marginal utility and m^H. However, although H needs to consume its entire nontraded endowment, it still is able to somewhat mitigate this shock by adjusting its traded good's intake. Indeed, in equilibrium, c_N^H drops (as we have seen earlier), which boosts the marginal utility and prevents m^H from falling all the way.[16] Therefore, this mechanism is driven by the indirect effect (i.e., through trades) and gives rise to the second term within the square brackets, which is reassuringly manifested by the presence of the taste coefficient ω_T associated with the trade. Altogether, the direct effect dominates the indirect,[17] and m^H unambiguously decreases with its own nontraded supply δ_N^H.

Third, country H's SDF decreases with its trade partners' nontraded endowments δ_N^F. Again, this is a consequence of equilibrium consumption allocation and trade effect. All else being equal, a surplus in F's nontraded supply prompts country F to curb, and country H to boost, its traded consumptions. As a result, H's marginal utility and m^H fall. The dependence of a country's stochastic discount factor on its trade partner's nontraded shock is an indirect relationship that arises only through sharing in the traded good.

Finally, the global supply of traded goods impacts all SDFs uniformly when countries have homogeneous preferences. Similar to the way in which the sizes of economies affect consumption allocations, the foreign nontraded endowment δ_N^F matters more for the home SDF m^H when size Λ^F is larger. The same holds for the home country; δ_N^H has greater impact on its own SDF m^H for the larger host country H because larger countries have less outside room to outsource their own nontraded output growth risk. Furthermore, we note that the coefficient associated with δ_N^H is invariably larger in m^H than in any other m^F, the latter is simply an indirect relationship (through trades). We recapitulate these findings in the following result.

Proposition 1 *In the current setting of the world economy, although the nontraded output shock of a country is priced by all of its trade-partner economies, the home nontraded output risk is always more dominant in the home SDF m^H than it is in foreign m^F;* $\left|\frac{\partial m^H}{\partial \delta_N^H}\right| > \left|\frac{\partial m^F}{\partial \delta_N^H}\right|$.

An immediate consequence of this proposition is that either a positive home nontraded supply shock $dZ_N^H > 0$ (or an adverse foreign shock $dZ_N^F < 0$) will decrease m^H more (or increase m^F

[16]Recall that we assume $\epsilon < \gamma$, an empirically reasonable relationship among the model's parameters, throughout.

[17]We note that $1 - \alpha(\gamma - \epsilon)\omega_T \left(1 - \frac{\Lambda^H}{\Lambda}\right) = \alpha\epsilon + \alpha(\gamma - \epsilon)\omega_T \frac{\Lambda^H}{\Lambda} > 0$ for all $\gamma > \epsilon > 0$.

less) than m^F, and thus widen the SDF differential $(m^F - m^H)$, i.e., the real exchange rate (see also (10)). Therefore, the asymmetry reported in the above proposition is the key to breaking the uncovered interest rate parity (UIP) and to generating carry trade profits in the model as will be shown in more detail in section 4.

3 Interest rates

In the current multi-country and multi-goods real setting, a country H's interest rate r^H (referred to hereafter as risk-free rate or short rate) is real and defined as the instantaneous return rate of any traded asset that is risk-free with respect to H's currency (i.e., one unit of consumption basket). A conceptually familiar risk-free asset is the consumption-based zero-coupon bond that delivers with certainty one unit of country's consumption basket at maturity. Before embarking on a formal solution and analysis, intuitions suffice to suggest the key role of nontradability on the magnitude of interest rates in the current model. We study settings with either frictionless or costly trades next.

3.1 Trades without frictions

For simplicity, we first assume that traded goods can be shipped worldwide without costs. The precautionary savings effects feature prominently in all consumption-risk aspects of interest rates. All else being equal, when an economy exhibits a higher level of uncertainty, the associated bond offering a sure payoff of one consumption unit becomes more valuable and interest rates drop. However, because the country-specific traded outputs are indifferently lumped together into the global supply of traded outputs, it is this global supply (but not the country-specific supplies of traded output) that matters for every country's interest rate. The more volatile the *global* traded output, the lower interest rates in *all* countries. Thus what causes interest rates to differ across countries must be the nontraded outputs. According to this logic, the volatility of a country's aggregate output, or GDP, is not wholly compounded in the level of interest rate. Thus, the presence of nontraded goods warrants a proper decomposition of GDP into traded and nontraded components, before deciphering the role of GDP movements on the interest rate and other returns.[18]

[18]Instead, the country's aggregate consumption and its volatility remain truthful indicators of a country's interest rate.

Volatile nontraded outputs either at home or abroad act to lower home interest rates. A foreign trade partner F with volatile nontraded output transmits its volatility to home country H by consuming highly uneven amount of traded goods. The larger country F is, the stronger is this impact, and the more aggressively H's interest rate decreases with F's nontraded volatility. In contrast, the larger home country H is, the less trading room it finds to outsource its volatility to its trade partners. Consequently, although r^H decreases with own nontraded volatility, such an inverse relationship is weaker when H is larger. All of these intuitions are confirmed by a more quantitative analysis, as presented below. Formally, the interest rate r^H can be determined from the respective SDF M^H through the pricing of the risk-free bond. This bond pays one unit of country's consumption basket in infinitesimal time dt into the future, and its current price is

$$e^{-r^H dt} = E_t\left[\frac{M^H(t+dt)}{M^H(t)}\right] \Longrightarrow r^H = \frac{1}{dt}\left(-E_t\left[dm^H\right] - \frac{1}{2}\text{Var}_t\left[(dm^H)^2\right]\right),$$

where the time subscript indicates conditional moments (expectation and variance). To simplify the exposition, we assume that countries' nontraded outputs are uncorrelated with one another and with the aggregate (global) traded output. This assumption naturally formalizes the stylized premise that nontraded shocks tend to be of an idiosyncratic nature across countries. The assumption simplifies our analysis considerably by separating and hence clearly identifying the role of nontradability on asset pricing. Section 6.2 empirically investigates the merit and implications of the assumption. Using the SDF m^H obtained in (5) yields an expression for risk free rates in equilibrium

$$\begin{aligned} r^H &= \rho + \gamma\omega_T\mu_T - \frac{1}{2}\gamma^2\omega_T^2\sigma_T^2 + \alpha\gamma(\gamma-\epsilon)\omega_T\omega_N\sum_{F=1}^{K}\frac{\Lambda^F}{\Lambda}\mu_N^F - \frac{1}{2}\alpha^2\gamma^2(\gamma-\epsilon)^2\omega_T^2\omega_N^2\sum_{F=1}^{K}\frac{(\Lambda^F)^2}{(\Lambda)^2}(\sigma_N^F)^2 \\ &+ \alpha\gamma\epsilon\omega_N\mu_N^H - \frac{1}{2}\alpha^2\gamma^2\epsilon^2\omega_N^2(\sigma_N^H)^2 - \alpha^2\gamma^2\epsilon(\gamma-\epsilon)\omega_T\omega_N^2\frac{\Lambda^H}{\Lambda}(\sigma_N^H)^2. \end{aligned} \quad (6)$$

All endowment expected growth rates μ's contribute to raising risk-free rates via intertemporal consumption smoothing effect. Given a fixed EIS $\frac{1}{\gamma}$, steadily growing outputs, either at home or abroad, and in either traded or nontraded sectors, always tend to encourage investors to consume more and save less, which causes risk free rates to surge. All endowment growth volatilities σ's act to suppress risk-free rates through the precautionary savings effect, as discussed intuitively above. In particular, the term $(\sigma_N^F)^2$ clearly shows that, in pricing bond H, home investors H

are concerned with the nontraded volatility of the trade partner country F's, knowing a shock in that seemingly unrelated sector will affect the traded consumption of F, and thus H itself. All terms containing coefficients $(\gamma - \epsilon)\omega_T$ arise in traded consumption sharing where ω_T characterizes investors' affection for the traded good (trade effect) and $(\gamma - \epsilon)$ their willingness to let nontraded shocks spill over to the traded sector by substituting these two consumption goods (substitution effect).

Interestingly, the first five terms (i.e., all terms in the first line of (5)) of risk-free rates are identical across countries, and what drives wedges between countries' real interest rates must have with country-specific nontraded sectors, as anticipated earlier.[19] Apparently, both the nontraded volatility and the size of the host country affect its own interest rate. However, the size contributes only because it influences in how the host country manages to outsource its nontraded shocks to its trade partners; a larger economy internalizes more of its nontraded shocks, which makes bonds more valuable against these uncertainties and depresses its interest rate. Finally, the interest rate (6) is derived by employing country-specific consumption basket as numeraire in each country and hence is different from the one obtained by Hassan (2010), who employs the common traded consumption good as numeraire for all countries.[20] Consequently, Hassan's results truly concern carry trade returns, but not interest rate differentials. Our risk free rate expression is more appropriate in the consumption-based setting and for tests using exclusive data on interest rates, as will be shown in section 6.2.

A hypothesis concerning interest rates

All findings presented so far paint two very different pictures for the implication of traded and nontraded growth risk on risk-free rates, which warrant a rigorous empirical investigation. Below, we formulate a testable hypothesis that concerns the distinct impact of nontraded output growth risk on the level of interest rate. The actual tests, which indeed confirm the hypothesis, are presented in section 6.2. Because *country-specific* traded output risk is internationalized and diversified by

[19] The interest rate differential is

$$\Delta r \equiv r^H - r^F = \alpha\gamma\epsilon\omega_N \Delta\mu_N - \frac{1}{2}\alpha^2\gamma^2\epsilon^2\omega_N^2 \Delta(\sigma_N)^2 - \alpha^2\gamma^2\epsilon(\gamma-\epsilon)\omega_T\omega_N^2 \left(\frac{\Lambda^H}{\Lambda}(\sigma_N^H)^2 - \frac{\Lambda^F}{\Lambda}(\sigma_N^F)^2\right)$$

.

[20] In particular, country nontraded output volatilities σ_N contribute to both interest rates and their differentials as stand-alone terms (i.e., they are not necessarily coupled to economic sizes).

means of trades and aggregation, its impact on asset returns should be relatively weak, and we contend the following.

Hypothesis 1: *All else being equal, the impact of country-specific nontraded output growth risk on home interest rate dominates that of the country-specific traded output growth risk.*

The key intuition underlying this hypothesis is the diversification principle, which is directly relevant to the market for traded goods. To see this, we concentrate on the explicit contributions of country-specific traded output volatilities σ_T^H to the interest rate (i.e., omitting terms unrelated to these volatilities)[21]

$$r^H = \# - \frac{1}{2}\gamma^2\omega_T^2\sigma_T^2 = \# - \frac{1}{2}\gamma^2\omega_T^2\frac{1}{dt}\left(\sum_{H=1}^{K}\frac{\Delta_T^H}{\Delta_T}\sigma_T^H dZ_T^H\right)^2.$$

Clearly, the contribution of country-specific traded shocks dZ_T^H is suppressed by the share of a country's traded output in the world $\frac{\Delta_T^H}{\Delta_T}$. Therefore, unless (i) the traded output shock of a country correlates almost perfectly with global (i.e., aggregate) traded output, or (ii) a country's traded output absolutely dominates the global traded output, home nontraded output volatility $(\sigma_N^H)^2$ affects home interest rate r^H [22] more strongly than $(\sigma_T^H)^2$ for all countries under a mild home bias (i.e., $\omega_N > \omega_T$) condition.[23] The empirical merit of this hypothesis is verified in section 6.2.

In a related study, Tian (2011)'s notes that a country's traded consumption growth should be less volatile than the country's traded output growth due to the diversification in the traded good market. Therefore, if the country-specific traded and nontraded output growths are highly correlated and equally volatile, a country-specific positive (negative) shock to these sectors tends to decrease (increase) the domestic relative value of nontraded goods. Consequently, prices of assets contingent on traded output should be more cyclical than those contingent on nontraded output. In the data, she finds that the earnings of traded-good producers are more volatile than those of nontraded-good producers (as many as five times). This result thus provides indirect evidences for the diversification in global market for traded goods.

[21] We recall that global (aggregate) traded output is the sum of the country-specific counterparts $\Delta_T = \sum_{H=1}^{K}\Delta_T^H$, and σ_T, $\{\sigma_T^H\}$ are their growth volatilities, respectively.

[22] The impact is characterized by the coefficients associated with $(\sigma_N^H)^2$ and $(\sigma_T^H)^2$.

[23] This condition is $\alpha\left[\epsilon + (\gamma-\epsilon)\omega_T\Lambda^H/\Lambda\right] > \frac{\omega_T}{\omega_N}\frac{\Delta_T^H}{\Delta_T}$.

3.2 Costly trades

The previous section's results are derived based on two assumptions, namely, goods are either perfectly traded or nontraded, and trades are frictionless. Consequently, traded goods can be perfectly aggregated globally, which then weakens the country-specific traded output growth risk and gives rise to Hypothesis 1 above. The introduction of trade costs in this section aims to relax both of these simplifications. In particular, the concept of (partial) *tradability* arises naturally by regulating the trade friction. A traded good can become a nontraded good when trade cost is sufficiently high. The tradability is the key to bringing our model to the data in section 6.2.

To model the frictions in trades, we adopt the "iceberg transport cost" approach and analysis of Samuelson (1954), Dumas (1992) and particularly Sercu et al. (1995). In this modeling approach, the commodity trade is not perfect because only a fraction of $\frac{1}{1+\theta}$ of the original traded good that leaves the exporting country arrives at the importing country, and the remainder disappears along the way as a result of this trade friction. To simplify the exposition, we first consider a single good shared by two countries $\{H, F\}$ of similar sizes.[24] The magnitude of θ directly regulates the amount of the good being exchanged (import and export) between countries, and thus determines the *tradability* of that good.[25] With this simplified setting in place, below we focus on the effect of output shocks on interest rates mediated solely by the varying degree of trade friction, while leaving other factors untouched.

The linearity in transport costs is a key modeling advantage because it keeps market completeness intact without further assumption. Consequently, the equilibrium is obtained by solving the static world optimization subject to appropriate global resource constraints,

$$\max_{\{C_H^H, C_F^H, C_H^F, C_F^F\}} U^H(C^H) + U^F(C^F) \equiv e^{-\rho t}\left[\frac{(C_H^H + C_F^H)^{1-\gamma}}{1-\gamma} + \frac{(C_H^F + C_F^F)^{1-\gamma}}{1-\gamma}\right]$$
$$\text{s.t.} \quad C_H^H + (1+\theta)C_H^F = \Delta^H; \quad C_H^F \geq 0; \quad C_F^F + (1+\theta)C_F^H = \Delta^F; \quad C_F^H \geq 0,$$

where $C^H = \{C_H^H, C_F^H\}$ are home consumption components that originate from home and foreign outputs, respectively (the counterpart notation $C^F = \{C_H^F, C_F^F\}$ is preserved for foreign consump-

[24]It is straightforward to add the transportation costs to the setting of the previous section to have all perfectly traded, partially traded and nontraded goods. Instead, we choose to work with this simplified setting here to concentrate on the role of partial tradability.

[25]Consequently, we drop the subscripts T, N throughout this subsection.

tion components). Thus, C_F^H is the import by H, which derives from the original amount $(1+\theta)C_F^H$ exported from F. Similarly, C_H^F, which is the import by H, derives from the original amount $(1+\theta)C_H^F$ exported from H. At all time, countries desire to trade to share risk stemming from their unrelated outputs. However, the transport cost hampers risk sharing. Intuitively, if the cost outweighs the benefit of risk sharing, countries opt not to trade and instead fully internalize their endowment shock; $C_H^F = C_F^H = 0$. To determine the conditions for commodity market freezing, we assume these conditions are currently not met and that trades take place. Because the shipping incurs a cost, the imported good is always more expensive than the locally endowed good, and countries always deplete their endowed resource before reaching out to the imported resource if they need it. In other words, conditional on trades taking place, there are two mutually exclusive alternatives:

$$\textbf{case 1}\text{: H imports, F exports,} \quad C_H^H = \Delta^H; \; C_F^H > 0; \; C_H^F = 0; \; C_F^F < \Delta^F,$$

$$\textbf{case 2}\text{: H exports, F imports,} \quad C_H^H < \Delta^H; \; C_F^H = 0; \; C_H^F > 0; \; C_F^F = \Delta^F.$$

By symmetry, it suffices to study case 1, in which the two FOCs associated with non-binding constraints and the market clearing condition for the home-endowed good establish the remaining equilibrium consumption allocations (i.e., apart from the binding constraints $C_H^H = \Delta^H$, $C_H^F = 0$)

$$C_F^H = \frac{\Delta^F - (1+\theta)^{\frac{1}{\gamma}}\Delta^H}{(1+\theta) + (1+\theta)^{\frac{1}{\gamma}}}; \qquad C_F^F = \frac{(1+\theta)^{\frac{1}{\gamma}}\left[\Delta^F + (1+\theta)\Delta^H\right]}{(1+\theta) + (1+\theta)^{\frac{1}{\gamma}}}. \tag{7}$$

It is apparent that the trades require net positive home import $C_F^H > 0$ and commodity market freezes otherwise. We analyze these two regimes in turn.

No-trade regime: Combining cases 1 and 2 yields the following no-trade condition for the commodity market:

$$\text{No-trade conditions:} \qquad (1+\theta)^{-1} < \left(\frac{\Delta^H}{\Delta^F}\right)^{\gamma} < (1+\theta).$$

Clearly, costly transport (large θ), similar outputs ($\frac{\Delta^H}{\Delta^F} \approx 1$), or low risk aversion (small γ) all discourage countries to share risk, and thus enforce the commodity market freeze. In this case, the single good becomes a legitimate nontraded good in any country. Moreover, each country's bond has no hedge power against others' shocks, and the risk-free rate solely reflects the respective country's output risk, as in the consumption-based CAPM. In other words, for each country, the nontraded output volatility is the only risk that matters here.

Costly trade regime: In contrast with the no-trade regime, when friction is moderate and home and foreign outputs are sufficiently different, countries choose to share output risk, although transport costs and trade flows take place in an appropriate direction. Without loss of generality, we continue with case 1 above, in which home is the importing country (or $C_F^H > 0$). Conditional on this being the case, $(1+\theta)^{-1} > \left(\frac{\Delta^H}{\Delta^F}\right)^{\gamma}$, the home unambiguously curbs its imports when transaction cost increases (C_F^H decreases in θ).[26] However, interestingly, the inverse holds for the exporting country F for all realistic values of transport cost and risk aversion. Contingent on trades taking place, the foreign country actually boosts its export $(1+\theta)C_F^H$ when θ increases to compensate for the increasing loss in the transition.[27] This is because, when home investors are risk averse, their net import C_F^H decreases less than linearly with the transport cost.

As long as trades take place, regardless of their "iceberg-melting" imperfect nature, marginal utilities are equalized across countries ($\frac{\partial U^H}{\partial C^H} = (1+\theta)\frac{\partial U^F}{\partial C^F}$), as are the interest rates in the current setting with a single good. We concentrate on the precautionary savings effect revealed in the interest rates, in which the interplay between output shocks and transport cost dominates.

$$r^H = r^F = \# - \frac{1}{2}\gamma(\gamma+1)\frac{(1+\theta)^2(\Delta^H)^2(\sigma^H)^2 + (\Delta^F)^2(\sigma^F)^2}{[(1+\theta)\Delta^H + \Delta^F]^2}. \tag{8}$$

As the transport cost increases, interest rates become increasingly sensitive to home output shocks and decreasingly sensitive to foreign output shocks; $\frac{\partial^2|r|}{\partial\theta\partial|(\sigma^H)^2|} > 0$, $\frac{\partial^2|r|}{\partial\theta\partial|(\sigma^F)^2|} < 0$. These behaviors, when combined with the earlier findings that $\frac{\partial C_F^H}{\partial\theta} < 0$ and $\frac{\partial\left[(1+\theta)C_F^H\right]}{\partial\theta} > 0$, precisely support our key thesis that when shocks are of a more nontraded nature (i.e., θ increases), they matter more to the country's asset prices. From the importing country H's perspective, a surge in trade cost coincides with a reduction in trades as its imports C_H^F drop. At the same time, the impact of the country's own volatility σ^H on its interest rate r^H increases while the impact of foreign volatility σ^F on r^H decreases, all of which is consistent with a reduction in the import in view of the above thesis. Likewise, from the exporting country F's perspective, a surge in trade cost coincides with a

[26]This is evident from the expression of C_F^H; conditional on trade taking place ($C_F^H > 0$), the numerator decreases and the denominator increases with τ.

[27] $\frac{\partial}{\partial\theta}\left[(1+\theta)C_F^H\right] = \frac{\gamma-1}{\gamma}(1+\theta)^{\frac{1-2\gamma}{\gamma}}\Delta^F - \frac{\gamma-1}{\gamma}(1+\theta)^{\frac{2-2\gamma}{\gamma}}\Delta^H - \frac{1}{\gamma}(1+\theta)^{\frac{1-\gamma}{\gamma}}\Delta^H - \frac{1}{\gamma}(1+\theta)^{\frac{2-2\gamma}{\gamma}}\Delta^H$. For all realistic values of γ and θ, the last two terms are negligible compared with the second term. Then, the trade condition $C_F^H > 0$ immediately implies that $\frac{\partial}{\partial\theta}\left[(1+\theta)C_F^H\right] > 0$.

boost in trades as its export $(1+\theta)C_H^F$ increases. At the same time, the impact of its own volatility σ^F on its interest rate r^F decreases, whereas the impact of partner's volatility σ^H on r^F increases, which is also consistent with a surge in the export according to the above thesis.[28]

Overall, by making a realistic and smooth transition between traded and nontraded extremes of goods market, the variation in trade frictions implies a structural relationship between nontradability and domestic asset prices. The former is naturally identified as the ratio of trades (import plus export) over output. A refined version of Hypothesis 1 in section 3 is

Hypothesis 1A: *All else being equal, a country-specific output growth volatility impacts the home risk-free rate more when the output is less tradable.*

In section 6.2, we will test this hypothesis empirically by employing several measures of nontradability, including countries' trade closedness, country-specific and global nontradability at the industry level. Here, we briefly discuss the generalization of the costly trade mechanism to a setting with arbitrary K countries, where subtleties arise because the import from a country does not unambiguously originate in the export of another. In this situation, conditional on trades taking place, each country H is classified into either an importing (I) or an exporting (E) group. Let C_H^H and C_{-H}^H denote country H's consumption components derived from its own and foreign outputs, respectively. Trades take place when $\{C_H^H < \Delta^H; C_{-H}^H = 0\}$ $\forall H \in \mathcal{E}$, and $\{C_H^H = \Delta^H; C_{-H}^H > 0\}$ $\forall H \in \mathcal{I}$. Because of the ambiguity mentioned above of global import-export source matching, there is now only a single market clearing condition, and the world optimization problem reads:

$$\max_{\{C_H^H, C_{-H}^H\}} \sum_H^K e^{-\rho t} \frac{(C_H^H + C_{-H}^H)^{1-\gamma}}{1-\gamma} \quad \text{s.t.} \quad \sum_{H\in\mathcal{E}} C_H^H + (1+\theta)\sum_{H\in\mathcal{I}} C_{-H}^H = \sum_{H\in\mathcal{E}} \Delta^H.$$

Combining FOCs associated with nonbinding constraints[29] and the market clearing condition yields the equilibrium consumption allocations.[30] Subject to trades taking place, mild conditions on

[28]Obviously, the interest is in the relationship between a country's risk-free rate and its trade volume (i.e., import and export goods that arrive at or leave a country's border). In contrast, the relationship between a country's risk-free rate and its trade partner's exports and imports is not of interest because a portion of these goods is lost in the transition.

[29]These FOCs arise from the partial derivatives $\frac{\partial}{\partial C_H^H}$ $\forall H \in \mathcal{E}$ and $\frac{\partial}{\partial C_{-H}^H}$ $\forall H \in \mathcal{I}$.

[30]Conditional on trades taking place, these allocations are

$$C_{-H}^H = \frac{(1+\theta)\sum_{I\in\mathcal{I}}\Delta^I + \sum_{E\in\mathcal{E}}\Delta^E}{(1+\theta)K_I + (1+\theta)^{\frac{1}{\gamma}}K_E} - \Delta^H \ \ \forall H \in \mathcal{I}; \qquad C_H^H = \frac{(1+\theta)^{\frac{1}{\gamma}}\left[(1+\theta)\sum_{I\in\mathcal{I}}\Delta^I + \sum_{E\in\mathcal{E}}\Delta^E\right]}{(1+\theta)K_I + (1+\theta)^{\frac{1}{\gamma}}K_E} \ \ \forall H \in \mathcal{E}.$$

where K_E and K_I are the numbers of exporting and importing countries, respectively.

the distribution of trades assure that when transport cost θ increases, country H's import C_{-H}^{H} decreases and its own output volatility σ^H matters more for the domestic risk-free rate r^H.

4 Carry trade returns

The underlying risk

Let us consider the typical carry trade strategy from the perspective of country H's investors, (i) at time t borrowing risk-free one unit of base (home) currency H at rate r^H; (ii) immediately converting this into foreign currency F and lending risk-free at rate r^F; and (iii) at time $t+dt$, liquidating the long position in currency F, immediately converting the proceeds into home currency and liquidating the short position in base currency H. It is then obvious that the return on carry trade strategies is beyond the simple difference between the two interest rates involved because the former also concerns the exchange rates. As risk free rates are known at t, in our real and rational setting, the uncertainty rests entirely with the exchange rate.[31] In other words, carry trades are bets on exchange rates, and the premia associated with the short-horizon strategies are rewards for bearing the exchange rate risk.

Let S_t denote the spot exchange rate. Our convention is that S_t units of foreign currency F exchange for one unit of home currency H. In the current complete market setting,[32] this exchange rate is $S_t = \frac{M_t^H}{M_t^F}$. The realized excess return (i.e., in excess of the base interest rate r^H) to this carry trade strategy, which shorts bond H and longs bond F, and its expected counterpart, respectively, are

$$\begin{aligned} XR_{t+dt}^{-H,+F} &= \tfrac{1}{dt}\left[\tfrac{M_{t+dt}^F}{M_{t+dt}^H}(1+r_t^F dt)\tfrac{M_t^H}{M_t^F} - (1+r_t^H dt)\right], \\ E_t\left[XR_{t+dt}^{-H,+F}\right] &= -\tfrac{1}{dt}Cov_t\left[dm^H, dm^F - dm^H\right]. \end{aligned} \tag{9}$$

[31] Our settings are real. In practice, there is risk associated with inflation. When we consider short-horizon carry trade strategies, which are rebalanced once every quarter or more frequently with new available risk-free rates, inflation risk is less important in practice.

[32] To illustrate this, we examine the current price (denominated in currency H) of bond H, which delivers one unit of currency H at $t+dt$. The pricing can either be done directly in currency H or in any other currency F with the help of exchange rates. The absence of arbitrage implies the law of one price, and thus

$$E_t\left[\frac{M_{t+dt}^H}{M_t^H}\right] = \frac{1}{S_t}E_t\left[\frac{M_{t+dt}^F}{M_t^F}S_{t+dt}\right] \Rightarrow S_t = \frac{M_t^H}{M_t^F}.$$

Reassuringly, the carry trade expected excess return is the premium associated with the exchange rate risk.[33]

The consumption volatilities contribute to the expected carry trade profits precisely because they perturb both SDFs m^H, m^F. Here our discussion is readily carried over from the previous section's analysis on the SDF. Because traded shocks spread uniformly to all countries, they do not affect exchange rates, and are not counted as risk to be compensated in the carry trades. In fact, they are canceled out in the difference $dm^H - dm^F$. This leaves nontraded volatilities as the sole sources of carry trade risk and return in the current rational setting. Indeed, the log exchange rate follows a simple diffusion process implied structurally from (5) in the model

$$d\log S_t = dm^H - dm^F = \#dt + \gamma\alpha\epsilon\omega_N\left(\sigma_N^H dZ_N^H - \sigma_N^F dZ_N^F\right). \tag{10}$$

On one hand, as a result of proposition 1 above, an adverse foreign nontraded shock $dZ_N^F < 0$ makes F's nontraded good scarce and suppresses the real exchange rate S (i.e., foreign currency appreciates), and therefore $m^F - m^H$ surges. On the other hand, $dZ_N^F < 0$ also forces F to consume more and H to consume less traded goods, and m^H surges. That is, the long bet on foreign currency pays off well when home investors highly value consumption. Therefore this carry trade strategy is a good hedge against foreign nontraded risk, and it commands high price and low expected return $E_t\left[XR^{-H,+F}\right]$ in equilibrium.

In contrast, an adverse home nontraded shock $dZ_N^H < 0$ directly boosts m^H. Moreover, it also leaves its trade partner F with less traded consumptions and thus also increases m^F to a lesser extent. Consequently, $m^F - m^H$ drops because the real exchange rate S increases (i.e., home currency appreciates). That is, the long bet on foreign currency pays off poorly when home investors highly value consumption. Therefore, this carry trade strategy is *not* a good hedge against home nontraded risk, and it carries a low price tag and offers a large expected return $E_t\left[XR^{-H,+F}\right]$ to compensate for the risk it cannot hedge in equilibrium.

The overall expected profit (or loss) of the carry trade is determined by whether home (or

[33]Indeed, in a currency long bet, a promised payoff of one unit of foreign currency at $t + dt$ yields S_{t+dt}^{-1} unit of home currency also at $t + dt$. The associated consumption-based Euler equation for this bet, under the perspective of country H's investors, produces identical premia above; $-Cov_t\left[\frac{M_{t+dt}^H}{M_t^H}, dS_{t+dt}^{-1}\right] = E_t\left[XR_{t+dt}^{-H,+F}\right]$. See also footnote 35.

foreign) nontraded risk dominates, as seen quantitatively in the following result.

Proposition 2 *The expected carry trade excess return to US investors is*

$$E_t\left[XR_{t+dt}^{-H,+F}\right] = \alpha^2\gamma^2\epsilon\omega_N^2\left\{\left[\epsilon + (\gamma-\epsilon)\omega_T\frac{\Lambda^H}{\Lambda}\right](\sigma_N^H)^2 - (\gamma-\epsilon)\omega_T\frac{\Lambda^F}{\Lambda}(\sigma_N^F)^2\right\}, \qquad (11)$$

where $\alpha \equiv (\gamma\omega_T + \epsilon\omega_N)^{-1}$ *is a weighted elasticity of consumption substitution* (4). *Consequently, the carry trade strategy offers the expected profit when either home nontraded risk dominates or trade effect is weak,*

$$\left[\epsilon + (\gamma-\epsilon)\omega_T\frac{\Lambda^H}{\Lambda}\right](\sigma_N^H)^2 > (\gamma-\epsilon)\omega_T\frac{\Lambda^F}{\Lambda}(\sigma_N^F)^2.$$

The intuitions underlying this result are as follows. First, we recall that the carry trade is a good (bad) hedge against the foreign (home) nontraded output growth risk. When home nontraded risk dominates, $(\sigma_N^H)^2 \gg (\sigma_N^F)^2$, this strategy is risky and necessarily offers high expected returns $E_t\left[XR_{t+dt}^{-H,+F}\right] > 0$, and vice versa. Second, when $(\gamma-\epsilon)\omega_T$ is positive but small, investors are not enthusiastic about substituting nontraded for traded consumption goods. This weakens the trade effect and makes home nontraded output risk even worse to home investors. Therefore, in this case, carry trades are also risky and tend to generate compensating profits in the expectation. A reflection on the behaviors of risk-free rates and carry trade returns reveals that the nontraded consumption risk is a factor behind the violation of uncovered interest rate parity, a prevailing puzzle observed in the international financial market.

Uncovered interest rate parity

The uncovered interest rate parity (UIP) puzzle (a.k.a. forward premium puzzle) is an empirical regularity in which appreciating currencies tend to be also associated with increasing interest rates (Hansen and Hodrick (1980), Fama (1984)). This pattern is puzzling because it appears that the appreciating currencies are more valuable, yet investors require higher premia (i.e., interest rates) to hold them. Carry trades, i.e., borrowing low-interest-rate currencies and lending high-interest-rate currencies, are a popular strategy to reap the profit from this regularity. In the current setting, a nontraded consumption risk offers a rationale behind this profit.

When the home country has volatile nontraded sector by nature (σ_N^H large), home risk-free bonds are very valuable as a safe asset, and home interest rates are low (r^H small). At the same time, carry trades returns tend to be high because these strategies are not a good hedge against this home nontraded volatility as asserted by proposition 2. In contrast, when the foreign nontraded sector is perceived to be of low-risk nature (σ_N^F small), foreign interest rates are high (r^F large), *and* the expected carry trade return to home investors also tends to be high.[34] All in all, the nontraded output risk, originated from either home or abroad, is a culprit behind the violation of the uncovered interest parity.

Examining a large set of countries, Lustig and Verdelhan (2007) document that the exchange rates (base currency being US dollar) of high interest rate currencies tend to positively correlate with the US's consumption growth. The study clearly identifies the interrelationship of the exchange rate risk and the consumption risk as the source of the currency bet's expected profits. Namely, the carry trades of selling US dollar and buying high interest rate currencies are risky to US investors because they pay poorly (i.e., foreign currencies depreciate) when investors value consumption the most (i.e., US consumption drops). Our investigation carries this line of rational reasoning a step further by explaining the positive correlation between home consumption growth and exchange rates, as observed for US by Lustig and Verdelhan (2007); it is the nontraded output risk that can not only perturb the two quantities but also push them in the same direction.

Whereas our analysis lends support for the widely-practiced carry trade strategy of shorting low-interest rate currencies and longing high-interest rate currencies, it also suggests the following novel currency bet, which is directly tied to the nontradability aspects of consumption risk. We examine empirically the merits of this macro-based strategy in section 6.3.

Hypothesis 2: *Borrowing currencies of countries with a volatile nontraded sector and lending currencies of countries with a stable nontraded sector generate positive expected returns.*

Linear factor analysis: Theory

Our finding that country-specific traded and nontraded shocks are priced very differently by the international market warrants a simple linear-factor pricing model in which the risk factors are

[34] See proposition 2. Intuitively, this is because the foreign nontraded risk against which carry trade strategies can hedge are perceived to be small.

country-specific traded and nontraded consumption growths.

$$f_T^H = \frac{dC_T^H}{C_T^H}; \qquad f_N^H = \frac{dC_N^H}{C_N^H}.$$

The exploration also emphasizes the difference between global (aggregate) traded output risk and the country-specific traded consumption risk. For illustration, carry trade portfolios are used as test assets in the discussion below and in the estimation process in section 6.3.2. As the risk factors are independent of test assets a priori, the discussion carries over to any other financial assets.

We consider the same carry trade return strategy of borrowing home and lending foreign currency. Again, its excess return to investor H and to be realized at $t + dt$ is (9): $XR_{t+dt}^{-H,+F} = \frac{1}{dt}\left[\frac{M_{t+dt}^F}{M_{t+dt}^H}(1 + r_t^F dt)\frac{M_t^H}{M_t^F} - (1 + r^H dt)\right]$. The factor analysis starts with the standard *unconditional*[35] consumption-based Euler equation for this carry trade return

$$E\left[\frac{M_{t+dt}^H}{M_t^H} XR_{t+dt}^{-H,+F}\right] = 0 \Longrightarrow E\left[XR_{t+dt}^{-H,+F}\right] = -\frac{1}{dt}Cov\left[1 + dm_{t+dt}^H - E[dm_{t+dt}^H], XR_{t+dt}^{-H,+F}\right].$$

Because home consumption is made of both traded and nontraded components, log-linearized SDF (5) immediately implicates that the carry trade is priced by the following linear two-factor model

$$E\left[XR_{t+dt}^{-H,+F}\right] = -Cov\left[b_T f_{T,t+dt}^H + b_N f_{N,t+dt}^H, XR_{t+dt}^{-H,+F}\right] \tag{12}$$

$$\begin{bmatrix} b_T \\ b_N \end{bmatrix} = \begin{bmatrix} -\gamma\omega_T \\ -\gamma\omega_N \end{bmatrix}; \quad \begin{bmatrix} f_T^H \\ f_N^H \end{bmatrix} = \begin{bmatrix} \frac{dC_T^H}{C_T^H} \\ \frac{dC_N^H}{C_N^H} \end{bmatrix} = \begin{bmatrix} d\delta_T - \alpha(\gamma - \epsilon)\omega_N \left(d\delta_N^H - \sum_{F=1}^K \frac{\Lambda^F}{\Lambda} d\delta_N^F\right) \\ d\delta_N^H \end{bmatrix}.$$

Several observations can be made here. First, this is a country-specific pricing model that prices the assets from the perspective of home investors. Accordingly, the risk factors $\{f_T^H, f_N^H\}$ are *home-specific* traded and nontraded consumption growths, because they are the only risks priced by home SDF m^H. By restricting the pricing to a country-specific perspective, we can conveniently pack other countries' nontraded outputs into a single home traded consumption factor to facilitate the accompanied empirical analysis.[36] Second, this is a factor pricing model in which the factor loadings

[35] In the conditional Euler equation approach, $E_t\left[XR_{t+dt}^{-H,+F}\right] = -\frac{1}{dt}Cov_t\left[1 + dm_{t+dt}^H - E_t[dm_{t+dt}^H], XR_{t+dt}^{-H,+F}\right] = -\frac{1}{dt}Cov_t\left[dm_{t+dt}^H, dm_{t+dt}^F - dm_{t+dt}^H\right]$, where the last equality confirms that the result here is indeed identical to the expected excess return computed by a more intuitive approach in the previous section.

[36] We can also construct an international factor model in which the global traded output growth is a stand-alone factor. However, this model inevitably needs to involve all other country-specific nontraded outputs, and it will

(b's) and risk factors (f's) are structurally determined and explicitly obtained. In particular, the loadings unambiguously increase with the tastes and risk aversion of investors. The factor f_T^H reveals all equilibrium effects established in previous sections, just as aggregate traded, trade partners' and country's own nontraded risk (respectively in δ_T, δ_N^F, δ_N^H) are all compounded in the home traded consumption allocation.

To better discern, both empirically and theoretically, the risk factors from the loadings of carry trade strategies on these risk types, we proceed to the beta-pricing version of the linear factor model.

$$E\left[XR_{t+dt}^{-H,+F}\right] = \lambda_T^H \beta_T^{H,F} + \lambda_N^H \beta_N^{H,F}, \tag{13}$$

$$\begin{bmatrix} \lambda_T^H \\ \lambda_N^H \end{bmatrix} = \left[Cov(\vec{f}^H, \vec{f}^H)\right] \begin{bmatrix} -b_T \\ -b_N \end{bmatrix}; \quad \begin{bmatrix} \beta_T^{H,F} \\ \beta_N^{H,F} \end{bmatrix} = \left[Cov(\vec{f}^H, \vec{f}^H)\right]^{-1} \begin{bmatrix} Cov(f_T^H, XR^{-H,+F}) \\ Cov(f_N^H, XR^{-H,+F}) \end{bmatrix},$$

where $\left[Cov(\vec{f}^H, \vec{f}^H)\right]$ denotes the 2×2 variance-covariance matrix of the factors $\{f_T^H, f_N^H\}$. As β are slope coefficients of returns linearly regressed on the risk factors, the magnitude of β quantifies the exposures of investment strategies to the two risk factors. In contrast, factor prices $\{\lambda_T^H, \lambda_N^H\}$ are the rewards (in the form of expected returns) to bear one notional unit of corresponding risk (i.e., as if $\beta = 1$), which are independent of assets.

How exactly is risk embedded in asset payoff priced by the home investors? The basic risk-return tradeoff picture is that any shock that moves asset payoff and home marginal utility (or SDF m^H) in opposite directions is perceived as risk (again, because these assets pay poorly when investors highly value the payoff), and the corresponding reward (factor price) is positive, and vice versa. We begin with the home nontraded consumption growth risk. Substituting the analytical expressions above for factors f's and loadings b's yields the following testable results.

Proposition 3 *The factor price associated with nontraded consumption growth risk is unambiguously positive,*

$$\lambda_N^H = \alpha\gamma\omega_N \left[\epsilon + (\gamma - \epsilon)\omega_T \frac{\Lambda^H}{\Lambda}\right] (\sigma_N^H)^2 > 0 \quad \forall H. \tag{14}$$

That is, the uncertainties in domestic nontraded consumption growth always pose as a risk to home

result in a multiple-factor model that would complicate the empirical analysis, requiring non-traded output data of all countries worldwide.

investors in all countries.

Because idiosyncratic nontraded outputs can only be consumed domestically, the price of nontraded consumption risk involves only the volatility σ_N^H. As smaller economies can better outsource this risk to their trade partners by flexibly adjusting their traded consumption, this risk is more severe for larger economies. We indeed see that the corresponding factor price λ_N^H is higher for larger size Λ^H. Section 6.3.2 obtains a positive and statistically significant estimate for the US nontraded consumption growth factor price, which thus lends empirical support for the current model. We now turn to the factor price associated with the country-specific traded consumption growth risk,

$$\begin{aligned} \lambda_T^H &= \gamma\omega_T(\sigma_T)^2 + \alpha^2\gamma(\gamma-\epsilon)^2\omega_T\omega_N^2 \sum_{F\neq H}^{K} \frac{(\Lambda^F)^2}{(\Lambda)^2}(\sigma_N^F)^2 \\ &- \alpha^2\gamma(\gamma-\epsilon)\omega_N^2\left(1-\frac{\Lambda^H}{\Lambda}\right)\left[\epsilon+(\gamma-\epsilon)\omega_T\frac{\Lambda^H}{\Lambda}\right](\sigma_N^H)^2. \end{aligned} \tag{15}$$

In sharp contrast with λ_N^H, the home traded consumption growth uncertainty is not necessarily a risk to home investors, which is manifested in the ambiguous sign of the associated factor price λ_T^H. This ambiguity arises because a country's traded consumption is endogenous in equilibrium. A surge in home traded consumption can be a consequence of either (i) a surge in global (aggregate) traded output (direct effect), (ii) a surge in trade partners' nontraded outputs (substitution and trade effects), or (iii) a *drop* in home nontraded output (substitution effect). Stating the last result inversely, a surge in home nontraded output acts to lower home traded consumption and boost home marginal utility. Consequently, from the perspective of the *endogenous* home traded consumption, home traded output shocks are not perceived as a risk, whereas shocks of global traded output and trade partners' nontraded outputs are, which explains the signs of all terms in λ_T^H. The overall sign of this home traded consumption growth factors depends on the relative contribution of these terms, and may vary from country to country.

Diversification benefits

Our consumption-risk framework not only delivers closed-form returns to carry trade strategies but also sheds light, both qualitatively and quantitatively, on the diversification benefits of the currency investment. In our setting, the key feature is that nontraded output risk of all countries enters the pricing of the carry trade return between any two countries. Consequently, forming currency portfolios facilitates the diversification among these sources of risk.[37] Previous literature[38] has found that forming equally weighted portfolios of currencies can substantially increase the Sharpe ratio of the carry trade investment strategies, although the underlying mechanism is not explicitly analyzed beyond the law of large number and ad-hoc mean-variance intuition.

Indeed, nontraded output shocks carry different weights, depending on the magnitude of their volatilities and the size of the economies of their origins, in the carry trade returns (11). This feature immediately offers a structural recipe that balances the above weights to achieve an optimal currency portfolio with maximal diversification. Let η^H denote market prices of risk from country H's perspective,[39] which is a vector in the face of multiple shocks priced by the H's SDF, M^H. Let us consider a generic carry trade portfolio that borrows home currency and lends several foreign currencies with weights $\{y_t^{HF}\}_F$ and $\sum_F y_t^{HF} = 1$.[40] The realized and expected excess returns of this portfolio are simply the weighted values of the pairwise carry trade realized excess returns,

$$PR_{t+dt} = \sum_F^K y_t^{HF} XR_{t+dt}^{-H,+F} = \frac{1}{dt}\sum_F^K y_t^{HF}\left[\eta_t^H\cdot\left(\eta_t^H - \eta_t^F\right)dt + \left(\eta_t^H - \eta_t^F\right)\cdot dZ_{t+dt}\right],$$
$$EPR_t \equiv E_t\left[PR_{t+dt}\right] = \sum_F^K y_t^{HF}\eta_t^H\cdot\left(\eta_t^H - \eta_t^F\right) = \eta_t^H\cdot\left(\eta_t^H - \sum_F^K y_t^{HF}\eta_t^F\right).$$

It is apparent that forming a portfolio is not about improving the expected excess returns; the return of a portfolio of high-return currency trades remains high and vice versa.[41] Risk-neutral investors, who care only about expected returns would stay only with the single currency that offers the highest expected carry trade profit. The diversification instead helps reduce the portfolio return fluctuation and thus is slated to generate a Sharpe ratio superior to any single-currency carry trade strategies. From the excess return follows the portfolio's Sharpe ratio (we conventionally set

[37] As long as the total number of countries K is finite, nontraded risk cannot be entirely diversified and expected returns on currency portfolios preserve spread; see footnote 41.

[38] The partial list includes Burnside et al. (2008), Burnside et al. (2011), Lustig and Verdelhan (2007), and Menkhoff et al. (2011).

[39] That is, $\frac{dM^H}{M^H} = -r^H dt - \eta^H\cdot dZ$ where notation $A\cdot B$ emphatically denotes the scalar product of vectors A and B.

[40] To simplify the notation, our convention is that this sum is over all K countries, including H. However, it is possible that investors take opposite positions in some pairwise carry trade strategies; i.e., y_t^{HF} can assume negative values.

[41] This statement holds, given the total number of countries K stays fixed and finite. When the number of countries K increases unbounded, however, all economies become atomistic $\frac{\Lambda^F}{\Lambda}\to 0$, and all pairwise expected carry trade returns converge because nontraded risk becomes less prominent in such a diluted world; see (11). This effect is related more to the dilution of economic scales than to the diversification of nontraded risk.

investment horizon $dt = 1$ for ease of exposition),

$$SR_t = \frac{E_t\left[PR_{t+dt}\right]}{\left(Var_t\left[PR_{t+dt}\right]\right)^{1/2}} = \frac{\eta_t^H \cdot \left(\eta_t^H - \sum_F y_t^{HF}\eta_t^F\right)}{\left\|\eta_t^H - \sum_F y_t^{HF}\eta_t^F\right\|} = \left\|\eta_t^H\right\| \cos\Theta,$$

where Θ is the angle between vectors η_t^H and $(\eta_t^H - \sum_F y_t^{HF}\eta_t^F)$ in the output innovation hyperspace. From the perspective of investor H, prices of risk η_t^H are fixed and the optimal portfolio (of highest Sharpe ratio) is characterized by weights $\{y_t^{HF}\}_F$ that deliver the highest value for $\cos\Theta$ (lowest value for Θ). That is, by forming a portfolio, we can align the price of risk vectors as much as possible. The intuition is simple. Independent noises optimally offset one another when they are of similar magnitude. Pairwise carry trade strategies do not offer this condition simply because nontraded output statistics are heterogeneous across countries and are priced differently by H. This can be seen most lucidly in the analytical expressions of the prices of risk

$$\forall H: \qquad \eta^H \cdot dZ = \left[dZ_T \;\; dZ_N^H \;\; \left\{dZ_N^F\right\}_{F\neq H}\right] \cdot \left[\begin{array}{c} -\gamma\omega_T\sigma_T \\ -\gamma\omega_N\left[1 - \alpha(\gamma-\epsilon)\omega_T\left(1 - \frac{\Lambda^H}{\Lambda}\right)\right]\sigma_N^H \\ \left\{-\gamma\omega_N\alpha(\gamma-\epsilon)\omega_T\frac{\Lambda^F}{\Lambda}\sigma_N^F\right\}_{F\neq H} \end{array}\right].$$

Accordingly, the optimal portfolio choices $\{y_t^{HF}\}_F$ place appropriate weights on $\{\eta^F\}_F$ to essentially undo these heterogeneities to maximally enhance the noise cancellation in the realized portfolio return. Simple geometric arguments immediately show that the minimum Θ is the angle between vector η_t^H and its projected image on the space generated by all other prices of risk vectors $\{\eta_t^F\}_{F\neq H}$.[42] Straightforward but tedious algebra then identifies analytically the optimal Θ, portfolio weights and the maximum Sharpe ratio.

5 Beyond benchmark model

The key intuition, developed alongside the basic setting of international finance in previous sections, is that the country-specific traded output risk should have a smaller impact on asset prices than the country-specific nontraded output risk because of the diversification in the traded good market. However, the basic model possesses several simplifications, including (i) homogeneous consumption

[42] One can show that the choice $\{y_t^{HF}\}$ that minimizes the angle between η_t^H and $(\eta_t^H - \sum_F y_t^{HF}\eta_t^F)$ also minimizes the angle between η_t^H and $-\sum_{F\neq H} y_t^{HF}\eta_t^F$.

taste for a single common traded good and (ii) complete financial markets worldwide. In this section, we relax these assumptions and verify and thus strengthen the above intuition to a more realistic and robust economic setting.

5.1 Arbitrary trade configuration

Generalized setup: In the current general setting, there are l varieties of traded goods and K types of nontraded goods, and each of the latter is consumed by one respective country. A particular type h of traded goods can be consumed only by some K_h countries, and similarly, a particular country H trades and consumes only some l^H varieties of traded goods. These features aim to capture the realistic and vastly different trade configurations among countries, as well as the vastly different popularity of different traded goods.[43] Moreover, countries can also have country-specific tastes for the traded goods ($\{\omega_h^H\}$) and nontraded good (ω_N^H) that they consume, subject to the conventional normalization $\omega_N^H + \sum_h^{l^H} \omega_h^H = 1$. We also assume that the financial market is complete because contingent claims on all outputs and countries' risk-free bonds are available investment instruments. Consequently, the world's static optimization problem can be used to study the equilibrium behaviors of consumption allocations and asset prices in this economy.

$$\max_{\{C_{h,T}^H\}} \sum_{H=1}^{K} \Lambda^H \frac{e^{-\rho t}}{1-\gamma} \left[\sum_h^{l^H} \omega_{h,T}^H (C_{h,T}^H)^{1-\epsilon} + \omega_N^H (\Delta_N^H)^{1-\epsilon} \right]^{\frac{1-\gamma}{1-\epsilon}} \quad \text{s.t.} \quad \sum_H^{K_h} C_{h,T}^H = \Delta_{h,T} \ \ \forall h = 1, \ldots, l.$$

Although a country may have different tastes for different goods that they consume, the substitutability between any two varieties, either traded or nontraded, is characterized by the same elasticity coefficient ϵ. It is apparent from the market market clearing conditions that only the aggregate outputs for traded good varieties directly enter the dynamic of the economy. However, the associated output shocks will have different impacts on different countries, depending on their country-specific trade configurations. The current complex setting calls for a quantitative analysis to shed light on the role of these shocks on consumption allocations and prices.

Equilibrium allocations: Combining log-linearization and iteration techniques yield the equilib-

[43] Examples include the oil consumed by all countries versus rare earth minerals, which are consumed only by the most advanced economies.

rium log consumption c_h^H of traded good h by country H,

$$\begin{aligned} c_h^H &= \delta_{h,T} - (\gamma-\epsilon)\alpha^H \omega_N^H \delta_N^H + (\gamma-\epsilon)\sum_J^{K_h} \alpha^J \frac{\Lambda^J}{\Lambda_h}\left(\omega_N^J \delta_N^J + \sum_j^{l^J} \omega_{j,T}^J \delta_{j,T}\right) \\ &- (\gamma-\epsilon)\alpha^H \sum_i^{l^H} \omega_{i,T}^H \left[\delta_{i,T} + (\gamma-\epsilon)\sum_I^{K_i} \alpha^I \frac{\Lambda^I}{\Lambda_i}\left(\omega_N^I \delta_N^I + \sum_k^{k^I} \omega_{k,T}^I \delta_{k,T}\right)\right], \end{aligned} \tag{16}$$

where in the current general setting,

$$\Lambda_i \equiv \sum_I^{K_i} \Lambda^I; \qquad \alpha^H \equiv \frac{1}{(1-\omega_N^H)\gamma + \omega_N^H \epsilon} > 0 \tag{17}$$

are the good-specific relative size of the aggregate economies (those that consume good i) and a country-specific measure of weighted elasticity of consumption substitution, respectively. It is plausible that in this entangled trade network, many outputs affect country H's consumption of good h. In leading orders of importance, these include h's global supply ($\delta_{h,T}$); H's nontraded output (δ_N^H); nontraded output (δ_N^J) and traded global supply ($\delta_{j,T}$) consumed by any other country $J \in K^h$ that also consumes h; global supply ($\delta_{i,T}$) of any other traded good $i \in l^H$ consumed by H; and finally, the nontraded output (δ_N^I) and global supply ($\delta_{k,T}$) of traded goods k consumed by any country $I \in K^i$ that also consumes i.

Similar to the simpler setting of section 2, a country's traded consumption allocation c_h^H increases with the global supply $\delta_{h,T}$, decreases with the host's nontraded output δ_N^H, and increases with nontraded output δ_N^J of all trade partners J in good h. As country H also consumes other traded variates $\{\delta_{i,T}\}_{i \in l^H}$, H's consumption c_h^H in good h tends to negatively correlate with shocks $dZ_{i,T}|_{i \neq h}$ through the substitution effect between any two traded goods. Furthermore, because the consumptions of all trade partners $J \in K_h$ in good h are tuned to the nontraded δ_N^J and traded global supplies $\{\delta_{j,T}\}_{j \in l^J}$ that they consume, these shocks are also positively compounded into c_h^H, again through trade (market clearing) and substitution effects.

Most interestingly, even in the current general trade network setting, the international transmission of output shocks follows a simple and intuitive quantitative pattern in the leading orders. That is, the transmission process involving trades in a good i with a mediating country I warrants a dampening coefficient,[44]

$$(\gamma-\epsilon)\alpha^I\frac{\Lambda^I}{\Lambda_i}=\frac{\frac{1}{\epsilon}-\frac{1}{\gamma}}{\frac{1-\omega_N^I}{\epsilon}+\frac{\omega_N^I}{\gamma}}\frac{\Lambda^I}{\Lambda_i}.$$

Here, $\frac{\Lambda^I}{\Lambda_i}$ characterizes the relative power of mediating country I in setting the global price for traded good i (through FOC), and $\left(\frac{1}{\epsilon}-\frac{1}{\gamma}\right)/\left(\frac{1-\omega_N^I}{\epsilon}+\frac{\omega_N^I}{\gamma}\right)$ quantifies how readily shocks in one consumption sector affect the others in a country.[45] Next, we examine the stochastic discount factors (SDFs) to explore how investors price the risk associated with these output shocks in different countries.

Equilibrium pricing: As shocks affect consumption allocations, they also move equilibrium prices accordingly to clear the market. The country H's log SDF is

$$\begin{aligned} m^H &= -\rho t-\gamma\sum_h^{l^H}\omega_{h,T}^H\delta_{h,T}-\gamma\omega_N^H\left[\delta_N^H-\sum_h^{l^H}(\gamma-\epsilon)\alpha^H\omega_{h,T}^H\left(1-\frac{\Lambda^H}{\Lambda_h}\right)\delta_N^H\right] \\ &- \gamma\sum_h^{l^H}\omega_{h,T}^H\sum_{J\neq H}^{K_h}(\gamma-\epsilon)\alpha^J\frac{\Lambda^J}{\Lambda_h}\left(\omega_N^J\delta_N^J+\sum_j^{l^J}\omega_{j,T}^J\delta_{j,T}\right) \end{aligned} \tag{18}$$

Reassuringly, all shocks that affect country H's consumptions are also priced by this stochastic discount factor. In particular, all traded and nontraded consumption shocks of H and any of it trade partners are compounded in m^H. As in the simpler case of section 2, up to taste coefficients, the traded shocks are fully internationalized (in the aggregate output $\delta_{h,T}$) and spread uniformly to all countries $I\in K_h$ that consume good h. As $\omega_{h,T}^H$ generally drops with the number l^H of varieties consumed by H,[46] the country-specific traded shock of a particular variety matters even less to its country of origin in the current setting of multiple traded goods. In contrast, nontraded shocks are internalized, but not fully. As the second term within the square brackets shows, country H can tunnel its own nontraded shock in δ_N^H through trades in all l^H channels in which H participates. The ability to mitigate this shock through a particular channel h clearly decreases with a country's

[44] As γ is (substantially) larger than ϵ, mild home bias conditions assure that $(\gamma-\epsilon)\alpha^I\frac{\Lambda^I}{\Lambda_i}<1$.

[45] Section 2 asserts that the difference $\frac{1}{\epsilon}-\frac{1}{\gamma}$ characterizes how willing a country is to substitute traded and nontraded consumptions to smooth its aggregate consumption. When this difference is large and positive as in the data, countries are flexible to make this substitution. As a result, a shock from one consumption sector is readily transmitted to the other sector. In the current setting, each country has one nontraded and several traded sectors, but all have the same pairwise substitution elasticity of ϵ.

[46] This is a consequence of the normalization condition $\omega_N^H+\sum_h^{l^H}\omega_{h,T}^H=1$.

relative size $\frac{\Lambda^H}{\Lambda_h}$ in the world trade market for good h. Under mild home bias condition, country-specific nontraded shocks still matter more to the country's pricing than do the traded counterparts. Finally, we also see that traded shocks (in $\delta_{j,T}$) affecting any trade partner J are also factored in m^H. When H does not consume these goods, $j \notin l^H$, their shocks to H are similar the purely nontraded shocks of partners J.

5.2 Incomplete market

In equilibrium, the complete financial markets equalize all countries' marginal utilities of the traded consumption and thus enable the optimal international risk sharing and consumption allocation. In reality, however, the financial markets of some countries are more developed than those of others, which should better facilitate these developed countries to manage their own as well as trade partners' output risk. Stylistically, because of either information asymmetry or lack of proper managerial enforcement, the equities associated with nontraded sectors of emerging economies are less marketable worldwide. It is interesting to explore the new qualitative implications of market incompleteness on international risk sharing and contrast them with those of the simplified complete market paradigm. To this end, we now analyze a stylized model in which nontraded output risk is the central factor behind the incompleteness in the financial markets.

Setup: We consider the world economy with perfect trades but an incomplete financial market. In the commodity sector, there are country-specific nontraded goods (one per country) and a single traded good (common to all countries). The traded good can be shipped globally without the friction, and thus only its aggregate output influences the pricing. Accordingly, we assume that the financial assets associated with the traded good sector are perfectly structured. That is, a stock S_T contingent on the aggregate output and a risk-free bond B_T paying one unit of traded good in the next period are available to investors worldwide. In contrast, the financial assets associated with nontraded sectors are incomplete. We assume that countries belong to either the "developed" or the "emerging" group. For any developed economy ($H \in \mathcal{D}$), the stock S_N^H contingent on the H's nontraded output and risk-free bond B_N^H paying one unit of H's nontraded good in the following period are also available to all investors. However, assets associated with nontraded sectors of emerging economies ($H \notin \mathcal{D}$) are not marketable and thus simply do not exist. In this framework, the world financial market is incomplete because there are more shocks than the available financial hedging instruments. To simplify the exposition, we assume a homogeneous size for all economies embedded in a two-period setting $\{t, t+1\}$, but maintain the heterogeneous consumption tastes $\{\omega_T^H, \omega_N^H\}_H$ across countries. Relaxing all of these assumptions is tedious but straightforward.

The most convenient choice for the numeraire in this setting is the traded good, which we adopt hereafter. Thus, in every period, all prices are in (contemporaneous term of) the traded good. Because the market is incomplete, we consider the optimization problem for each country.[47] Let $x_T^{HS}, x_T^{HB}, x_N^{HFS}, x_N^{HFB}$ denote the holdings of H's investor, respectively, in world stock S_T, world bond B_T, F's stock S_N^F, and F's bond B_N^F.

$$\max_{C_{T,t}^H, x_{T,t}^{HS}, x_{T,t}^{HB}, \{x_{N,t}^{HFS}, x_{N,t}^{HFB}\}_{F\in\mathcal{D}}} U^H(C_t^H) + e^{-\rho} E_t \left[U^H(C_{t+1}^H) \right],$$

subject to the market clearing and budget constraints

$$\begin{array}{ll} \sum_H x_{T,t}^{HS} = \sum_H x_{T,t+1}^{HS} = 1; & \sum_H x_{T,t}^{HS} = \sum_H x_{T,t}^{HS} = 0 \\ \sum_H x_{N,t}^{HFS} = \sum_H x_{N,t+1}^{HFS} = 1; & \sum_H x_{N,t}^{HFS} = \sum_H x_{N,t+1}^{HFS} = 0 \quad \forall F \in \mathcal{D} \end{array}$$

$$C_{T,t}^H + \Delta_{N,t}^H P_{N,t}^H \mathbb{1}_{H\in\mathcal{D}} + S_{T,t} x_{T,t}^{HS} + B_{T,t} x_{T,t}^{HB} + \sum_{F\in\mathcal{D}} S_{N,t}^F x_{N,t}^{HFS} + \sum_{F\in\mathcal{D}} B_{N,t}^F x_{N,t}^{HFB} \leq W_t^H$$

$$C_{T,t+1}^H + \Delta_{N,t+1}^H P_{N,t+1}^H \mathbb{1}_{H\in\mathcal{D}} \leq x_{T,t}^{HS} \Delta_{T,t+1} + x_{T,t}^{HB} + \sum_{F\in\mathcal{D}} x_{N,t}^{HFS} \Delta_{N,t+1}^F + \sum_{F\in\mathcal{D}} x_{N,t}^{HFB} P_{N,t+1}^F$$

where $C^H = \{C_T^H, C_N^H\}$ denotes the standard CES consumption aggregator as in section 2, U^H denotes the power utility function of C^H, and W_t^H denotes investor H's initial wealth. Identity operator $\mathbb{1}_{F\in\mathcal{D}}$ equals one if F is an developed country and zero otherwise, which simply reflects the fact that investors can invest in financial assets paying nontraded goods and convert these payoffs into units of traded good at the respective nontraded price P_N^F, $\forall F \in \mathcal{D}$ for their consumption purpose. In contrast, no assets paying nontraded goods of emerging markets exist, and consequently no investors, domestic or otherwise, ever need to convert these goods into the traded good and back. In other words, in the current incomplete market setting, nontraded shocks are identical to preference shocks. Furthermore, we note that by summing all countries, the above budget constraints and market clearing conditions automatically imply the resource constraints $\sum_H C_{T,t}^H =$

[47] With an incomplete market, the centralized optimization can also be formulated as in Pavlova and Rigobon (2008) using the convex duality technique (Cvitanic and Karatzas (1992)). However, this approach offers an exact and analytical solution only for the special case of log utility.

$\Delta_{T,t}$, $\sum_H C^H_{T,t+1} = \Delta_{T,t+1}$ in both periods.

First order conditions corresponding to variations about optimal holding positions x^{HS}_T, x^{HB}_T, x^{HFS}_N, x^{HFB}_N, respectively, generate pricing equations for all available financial assets,

$$S_{T,t} = E_t\left[\frac{M^H_{T,t+1}}{M^H_{T,t}}\Delta_{T,t+1}\right]; \qquad B_{T,t} = E_t\left[\frac{M^H_{T,t+1}}{M^H_{T,t}}\right] \qquad \forall H,$$
$$S^F_{N,t} = E_t\left[\frac{M^H_{T,t+1}}{M^H_{T,t}}\Delta^F_{N,t+1}P^F_{N,t+1}\right]; \qquad B^F_{N,t} = E_t\left[\frac{M^H_{T,t+1}}{M^H_{T,t}}P^F_{N,t+1}\right] \qquad \forall F\in\mathcal{D},\ \forall H.$$

where $M^H_{T,t} = \frac{\partial U^H}{\partial C^H_{T,t}}$ is the *country-specific* marginal utility of the *traded* consumption[48]

In the complete market setting, the marginal utilities are necessarily equalized across countries $\frac{M^H_{T,t+1}}{M^H_{T,t}} = \frac{M^F_{T,t+1}}{M^F_{T,t}}$ $\forall\{H,F\}$, which together with market clearing conditions, then establishes directly the equilibrium consumption allocations. In the incomplete market, the marginal utilities are indirectly connected to one another only through the pricing of available assets. Accordingly, the solution approach here is very different. In sequence, we first conjecture a solution for the consumption allocations, solve for the asset prices, and verify that these prices support the conjectured consumptions in equilibrium. As before, we log-linearize the above first order conditions for all countries H and all developed countries $F\in\mathcal{D}$[49]

$$\log\left(\frac{S_{T,t}}{B_{T,t}}\right) = Cov_t\left[dm^H_{T,t+1}, \delta_{T,t+1}\right]; \qquad \log\left(\frac{S^F_{N,t}}{B^F_{N,t}}\right) = Cov_t\left[dm^H_{T,t+1}, \delta^F_{N,t+1}\right]; \tag{19}$$
$$\log B_{T,t} = E_t\left[dm^H_{T,t+1}\right] + \frac{1}{2}Var_t\left[dm^H_{T,t+1}\right]; \qquad \log\left(\frac{B^F_{T,t}}{B_{T,t}}\right) = Cov_t\left[dm^H_{T,t+1}, P^F_{N,t+1}\right],$$

where dm^H denotes the log-linearized stochastic discount factor (recall from (4) that $\alpha^H \equiv \frac{1}{\gamma\omega^H_T + \epsilon\omega^H_N}$),

$$dm^H_{T,t+1} \equiv m^H_{T,t+1} - m^H_{T,t} = \log\left(\frac{M^H_{T,t+1}}{M^H_{T,t}}\right) = -(\gamma-\epsilon)\omega^H_N d\delta^H_{N,t+1} - \frac{1}{\alpha^H}dc^H_{T,t+1}. \tag{20}$$

Equilibrium: Consistent with the log-linearization approximation scheme, we look for the equi-

[48] We recall that the current numeraire is the traded good, and therefore $M^H_{T,t} = e^{-\rho t}\omega^H_T(C^H_{N,t})^{-\epsilon}\left[\omega^H_T(C^H_{T,t})^{1-\epsilon} + \omega^H_N(C^H_{N,t})^{1-\epsilon}\right]^{\frac{-\gamma+\epsilon}{1-\epsilon}}$ is the country H's pricing kernel with respect to this numeraire.

[49] Although the log-linearization technique remains useful to obtain an approximate closed-form solution, it does not address the possible multiplicity and stability of the equilibrium.

librium consumption allocations in the following most general log-linear form,

$$dc^H_{T,t+1} \equiv \log\left(\frac{C^H_{T,t+1}}{C^H_{T,t}}\right) = g^H + a^H d\delta_{T,t+1} + \sum_F b^{HF} d\delta^F_{N,t+1} \qquad \forall H, \tag{21}$$

and g's, a's, b's are constant parameters to be determined, and $d\delta$'s denote the changes in log outputs, i.e., output growths ($dt = 1$)

$$d\delta_{T,t+1} \equiv \delta_{T,t+1} - \delta_{T,t} = \mu_T dt + \sigma_T dZ_T; \qquad d\delta^H_{N,t+1} \equiv \delta^H_{N,t+1} - \delta^H_{N,t} = \mu^H_N dt + \sigma^H_N dZ^H_N$$

This choice renders a log-linear SDF dm^H in the approximation and greatly simplifies the pricing of financial assets in the incomplete market settings (Weil (1994)). Indeed, substituting the above conjectured consumptions and SDFs into the pricing equations and the market clearing conditions readily yields the following consumption allocations (derived in appendix D),[50] where we recall that $\alpha^I \equiv \frac{1}{\gamma\omega^I_T + \epsilon\omega^I_N} > 0$ denotes the country-specific weighted elasticity of consumption substitution.

- incomplete market: H is an emerging economy ($H \notin \mathcal{D}$)

$$c^H_{T,t} = g^H t + \frac{K\alpha^H}{\sum_I \alpha^I}\delta_{T,t} - \alpha^H \sum_{F\notin\mathcal{D}} \delta^F_{N,t} + \frac{(\gamma-\epsilon)\alpha^H}{\sum_I^K \alpha^I} \sum_{F\in\mathcal{D}} \alpha^F \omega^F_N \delta^F_{N,t}. \tag{22}$$

- incomplete market: H is a developed economy ($H \in \mathcal{D}$)

$$\begin{aligned} c^H_{T,t} &= g^H t + \frac{K\alpha^H}{\sum_I \alpha^I}\delta_{T,t} + \frac{\alpha^H \sum_{I\notin\mathcal{D}} \alpha^I}{\sum_{J\in\mathcal{D}} \alpha^J} \sum_{F\notin\mathcal{D}} \delta^F_{N,t} \\ &+ \frac{(\gamma-\epsilon)\alpha^H}{\sum_I^K \alpha^I} \sum_{F\in\mathcal{D}} \alpha^F \omega^F_N \delta^F_{N,t} - (\gamma-\epsilon)\alpha^H \omega^H_N \delta^H_{N,t}, \end{aligned} \tag{23}$$

where g^H's are country-specific parameters. These parameters help to enforce, and thus can be found from the market clearing conditions (see appendix D), but because they are deterministic factors, they do not enter the analysis below. To verify these equilibrium consumptions, we substitute them back into the above pricing equations to compute all available asset prices $\{S_{T,t}, B_{T,t}\}$, $\{S^F_{N,t}, B^F_{N,t}\}_{F\in\mathcal{D}}$, which finance these consumptions by the construction of the solution. This config-

[50] Specifically, the pricing equations $\log(S_{T,t}/B_{T,t})$'s determine coefficients $\{a^H\}_{\forall H}$, $\log(S^F_{N,t}/B^F_{N,t})$'s determine $\{b^{HF}\}_{\forall F\in\mathcal{D},\forall H}$, $\log B_{T,t}$'s determine $\{b^{HF}\}_{\forall F\notin\mathcal{D},\forall H}$, and $\log(B^F_{T,t}/B_{T,t})$'s determine the nontraded prices of developed countries $\{P^F_{N,t+1}\}_{\forall F\in\mathcal{D}}$, see appendix D.

uration is in equilibrium,[51] because, for each available asset, the associated price is identical under all investors' perspectives in the construction. Compared with the counterpart complete market setting with a single traded good, in which the consumption allocations are[52]

- complete market: $c^H_{T,t} = g^H t + \frac{K\alpha^H}{\sum_I^K \alpha^I}\delta_{T,t} + \frac{(\gamma-\epsilon)\alpha^H}{\sum_I^K \alpha^I}\sum_F \alpha^F \omega^F_N \delta^F_{N,t} - (\gamma-\epsilon)\alpha^H\omega^H_N\delta^H_{N,t}$,

the incomplete market allocations are markedly different in several aspects.[53] First, the traded shock impacts stay the same in both market configurations. This is because even when the market is incomplete, the equity and bond on the traded output δ_T are available to all investors, who then are able to mitigate these shocks as optimally as possible by trading these financial assets. When combined with the force of cross-country diversification in the traded sector, this result implies that country-specific traded output risks remain relatively less material to countries' risk free rates, compared with the nontraded output risk.

Second, the nontraded output shocks (in δ^F_N) of a developed country $F \in \mathcal{D}$ affect the traded consumption c^H_T of *all* other countries H similarly, regardless of the market's completeness. Because investors can trade the financial assets contingent on these nontraded shocks, their associated risk can be shared effectively. In particular, all else being equal, a surge in developed country F's nontraded output prompts F to trim its traded consumption and boosts other countries' traded consumption by forces of trades and market clearings. Similar to the complete market settings, under a mild degree of home biases, a country's own nontraded shocks matter quantitatively more to a developed country's consumption allocation than do the nontraded shocks of their *developed* trade partners.

Third, the nontraded output shocks (in δ^F_N) of an emerging country $F \notin \mathcal{D}$ are uniformly compounded in the consumptions c^H_T of all developed countries $H \in \mathcal{D}$.[54] This feature is intuitive. In the absence of financial assets in emerging markets, these shocks cannot be properly hedged. The developed investors instead opt to simply pool their consumptions uniformly to cope with

[51] Although this is not necessarily the unique equilibrium.

[52] This is a straightforward generalization of (3) (in the basic model) to the setting where countries have heterogeneous consumption tastes (but countries' sizes are homogeneous). In the current case, the log-linearization of FOC implies $m_T = -\rho t + \omega^H_T - (\gamma-\epsilon)\omega^H_N\delta^H_N + \frac{1}{\alpha^I}c^H_T$. Combining this FOC with the (log-linearized) market clearing condition (27) for traded good yields this log consumption c^H_T in complete market. See further details in appendix B.

[53] In light of the possible existence of other incomplete market settings and multiple equilibria, our discussion here pertains to the specific incomplete market setup and the associated equilibrium presented earlier in this section.

[54] That is, $\frac{\partial c^H_T}{\partial \delta^F_N}$ is same for all $F \notin \mathcal{D}$, $H \in \mathcal{D}$.

the associated risk. Risk sharing is still feasible, albeit imperfect, because it is evident from the equilibrium allocation that a surge in the nontraded output from an *emerging* economy boosts traded consumptions of all *developed* economies. The coefficient characterizing this relationship, $\frac{\sum_{I\notin\mathcal{D}}\alpha^I}{\sum_{J\in\mathcal{D}}\alpha^J}$, increases (decreases) with the number of emerging (developed) economies. That is, the significance of the unhedged risk on consumption allocations is larger when the financial market is less complete in this pooling equilibrium.

Fourth, the incomplete market has a strong and surprising impact on risk sharing between two emerging economies. Possessing no financial assets directly tied to the nontraded output shocks of their own or those of their emerging trade partners, the emerging economies also pool their traded consumption in equilibrium to uniformly share nontraded risk. Emerging country H's traded consumption c_T^H decreases with not only its own nontraded good endowment δ_N^H but also with other emerging countries' nontraded output δ_N^F. The latter behavior, which is the inverse of a perfect financial market, signals that the risk sharing is most severely hampered between emerging trade partners. This is indeed the group of countries whose nontraded output risk is the least hedgeable because of the incompleteness of the market.

The incomplete market setting, as formulated in this section and pertaining to the pooling equilibrium, does not qualitatively change the risk sharing behaviors, and thus prices, among developed economies. Any sizable effects stemming from market incompleteness instead arise in the group of emerging countries whose financial markets are the least developed in the setting.

6 Empirical results

The principal assertion of this paper, motivated by theoretical considerations in preceding sections, is that nontraded output risk is a key factor determining asset prices and price differentials in international markets. This section investigates this assertion empirically and provides supportive evidence. We implement various tests on interest rates and carry trade returns. Our empirical analysis involves OECD countries[55] plus Eurozone (i.e., Economic and Monetary Union, available after 1998), which are more developed economies and economic and financial data series of which

[55]In our notation, before the German reunification in 1990 (and including that year), the Federal Republic of Germany (FRG) is referred to as West Germany. From 1991 onward, the (reunified) Federal Republic of Germany is referred to as Germany.

are reasonably expected to be more complete and of higher quality. Our main empirical tests exclude three possible outlier countries (Estonia, Iceland, and Turkey) for the reasons presented in the next section on stylized facts of nontraded output risk. All nominal macroeconomic output series are first transformed into real series and then detrended using Hodrick-Prescott (HP) filter.[56] All employed data series are cited in double quotes, and their original sources and other details are listed in the data appendix.

6.1 Stylized facts concerning nontraded output risk

We identify "services" as nontraded sectors in all countries, following the standard classification in the literature (see, e.g., Stockman and Tesar (1995)). Key components of services sectors include wholesale and retail trade, hotels and restaurants, financial intermediation, real estate, business activities and construction services.

To obtain some idea about the size of nontraded sectors in the economies worldwide, figure 1 plots the ratio of real services output over real GDP, averaged over the period 1971-2010, for all OECD countries plus Eurozone. Output data are from "Aggregate National Accounts: Gross domestic product," and services are computed as the sum of (i) wholesale and retail trade, repairs, hotels and restaurants, and transport; (ii) financial intermediation, real estate, renting and business activities; (iii) construction; and (iv) other service activities. Figure 1 shows that nontraded outputs constitute a substantial fraction of the total GDP in all OECD countries, ranging from 0.5 (Iceland) to 0.7 (US). Among others, this figure thus re-documents a known fact that services sectors carry a huge weight of the US economy.

To justify the identification of services as a nontraded sector, Table 1 lists the country-specific tradability and size of financial services, construction services, and other services for a representative set of 13 OECD countries (see data appendix for classification details). Tradabilities and sizes are averaged over the period 1971-2010. The country-tradability of services is (one half of) the ratio of total exports and imports over the total output of these services by the country (see (26)). The economic size of services is the ratio of total domestic output of these services over the country's GDP. Countries' export and import series are from OECD's "Trade in Services" data

[56] We use smoothing parameters $\lambda = 1600$ for quarterly time series, as in Hodrick and Prescott (1997), and $\lambda = 6.25$ for annual time series, as in Ravn and Harald (2002).

base. Countries' services output series[57] are from OECD's "Aggregate National Accounts: Gross domestic product." The table shows that, whereas the tradabilities and sizes of the same services vary considerably across OECD economies, their tradabilities are indeed small (in the order of few percentage points, and never exceeding 20%). In particular, financial services are a substantial part of GDP in all countries (ranging from 14.7% for the Czech Republic to 27.7% for the US), yet their tradabilities are very low (ranging from .21% for Japan to 7.5% for Switzerland). Similarly, Table 2 lists the 15 most traded industries in the US, along with their two measures of tradability. The US-specific tradability of an industry is computed similarly to the above country-specific tradability (26). In the determination of OECD tradability (see (25)), export, import and output are OECD-aggregate quantities. These industry-level macro series are from the "OECD Structural Analysis (STAN)" database. Table 2 shows that all of the top 15 traded industries in the US belong to the manufacturing sector. In either measure, their tradabilities are substantially higher than those in the services sectors listed in Table 1, which justifies the classification of traded and nontraded goods adopted in the literature as well as in the current paper. The table also shows that country-specific tradabilities do not necessarily and quantitatively coincide with their OECD counterparts because countries are heterogeneous in their consumption and production to a certain extent. For the sake of robustness, our tests presented in the next section will employ both of these tradability measures.

To have a sense of the level of nontraded output risk across countries, figure 2 plots the volatility of per-capita nontraded output growth for each OECD country. The volatility is computed as the standard deviation of these nontraded output growth series over the entire period of 1971-2010. Per-capita quantities are computed using the World Bank's "Total Population" series. This figure shows that the level of fluctuation of nontraded output varies widely across OECD countries. In particular, Estonia is the second smallest economy among OECD member states (Iceland is the smallest economy),[58] yet its per-capita nontraded output growth is subtantially more volatile than any other country (approximately ten times more volatile than Germany, France and the US). We therefore exclude Estonia and Iceland from empirical tests. When countries' nontraded output volatilities are computed for each ten-year period, Turkey exhibits an extremely unstable volatility pattern over time. We thus also drop Turkey from the tests.

[57] Specifically, these series are B1GF (Construction), B1GJ_K (Financial intermediation, real estate, renting and business activities), and B1GL_P (Other service activities).

[58] Estonia's GDP is approximately 20 Bln USD for the year of 2010, or less than 0.05% of the aggregate GDP of OECD group. Iceland GDP is 12 Bln USD for the same year.

To have a sense of the level of trade "openness" of OECD countries, figure 3 plots the ratio of each country's total exports and imports over its GDP (see also (24)), averaged over the period 1971-2010. These ratios are from OECD's "Trade-to-GDP ratio" annual series. The figure shows that trade openness is markedly heterogeneous across OECD countries, ranging from 0.17 for Japan to 2.08 for Luxembourg. It is known that this ratio can be biased downward for larger economies, and hence a low value of the openness for a country does not necessarily imply high (tariff or non-tariff) obstacles to foreign trade. Rather, the low value of the openness can be a measure of either weak reliance of domestic producers on foreign supplies and markets or of the country's geographic remoteness from potential trading partners. Any of these possible causes are consistent with our notion that the output growth risk of the more closed economies is internalized by home countries to a larger extent.

6.2 Interest rates

In reality, no goods are either perfectly nontraded or perfectly traded. Even if some goods were, macro output series are inevitably subject to measurement errors. Furthermore, costs in trades also affect the structural relation between nontraded output risk and asset prices. In this section, we investigate the empirical relationship between nontraded output volatility and the level of real interest rate across OECD countries, taking into account these practical regularities. Specifically, we devise four tests based on the various classifications of nontradability, in order of increasing sophistication. These regression-based tests involve (i) the closedness of an economy, (ii) the brute-force cutoff dummy of nontradability at the industry level, (iii) the global nontradabilities at industry level, and (iv) country-specific nontradabilities at the industry level, respectively.

6.2.1 Tests using countries' trade closedness

The hypothesis to be examined here is that when an economy is exposed more to international trades, its nontraded risk can be better mitigated through trades and the substitution between traded and nontraded consumption. This assertion is a specific form of Hypothesis 1 (section 3) and Hypothesis 1A (section 3.2), and is motivated by the structural model with trade friction presented in previous sections. The basic regression test of this relationship reads

$$r_t^H = \alpha + \beta_\sigma (\sigma_t^H)^2 + \beta_C \mathcal{C}_t^H + \beta_{\sigma C} (\sigma_t^H)^2 \mathcal{C}_t^H + \beta_x X_t^H + \epsilon_t^H,$$

where σ^H denotes the per-capita GDP growth volatility and X's the various control variables. We adopt the common definition of a country's trade openness $\mathcal{O}^H$ as trade-to-GDP ratio (trade being the sum of export and import), from which also follows the closedness $\mathcal{C}^H$

$$\mathcal{O}^H = \frac{\mathrm{IM}^H + \mathrm{EX}^H}{\mathrm{GDP}^H}; \qquad \mathcal{C}^H = 1 - \frac{\mathrm{IM}^H + \mathrm{EX}^H}{\mathrm{GDP}^H}. \tag{24}$$

Table 3 reports the results associated with this regression. National output data are from "Aggregate National Accounts: Gross domestic product" and trade openness from "Trade-to-GDP ratio." We compute the volatility of per-capita GDP growth either over the entire period of 1971-2010 (in which case, the above time index t should be dropped), or over each of four non-overlapping 10-year periods, and the mean of interest rates (dependent variable) over exactly the same periods. Control variables include per-capita GDP mean growth, GDP size (or the ratio of countries' GDP over the aggregate GDP of OECD group), and inflation volatility.[59] The last control variable aims to address the fact that the model is real and thus does not capture the possible effects from inflation risk.

The key observation from table 3 is that the slope coefficients associated with the interaction term (variance $\times$ closedness) are always negative. These coefficients are statistically significant when we take into account the GDP growth (which contributes through the intertemporal smoothing desires of investors), economy size, and inflation risk effects, for either the entire period (i.e., in the cross sectional data) or for four 10-year periods (i.e., in the panel data). This negative sign is consistent with the model's central economic rationale that when a country is less open to trade and all else is equal, the country's output shock tends to be more internalized, and to have stronger impacts on lowering country's real interest rate through the precautionary savings mechanism.

[59] Inflation is computed as the year-to-year percentage change of the consumer price index, and the latter is sourced from IMF's CPI series. Furthermore, inflation volatility is computed as standard deviation of the inflation growth.

6.2.2 Tests using multiple industry outputs and their nontradability dummies

Another form of Hypothesis 1 and Hypothesis 1A (sections 3.1, 3.2, respectively) to be examined in this section is as follows. Controlling for anything else, a country's output growth risk of nontraded industries tends to have a stronger impact on domestic interest rate than its output growth risk of traded industries. Intuitively, this is because country-specific traded risk can be diversified in the global pool of traded goods before it affects prices in any country. The basic regression testing this relationship employs national output data at the industry level. We use binary dummies to classify the nontradability of the industries.

$$r_{i,t}^H = \alpha + \beta_\sigma (\sigma_{i,t}^H)^2 + \beta_d d_{i,t} + \beta_{\sigma d} (\sigma_{i,t}^H)^2 d_{i,t} + \beta_x X_{i,t}^H + \epsilon_{i,t}^H,$$

where $r_{i,t}^H = r_t^H$ is country H's interest rate and thus independent of industry type i, $d_{i,t}$ is nontradability dummy ($d_{i,t} = 1$ for nontraded industries and 0 otherwise, as we explain below). Table 4 reports the results associated with this regression. Countries' real annual industry-level outputs are constructed from the "OECD Structural Analysis (STAN)" database. An industry i is classified as nontraded ($d_{i,t} = 1$) if it belongs to one of the following ISIC classes[60] (see further details in data appendix): 40-41 (electricity gas and water supply); 45 (construction); 50-55 (wholesale and retail trade, restaurant and hotels); 60-64 (transport storage and communications); 65-74 (finance insurance real estate and business services); 75-99 (community social and personal services). Other industries are taken as traded ($d_{i,t} = 0$). We divide the entire time period 1971-2010 into four 10-year periods, and the volatility of per-capita output growth for each industry is computed as the respective standard deviation over each period. As before, the control variables include per-capita GDP mean growth, GDP size, and inflation volatility.

The key observation from table 4 is that the slope coefficients associated with the interaction term (variance × dummy) are always negative. When we take into account the GDP growth, economy size, and inflation risk effects, these coefficients are statistically significant either for robust or between-effect standard errors.[61] The negative sign precisely fits the basic economic intuition that the output growth risk is more serious to the economy than that of the traded output. Consequently, the output risk enhances the value of risk-free bonds, and depresses risk-free

[60] ISIC stands for International Standard Industrial Classification of All Economic Activities

[61] Due to limited data, the choice of between-effect model is appropriate.

rate more aggressively when the risk comes from a nontraded industry.

6.2.3 Tests using multiple industry outputs and their global nontradabilities

Some industries are not clear-cut traded or nontraded as depicted by a binary dummy of the above regression. In this section, we use continuous-valued global nontradability at industry level to account for this fine distinction. The hypothesis to be examined here is the same as above, namely all else being equal, output risk of nontraded industries matter more to country's interest rate than that of traded industries. The basic regression testing this relationship reads

$$r_{i,t}^H = \alpha + \beta_\sigma (\sigma_{i,t}^H)^2 + \beta_\tau \tau_{i,t} + \beta_{\sigma\tau} (\sigma_{i,t}^H)^2 \tau_{i,t} + \beta_x X_{i,t}^H + \epsilon_{i,t}^H,$$

where τ_i is a global measure of nontradability of industry i. We adopt the standard definition of tradability as the ratio OECD aggregate trade over OECD aggregate output of the industry i, and nontradability is the complement to tradability

$$\tau_i = 1 - \frac{\sum_{\text{OECD countries}} [i\text{'s import} + i\text{'s export}]}{2 \times \sum_{\text{OECD countries}} i\text{'s output}}. \tag{25}$$

Table 5 reports the results associated with this regression. Data sources are identical to those employed in the above regression. We use country-specific output series to compute country-specific industry i's growth volatility over each of four 10-year periods. We aggregate these series to compute the global tradability and nontradability for each of good i.

The key observation from table 5 is that the slope coefficients associated with the interaction term (variance $\times$ nontradability) are always negative. When we take into account the GDP growth, economy size, and inflation risk effects, these coefficients are statistically significant either for robust or between-effect standard errors. The negative sign precisely fits the basic economic intuition that as countries mostly internalize their own nontraded shocks, the fluctuations in nontraded industries are more serious risk to the economy than those of the traded ones. Furthermore, output volatility act to lower risk-free rate. Consequently, risk-free rate is more sensitive (and negatively related) to output risk of industries of higher nontradabilities.

6.2.4 Tests using multiple industry outputs and their country-specific nontradabilities

In some situation, global measure of tradability does not exactly reflect the tradability of an industry at country level. This happens, for e.g., when the trade levels are highly heterogeneous across countries in certain industries. To account for this fine distinction, in this section, we use continuous-valued country-specific nontradability at industry level. The hypothesis to be examined here is the same as above, namely all else being equal, output risk of nontraded industries matter more to country's interest rate than that of traded industries. The basic regression testing this relationship reads

$$r_{i,t}^H = \alpha + \beta_\sigma (\sigma_{i,t}^H)^2 + \beta_\tau \tau_{i,t}^H + \beta_{\sigma\tau} (\sigma_{i,t}^H)^2 \tau_{i,t}^H + \beta_x X_{i,t}^H + \epsilon_{i,t}^H,$$

where τ_i^H is a country-specific measure of nontradability of industry i. We adopt the standard definition of tradability as the ratio of national trade over national output of the industry i, and nontradability is the complement to tradability

$$\tau_i^H = 1 - \frac{[i\text{'s import} + i\text{'s export}] \text{ by country } H}{[2 \times i\text{'s output}] \text{ by country } H}. \tag{26}$$

Table 6 reports the results associated with this regression. Data sources are identical to those employed in the above regression. We use country-specific output series to compute both country-specific industry i's growth volatility over each of four 10-year periods and i's country-specific tradability and nontradability.

The key observation from table 6 is that the slope coefficients associated with the interaction term (variance $\times$ nontradability) are always negative. When we take into account the GDP growth, economy size, and inflation risk effects, these coefficients are statistically significant either for robust or between-effect standard errors. The negative sign precisely fits the basic economic intuition that as countries mostly internalize their own nontraded shocks, the fluctuations in nontraded industries are more serious risk to the economy than those of the traded ones. Furthermore, output volatility act to lower risk-free rate. Consequently, risk-free rate is more sensitive (and negatively related) to output risk of industries of higher nontradabilities.

6.3 Carry trade returns

The evidences above shows that nontraded risk is a key factor behind national asset returns. This is very intuitive because national asset prices are country-specific measures and nontraded shocks are mostly internalized by countries. Taking a step further, as every *international* investment strategy is exposed to nontraded risk of all countries involved, the associated compensating profits should reflect the interplay of these risk factors. In this section, we investigate the empirical relationship between carry trade expected returns and nontraded output volatilities of the countries involved. Specifically, we devise two tests which involve (i) forming currency portfolios based on countries's nontraded volatility and size, and (ii) constructing nontraded and traded consumption risk factors to price carry trades. The valuation of all carry trades is exclusively from the perspective of US investors, for whom the ultimate profits are in term of US dollars.

6.3.1 Forming portfolios based on the nontraded output growth volatilities and economy sizes

The theoretical analysis of section 4 clearly indicates that[62] controlling for all else, carry trades with partner countries of smaller sizes and less volatile nontraded outputs yield higher expected returns to US investors.[63] To directly verify this structural mechanism, stated in Hypothesis 2 (section 4), we construct portfolios of currencies based mainly on the volatilities of nontraded output as suggested by the theory. As argued by Lustig and Verdelhan (2007), forming portfolios helps filter out the noises in individual currency returns, and delivers large and stable return spreads between portfolios by means of frequent rebalancing. Burnside et al. (2008) document and the current paper's section 4 theoretically shows sizable benefits of diversification in portfolio construction.

We consider carry trade returns from US investors' perspectives. For each country, we identify the nontraded consumption as the expenditure on services (a component of the expenditure on total private consumption in the expenditure approach to GDP). These consumption expenditure series are available only at quarterly (or lower) frequencies, and sourced from OECD's "Quarterly

[62]Expected returns of the carry trades to US investors have been computed in section 4, $E_t\left[XR_{t+dt}^{-H,+F}\right] = \alpha^2\gamma^2\epsilon\omega_N^2\left\{\left[\epsilon+(\gamma-\epsilon)\omega_T\frac{\Lambda^H}{\Lambda}\right](\sigma_N^H)^2-(\gamma-\epsilon)\omega_T\frac{\Lambda^F}{\Lambda}(\sigma_N^F)^2\right\}$.

[63]All carry trades involve shorting US dollars and longing foreign currencies.

National Accounts" database.[64] At the beginning of each quarter t, countries are sorted into four (quartile) portfolios based on the value of country-specific product of per-capita[65] nontraded consumption growth variance and relative GDP size. For each country, the product is computed over the previous eight-quarter period, and thus the portfolios are quarterly rebalanced on rolling basis. Portfolio 1 contains countries with lowest value of the above product, and portfolio 4 the highest. After portfolios' currency compositions are known at the beginning of quarter t, US investors short US dollars and long equally weighted portfolios P of foreign currencies F to earn the quarterly returns $XR_{t+1}^{-US,P}$ realized at the beginning of quarter $t+1$

$$XR_{t+1}^{-US,+F} = \frac{S_t^F}{S_{t+1}^F}(1+\frac{r_t^F}{4}) - (1+\frac{r_t^{US}}{4}); \qquad XR_{t+1}^{-US,+P} = \sum_{F\in P}^{K_P} \frac{1}{K^P} XR_{t+1}^{-US,+F}.$$

By the convention adopted here, spot exchange rate S_t^F is the number of foreign currency units per US dollar. These spot exchange rates are sampled simultaneously with the above interest rates.[66] To compute the real carry trade returns to US investors, we subtract US inflation from the above nominal returns $XR_{t+1}^{-US,+P}$. The US inflation is constructed as percentage change of "US quarterly consumer price index (CPI) series". Finally, the annualized real carry trade returns for each portfolio are obtained by compounding the quarterly counterpart values.[67] We note that because OECD's "Quarterly National Accounts" database is unbalanced (data start at different times for different countries, see data appendix), when we match it to IMF's IFS dataset, not all OECD countries are available at the same time for the purpose of portfolio sorting.

Figure 4 plots the mean annualized returns and Sharpe ratios on four equally weighted carry trade portfolios. The figure shows a monotonically inverse relationship between mean returns and the values of product of nontraded output variance and size across portfolios. Portfolio 1 earns a mean annual return of 2.33% (Sharpe ratio of 14%), and portfolio 4 a return of -.47% (Sharpe ratio of -4%) to US investors. Thus a long-short portfolio strategy (long portfolio 1, short portfolio 4) earns

[64] To obtain a more extensive historical data, however, US quarterly consumption expenditure series are sourced from US Bureau of Economic Analysis. See data appendix for further details.

[65] Since the population time series are not available at quarterly frequency, they are constructed from the annual population by intrapolation, assuming constant population growth within each year. Annual population data are from World Bank's "Total Population series".

[66] Both nominal interest rate series r_t and spot exchange rate series S_t^F are sampled at quarterly frequency from IMF's International Financial Statistics (IFS) database.

[67] Because portfolios are rebalanced quarterly, the currency compositions of portfolios do not necessarily stay fixed over the course of any year.

mean annual return of 2.8%, and Sharpe ratio of around 20%. This empirical inverse relationship is supported by our rational theory concerning nontraded risk as summarized in Hypothesis 2 (section 4). The intuition is that, partner countries' risk-free bonds, as insurance instruments, are relatively less valuable when their domestic economic environments are more stable, and offer larger interest rates to benefit the carry trade investors. However, high-return portfolios' payoffs tend to go up and down together with US nontraded endowment. They thus pose a consumption risk to US investors and necessarily pay superior expected returns to stay attractive in equlibrium. Sorting portfolio based directly on nontraded output volatilities (coupled with sizes) provide direct empirical supports for the key role of nontraded risk in the current rational approach to intrenational asset pricing.

6.3.2 Linear factor analysis: Empirics

The theoretical analysis of section 4 suggests another very intuitive way to consider nontraded and traded consumption risk as two key pricing factors. From US investors' perspectives, fluctuations in US traded and nontraded consumption are risk, and payoffs that correlate with these consumptions are priced, and carry risk premium accordingly. In this section we use currency portfolios sorted on interest rates as test assets to estimated the prices of risk associated with these two consumption risk factors. We do not sort currency portfolios on the nontraded output volatilities because doing so amounts to replicating the empirical exercise of the previous section, which already offers evidences that US investors price the nontraded risk of carry-trade partner countries. Instead, the choice of currency portfolios sorted on interest rates aims to relate the consumption risk to the violation of uncovered interest rate parity, which has been most robustly observed in these interest-rate-sorted currency portfolios. Below we discuss, in order, the estimation procedure, the data, and estimation results.

We empirically identify the US traded and nontraded consumption variations as risk factors for US investor; $f^{US}_{T,t+1} = \frac{C^{US}_{T,t+1} - C^{US}_{T,t}}{C^{US}_{T,t}}$, $f^{US}_{N,t+1} = \frac{C^{US}_{N,t+1} - C^{US}_{N,t}}{C^{US}_{N,t}}$. Using carry trade portfolio excess returns $XR^{-US,+P}_{t+1}$ as test assets, the fundamental Euler pricing equation (see section 4) can be written as[68]

$$E_t\left[\left\{1-b_T\left(f^{US}_{T,t+1}-\mu^{US}_T\right)-b_N\left(f^{US}_{N,t+1}-\mu^{US}_N\right)\right\}XR^{-H,+F}_{t+1}\right]=0,$$

where $\mu^{US}_T\equiv E\left[f^{US}_{T,t+1}\right]$, $\mu^{US}_N\equiv E\left[f^{US}_{N,t+1}\right]$ are *unconditional* means of the factors. The latter form readily suits a GMM process to estimate the factor loadings $\{b_T, b_N\}$. Consequently, follow the factor prices $\{\lambda^{US}_T, \lambda^{US}_N\}$ of the traded and nontraded risk, and the exposures $\{\beta^{US,P}_T, \beta^{US,P}_N\}$ of currency portfolios P to the US traded and nontraded consumption risk (see section 4)

$$\begin{bmatrix}\lambda^{US}_T\\ \lambda^{US}_N\end{bmatrix}=\left[Cov(\vec{f}^{US},\vec{f}^{US})\right]\begin{bmatrix}b_T\\ b_N\end{bmatrix};\quad\begin{bmatrix}\beta^{US,P}_T\\ \beta^{US,P}_N\end{bmatrix}=\left[Cov(\vec{f}^{US},\vec{f}^{US})\right]^{-1}\begin{bmatrix}Cov(f^{US}_T, XR^{-US,+P})\\ Cov(f^{US}_N, XR^{-US,+P})\end{bmatrix}$$

where $\left[Cov(\vec{f}^{US},\vec{f}^{US})\right]$ is the covariance matrix of risk factors. Thus the GMM procedure employed to estimate factor loading b's also estimates factor prices λ's and portfolio risk exposures β's.

Currencies are sorted into four portfolios based on previous nominal interest rates in a procedure similar to the one presented in the above section. Portfolio 1 contains currencies associated with the lowest interest rates, portfolio 4 the highest rates. For this sorting, we use current quarter's nominal interest rates sourced from IMF. The quarterly carry trade excess returns $XR^{-US,+P}_{t+1}$ to US investors are computed over the next three-month periods. This return computation is identical to that of above section. The risk factors f^{US}_T, f^{US}_N are computed as quarter-to-quarter percentage changes of per-capita real US traded and nontraded consumption respectively. The US consumption and CPI series are from US Bureau of Economic Analysis' "Quarterly US consumption expenditures and price indexes". We identify the personal consumption expenditures on "services" as nontraded consumption, and on "goods" as traded consumption (see data appendix for further details).[69] After having constructed the quarterly series of portfolio returns $XR^{-US,+P}_t$ and factors $f^{US}_{T,t}$, $f^{US}_{N,t}$, we employ a two-stage GMM procedure on the above Euler equation to estimates factor loadings b_T, b_N jointly with the first moments μ^{US}_T, μ^{US}_N of the factors, as detailed in Menkhoff et al. (2011).[70] Finally, traded and nontraded factor prices λ^{US}_T, λ^{US}_N and portfolio risk exposures $\beta^{US,P}_T$,

[68]This equation results from the standard Euler equation $E_t\left[\left(1+dm^{US}_{t+1}-E[dm^{US}_{t+1}]\right)XR^{-US,+P}_{t+1}\right]=0$ and the linear factor pricing specification $\log\frac{M^{US}_{t+1}}{M^{US}_t}\equiv dm^{US}_{t+1}=b_T f^{US}_{T,t+1}+b_N f^{US}_{N,t+1}$. See section 4.

[69]It is important to note that we should not use US output series (in the output approach to GDP) for the current factor analysis. This is because for traded component, due to trades, the US traded output is not the same as US traded consumption. And in the theory being tested, it is the consumption risk that matters for the pricing.

[70]We also use lagged values of the carry trade portfolio returns as instruments.

$\beta_N^{US,P}$ are deduced from the above simple matrix operation. Their standard errors are determined from GMM-generated standard errors of factor loading b's and the delta method, as suggested by Burnside et al. (2011).

Figure 5 plots the mean annualized returns and Sharpe ratios on four equally weighted carry trade portfolios. The figures show a monotonic relationship between mean returns in carry trades and the values of mean interest rates across portfolios. This in essence exhibits the violation of UIP and have been widely documented in the literature.[71] It is this monotone that qualifies these four carry trade portfolios as test assets for the empirical analysis of the current linear factor model. Accordingly, Table 7 reports the estimated factor prices. Both factor prices for traded and nontraded risk are positive and significant. Quantitatively, one additional "unit" of exposure to US nontraded consumption risk (i.e., $\Delta\beta_N = 1$) boosts the expected return on the strategy by 32 basis points. The corresponding figure for US traded consumption risk is 34 basis points. Most importantly, the positive nontraded factor price well suits the rational implication of nontraded risk.[72] As nontraded output are largely confined and consumed within country's border, fluctuations in nontraded consumption growths are perceived as risk by all host countries. The proposition 3 then asserts that nontraded factor price λ_N^H should always be positive for all countries H. The results reported in table 7 thus empirically confirms this assertion from US investors' perspective. Beyond that, the results also show that fluctuations in US traded consumption are perceived as a risk by US investors. Table 7 also reports the estimated consumption betas for four currency portfolios. Values of betas vary across portfolios implying that foreign countries with different interest rate levels correlate differently with US traded and nontraded consumption growth. While the current two-factor model most likely leaves out other risk factors,[73] the movements in US traded and nontraded consumption growth are statistically significant sources of risk being priced in the currency market.

[71] For recent related work on UIP violation at protfolio level, see e.g. Burnside et al. (2011), Lustig et al. (2011), Menkhoff et al. (2011).

[72] In the current factor pricing model, the expected excess return on any asset is $E[XR] = \lambda_T\beta_T + \lambda_N\beta_N$. The positive factor price $\lambda_N > 0$ implies that any payoff positively correlated with nontraded consumption growth, $\beta_N > 0$, commands a positive expected return components. In other words, nontraded consumption growth volatility is a risk to investor.

[73] We can infer from table 7 that these two risk factors account for about 15% of the expected carry trade return to US investors.

7 Conclusion

This paper points out the effects of nontraded output growth risk on national asset and international investment returns. Nontraded output growth risk is particularly impactful, because this output makes a large share of GDP and is consumed almost entirely by home population. In contrast, country-specific traded output growth risk can be diversified by means of commodity trades. Hence our analysis calls for a careful decomposition of GDP into traded and nontraded output components before assessing its role on the determination of asset prices.

Nontraded output shocks are nevertheless not entirely internalized by home countries because countries engage in trades in other goods as well. While, to a certain extent, trades weaken the impact of nontraded output risk on the home country, trades also transmit and thus broaden the impact of home nontraded output shocks to all trade partners of the home country. This mechanism is behind the profits of all international strategies, including carry trades. This is because the global traded output risk spreads fairly equally across countries, and thus drops out of strategies involving off-setting positions in different national markets.

The frameworks in which a risk, apparently intrinsic to only one party, actually affects other parties are pervasive in the real world. Examples include any social network settings, financial institutions, or interbank systems featuring counter-party risk. The asset pricing analysis presented here for the international finance setting, especially in regards to transaction costs and incomplete markets, would help shed light on other interesting frameworks just mentioned. We hope to address these frameworks in future work.

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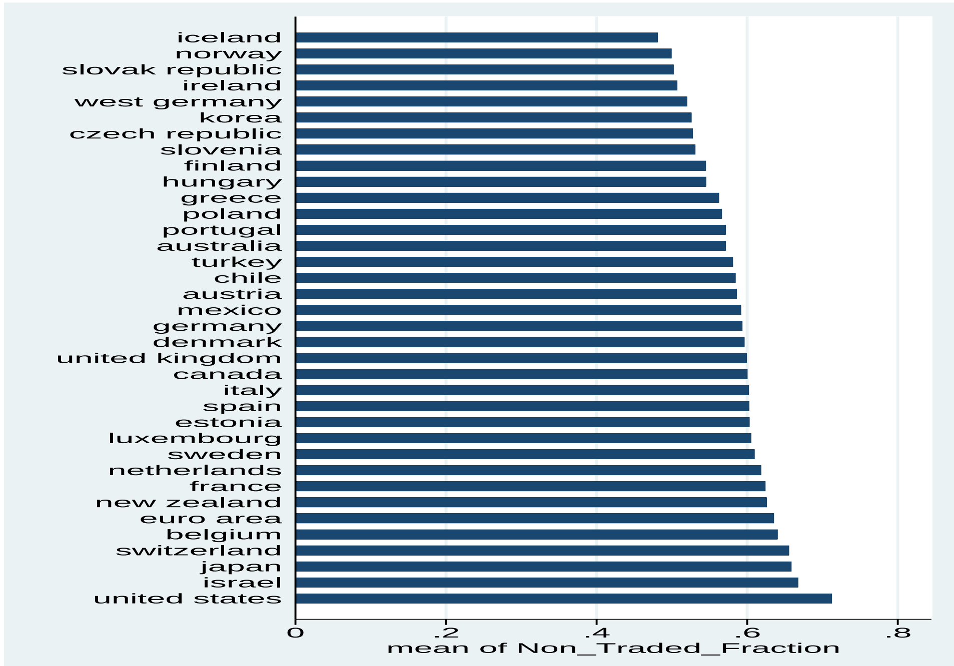


Figure 1: Mean of nontraded output-over-GDP ratio, 1971-2010, for OECD countries

Table 1: Services' tradabilities, 1971-2010

Country	Measure	Financial services	Other services	Construction services
Australia	Tradability (%)	0.36	2.02	0.09
	Fraction of GDP (%)	22.31	16.07	6.28
Canada	Tradability (%)	0.69	3.94	0.34
	Fraction of GDP (%)	21.40	20.72	5.67
Czech Republic	Tradability (%)	2.68	18.88	4.47
	Fraction of GDP (%)	14.72	14.44	6.45
Denmark	Tradability (%)	0.67	12.41	2.43
	Fraction of GDP (%)	18.58	24.38	5.58
Hungary	Tradability (%)	1.70	17.01	7.05
	Fraction of GDP (%)	18.15	18.43	3.98
Japan	Tradability (%)	0.21	2.51	1.87
	Fraction of GDP (%)	23.51	23.72	9.78
New Zealand	Tradability (%)	0.22	5.82	0.67
	Fraction of GDP (%)	26.17	17.13	4.83
Norway	Tradability (%)	0.89	9.28	1.29
	Fraction of GDP (%)	14.76	19.71	4.47
Poland	Tradability (%)	0.72	6.95	5.36
	Fraction of GDP (%)	15.94	16.43	06.37
Sweden	Tradability (%)	0.99	14.41	10.16
	Fraction of GDP (%)	20.52	25.17	4.58
Switzerland	Tradability (%)	7.46	2.69	N/A
	Fraction of GDP (%)	19.31	24.88	N/A
United Kingdom	Tradability (%)	2.93	7.77	1.17
	Fraction of GDP (%)	20.39	20.74	5.30
United States	Tradability (%)	0.46	1.43	0.23
	Fraction of GDP (%)	27.65	26.96	5.23

Notes: This table lists the mean of country-specific tradabilities and sizes of financial, construction, and other services for a representative set of 13 OECD countries, 1971-2010. Tradability of services is (one half of) the ratio of total export and import over total output of these services by a country (see (26)). Fraction of GDP (or size) of services is the ratio of total output of these services over the GDP of a country. See section 6.1 and data appendix for further details.

Table 2: Top-15 (ISIC rev. 3) US traded industries, 1971-2010

	ISIC rev. 3 designation	Industries	US-specific tradability (%)	OECD tradability (%)
1	19	leather, leather products and footwear	379.10	173.16
2	30	office, accounting and computing machinery	188.51	247.59
3	18	wearing apparel, dressing and dying of fur	135.52	105.76
4	34	motor vehicles, trailers and semi-trailers	97.98	128.61
5	272+2732	non-ferrous metals	93.10	149.44
6	32	radio, television and communication equipment	88.05	105.83
7	31	electrical machinery and apparatus, n.e.c.	66.99	82.14
8	33	medical, precision and optical instruments	66.83	106.44
9	29	machinery and equipment, n.e.c.	65.42	83.14
10	353	aircraft and spacecraft	60.00	104.28
11	352+359	railroad equipment and transport equipment, n.e.c.	58.33	111.70
12	17	textiles	56.39	99.83
13	24ex2423	chemicals excluding pharmaceuticals	50.20	108.05
14	23	coke, refined petroleum products and nuclear fuel	44.02	101.03
15	271+2731	iron and steel	41.06	74.31

Notes: This table lists 15 most traded industries in the US, along with their US-specific and OECD tradabilities. The industries are classified by ISIC Revision 3. US-specific tradability is (one half of) the ratio of total export and import over total output by the US of the industry (see (26)). OECD tradability for a industry is defined similarly, but with export, import, and output replaced by total-OECD counterparts (see (25)). See section 6.1 and data appendix for further details.

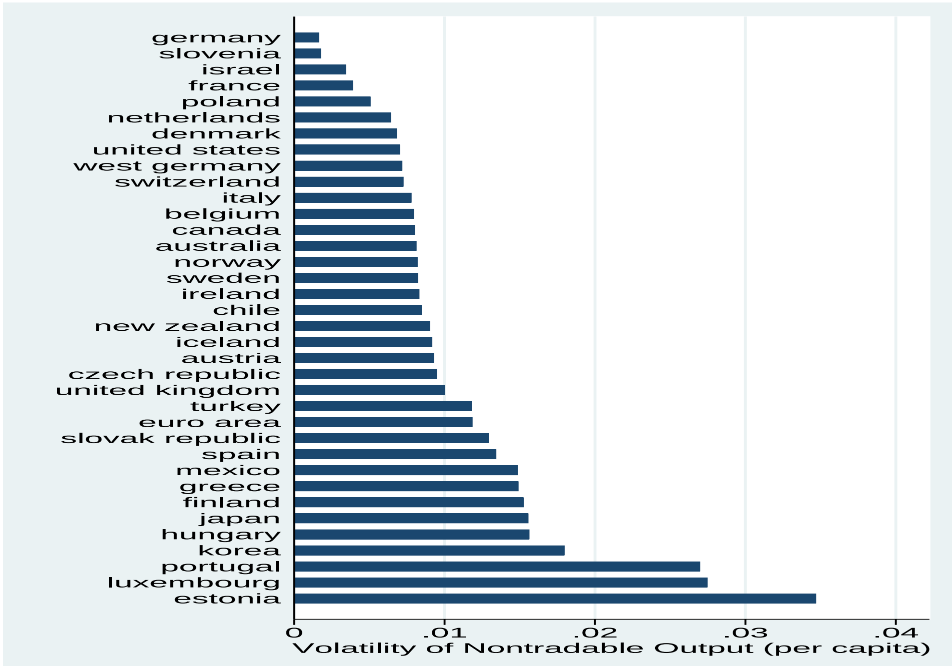


Figure 2: Volatility of per-capita nontraded output growth, 1971-2010, for OECD countries

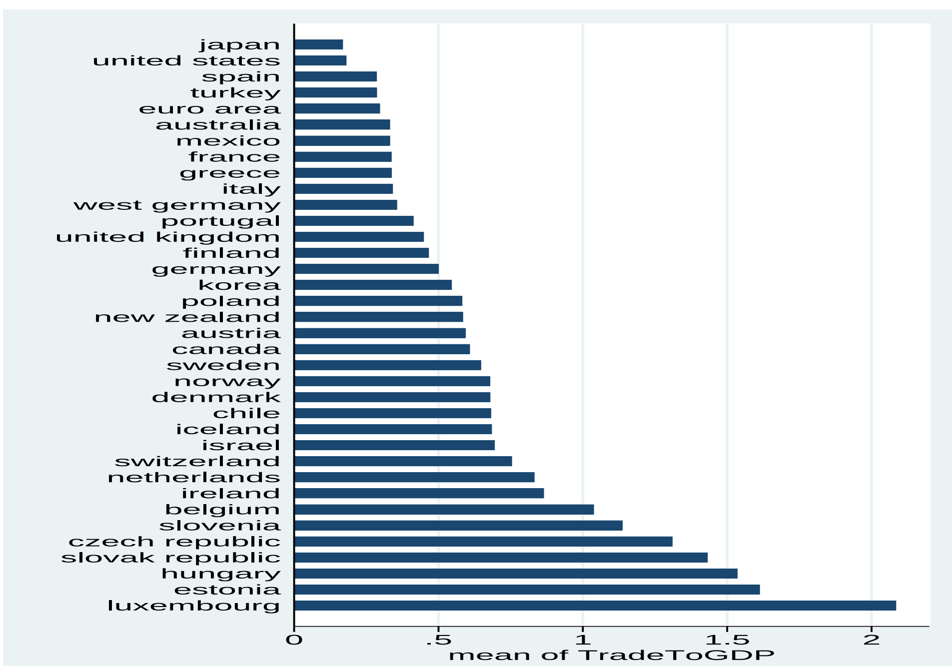
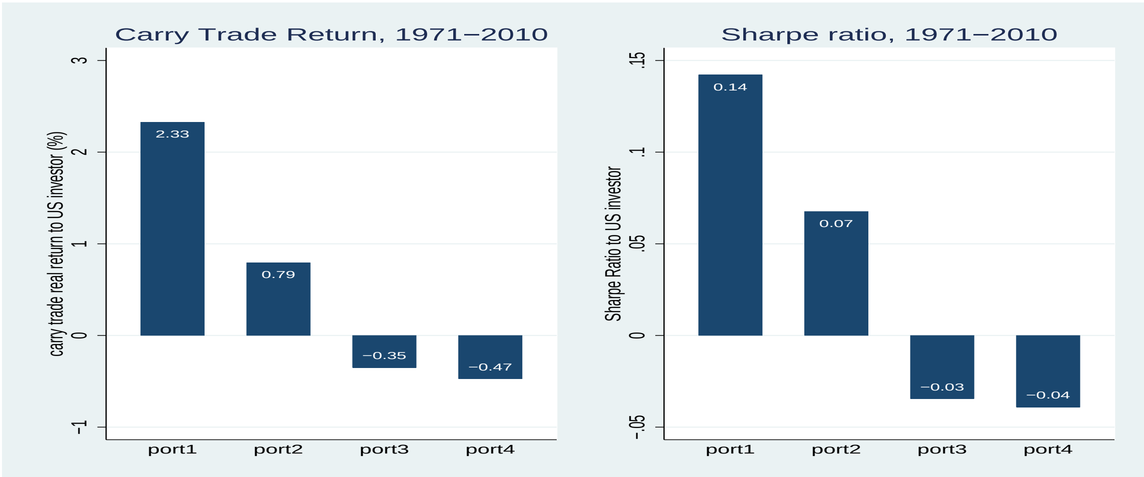


Figure 3: Mean of Trade-to-GDP ratio (i.e., openness), 1971-2010, for OECD countries. Trade is defined as the sum of export and import of the country.

Figure 4: Carry trade excess returns and Sharpe ratios for portfolios sorted on nontraded output risk



This figure presents means and Sharpe ratios of real excess returns on four quarterly rebalanced currency portfolios to US investors. The sample consists of quarterly data series for period 1971-2010. The portfolio are constructed by sorting currencies into four groups at beginning of quarter t based on the value of nontraded variance $\times$ gdp's size over the previous 8 quarters. Portfolio 1 contains currencies with the lowest value of nontraded variance $\times$ gdp's size, portfolio 4 the highest. Due to unbalances in macro-data series, countries' data become available at different times, and number of countries changes over time. See data appendix for further details.

Table 3: Trade-closedness regression, 1971-2010

	Panel A: Four 10-year Periods				Panel B: Entire Period			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
growth variance	-36.245	-39.245	-37.266	-36.449	6.769	2.9643	19.305	18.233
	(22.249)	(23.685)	(23.92)	(24.666)	(27.713)	(29.912)	(28.615)	(30.002)
closedness	-.01246	-.0118	-.00818	-.00842	.01565	.02349	.0436**	.03991**
	(.00892)	(.00889)	(.01046)	(.01039)	(.01458)	(.01706)	(.01809)	(.01918)
variance × closedness	-44.26	-43.553	-51.34*	-52.343*	-93.324*	-113.38**	-167.21***	-159.61***
	(30.133)	(29.686)	(30.984)	(32.282)	(45.975)	(52.996)	(53.36)	(56.442)
growth mean		.1529	.13368	.1205		.39029	.34268	.26642
		(.28422)	(.29327)	(.30318)		(.277)	(.28719)	(.3311)
gdp size			-.03433	-.03288			-.08515***	-.08045***
			(.03625)	(.03593)			(.02984)	(.0277)
inflation volatility				-.00054				-.00124
				(.0007)				(.00082)
constant	.02892***	.02537***	.02596***	.02682***	.01781**	.0074	.00492	.00944
	(.00426)	(.00721)	(.00742)	(.0079)	(.00731)	(.01071)	(.01071)	(.01271)
N	98	98	98	98	33	33	33	33
adj. R^2	0.103	0.097	0.093	0.085	0.082	0.120	0.228	0.228

Notes: OLS regressions with robust standard errors in parentheses: $r_t^H = \alpha + \beta_\sigma^H(\sigma_t^H)^2 + \beta_C^H \mathcal{C}_t^H + \beta_{\sigma C}^H(\sigma_t^H)^2\mathcal{C}_t^H + \beta_x X_t + \epsilon_t^H$ to examine the effects of output volatility σ^H and trade closedness $\mathcal{C}^H$ on interest rate r^H. Panel A reports results when the variance of GDP growth is computed for each of 10-year non-overlapping periods, from 1971 to 2010. Panel B reports results when the variance of GDP growth is computed for the entire period from 1971 to 2010. The sample consists of annual data series for OECD countries 1971-2010, excluding Estonia, Iceland and Turkey. Current members of European Monetary Union are dropped from the sample at the moment they joined the Union, and replaced by a single observation for Eurozone. Growth variance is the annualized variance of growth rate of per-capita real GDP over corresponding period. closedness is one subtracted by the ratio of country's total trade over country's GDP (see (24)). Growth mean is the annualized mean of growth rate of per-capita real GDP over corresponding period. GDP size is the ratio of country's real GDP over total real GDP of OECD member states. Inflation volatility is the standard deviation of country's consumption price index over the corresponding period. See data appendix for further details.

Table 4: Multi-industry nontradability-dummy regression, 1971-2010

	Panel A: Pooled OLS Regression				Panel B: Panel Regression			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
growth variance	-.02793***	-.02659***	-.03036***	-.02558***	-.03765	-.03676	-.04416*	-.03697*
	(.00767)	(.00674)	(.00831)	(.0097)	(.02311)	(.02342)	(.02303)	(.02179)
nontradability dummy	.00347	.00339	.00291	.00334	.00471**	.0047**	.0044**	.00494**
	(.00237)	(.00236)	(.00233)	(.00228)	(.00222)	(.00222)	(.00218)	(.00206)
variance × dummy	-.36047**	-.44973***	-.47498***	-.52871***	-1.5183	-1.5324	-1.5814*	-1.6366*
	(.16142)	(.16597)	(.17103)	(.17824)	(.94835)	(.95064)	(.93323)	(.88259)
growth mean		.01232**	.00932*	.00891		.00172	-.00299	-.0045
		(.00564)	(.00562)	(.00558)		(.00716)	(.00707)	(.00669)
gdp size			-.04086***	-.03773***			-.04019***	-.0346***
			(.00479)	(.00459)			(.00675)	(.0064)
inflation volatility				-.00096***				-.00189***
				(7.1e-05)				(.00018)
constant	.02555***	.02535***	.02716***	.02824***	.02621***	.02618***	.02785***	.03006***
	(.0006)	(.00061)	(.00069)	(.00072)	(.00051)	(.00052)	(.00059)	(.00059)
N	2026	2026	2026	2026	2026	2026	2026	2026
adj. R^2	0.000	0.001	0.016	0.031	0.006	0.005	0.041	0.143

Notes: OLS regressions $r^H_{i,t} = \alpha + \beta_\sigma(\sigma^H_{i,t})^2 + \beta_d d_{i,t} + \beta_{\sigma d}(\sigma^H_{i,t})^2 d_{i,t} + \beta_x X^H_{i,t} + \epsilon^H_{i,t}$ to examine the effects of industry-level output volatility σ^H_i and its dummy nontradability d_i on interest rate r^H. Panel A reports results with robust standard errors in parentheses. Panel B reports results with between-effect standard errors in parentheses. Dependent variable is the annualized real interest rate, proxied by the short-term Treasury bill rate, averaged over the corresponding period. The sample consists of annual data series for OECD countries 1971-2010, excluding Estonia, Iceland and Turkey. Current members of European Monetary Union are dropped from the sample at the moment they joined the Union, and replaced by a single observation for Eurozone. Growth variance is the annualized variance of growth rate of per-capita country-specific industries' real output over each of four 10-year periods, from 1971 to 2010. Nontradability dummies are at industry level; they assume value 1 for industries classified as nontraded sectors (Electricity gas and water supply, Construction, Wholesale and retail trade, restaurant and hotels, Transport storage and communications, Finance insurance real estate and business services, Community social and personal services), and 0 otherwise. Growth mean is the annualized mean of growth rate of per-capita country-specific industries' real output over the corresponding period. GDP size is the ratio of country's real GDP over total real GDP of OECD member states. Inflation volatility is the standard deviation of country's consumption price index over the corresponding period. See data appendix for further details.

Table 5: Multi-industry global nontradability regression, 1971-2010

	Panel A: Pooled OLS Regression				Panel B: Panel Regression			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
growth variance	-.05941**	-.05469**	-.06901**	-.07466***	-.10134	-.10093	-.12434*	-.14698**
	(.02778)	(.02575)	(.02752)	(.02809)	(.06564)	(.06573)	(.0646)	(.06122)
global nontradability	.00133**	.00128**	.00127**	.001*	.0014**	.0014**	.00137**	.00085
	(.00052)	(.00053)	(.00052)	(.00051)	(.00068)	(.00068)	(.00066)	(.00063)
variance × nontradability	-.00608*	-.0055*	-.00706**	-.00829**	-.01099	-.01101	-.01334	-.01707**
	(.00349)	(.00325)	(.00346)	(.00353)	(.00904)	(.00905)	(.00889)	(.00842)
growth mean		.01139**	.00821	.00787		.00109	-.00371	-.00498
		(.00569)	(.00564)	(.00555)		(.00715)	(.00706)	(.00668)
gdp size			-.04127***	-.03829***			-.04075***	-.03539***
			(.00473)	(.00455)			(.00675)	(.00641)
inflation volatility				-.00094***				-.00188***
				(6.9e-05)				(.00018)
constant	.02555***	.02535***	.02719***	.02833***	.02634***	.02632***	.02803***	.03039***
	(.00057)	(.00058)	(.00066)	(.00069)	(.00052)	(.00054)	(.0006)	(.00061)
N	2026	2026	2026	2026	2026	2026	2026	2026
adj. R^2	0.002	0.002	0.018	0.032	0.006	0.005	0.042	0.141

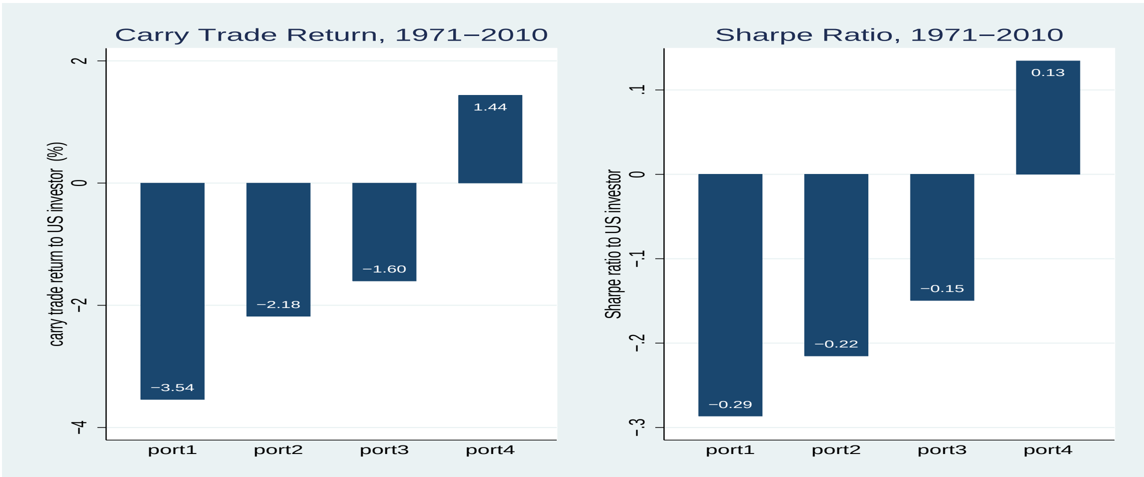
Notes: OLS regressions $r^H_{i,t} = \alpha + \beta_\sigma(\sigma^H_{i,t})^2 + \beta_\tau \tau_{i,t} + \beta_{\sigma\tau}(\sigma^H_{i,t})^2\tau_{i,t} + \beta_x X^H_{i,t} + \epsilon^H_{i,t}$ to examine the effects of industry-level output volatility σ^H_i and its global nontradability τ_i on interest rate r^H. Panel A reports results with robust standard errors in parentheses. Panel B reports results with between-effect standard errors in parentheses. Dependent variable is the annualized real interest rate, proxied by the short-term Treasury bill rate, averaged over the corresponding period. The sample consists of annual data series for OECD countries 1971-2010, excluding Estonia, Iceland and Turkey. Current members of European Monetary Union are dropped from the sample at the moment they joined the Union, and replaced by a single observation for Eurozone. Growth variance is the annualized variance of growth rate of per-capita country-specific industries' real output over each of four 10-year periods, from 1971 to 2010. Here nontradability is a global measure and at industry level; it is one subtracted by the ratio of global total trade (i.e., import plus export) in an industry over the global total output in that industry (see (25)). Growth mean is the annualized mean of growth rate of per-capita country-specific industries' real output over the corresponding period. GDP size is the ratio of country's real GDP over total real GDP of OECD member states. Inflation volatility is the standard deviation of country's consumption price index over the corresponding period. See data appendix for further details.

Table 6: Multi-industry country-specific nontradability regression, 1971-2010

	Panel A: Pooled OLS Regression				Panel B: Panel Regression			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
growth variance	-.03498	-.03985	-.05086*	-.06099**	-.05515	-.05372	-.07389	-.11651*
	(.0269)	(.02707)	(.02697)	(.028)	(.07436)	(.07465)	(.07329)	(.06955)
nontradability	6.3e-05***	4.5e-05*	5.6e-05**	4.2e-05*	9.4e-05*	9.7e-05*	.0001**	6.1e-05
	(2.0e-05)	(2.5e-05)	(2.4e-05)	(2.4e-05)	(5.0e-05)	(5.2e-05)	(5.1e-05)	(4.8e-05)
variance× nontradability	-6.9e-05**	-5.9e-05*	-7.7e-05**	-8.2e-05***	-.00013	-.00013	-.00016**	-.00016**
	(2.8e-05)	(3.0e-05)	(3.0e-05)	(3.1e-05)	(8.1e-05)	(8.1e-05)	(8.0e-05)	(7.5e-05)
growth mean		.00969	.00597	.0063		-.00172	-.00678	-.00659
		(.00626)	(.00623)	(.00614)		(.00739)	(.0073)	(.00691)
gdp size			-.04162***	-.03852***			-.04109***	-.03557***
			(.0048)	(.0046)			(.00675)	(.00642)
inflation volatility				-.00095***				-.00187***
				(7.0e-05)				(.00018)
constant	.02583***	.02564***	.02751***	.02859***	.02664***	.02667***	.0284***	.03061***
	(.00059)	(.00061)	(.0007)	(.00073)	(.00051)	(.00053)	(.00059)	(.0006)
N	2026	2026	2026	2026	2026	2026	2026	2026
adj. R^2	0.001	0.001	0.017	0.031	0.005	0.004	0.042	0.140

Notes: OLS regressions $r^H_{i,t} = \alpha + \beta_\sigma(\sigma^H_{i,t})^2 + \beta_\tau\tau^H_{i,t} + \beta_{\sigma\tau}(\sigma^H_{i,t})^2\tau^H_{i,t} + \beta_x X^H_{i,t} + \epsilon^H_{i,t}$ to examine the effects of industry-level output volatility σ^H_i and its country-specific nontradability τ^H_i on interest rate r^H. Panel A reports results with robust standard errors in parentheses. Panel B reports results with between-effect standard errors in parentheses. Dependent variable is the annualized real interest rate, proxied by the short-term Treasury bill rate, averaged over the corresponding period. The sample consists of annual data series for OECD countries 1971-2010, excluding Estonia, Iceland and Turkey. Current members of European Monetary Union are dropped from the sample at the moment they joined the Union, and replaced by a single observation for Eurozone. Growth variance is the annualized variance of growth rate of per-capita country-specific industries' real output over each of four 10-year periods, from 1971 to 2010. Nontradability is a country-specific measure and at industry level; it is one subtracted by the ratio of country's trade (i.e., import plus export) in an industry over the country's output in that industry (see (26)). Growth mean is the annualized mean of growth rate of per-capita country-specific industries' real output over the corresponding period. GDP size is the ratio of country's real GDP over total real GDP of OECD member states. Inflation volatility is the standard deviation of country's consumption price index over the corresponding period. See data appendix for further details.

Figure 5: Carry trade excess returns and Sharpe ratios for portfolios sorted on nominal interest rates



This figure presents means and Sharpe ratios of real excess returns on four quarterly rebalanced currency portfolios to US investors. The sample consists of quarterly data series for period 1971-2010. The portfolio are constructed by sorting currencies into four groups at beginning of quarter t based on the value of nominal interest rate available then. Portfolio 1 contains currencies with the lowest nominal interest rates, portfolio 4 the highest. Due to unbalances in interest rate and spot exchange rate series, countries' data become available at different times, and number of countries changes over time. See data appendix for further details.

Table 7: Estimation of factor prices in linear factor models

		Nontraded consumption	Traded consumption
Factor prices (%)		.32*** (.02)	.34*** (.07)
beta's	port. 1	-1.92	.49
	port. 2	-1.61	.41
	port. 3	-1.51	.87
	port. 4	-1.87	2.31

Note: Upper panel reports the GMM annualized estimates of the factor prices (in percentage points), lower panel reports the estimates of the portfolios' exposures to risk factors (i.e. beta's) in the carry trade linear factor model using four quarterly rebalanced currency portfolios as test assets. HAC standard errors for the factor prices are obtained by two-stage GMM procedure using constant and lagged carry trade portfolio returns as instruments, and are reported in parenthesis. The currencies are sorted based on interest rates. The sample consists of quarterly data series for the period 1971-2010.

A Data sources

The empirical part of the current paper concerns only countries that belong to the Organisation for Economic Co-operation and Development (OECD) principally because we reasonably expect that data quality for these developed economies should be higher than the rest of the world.

OECD countries: currently, there are 34 OECD member states listed as follows; Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Korea, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States.

Eurozone countries: among OECD states, the following 15 belong to the Economic and Monetary Union (a.k.a., Eurozone or Euro area) with respective adopting date in the parenthesis;[74] Austria (01/01/1999), Belgium (01/01/1999), Estonia (01/01/2011), Finland (01/01/1999), France (01/01/1999), Germany (01/01/1999), Greece (01/01/2001), Ireland (01/01/1999), Italy (01/01/1999), Luxembourg (01/01/1999), Netherlands (01/01/1999), Portugal (01/01/1999), Slovak Republic (01/01/2009), Slovenia (01/01/2007), Spain (01/01/1999).

"Aggregate National Accounts: Gross domestic product" contains the following annual real output series available either in national currency or USD, constant prices of OECD base year 2000 (output approach to GDP): Gross domestic product (B1_GA); Wholesale and retail trade, repairs, hotels and restaurants, transport (B1GG_I); Financial intermediation, real estate, renting and business activities (B1GJ_K); Construction (B1GF); Other service activities (B1GL_P), sourced from OECD.org, downloadable via "OECD.Stat Extracts".

"Total Population series" contains annual data on population, sourced from World Bank World Development Indicators (WDI).

"IMF Exchange Rates and short-term Treasury Bill Rates" provide spot exchange rates and nominal interest rates sourced from IMF International Financial Statistics (IFS) at both quarterly and annually frequencies. Treasury Bill Rates are associated with maturities varying from one to three months. For those countries where the short-term Treasury Bill Rates are not available, we use Money Market Rates from the same sources. Consumer price index (CPI) series is also provided by IFS (at quarterly and annual frequencies). Inflation then is computed as the period-to-period percentage change of the consumer price index.

"Trade-to-GDP ratio" (i.e., trade openness) contains the value of ratio of nominal national total import plus export over national GDP, sourced from OECD Trade Indicators database, downloadable via "OECD.Stat Extracts".

"OECD exchange rate series" contain the exchange rates, in national units per USD (USD monthly average), for all OECD countries. The series is sourced from OECD Main Economic Indicators (MEI), downloadable via "OECD.Stat Extracts".

"Three-month nominal interest rate series" of OECD countries are provided by Data Stream. These original daily series consist of the bid, ask (i.e., offered) and mid quotes for 3-month Eurocurrency-deposit interest rates (end-

[74] Only two other Eurozone states are Cyprus and Malta, but they do not belong to OECD and are not considered in the empirical analysis of the current paper.

of-day quotes from London market). This dataset is unbalanced; Australia's series starts in 1997, Greece in 1994, New Zealand in 1997, Norway in 1997, Portugal in 1993, Spain in 1992, Sweden in 1997. Other OCED countries's series start earlier (before 1984, the date when the spot exchange rate series start, and hence this date 1984 does not pose further data limit constraints for the computation of carry trade returns).

"US quarterly consumer price index (CPI) series" is sourced from OECD Main Economic Indicators (MEI), downloadable via "OECD.Stat Extracts".

"Quarterly National Accounts" database contains quarterly series on expenditure on services ("P314B: Services") for individual OECD countries. This is a component of the expenditure on total private consumption, in the expenditure approach to GDP. For those OECD countries where these series on services expenditure are not available, we substitute them by the quarterly services output series ("B1GG_P-Services"). These quarterly dataset is quite unbalanced, namely available data of different countries start at quite different time. Quarterly US consumption data series are very limited, being available only from 1995 onward. Consequently, the Quarterly US consumption data will be sourced from the US Bureau of Economic Analysis (see next).

"Quarterly US consumption expenditures and price indexes" are series from US Bureau of Economic Analysis. Table 2.3.5. therein contains "Personal Consumption Expenditures by Major Type of Product". Table 2.3.4. contains "Price Indexes for Personal Consumption Expenditures by Major Type of Product". We identify the personal consumption expenditures on services (i.e., the component "Services" listed in these tables) as the US nontraded consumption. We identify the personal consumption expenditures on other goods (i.e., the component "Goods" listed in these tables) as the US traded consumption. These quarterly series start well before 1971 (all our empirical studies in the current paper concern periods starting in 1971 or later).

"Trade in Services" is from OECD's International Trade and Balances of Payments database. This dataset includes the export and import series (in the transactions between residents and non-residents), in unit of countries' currencies and at annual frequency, of financial services, construction services and other services. Financial services cover financial intermediary and auxiliary services (except those of insurance enterprises and pension funds) conducted between residents and non-residents. Included are intermediary service fees, such as those associated with letters of credit, bankers' acceptances, lines of credit, financial leasing, and foreign exchange transactions. Construction services cover work performed on construction projects and installations by employees of an enterprise in locations outside the economic territory of the enterprise. Other business services cover various categories of service transactions between residents and non-residents. They include (i) merchanting and other trade-related services, (ii) operational leasing services (rental) without operators, (iii) legal, accounting, management consulting, and public relation services, (iv) advertising, market research and public opinion polling services transacted between residents and non-residents (v) research and development services, (vi) architectural, engineering and other technical services, (vii) agricultural, mining and on-site processing services, (viii) other miscellaneous business, professional and technical services. See original data source for further details.

"OECD Structural Analysis (STAN)" database provides, for each OECD country, the annual nominal output series (in national currency) and the corresponding deflator series (of OECD base year 2000) for various industries. It also provides country-specific annual nominal import and export series (in national currency) and the corresponding deflator series (of OECD base year 2000) for these industries. We construct the real output series by dividing the nominal series by the respective deflator series. The constructed real output series are thus in national currency, constant price of base year 2000. All real output series are detrended using Hodrick-Prescott filter. The following non-nested industries are listed in STAN, with International Standard Industrial Classification of All Economic Activities (ISIC) Rev. 3 identification given in parenthesis: Agriculture hunting and related service activities (01); Forestry logging and related service activities (02); Fishing; fish hatcheries; fish farms and related services (05); Mining of coal and lignite extraction of peat (10); Extraction of crude petroleum and natural gas and related services (11); Mining of uranium and thorium ores (12); Mining of metal ores (13); Other mining and quarrying (14); Food products and beverages (15); Tobacco products (16); Textiles (17); Wearing apparel, dressing and dying of fur (18); Leather, leather products and footwear (19); Wood and products of wood and cork (20); Pulp, paper and paper products (21); Printing and publishing (22); Coke, refined petroleum products and nuclear fuel (23); Chemicals and chemical products (24ex2423); Pharmaceuticals (2423); Rubber and plastics products (25); Other non-metallic mineral products (26); Iron and steel (271+2731); Non-ferrous metals (271+2732); Fabricated metal products, except machinery and equipment (28); Machinery and equipment n.e.c. (29); Office, accounting and computing machinery (30); Electrical machinery and apparatus n.e.c. (31); Radio, television and communication equipment (32); Medical, precision and optical instruments (33); Motor vehicles, trailers and semi-trailers (34); Building and repairing of ships and boats (351); Aircraft and spacecraft (353); Railroad equipment and transport equipment n.e.c. (352+359); Manufacturing nec (36); Recycling (37); Electricity, gas, steam and hot water supply (40); Collection, purification and distribution of water (41); Construction (45); Sale, maintenance and repair of motor vehicles; retail sale of fuel (50); Wholesale, trade & commission excl. motor vehicles (51); Retail trade excl. motor vehicles; repair of household goods (52); Hotels and restaurants (55); Land transport, transport via pipelines (60); Water transport (61); Air transport (62); Supporting and auxiliary transport activities (63); Post and telecommunications (64); Financial intermediation except insurance and pension funding (65); Insurance and pension funding, except compulsory social security (66); Activities related to financial intermediation (67); Real estate activities (70); Renting of machinery and equipment (71); Computer and related activities (72); Research and development (73); Other business activities (74); Public administration and defense compulsory social security (75); Education (80); Health and social work (85); Sewage and refuse disposal sanitation and similar activities (90); Activities of membership organization n.e.c. (91); Recreational cultural and sporting activities (92); Other service activities (93); Private households with employed persons (95); Extra-territorial organizations and bodies (99); High technology manufactures (N/A); Medium-high technology manufactures (N/A); Medium-low technology manufactures (N/A); Low technology manufactures (N/A).

B Derivations and proofs: Basic model

In the basic model with complete market and no trade friction, in equilibrium the marginal utilities of traded consumption equal across countries, which give K FOCs; $M_T = \frac{\partial U^H}{\partial C_T^H}$ $\forall H = 1, \ldots, K$. The market clearing condition for traded good presents another equation to solve for $K+1$ unknowns; $\{C_T^H\}_{H=1}^K$ and M_T. We log-linearize the system to obtain approximative solution in closed form.

Equilibrium log consumption (3): Plugging the expression (1) for U^H into the FOC (2), and log-linearizing this FOC around the steady state corresponding to the symmetric configuration $\left\{\delta_T^H = \delta_N^H;\ \delta_T^H/\Lambda^H = \delta_T^F/\Lambda^F\right\}$ yield an approximate equation[75]

$$m_T \approx \lambda^H - \rho t + (\epsilon - \gamma)(\omega_T c_T^H + \omega_N \delta_N^H) - \epsilon c_T^H + \log \omega_T.$$

Similarly, log-linearizing the traded good market clearing equation yields (where $\lambda = \log \Lambda = \log \sum_H^K \Lambda^H$)

$$\sum_H^K \frac{\Lambda^H}{\Lambda} c_T^H = \delta_T + \sum_H^K \frac{\Lambda^H}{\Lambda} \lambda^H - \lambda. \tag{27}$$

Substituting c_T^H from the first equation above into the second equation gives m_T, and then c_T^H in (3).

Country-specific stochastic discount factor (5): In pricing country-specific financial assets, the appropriate measures are country-specific consumption baskets (i.e., national currencies in the current consumption-based setting). A country-specific consumption basket is the lowest-cost bundle of traded and nontraded consumption that delivers a unit of country's utility, given the consumption goods' prices $\{P_T^H \equiv 1, P_N^H\}$ (in term of traded goods). The basket's composition $\{C_T^H, C_N^H\}$ and value P^H solve $\min_{C_T^H, C_N^H} P^H \equiv C_T^H + C_N^H P_N^H$ subject to $\left[\omega_T (C_T^H)^{1-\epsilon} + \omega_N (C_N^H)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} = 1$. Then follows the value of consumption basket in term of traded good

$$P_t^H = \left[\omega_T^{\frac{1}{\epsilon}} + \omega_N^{\frac{1}{\epsilon}} (P_N^H)^{\frac{1-\epsilon}{-\epsilon}}\right]^{\frac{-\epsilon}{1-\epsilon}};$$

From this and M_T above follows the identity in equilibrium $M_t P_t^H = M_t^H$, where $M_t^H \equiv \frac{\partial U^H}{\partial C^H} = e^{-\rho t}(C^H)^{-\gamma}$ and C^H is the country-specific consumption aggregator.[76] The current price of the country-specific risk-free bond (that pays one unit of country-specific consumption basket at time s) is

$$B_{t,s}^H = \frac{1}{P_t^H} E_t \left[\frac{M_s}{M_t} P_s^H\right] = E_t \left[\frac{M_s^H}{M_t^H}\right].$$

It is this pricing equation that establishes the above M_t^H as the country-specific SDF of country H. That is, prices computed using this SDF are in unit of country-specific consumption basket. Log-linearizing $m^H = \log M^H = -\rho t - \gamma C^H$ and using log equilibrium traded consumption c_T^H in (3) yield country-specific log SDF (5).

Costly trades: Suppose that home country is an importer (case 1) and trades take place, the variation of

[75]We recall that lower-case letters always denote logarithms; $m \equiv \log M$, $\lambda \equiv \log \Lambda$, $c = \log C$, $\delta = \log \Delta$ and so on.

[76]In contrast with the country-specific M^H, M_T is the marginal utility with respect to traded good and is same for all countries in complete market settings.

social planner's Lagrangian with respect to non-binding consumptions $\frac{\partial}{\partial C_F^H}$, $\frac{\partial}{\partial C_F^F}$ produces FOC $\left(C_H^H + C_F^H\right)^{-\gamma} = (1+\theta)\left(C_H^F + C_F^F\right)^{-\gamma}$. Combining this with binding consumption $C_H^H = \Delta^H$, $C_F^H = 0$, and market clearing condition $C_F^F + (1+\theta)C_F^H = \Delta^F$ yields (7). From this we can also find home SDP $M^H = e^{-\rho t}\left(\Delta^H + C_F^H\right)^{-\gamma}$. The risk-free rate r^H is the opposite to expected growth rate of M^H; $r^H = -\frac{1}{dt}E_t\left[\frac{dM^H}{M^H}\right]$. Plugging equilibrium consumption solutions (7) into M^H, and an application of Ito lemma yields (assuming independent endowments Δ^H, Δ^F)

$$r^H = \rho + \gamma\frac{(1+\theta)\mu^H\Delta^H + \mu^F\Delta^F}{(1+\theta)\Delta^H + \Delta^F} - \frac{1}{2}\gamma(\gamma+1)\frac{(1+\theta)^2(\sigma^H)^2(\Delta^H)^2 + (\sigma^F)^2(\Delta^F)^2}{[(1+\theta)\Delta^H + \Delta^F]^2}$$

which is a more explicit version of (8).

Proof of Proposition 1. From (5) follow the partial derivatives $\frac{\partial m^H}{\partial \delta_N^H} = -\gamma\omega_N\left[1 - \alpha(\gamma-\epsilon)\omega_T\left(1 - \frac{\Lambda^H}{\Lambda}\right)\right]$ and $\frac{\partial m^F}{\partial \delta_N^H} = -\gamma\omega_N\left[\alpha(\gamma-\epsilon)\omega_T\frac{\Lambda^H}{\Lambda}\right]$. Evidently, $\left|\frac{\partial m^H}{\partial \delta_N^H}\right| > \left|\frac{\partial m^F}{\partial \delta_N^H}\right|$ because $\gamma - \epsilon > 0$ (assumption 1, section 2). ∎

Proof of eq. (9) **and Proposition 2.** We start with the differential representation for SDF M^H

$$\frac{dM^H}{M^H} = -r^H dt - \eta^H dZ^H; \quad m^H = \log m^H \Longrightarrow dm^H = -\left(r^H + \frac{1}{2}(\eta^H)^2\right) - \eta^H dZ^H.$$

where η^H is the home market price of risk. Similar relations hold for M^F and m^F. Plugging these into the realized carry trade excess return $XR_{t+dt}^{-H,+F}$ (upper equation in (9)), applying Ito's lemma and taking the conditional expectation yield

$$\begin{aligned}
E_t\left[XR_{t+dt}^{-H,+F}\right] &= E_t\left[\frac{1+\frac{dM^F}{M^F}}{1+\frac{dM^H}{M^H}}(1+r^F dt) - (1+r^H dt)\right] \\
&= E_t\left[\left(1 + dm^F + \frac{1}{2}(dm^F)^2\right)\left(1 - dm^H + \frac{1}{2}(dm^H)^2\right)(1+r^F dt) - (1+r^H dt)\right] \\
&= E_t\left[dm^F + \frac{1}{2}(dm^F)^2 - dm^H + \frac{1}{2}(dm^H)^2 - dm^H dm^F + r^F dt - r^H dt\right] \\
&= (\eta^H)^2 - \eta^H\eta^F = -Cov_t\left[dm^H, dm^F - dm^H\right],
\end{aligned}$$

which is (9). Next, combining (5) and (10) implies the key expression for expected carry trade excess return (11) of Proposition 2. ∎

Proof of Proposition 3. We first develop (13) to obtain more explicit expressions for λ_T and λ_N

$$\lambda_T^H = Var(f_T^H)b_T + Cov(f_T^H, f_N^H)b_N; \qquad \lambda_N^H = Cov(f_T^H, f_N^H)b_T + Var(f_N^H)b_N.$$

Plugging $\{b_T, b_N\}$ and $\{f_T^H, f_N^H\}$ from (12) into above expressions yields (14) of Proposition 3 and (15) for factor prices associated with nontraded and traded consumption growth risk respectively. ∎

C Derivations and proofs: Arbitrary trade configurations

This appendix presents technical derivations of the results concerning arbitrary trade configurations of section 5.1. Here, there are K countries and l different types of traded goods. A (generic) traded good of type h is consumed by some subset of K_h countries, and a (generic) country H consume l^H types of traded goods (apart from the country's intrinsic nontraded good). Consumption tastes $\left\{\{\omega^H_{h,T}\}_{h=1,\dots,K_h}, \omega^H_N\right\}$ (with normalization $\sum_h^{l^H} \omega^H_{h,T} + \omega^H_N$) are heterogeneous across countries. Country-good count and good-country count are necessarily identical

$$\sum_h^l K_h = \sum_H^K l^H. \tag{28}$$

The assumption of complete financial market is maintained here and implies that marginal utilities of a traded good are equalized across counries that consume this traded good in equilibrium (this is a FOC in the social planner's optimization problem). Furthermore, the physical market for this traded good h is also cleared among K_h countries,

$$M_h = \Lambda^H \frac{\partial U^H}{\partial C^H_h}; \qquad \sum_{H \in K_h} C^H_h = \Delta_{h,T}, \qquad \forall h;\ \forall H \in K_h$$

Thus, in total we have $\sum_h K^h$ equations[77] and $\sum_H l^H$ unknowns consumptions $\{C^H_h\}^{H=1,\dots,K}_{h \in l^H}$. By virtue of (28), in principle, the social planner's optimization alone is sufficient to determine all equilibrium traded consumption allocations $\{c^H_h\}$. In practice, however, the above system is highly nonlinear for CES utilities (1). To obtain approximate solution we log-linearize above system, which yields a set of $\sum_h K^h$ linear equations and that same number of unknowns,

$$\left\{ \begin{array}{l} m_h = \lambda^H - \rho t + \log \omega^H_{h,T} + (\epsilon - \gamma) \left(\sum_j^{l^H} \omega^H_{j,T} c^H_j + \omega^H_N \delta^H_N \right) - \epsilon c^H_h, \\ \sum_{H \in K_h} \frac{\Lambda^H}{\Lambda_h} c^H_h = \delta_{h,T} + \sum_{H \in K_h} \frac{\Lambda^H}{\Lambda_h} \lambda^H - \lambda_h; \quad \Lambda_h \equiv \sum_H^{K_h} \Lambda^H;\ \lambda^H \equiv \log \Lambda^H;\ \lambda_h \equiv \log \Lambda_h; \end{array} \right. \quad \forall h;\ \forall H \in K_h. \tag{29}$$

Albeit linearity, this system is (almost arbitrarily) large due to arbitrary trade configuration. We first note that we can always reduce this system to l equations and l unknowns. Multiplying both sides of above eq for m_h by ω^H_h, then summing over $h \in l^H$ (while keeping H fixed) generate a relation between $\sum \omega^H_{h,T} m_h$ and $\sum \omega^H_{h,T} c^H_h$,

$$\begin{aligned} \sum_h^{l^H} \omega^H_{h,T} m_h &= \left(\lambda^H - \rho t + \log \omega^H_{h,T}\right)(1 - \omega^H_N) + (\epsilon - \gamma)(1 - \omega^H_N)\omega^H_N \delta^H_N - \left[\epsilon \omega^H_N + \gamma(1 - \omega^H_N)\right] \sum_h^{l^H} \omega^H_{h,T} c^H_h \\ &= \left(\lambda^H - \rho t + \log \omega^H_{h,T}\right)(1 - \omega^H_N) + (\epsilon - \gamma)(1 - \omega^H_N)\omega^H_N \delta^H_N - \frac{1}{\alpha^H} \sum_h^{l^H} \omega^H_{h,T} c^H_h, \end{aligned} \tag{30}$$

where we have used the consumption tastes normalization, $\sum_h^{l^H} \omega^H_{h,T} + \omega^H_N = 1$ and the definition (17) of weighted elasticity of substitution

$$\alpha^H \equiv \frac{1}{\epsilon \omega^H_N + \gamma(1 - \omega^H_N)}.$$

This relation is the key bridge that connects the country-specific SDF M^H (or marginal utilities of consumption aggregator) to the marginal utilities of traded goods M_h. Indeed, by log-linearizing $m^H \equiv \log M^H_t = \log \frac{\partial U^H}{\partial C^H}$ we obtain

$$m^H = -\rho t - \gamma \left(\sum_h^{l^H} \omega^H_{h,T} c^H_h + \omega^H_N \delta^H_N \right) = \# - \epsilon \gamma \alpha^H \omega^H_N \delta^H_N + \gamma \alpha^H \sum_h^{l^H} \omega^H_{h,T} m_h \tag{31}$$

[77] For each traded good h, we have one market clearing equation and $(K_h - 1)$ FOCs (because M_h is not known a priori).

where we have omitted the deterministic terms (which are independent of stochastic endowments δ's). Backing out $\sum \omega_{h,T}^H c_h^H$ in term of $\sum \omega_{h,T}^H m_h$ from (30) and substituting it into upper equation of (29) give an consumption allocation c_h^H in term of $\{m_j\}$,

$$c_h^H = \alpha^H \left(\lambda^H - \rho t + \log \omega_{h,T}^H\right) - (\gamma - \epsilon)\alpha^H \omega_N^H \delta_N^H - \frac{m_h}{\epsilon} + \frac{(\gamma-\epsilon)\alpha^H}{\epsilon} \sum_j^{l^H} \omega_{j,T}^H m_j. \tag{32}$$

Multiplying both sides of this equation by $\frac{\Lambda^H}{\Lambda_h}$, summing over H, and plugging it into market clearing conditions (lower equation of (29)) indeed yield l linear equations (i.e., $h = 1, \ldots, l$)

$$\begin{aligned} m_h = \quad & -\epsilon \left(\sum_{H \in K_h} \frac{\Lambda^H}{\Lambda_h} \lambda^H - \lambda_h \right) + \epsilon \sum_{H \in K_h} \frac{\Lambda^H \alpha^H}{\Lambda_h} \left(\lambda^H - \rho t + \log \omega_h^H\right) - \epsilon(\gamma - \epsilon) \sum_{H \in K_h} \frac{\Lambda^H}{\Lambda_h} \alpha^H \omega_N^H \delta_N^H \\ & -\epsilon \delta_{h,T} + (\gamma - \epsilon) \sum_{H \in K_h} \frac{\Lambda^H}{\Lambda_h} \alpha^H \sum_j^{l^H} \omega_{j,T}^H m_j, \qquad \forall h = 1, \ldots, l, \end{aligned} \tag{33}$$

for l unknowns $\{m_h\}$. We next solve this system and the equilibrium consumption (approximately) by iteration method. The procedure consists of 4 steps.

step 1: (Zeroth order of m_h) We conjecture that the global (aggregate) endowment $\delta_{h,T}$ of traded good type h dominates other endowment $\{\delta_{j,T}\}_{j \neq h}$ in the contribution to m_h. We then can decouple the above system and solve for each m_h separately in zeroth order. We also note that the term m_h on the right-hand side of above equation is negligible compared to term m_h on the left-hand side. Thus, in zeroth order, $\forall h = 1, \ldots, l$,

$$m_h^{(0)} = -\epsilon \left(\sum_{H \in K_h} \frac{\Lambda^H}{\Lambda_h} \lambda^H - \lambda_h \right) + \epsilon \sum_{H \in K_h} \frac{\Lambda^H \alpha^H}{\Lambda_h} \left(\lambda^H - \rho t + \log \omega_h^H\right) - \epsilon(\gamma - \epsilon) \sum_{H \in K_h} \frac{\Lambda^H}{\Lambda_h} \alpha^H \omega_N^H \delta_N^H - \epsilon \delta_{h,T}$$

step 2: (First order of m_h) We substitute the zeroth-order $m_h^{(0)}$ above into right-hand size of (33) to obtain first-order expression for m_h (we again omit all deterministic terms, which are independent of stochastic endowments δ's)

$$\begin{gathered} m_h^{(1)} = \# - \epsilon \left(\delta_{h,T} + (\gamma-\epsilon) \sum_{H \in K_h} \tfrac{\Lambda^H}{\Lambda_h} \alpha^H \sum_j^{l^H} \omega_{j,T}^H \delta_{j,T}\right) \\ -\epsilon(\gamma-\epsilon) \sum_{H \in K_h} \tfrac{\Lambda^H}{\Lambda_h} \alpha^H \left(\omega_N^H \delta_N^H + (\gamma-\epsilon) \sum_j^{l^H} \omega_{j,T}^H \sum_{J \in K_j} \tfrac{\Lambda^J}{\Lambda_j} \alpha^J \omega_N^J \delta_N^J\right) \end{gathered}$$

The coefficient associated with $\delta_{j,T}|_{j \neq h}$ is $(\gamma - \epsilon) \sum_{H \in K_h \cap K_j} \frac{\Lambda^H}{\Lambda_h} \alpha^H \omega_{j,T}^H$, so endowment of traded good of type j contributes more the marginal utility m_h of good h when there are more countries H that consume both goods. To be consistent with the log-linearization approximation, we do not need to go beyond the iteration's first order.

step 3: (Traded consumption allocation c_h^H) Substituting the first-order $m_h^{(1)}$ above into (32) yields equilibrium consumption[78] c_h^H in (16).

step 4: (Country-specific log SDF m^H) Substituting the first-order $m_h^{(1)}$ above into (31) yields equilibrium consumption c_h^H in (18).

[78] We note that the log-linearization approximation is accurate up to terms of order $\mathcal{O}(\omega_N)$, $\mathcal{O}(\omega_T)$. Consequently, we disregard all terms of order $\mathcal{O}((\omega_N)^2)$, $\mathcal{O}((\omega_T)^2)$, $\mathcal{O}(\omega_T \omega_N)$.

D Derivations and proofs: Incomplete markets

This appendix presents technical derivations of results concerning incomplete financial markets of section 5.2, in particular the equilibrium consumptions (22), (23). Substituting the conjectured consumption allocation (21) into (20) yields a more explicit expression for the log-linearized SDF

$$dm_T^H = -\frac{g^H}{\alpha^H} - \frac{a^H}{\alpha^H} d\delta_T - \left[(\gamma - \epsilon)\omega_N^H + \frac{b^{HH}}{\alpha^H}\right] d\delta_N^H - \sum_{F \neq H} \frac{b^{HF}}{\alpha^H} d\delta_N^F. \tag{34}$$

In the current setting (K countries of homogeneous size with a single traded good), the market clearing condition in log-linearized form is a special case of (27) (where all Λ^H are identical) and reads $\sum_{H=1}^K c_T^H = \sum_{H \in \mathcal{D}} c_T^H + \sum_{H \notin \mathcal{D}} c_T^H = K\delta_T - K \log K$, which implies

$$\sum_{H=1}^K dc_T^H = \sum_{H \in \mathcal{D}} dc_T^H + \sum_{H \notin \mathcal{D}} dc_T^H = K d\delta_T.$$

where d denotes the difference operator acting between t and $t+1$. Substituting the conjectured equilibrium consumption allocations (21) in above equation yields the a set of constraints for the solution parameters

$$\sum_H a^H = K; \qquad \sum_H g^H = 0; \qquad \sum_H b^{HF} = 0, \ \ \forall F. \tag{35}$$

On the other hand, substituting dm^H in (34) into the law of one price (19) for $\frac{S_{T,t}}{B_{T,t}}$ implies that the following expression is the same for all H,

$$\log\left(\frac{S_{T,t}}{B_{T,t}}\right) = Cov_t\left[dm_{T,t+1}^H, \delta_{T,t+1}\right] = -\frac{a^H}{\alpha^H}\sigma_T, \quad \forall H.$$

Combining above two equations immediately yields

$$a^H = \frac{K\alpha^H}{\sum_{H=1}^K \alpha^H}; \qquad \alpha^H \equiv \frac{1}{\gamma\omega_T^H + \epsilon\omega_N^H}. \tag{36}$$

Similarly, the law of one price (19) for $\frac{S_{T,t}^F}{B_{T,t}^F}$ implies that $\log\left(\frac{S_{N,t}^F}{B_{N,t}^F}\right) = Cov_t\left[dm_{T,t+1}^H, \delta_{N,t+1}^F\right]$ is identical for each developed country $F \in \mathcal{D}$ and all countries H. Using (34), we have

$$(\gamma - \epsilon)\omega_N^F + \frac{b^{FF}}{\alpha^F} = \frac{b^{HF}}{\alpha^H} \qquad \text{for each } F \in \mathcal{D}, \ \text{ for all } H \neq F.$$

When combined with the constraint (35) above, this yields

$$\begin{cases} b^{FF} = -(\gamma - \epsilon)\omega_N^F \alpha^F \left(1 - \frac{\alpha^F}{\sum_{I=1}^K \alpha^I}\right), & \forall F \in \mathcal{D} \\ b^{HF} = (\gamma - \epsilon)\omega_N^F \alpha^F \alpha^H \frac{1}{\sum_{I=1}^K \alpha^I}, & \forall F \in \mathcal{D} \quad \forall H \neq F \end{cases} \tag{37}$$

In particular, given a choice of $F \in \mathcal{D}$, we note that $\frac{b^{HF}}{\alpha^H}$ is the same for all $H \neq F$.

Next, first substituting conjectured solution (21) into (34), and then into the law of one price (19) for bond $B_{T,t}$

imply that

$$A \equiv \frac{g^H}{\alpha^H} + (\gamma-\epsilon)\omega_N^H \mu_N^H + \frac{1}{\alpha^H}\sum_{F=1}^{K} b^{HF}\mu_N^F - \frac{1}{2}\frac{1}{(\alpha^H)^2}\sum_{F=1}^{K}\left(b^{HF}\right)^2 (\sigma_N^F)^2 - \frac{1}{2}(\gamma-\epsilon)^2(\omega_N^H)^2(\sigma_N^H)^2 - (\gamma-\epsilon)\omega_N^H \frac{b^{HH}}{\alpha^H}(\sigma_N^H)^2,$$

is the same for all H. Using (37), we separately rewrite the above expression for emerging and developed economies,

$$\begin{aligned} H \notin \mathcal{D}: \quad A &= \frac{g^H}{\alpha^H} + (\gamma-\epsilon)\omega_N^H\mu_N^H + \sum_{F\notin\mathcal{D}} \frac{b^{HF}}{\alpha^H}\mu_N^F - \frac{1}{2}\sum_{F\notin\mathcal{D}}\left(\frac{b^{HF}}{\alpha^H}\right)^2 (\sigma_N^F)^2 - \frac{1}{2}(\gamma-\epsilon)^2(\omega_N^H)^2(\sigma_N^H)^2 \quad (38) \\ &- (\gamma-\epsilon)\omega_N^H\frac{b^{HH}}{\alpha^H}(\sigma_N^H)^2 + \left[\sum_{F\in\mathcal{D}} \frac{(\gamma-\epsilon)\omega_N^F\alpha^F}{\sum_{I=1}^K \alpha^I}\mu_N^F - \frac{1}{2}\sum_{F\in\mathcal{D}}\left(\frac{(\gamma-\epsilon)\omega_N^F\alpha^F}{\sum_{I=1}^K\alpha^I}\right)^2(\sigma_N^F)^2\right] \\ H \in \mathcal{D}: \quad A &= \frac{g^H}{\alpha^H} + \sum_{F\notin\mathcal{D}}\frac{b^{HF}}{\alpha^H}\mu_N^F - \frac{1}{2}\sum_{F\notin\mathcal{D}}\left(\frac{b^{HF}}{\alpha^H}\right)^2(\sigma_N^F)^2 \quad (39) \\ &+ \left[\sum_{F\in\mathcal{D}} \frac{(\gamma-\epsilon)\omega_N^F\alpha^F}{\sum_{I=1}^K \alpha^I}\mu_N^F - \frac{1}{2}\sum_{F\in\mathcal{D}}\left(\frac{(\gamma-\epsilon)\omega_N^F\alpha^F}{\sum_{I=1}^K\alpha^I}\right)^2(\sigma_N^F)^2\right]. \end{aligned}$$

We note that the expressions within the square brackets are identical (i.e., independent) for all countries H (either $H \in D$ or $H \notin D$), and thus can be disregarded. The above requirement imposed by the law of one price on bond $B_{T,t}$ thus becomes

$$\left\{\frac{g^H}{\alpha^H} + (\gamma-\epsilon)\omega_N^H\mu_N^H - \tfrac{1}{2}(\gamma-\epsilon)^2(\omega_N^H)^2(\sigma_N^H)^2 - (\gamma-\epsilon)\omega_N^H\frac{b^{HH}}{\alpha^H}(\sigma_N^H)^2 + \sum_{F\notin\mathcal{D}}\frac{b^{HF}}{\alpha^H}\mu_N^F - \tfrac{1}{2}\sum_{F\notin\mathcal{D}}\left(\frac{b^{HF}}{\alpha^H}\right)^2(\sigma_N^F)^2\right\}\Bigg|_{\forall H\notin\mathcal{D}}$$

$$= \left\{\frac{g^H}{\alpha^H} + \sum_{F\notin\mathcal{D}}\frac{b^{HF}}{\alpha^H}\mu_N^F - \tfrac{1}{2}\sum_{F\notin\mathcal{D}}\left(\frac{b^{HF}}{\alpha^H}\right)^2(\sigma_N^F)^2\right\}\Bigg|_{\forall H\in\mathcal{D}}. \quad (40)$$

This system has the following simple solution (of pooling type within developed economies, and within emerging economies), that also satisfies the constraint $\sum_H b^{HF} = 0$ in (35),

$$\begin{cases} b^{HF} = -\alpha^H, & \forall H \notin \mathcal{D}, F \notin \mathcal{D} \\ b^{HF} = \frac{\sum_{I\notin D}\alpha^I}{\sum_{J\in D}\alpha^J}\alpha^H, & \forall H \in \mathcal{D}, F \notin \mathcal{D} \end{cases} \quad (41)$$

and the appropriate country-specific parameters g^H to assure all equalities in 40. Finally, substituting the solution parameters in (36), (37), (41) into (21) we obtain the equilibrium consumption allocations (22), (23) for emerging and developed economies, respectively.