# Microprudential Regulation in a Dynamic Model of Banking[1]

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## ABSTRACT

This paper studies the quantitative impact of microprudential bank regulations on bank lending and value metrics of efficiency and welfare in a dynamic model of banks that are financed by debt and equity, undertake maturity transformation, are exposed to credit and liquidity risks, and face financing frictions. We show that: (a) there exists an inverted U–shaped relationship between bank lending, welfare, and capital requirements; (b) liquidity requirements unambiguously reduce lending, efficiency and welfare; and (c) resolution policies contingent on observed capital, such as prompt corrective action, dominate in efficiency and welfare terms (non–contingent) capital and liquidity requirements.

**Keywords**: Bank Regulation, Dynamic Banking Model

**JEL codes**: G21, G28, G33

# I. Introduction

The 2007–09 financial crisis has been a catalyst for significant bank regulation reforms, as the pre-crisis *microprudential* regulatory framework has been judged inadequate to cope with large financial shocks.[1] The new Basel III framework envisions a raise in bank capital requirements and the introduction of new liquidity requirements. At the same time, there is an active debate regarding how to make prompt corrective action (PCA) policies and related bank closure rules more effective in reducing the costs of government's bank bailouts under deposit insurance.[2] Yet, the relatively large literature on microprudential bank regulation presents several models that are mostly static and seldom evaluate microprudential regulation policies in terms of efficiency or welfare criteria.[3] To our knowledge, there is no study offering a *joint* assessment of capital regulation, liquidity requirements, and PCA policies in a dynamic model of banks which perform a well-defined intermediation function and deposits are insured. Formulating such a model is the main contribution of this paper.

We study a partial equilibrium model in the tradition of the valuation approach pioneered by Merton (1977) and Kareken and Wallace (1978), which we design consistently with standard corporate finance set-ups adapted to the specifics of banks, as advocated in Flannery (2012). The economy is driven by a macroeconomic (systematic) risk factor and financial markets are in equilibrium. The banking system is composed of banks exposed to a systematic risk factor and an idiosyncratic risk component. Investors' preferences are represented by a stochastic discount factor that is parameterized as in Jones and Tuzel (2013), which delivers countercyclical risk

[1] We follow Brunnermeier, Crockett, Goodhart, Persaud, and Shin (2009) in referring to microprudential regulation as regulation that concerns the stability of individual financial institutions, as opposed to macroprudential regulation, which refers to regulation that concerns the stability of the financial system as a whole.

[2] The new Basel III framework is detailed in *Basel III: A global regulatory framework for more resilient banks and banking systems*, Bank for International Settlements, Basel, June 2011. The reform would increase the minimum common equity requirement from 2% to 4.5%. The Tier 1 capital requirement will increase from 4% to 6%. In addition, banks will be required to hold a "capital conservation buffer" of 2.5% to withstand future periods of stress bringing the total common equity requirements to 7%. Two new liquidity requirements are planned to be introduced: a short–term liquidity coverage ratio, meant to ensure the survival of a bank for one month under stressed funding conditions, and a long–term so–called net stable funding ratio, designed to limit asset and liabilities mismatches. For a discussion of PCA reforms, see Government Accountability Office, Bank Regulation: Modified Prompt Corrective Action Framework Would Improve Effectiveness, Washington D.C., June 2011.

[3] For a review of the literature, see Van Hoose (2007) and Freixas and Rochet (2008). Hellwig (2010) recently observed that "the current (regulatory) system has no theoretical foundation, its objectives are ill–specified, and its effects have not been thought through, either for the individual bank or for the system."

premia. With this pricing kernel, we evaluate banks' securities, and derive measures of bank efficiency and welfare.

Four features characterize our model. First, we analyze banks that dynamically transform short–term liabilities into longer–term partially illiquid assets whose returns are uncertain. This feature is consistent with banks' special role in liquidity transformation, as banks in our model can be viewed as a dynamic infinite–horizon version of the intermediaries studied by Diamond and Dybvig (1983) and Allen and Gale (2007). Second, we consider banks whose deposits are insured. Deposit insurance is introduced since a key asserted role of capital regulation is the abatement of the excessive bank risk–taking arising from moral hazard under partial or total insurance of its liabilities. Thus, there is a potential role for capital requirements, and their effectiveness in abating banks' probability of default can be assessed. Third, banks can be in financial distress. Such distress can be viewed as arising from market incompleteness of the type analyzed by Allen and Gale (2004) as well as from asymmetric information frictions that make equity issuance costly and make banks unable to raise uncollateralized debt. These assumptions are meant to capture an environment in which liquidity requirements may in principle have a role in minimizing the risk that illiquid but solvent banks become insolvent, and face increased costs to raise funding in case of financial distress. Lastly, we assess the impact of microprudential bank regulation in terms of value metrics of bank efficiency and welfare. The first metric is enterprise value, which is interpreted as the efficiency with which the bank carries out its maturity transformation function. The second metric, called "social value," proxies the contribution to welfare associated with banking activities, as measured by the discounted expected value of banks to all bank stakeholders and the government. In the sequel, we refer to this metric interchangeably as social value or welfare.

The optimal polices and metrics of efficiency and welfare of unregulated banks are our benchmarks. Relative to these benchmarks, we compare policies and value metrics of efficiency and welfare of banks subject to: a) capital requirements resembling risk–based Basel II-type capital requirements; b) capital requirements *and* liquidity requirements resembling Basel-III liquidity coverage ratios, and c) a PCA provision which implements capital requirements through restrictions on banks' payouts and a bank closure rule, both *contingent* on observed levels of capital. We assess the impact of these bank regulations quantitatively by simulating the model under a set of calibrated parameters, with regulatory parameters mimicking current regulations.

Our study contributes to a sparse literature analyzing microprudential bank regulation in the context of dynamic models of banking. Calem and Rob (1999) study a model of a bank of fixed size that chooses its return distribution, but is not allowed to issue equity. Bhattacharya, Plank, Strobl, and Zechner (2002) and Peura and Keppo (2006) consider capital regulation with bank closure rules implemented under banks' random audits. Estrella (2004) considers bank regulatory restrictions on value at risk, while Elizalde and Repullo (2007) and Zhu (2008)) consider capital regulation under simple bank closure rules. However, differing from our work, none of these papers analyze bank regulations in a unified framework where banks undertake maturity transformation under deposit insurance, can issue debt and equity, are exposed to multiple sources of risks, and value metrics of efficiency and welfare are evaluated quantitatively.

Our study of microprudential regulation focuses on the properties of bank optimal policies as related to the systematic risk factor, whose evolution proxies the business cycle, and on optimal policies and metrics of efficiency and welfare in steady state.

The analysis of bank optimal policies along the business cycle yields two main results. First, under a mild capital requirement and the PCA, bank lending, debt, and capital ratios are positively correlated with the systematic risk factor, while liquidity ratios are negatively correlated with this factor. These correlations turn out not significantly different from the ones exhibited by unregulated banks. In particular, these results suggest that risk–based capital regulation does not necessarily enhance the pro–cyclicality of bank lending. Second, when liquidity requirements are added to capital requirements, the positive correlation between capital ratios and the systematic risk factor increases especially in upturns. This happens because liquidity requirements force banks to use retained earnings to build up liquidity buffers rather than invest in lending. As a result, capital ratios become inflated in an upturn, but they are not different from those of banks subject to capital regulation only in a downturn. In essence, capital regulation and the PCA provide banks with incentives to create liquidity buffers in downturns. With mandatory liquidity requirements, however, banks are forced to build these buffers during upturns as well, thereby depressing lending.

The steady state (or unconditional) analysis of the model delivers three key results. First, there exists an inverted U–shaped relationship between bank lending and the stringency of capital requirements. Such relationship translates into an inverted U–shaped relationship between welfare and the stringency of capital requirements. When capital requirements are mild, banks find it optimal to fulfill them through an increase in lending. This increase allows banks to build up capital through increased revenues and retained earnings. Relative to unregulated banks, the quantitative increase in bank lending due to mild capital requirement is a notable 15% in our calibration. Banks' probability of default is also lower than that of their unregulated counterparts, indicating that capital requirements are successful in reducing banks' failures. However, if capital requirements become too stringent, then banks find it optimal to fulfill them through a reduction of lending, since lending exhibits decreasing returns, and building up equity through increased revenues and retained earnings becomes too costly. These novel findings suggest the existence of optimal levels of bank–specific regulatory capital under deposit insurance.

Second, liquidity requirements reduce bank lending, efficiency, and welfare, with these reductions increasing monotonically with their stringency. This occurs because liquidity requirements severely hamper banks' maturity transformation, forcing banks to use retained earnings to increase bond holdings or reduce indebtedness, rather than investing them in lending. Moreover, when liquidity requirements are added to capital requirements, they also destroy the efficiency and welfare benefits of mild capital requirements, since bank lending, efficiency and social values are reduced relative to the bank subject to capital regulation only. Quantitatively, the declines in bank lending and value metrics of efficiency and welfare associated with liquidity requirements are large, namely in the order of 20%–25% in our calibration.

Third, a resolution procedure contingent on observed bank capitalization such as the PCA dominates both capital and liquidity requirements in efficiency and welfare terms. Recall that deposit insurance introduces incompleteness in deposit contracts, as deposit payments are not contingent on the realization of states of nature. This is inefficient, as these contingencies may be instrumental in attaining optimality in banking environments similar to ours (see, e.g., De Nicolò (1996) and Allen and Gale (1998)). Non-contingent capital and liquidity requirements are insufficient for, or even detrimental to, attaining optimality. By contrast, the PCA introduces contingencies based on observed equity. These contingencies substitute for the missing contingencies in deposit payments due to deposit insurance. Thus, resolution procedures such as the PCA appear a necessary tool to achieve optimality of bank regulation.

Importantly, our results arise from the full dynamic modeling of bank management of portfolio allocations and of retained earnings. Static or finite horizon models used extensively in the banking literature can only partially capture how banks make their policy decisions subject to the true shadow costs associated with regulatory requirements. This explains why our results differ in some important respects from those obtained by static or finite short–horizon models used extensively in the banking literature.

The remainder of this paper comprises four sections. Section II describes the model and the decision problem of unregulated banks. Section III introduces microprudential bank regulations. Section IV illustrates some basic trade-offs on optimal policies through a three–period version of the model. Section V details the results of our simulation analysis of optimal policies and value metrics of efficiency and welfare along the business cycle as well as in steady state. Section VI concludes. The Appendix describes some properties of the bank's dynamic program, the computational procedures used to map systematic and idiosyncratic risk factors onto banks' credit and liquidity risks, and the algorithm used to solve the model.

## II. The model

Time is discrete, the horizon is infinite, and a systematic (macroeconomic) risk drives risk–premia in a financial market equilibrium. The systematic risk is denoted by $u$ and follows an autoregressive process

$$u_t = \kappa_u u_{t-1} + \sigma_u \varepsilon_t^u, \tag{1}$$

where $\varepsilon_t^u$ is i.i.d. with a truncated standard Normal distribution, $\kappa_u$ is the autocorrelation parameter such that $|\kappa_u| < 1$, and $\sigma_u$ is the conditional standard deviation.

Following Jones and Tuzel (2013), the preferences of the representative investor are summarized by a one–period stochastic discount factor. Given the transition to state $u_{t+1}$ from the current state, $u_t$, the stochastic discount factor is

$$M(u_t, u_{t+1}) = \beta e^{-g_t \varepsilon_{t+1}^u - \frac{1}{2} g_t^2 \sigma_u^2}, \tag{2}$$

where the state–dependent coefficient of risk–aversion is defined as $g_t = g(u_t) = \exp(\gamma_1 + \gamma_2 u)$, with $0 < \beta < 1$, $\gamma_1 > 0$ and $\gamma_2 < 1$. Using this pricing kernel, we will later on evaluate banks' securities.[4]

There is a finite set of heterogeneous banks indexed by $j$. Bank managers maximize shareholders' value, so there are no managerial agency conflicts. Banks receive a random stream of short–term insured deposits, can issue risk–free short–term collateralized debt, and invest in longer term risky productive assets and short–term bonds. Thus, banks are exposed to credit and liquidity risks. As detailed below, these risks are assumed to be affine functions of a systematic risk factor and a bank idiosyncratic risk factor. The bank idiosyncratic risk factor $v^j$ follows an autoregressive process

$$v_t^j = \kappa_v v_{t-1}^j + \sigma_v \varepsilon_t^j, \tag{3}$$

where $\varepsilon_t^j$ is i.i.d. with truncated standard Normal distribution, and $|\kappa_v| < 1$. We assume that $\varepsilon_{t+1}^u$ is independent of $\varepsilon_{t+1}^j$ for all $j$'s, and that the latter is independent across banks. The random vector $s_t = (u_t, v_t)$ evolves according to a stationary and monotone Markov transition function $Q(s_{t+1} \mid s_t)$ jointly defined by Equations (1) and (3). We denote $\mathcal{S}$ the state space of $s$, where $\mathcal{S}$ is compact.[5] In the remainder of this section, we drop the superscript $j$, as we will be solving a representative bank problem.

## A. Bank's balance sheet

On the asset side, a bank can invest in a liquid, one–period bond (a T-bill), which yields a constant rate $r_f$, and in a portfolio of risky assets, called loans. We denote with $B_t$ the face value of the risk–free bond, and with $L_t \geq 0$ the nominal value of the stock of loans outstanding in period $t$ (i.e., in the time interval $(t-1, t]$). Note that since $B_t$ can have unrestricted sign,

[4] Other partial equilibrium approaches based on a reduced–form stochastic discount factor are Berk, Green, and Naik (1999) and Zhang (2005). Differently from other functional forms of stochastic discount factors, the one assumed by Jones and Tuzel (2013) has the convenient property of having a state–independent risk–free discount factor: in our case, the gross yield of a risk–free zero coupon bond is $1/\mathbb{E}_t\left[M_{t+1}\right]$, where $\mathbb{E}_t\left[M_{t+1}\right] = \beta e^{-\frac{1}{2}g_t^2\sigma_u^2} \cdot \mathbb{E}_t\left[e^{-g_t\varepsilon_{t+1}^u}\right] = \beta$.

[5] As detailed in Appendix C, the support of each state variable $u$ and $v$ is a compact set discretized using Rouwenhorst (1995) approach.

we assume it is the net position in bonds, thereby allowing the bank to simultaneously borrow or lend at the rate $r_f$. Similarly to Zhu (2008), we make the following assumptions.

*Assumption* 1 (Revenue function). The total revenue from loan investment is given by $Z_t\pi(L_t)$, where $\pi(L_t)$ satisfies $\pi(0) = 0$, $\pi > 0$, $\pi' > 0$, and $\pi'' < 0$.

This assumption is empirically supported, as there is evidence of decreasing return to scale of bank investments.[6] In our model loans can be viewed as including traditional loans as well as risky securities. $Z_t = Z(u_t, v_t)$ is a random *credit shock* realized on loans in the same time period, which captures variations in banks' total revenues as determined by the systematic and idiosyncratic shocks. Note that the choice variables $B_t$ and $L_t$ are set at the beginning of the period, while $Z_t$ is realized only at the end of the period.

The maturity of deposits is set to one period. Bank maturity transformation is introduced with the following

*Assumption* 2. A constant proportion $\delta \in (0, 1/2)$ of the existing stock of loans at $t$, $L_t$, becomes due at $t + 1$.

The parameter $\delta < 1/2$ indexes the average maturity of the existing stock of loans, which is $1/\delta - 1 > 1$.[7] Thus, the bank is engaging in maturity transformation of short–term liabilities into longer–term investments, as in Diamond and Dybvig (1983). Under Assumption 2, the law of motion of $L_t$ is

$$L_t = L_{t-1}(1 - \delta) + I_t, \tag{4}$$

where $I_t$ is the investment in new loans if it is positive, or the amount of cash obtained by liquidating loans if it is negative.

Convex asymmetric loan adjustment costs as in the Q-theory of investment (see, e.g., Abel and Eberly (1994)) are introduced to capture banks' information production costs about credit quality through long–term banking relationships, with the following

[6] See for instance, Berger, Miller, Petersen, Rajan, and Stein (2005), Carter and McNulty (2005), Cole, Goldberg, and White (2004).

[7] The (weighted) average maturity of existing loans at date $t$, assuming the bank does not default nor it makes any adjustments on the current investment in loans, is $\sum_{s=0}^{\infty} s \frac{\delta L_{t+s}}{L_t} = \frac{1}{\delta} - 1$, as the residual loans outstanding at date $t + s$, $s \geq 0$, is $L_{t+s} = L_t(1 - \delta)^s$.

*Assumption* 3 (Loan Adjustment Costs). The adjustment–costs function for loans is quadratic:

$$m(I_t) = |I_t|^2 \left(\chi_{\{I_t>0\}} \cdot m^+ + \chi_{\{I_t<0\}} \cdot m^-\right), \tag{5}$$

where $\chi_{\{\mathcal{A}\}}$ is the indicator of event $\mathcal{A}$, $m^+ > 0$ and $m^- > 0$ are the unit cost parameters, and costs are deducted from profits.

According to Assumption 3, a bank incurs screening and monitoring per–unit costs $m^+$ when it increases lending, and per–unit liquidation costs $m^-$ when loans are reduced. If $m^- > m^+$, then there is costly reversibility, since a bank would face higher costs to liquidate investments rather than expanding them. This would be consistent with costs of breaking bank relationships higher than those associated with an expansion of lending to old as well as new customers.[8] The magnitude of these loan adjustment costs will be pinned down in our calibration.

On the liability side, the bank receives a random amount of one-period deposits $D_{t+1} = D(u_t, v_t)$ at the beginning of period $t+1$, and this amount remains outstanding during the period. Considering random and exogenous short–term deposits is an assumption that simplifies banks' optimal problem.[9] The interest rate on deposits is $r_d \leq r_f$, where the difference between the rate on bonds and the remuneration of deposits captures implicit costs of payment services associated with deposits. The change $D_{t+1} - D_t$ is an exogenous *liquidity shock* driven by the realizations of systematic and idiosyncratic shocks.

Deposits are insured according to the following

*Assumption* 4 (Deposit insurance). The deposit insurance agency insures all deposits. In the event that a bank defaults on deposits and on the related interest payments, depositors are paid interest and principal by the deposit insurance agency, which absorbs the relevant loss.

Under this assumption, with no change in the model, the depositor can be viewed as the deposit insurance agency itself, whose claims are risky, while deposits are effectively risk–free from depositors' standpoint. Thus, the difference between the ex–ante yield on deposits and

[8] For a review of bank relationships, see Boot (2000). For the costs associated with breaking up bank relationships as forgone monopoly rents due to holdup problems, see Rajan (1992) and Sharpe (1990). A portion of liquidation costs $m^-$ could also be viewed as capturing fire sales costs arising from financial distress (see, e.g., Acharya, Shin, and Yorulmazer (2011) and Hanson, Kashyap, and Stein (2011)).

[9] Banks' increasing reliance on market funding in the past decade may suggest that modeling them as active seekers of liquidity may be an important extension of the current model, as mentioned in our conclusions.

the rate on bonds can be viewed as including a subsidy that the agency provides to the bank, as the cost of this insurance is not charged to either banks or depositors.

To fund operations, a bank can issue one–period bonds and equity. Following Hennessy and Whited (2005), we assume that a bank is constrained to issue fully collateralized bonds, so that the bond yield is the rate $r_f$. We denote $B_t < 0$ the notional amount of the bond issued at $t-1$ and outstanding until $t$. The collateral constraint is described below.

To summarize, at $t-1$ (i.e., at the beginning of period $t$), after the investment and financing decisions have been made, a bank's balance sheet equation is

$$L_t + B_t = D_t + K_t, \tag{6}$$

where $K$ denotes the ex–ante book value of equity, or *bank capital*. Note that $B > 0$ denotes a positive risk–free investment (net of issued bonds), whereas $B < 0$ denotes the face value of issued bond (net of risk–free investment).

## B. Bank cash flow

Once $Z_t$ and $D_{t+1}$ are realized at $t$, the current state (before a decision is made) is summarized by the vector $x_t = (L_t, B_t, D_t, u_t, v_t)$, as a bank enters date $t$ with loans, bonds and deposits in amounts $L_t$, $B_t$, and $D_t$, respectively. *Prior* to investment, financing and cash distribution decisions, the total internal cash available to a bank is

$$W_t = W(x_t) = y_t - \mathcal{T}(y_t) + B_t + \delta L_t + (D_{t+1} - D_t). \tag{7}$$

Equation (7) says that total internal cash $W_t$ equals earnings before taxes (EBT),

$$y_t = y(x_t) = \pi(L_t)Z_t + r_f B_t - r_d D_t, \tag{8}$$

minus corporate taxes $\mathcal{T}(y_t)$, plus the principal of one–year investment in bond maturing at $t$, $B_t > 0$ (or alternatively the amount of maturing one–year debt, $B_t < 0$), plus the repayment of maturing loans $\delta L_t$, plus the net change in deposits, $D_{t+1} - D_t$.

Consistently with current dynamic models of a non–financial firm (see, e.g., Hennessy and Whited (2007)), corporate taxation is introduced with the following

*Assumption* 5 (Taxation). Corporate taxes are paid according to a convex function of EBT:

$$\mathcal{T}(y_t) = \tau^+ \max\{y_t, 0\} + \tau^- \min\{y_t, 0\}, \tag{9}$$

where $\tau^-$ and $\tau^+$, $0 \leq \tau^- \leq \tau^+ < 1$, are the marginal corporate tax rates in case of negative and positive EBT, respectively.

The assumption $\tau^- \leq \tau^+$ is standard in the literature, as it captures a reduced tax benefit from loss carryforward or carrybacks. Note that convexity of the corporate tax function creates an incentive to manage cash flow risk, as noted by Stulz (1984).

Given the available cash $W_t$ as defined in Equation (7) and the residual loans, $L_t(1-\delta)$, a bank chooses the new level of investment in loans, $L_{t+1}$ and the amount of risk–free bonds $B_{t+1}$ (purchased if positive, issued if negative). As a result, Equation (6) applies to $B_{t+1}$, $L_{t+1}$, $D_{t+1}$, and both $L_{t+1}$ and $B_{t+1}$ remain constant until the next decision date, $t+1$.

These choices may differ according to whether a bank is, or is not, in financial distress. If total internal cash $W_t$ is *positive*, it can be retained or paid out to shareholders. If $W_t$ is *negative*, a bank is in *financial distress*, since absent any action, it would be unable to honor part, or all, of its obligations towards either the tax authority, or depositors, or bondholders. When in financial distress, a bank can finance the shortfall $W_t$ by liquidating loans, by issuing bonds ($B_{t+1} < 0$), or by injecting equity capital. Overcoming this shortage of liquidity is expensive because all these transactions generate either explicit of implicit costs. In liquidating loans, a bank incurs the downward adjustment cost defined by Equation (5), bond issuance is subject to a collateral restriction that can limit bank debt capacity, and underwriting costs are paid when seasoned equity is offered. We now present these latter two restrictions on the financial channels of a bank.

Bond issuance is constrained by the following

*Assumption* 6 (Collateral constraint). If $B_t < 0$, the amount of bond issued by the bank must be fully collateralized. In particular, the constraint is

$$L_t - m(-L_t(1-\delta)) + \pi(L_t)Z_d - \mathcal{T}(y_t^{\min}) - r_d)D_t + B_t(1+r_f) + D_d - D_t \geq 0, \tag{10}$$

where $Z_d$ is the worst possible credit shock (i.e., the lower bound of the support of $Z$), $D_d$ is the worst case scenario flow of deposits, and $y_t^{\min} = \pi(L_t)Z_d + r_f B_t - r_d D_t$ is the EBT in the worst case end–of–period scenario for current $L_t$, $B_t$ and $D_t$.

The constraint in Equation (10) reads as follows: the end–of–period amount $B_t(1 + r_f)$ that the bank has to repay must not be higher than the after–tax operating income, $\pi(L_t)Z_d - r_d D_t - \mathcal{T}(y_t^{\min})$ in the worst case scenario, plus the total available cash obtained by liquidating the loans, $L_t - m(-L_t(1-\delta))$, plus the flow of new deposits in the worst case scenario, $D_d$, net of the claim of current depositors, $D_t$. Available cash would then be the sum of the loans that will become due, $L_t\delta$, plus the amount that can be obtained by a forced liquidation of the loans, $L_t(1-\delta)$ net of the adjustment cost $m(-L_t(1-\delta))$, as per Equation (5).[10] An obvious implication of this constraint is that a bank's indebtedness will be always bounded above.

We denote with $\Gamma(D_t)$ the feasible set for a bank when the current deposit is $D_t$, defined as the set of $(L_t, B_t)$ such that condition (10) is satisfied if $B_t < 0$, with no restrictions being imposed when $B_t \geq 0$:

$$\begin{aligned} \Gamma(D_t) = \{&(L_t, B_t) \mid \\ &\frac{L_t - m(-L_t(1-\delta)) + D_d + \pi(L_t)Z_d(1-\tau_t^{\min})}{1 + r_d(1-\tau_t^{\min})} + B_t \frac{1 + r_f(1-\tau_t^{\min})}{1 + r_d(1-\tau_t^{\min})} \geq D_t, \\ &B_t < 0\} \cup \{B_t \geq 0\}, \end{aligned} \quad (11)$$

where $\tau_t^{\min} = \tau^+$ if $y_t^{\min} > 0$ and $\tau_t^{\min} = \tau^-$ if $y_t^{\min} < 0$.

Similarly to Cooley and Quadrini (2001), we assume that issuing equity is costly (for instance because of underwriting fees):

*Assumption* 7 (Equity issuance costs). A bank raises capital by issuing seasoned shares incurring a proportional cost $\lambda > 0$ on the value of new equity issued.

Observe that when banks are not in financial distress, total costs of equity issuance will be just a proportion of the total amount of equity issued. When banks are in financial distress, however, the cost of equity issuance will be increased by the additional amount that the bank

[10]By Assumption 9 introduced below, the support for deposits and credit shock processes is compact, implying that the collateral constraint is well defined.

has to raise owing to loan liquidation costs. Therefore, the more severe the financial distress, the larger is the transaction cost incurred to raise equity financing.[11]

As a result of the choice of $(L_{t+1}, B_{t+1})$, the residual cash flow to shareholders at date $t$ is

$$U_t = U(x_t, L_{t+1}, B_{t+1}) = W_t - B_{t+1} - L_{t+1} + L_t(1-\delta) - m(I_{t+1}). \tag{12}$$

If $U_t$ is positive, it is distributed to shareholders (as either dividends or stock repurchases). If $U_t$ is negative, it equals the amount of newly issued equity inclusive of the higher cost due to underwriting fees. Hence, the actual cash flow to equity holders is

$$e_t = e(x_t, L_{t+1}, B_{t+1}) = \max\{U_t, 0\} + \min\{U_t, 0\}(1+\lambda). \tag{13}$$

Figure 1 depicts the evolution of the state variables and the related bank's decisions when the bank is solvent.

Lastly, bank's insolvency occurs according to the following

*Assumption* 8 (Insolvency). In the case of default, bank shareholders exercise the limited liability option (i.e., equity value is zero), and the assets are transferred to the deposit insurance agency, net of bankruptcy costs in proportion $\eta > 0$ of the size of a bank, proxied by the face value of deposits, $D_t$. Right after default a bank is reorganized as a new entity endowed with deposits $D_{t+1}$ and new capital $K_{t+1} = D_u - D_{t+1} \geq 0$, where $D_u$ is the upper bound of deposit process. The restructured bank invests initially only in risk–free bonds, $B_{t+1} = D_u$, so that $L_{t+1} = 0$. The capital injected by the government in the new bank is financed with general tax proceeds.

Assumption 8 embeds three features. First, since default is irreversible, a new bank financed with initial public capital is formed to replace a defaulted bank in order to preserve intermediation services. Second, when the government intervenes and sets up a new bank, it does not incur any underwriting cost, since no new shares are issued in the open market.

[11] Note that even though we make the simplifying assumption that the costs of debt and equity issuance costs are independent, thus assuming some segmentation of equity and debt markets, the shadow costs associated with these two forms of financing are not necessarily independent, since the shadow cost of debt will be determined by the extent to which banks have spare debt capacity, while, as observed, total equity issuance costs are increasing in the degree of financial distress.

Lastly, the government is assumed to be able to finance any recapitalization of an individual bank with general tax proceeds.[12]

The mapping of systematic and idiosyncratic risk factors onto credit and liquidity risks is defined by the following

*Assumption* 9. The random vector $X_t = (Z_t, \log D_{t+1})$ is an affine function of the state variable $s_t = (u_t, v_t)$:

$$X_t = \mu + N s_t, \tag{14}$$

for given two–dimensional vector $\mu$ and two–times–two non–singular matrix $N$.

In particular, this assumption translates into the following law of motion for $(Z_t, \log D_{t+1})$:

$$\begin{aligned} Z_t &= (1-\kappa_Z)\overline{Z} + \kappa_Z Z_{t-1} + \xi_t^Z \\ \log D_{t+1} &= (1-\kappa_D)\log \overline{D} + \kappa_D \log D_t + \xi_t^D. \end{aligned} \tag{15}$$

In the above equations, $\kappa_Z$ is the persistence parameter, $\sigma_Z$ is the conditional volatility, and $\overline{Z}$ is the long–term average of the credit shock; $\kappa_D$ is the persistence parameter for the deposit process, $\sigma_D$ is the conditional volatility, and $\overline{D}$ is the long–term level of deposits. The error terms $(\xi_t^Z, \xi_t^D)$ are related to $(\varepsilon_t^u, \varepsilon_t)$ and have correlation coefficient $\rho$. The parameters of the process $(Z_t, D_{t+1})$ are estimated on data. In Appendix B we detail how the parameters of the process $(u_t, v_t)$ are related to the parameters of the process $(Z_t, D_{t+1})$.

[12] Observe that no default can occur if $B_t < 0$ owing to the collateral constraint. From Equation (12), the cash flow to shareholders is

$$[L_t - m(I_{t+1}) + \pi(L_t)Z_t - \mathcal{T}(y_t) + B_t(1+r_f) - D_t(1+r_d) + D_{t+1}] - B_{t+1} - L_{t+1}.$$

The collateral constraint implies

$$L_t - m(-L_t(1-\delta)) + \pi(L_t)Z_d - \mathcal{T}(y_t^{\min}) + B_t(1+r_f) - D_t(1+r_d) + D_d \geq 0,$$

and therefore, the part in square brackets in the cash flow expression is non–negative. Hence, the cash flow to equity holders can be negative only if $L_{t+1}$ is significantly higher than $B_{t+1}$. However, if the bank defaults, such a policy will be undone (and set at levels $B_{t+1} = D_u$, $L_{t+1} = 0$, as specified in Assumption 8), and shareholders will avoid default to get a positive value of their claim. Therefore, in the model, default of a non–regulated bank may occur only when $B_t \geq 0$.

## C. The unregulated bank program and the valuation of securities

Let $E$ denote the market value of bank's equity. Given the state, $x_t = (L_t, B_t, D_t, u_t, v_t)$, bank's equity value is the result of the following program

$$E(x_t) = \max_{\{(L_{i+1}, B_{i+1}) \in \Gamma(D_{i+1}), i=t,\ldots,T\}} \mathbb{E}_t \left[ \sum_{i=t}^{T} e(x_i, L_{i+1}, B_{i+1}) \prod_{j=t}^{i} M(x_{j-1}, x_j) \right], \tag{16}$$

where $\mathbb{E}_t[\cdot]$ is the expectation operator conditional on $D_t$, on the state variables at $t$, $(u_t, v_t)$, and on the decision $(L_{t+1}, B_{t+1})$; $M(x_t, x_{t+1})$ is a discount factor defined in Equation (2) (which depends only on the component $u_t$ of $x_t$), such that $M(x_{t-1}, x_t) = 1$ at $t$; $(L_{i+1}, B_{i+1})$ is the decision at date $i$, for $i = t, \ldots$, and $T$ is the default date. Because the model is stationary and the Bellman equation involves only two dates (the current, $t$, and the next one, $t+1$), we can drop the time index $t$ and use the notation without a prime for the current value of the variables, and with a prime to denote next–period value of the variables. The value of equity satisfies the following Bellman equation

$$E(x) = \max \left\{ 0, \max_{(L', B') \in \Gamma(D')} \left\{ e(x, L', B') + \mathbb{E}\left[ M(x, x') E(x') \right] \right\} \right\}. \tag{17}$$

Compactness of the feasible set of a bank and standard properties of the value function are described in Appendix A.

When a bank is solvent, the value of equity satisfies the following Bellman equation:

$$E(x) = \max_{(L', B') \in \Gamma(D')} \left\{ e(x, L', B') + \mathbb{E}\left[ M(x, x') E(x') \right] \right\}. \tag{18}$$

We denote with $(L^*(x), B^*(x))$ the optimal policy when the bank is solvent. When it is insolvent, shareholders exercise the limited liability option, which puts a lower bound on $E$ at zero. The default indicator function is denoted $\Delta(x)$.

We solve Equation (17) to determine the value of equity and the optimal policy, including the optimal default policy, $\Delta$, as functions of the current state, $x$. We denote $\varphi$, the state transition function based on the optimal policy:

$$\varphi(x) = \begin{pmatrix} L^* \\ B^* \\ D' \end{pmatrix} (1 - \Delta) + \begin{pmatrix} 0 \\ D_u \\ D' \end{pmatrix} \Delta. \tag{19}$$

Equation (19) says that the new state is $(L^*, B^*, D')$ if the bank is solvent, and $(0, D_u, D')$ if the bank defaults. In case of default, a new bank is started endowed with seed capital $D_u - D'$ and deposits, $D'$, and a cash balance $D_u$, and no loans. This new bank will revise its investment (together with the financing) policy in the following decision dates.

The end–of–year cash flow from current deposits, $D_{t+1}$, for a given realization of the exogenous state variables, $(Z_{t+1}, D_{t+2})$, and on the related optimal policy, is

$$f(x_{t+1} \mid \varphi(x_{t+1})) = D_{t+1}(1 + r_d)(1 - \eta\Delta(x_{t+1})). \tag{20}$$

Hence, the ex–ante fair value of newly issued deposits at $t$, from the viewpoint of the deposit insurance agency (i.e, incorporating the risk of bank's default), is

$$F(x_t) = \mathbb{E}_t \left[M(x_t, x_{t+1}) f(x_{t+1} \mid \varphi(x_{t+1}))\right] = D_{t+1}(1 + r_d) \left(1 - \eta P(x_t)\right), \tag{21}$$

where $P(x_t) = \mathbb{E}_t \left[M(x_t, x_{t+1})\Delta(x_{t+1})\right]$ is the price of the relevant default contingent claim. Dropping the dependence on the calendar date,

$$F(x) = D'(1 + r_d) \left(1 - \eta P(x)\right). \tag{22}$$

## D. Value metrics of bank efficiency and welfare

A standard valuation concept is the market value of bank assets $E(x)+f(x)-B^-$, where $f(x) = D(1+r_d)(1-\eta\Delta(x))$ from Equation (20), $B^- = \min\{B, 0\}$, which *includes* the contribution of (cash–equivalent) one-period bonds $B$ to bank's value, since bond investment helps reducing the potential costs triggered by high cash flows volatility. However, this definition of market value does not capture the role of banks as maturity transformers of liquid liabilities into longer–term *productive* assets (loans). One of the key economic contributions of banks identified in the literature is their role in efficiently intermediating funds toward their best productive use (see, e.g., Diamond (1984) and Boyd and Prescott (1986)). Banks play no such role if they just raise funds to acquire risk–free (cash–equivalent) bonds. A suitable metric of bank efficiency is the *enterprise value* of a bank, defined as $EV(x) = (E(x) + f(x) - B^-) - B^+$, with $B^+ = \max\{B, 0\}$. The enterprise value is thus the market value of equity plus the value of deposits net of cash balances, or plus short–term debt, capturing a bank's ability to create "productive" intermediation.[13] Because $B = B^+ + B^-$, the enterprise value of the bank can be also written as $EV(x) = E(x) + f(x) - B$.

Our welfare metric of bank activities, called "social value," is defined as the sum of the values of banks' activities to the government and to all banks' stakeholders. In essence, this metric measures the *contribution* to welfare of bank activities. The welfare metric is given by:

$$SV(x) = E(x) + D(1 + r_d) - B + G(x), \tag{23}$$

where $D(1 + r_d)$ is the book value of current deposits, and $G(x)$ is the value of the net payoff to the government, defined by the recursive equation

$$G(x) = (1 - \Delta(x))\left(\mathcal{T}(y') + \mathbb{E}[M(x, x')G(x')]\right) - \Delta(x)\left(\eta D(1 + r_d) + K'\right) \tag{24}$$

with $\Delta(x)$ denoting the default indicator at $x$. Equation (24) reads as follows: so long as the bank is solvent ($\Delta(x) = 0$), taxes are collected, where $\mathbb{E}[M(x, x')G(x')]$ is the present discounted value of future tax proceeds. If the bank is insolvent ($\Delta(x) = 1$), then the government incurs direct bankruptcy costs $\eta D$, and injects new equity capital $K' = D_u - D'$.[14]

[13] For the use of enterprise value as a metric of efficiency in the context of dynamic models of non–financial firms see, e.g., Gamba and Triantis (2008) and Bolton, Chen, and Wang (2011).

[14] Note that if the net payoff to the government is positive along a given path, the tax proceeds collected from a bank in the past are sufficient to cover the recapitalization of a new bank. Otherwise, the shortfall of the government will be covered by tax proceeds raised from other agents in the economy. In either case the government value metrics of Equation (24) captures the net cost of banks to the government. Equivalently, the welfare metric is the sum of enterprise value *gross* of bankruptcy costs, plus $G(x)$, which is net of bankruptcy costs incurred by the government.

## III. Bank regulation

In this section we define capital requirements, liquidity requirements, and a policy of prompt corrective action, illustrating their implications for the feasible choice set of a bank.

### A. Capital requirement

In our model the capital ratio is the ratio of the book value of capital over the book value of loans. Basel II–type capital regulation establishes a lower bound $K_d$ on the book value of equity, set by the regulator as a function of bank's risk exposure at the beginning of the period. In particular, this requirement is a weighted average of risks associated to a bank's exposure of assets of different riskiness.[15] Since our model has just one composite risky asset, we set the weight applied to loans equal to 100%. Thus, in our setting the *required* capital $K_d$ is at least a proportion $k$ of loans outstanding at the beginning of the period, $L$, or $K_d = kL$. This requirement is equivalent to constraining net worth to be positive ex–ante. Given the definition of bank capital in Equation (6), under the capital requirement, the bank's feasible choice set is

$$\Theta(D) = \{(L, B) \mid (1-k)L + B \geq D\} . \tag{25}$$

When we compare the feasible choice set under the collateral constraint in Equation (11) with the feasible set under the capital requirement, in general neither $\Gamma(D) \subset \Theta(D)$ nor $\Theta(D) \subset \Gamma(D)$ in a proper sense. Figure 2 shows how the capital requirement is related to the collateral constraint for a specific set of parameters.

If a bank is short–term borrowing, $B < 0$. For a given $D$, the capital constraint results in a restriction of the bank's choice set if $\Theta(D) \subset \Gamma(D)$. Alternatively, if a bank is short–term lending, $B \geq 0$. Then, the capital requirement restricts the choice set if $L < D/(1-k)$ because it forces the bank to have a fairly large cash balance $B$, while the constraint is not binding if $L \geq D/(1-k)$.

The Bellman equation for the equity value of a currently solvent bank under a capital requirement is given by Equation (18), the only difference being a feasible set $\Gamma(D') \cap \Theta(D')$

[15] The specific norm is in *International Convergence of Capital Measurement and Capital Standards*, Bank for International Settlements, Basel, June 2006, Part 2: The First Pillar - Minimum Capital Requirements, Ia The Constituents of Capital A. Core Capital (Basic Equity or Tier 1).

in place of $\Gamma(D')$, since the bank is forced to comply ex–ante with the capital requirement. However, at the end of each period, when the credit shock on existing loans $Z'$ and the new deposit $D'$ are realized, a bank may still face default risk if the innovations of the state variables are particularly unfavorable.

## B. Liquidity requirement

Current Basel III regulation introduces a mandatory *liquidity coverage ratio* as a defense against the potential onset of severe liquidity stress. According to this requirement, banks should hold a stock of high quality liquid assets such that the ratio of this stock over the predicted net cash outflows over a 30-day time period in the case of acute stress–as defined by the regulator–is not lower than a certain percentage threshold.

In our model, the stock of high quality liquid assets over the net cash outflows over a bank's planning period is given by the total cash available at the end of the period over the total net cash flow in the worst case scenario for both credit and liquidity shocks. Formally, this liquidity coverage ratio should be not lower than a level $\ell$ defined by the regulator, or

$$\frac{\delta L + Z_d \pi(L) - \mathcal{T}(y^{\min}) + B(1 + r_f)}{D(1 + r_d) - D_d} \geq \ell. \tag{26}$$

Hence, the feasible set for a bank complying with the liquidity requirement is

$$\Lambda(D) = \left\{ (L, B) \mid \frac{\delta L + \ell D_d + Z_d \pi(L)(1 - \tau^{\min})}{\ell(1 + r_d) - \tau^{\min} r_d} + B \frac{1 + r_f(1 - \tau^{\min})}{\ell(1 + r_d) - \tau^{\min} r_d} \geq D \right\}. \tag{27}$$

Thus, the liquidity ratio is the end–of–period total cash available in the worst case scenario over the end–of–period net cash outflows due to a variation in deposits.

Figure 2 shows a comparison of the liquidity requirement to the collateral constraint for a specific choice of parameters. The liquidity constraint may turn out to restrict the bank's feasible choice set relative to the collateral constraint for a wide range of parameter values. It is apparent that when considered together, capital and liquidity constraints may create considerable restrictions on a bank's feasible choices.

## C. Prompt Corrective Action (PCA)

An important objective of bank regulation is the minimization of losses of the deposit insurance fund. This is achieved by PCA policies, which force banks to liquidate assets or suspend payouts contingent on pre–specified observed levels of capitalization, or close a distressed bank if capitalization is lower than a given threshold.

Correspondingly, we define a PCA rule contingent on observed ex–post bank capital. Formally, ex–post bank capital in period $t$ (as opposed to $K$, which is the ex–ante bank capital in period $t$), is

$$V = L + B - D + y - \mathcal{T}(y) = K + y - \mathcal{T}(y).$$

The regulator implements PCA according to the following

*Assumption* 10 (PCA). If at time $t$ the ex–post bank capital satisfies the ex-ante capital ratio, i.e., $V \geq kL$, no action is taken. If the ex-post bank capital is such that $0 < V < kL$, the deposit insurer forces the distressed bank to restore capital in the current period by replenishing the shortfall $kL - V$ *and* to satisfy the regulatory ratio next period. If the ex–post bank capital is negative ($V \leq 0$), the bank is closed at time $t$ and taken over by the deposit insurance agency.

The PCA thus establishes that banks with a positive ex–post capital, but lower than that satisfying the regulatory ratio, are forced to finance the shortfall by liquidating assets, raising collateralized debt, suspending payouts or issuing new equity. From Equation (12) we have

$$U = y - \mathcal{T}(y) + K - K' - m(I) = V - m(I) - K'.$$

When $V < kL$, a bank is forced to finance the shortfall $kL - V$, as well as to set the capital for next period to the required level, so that: $K' = V - m(I) - U \geq kL' + (kL - V)$. On the other hand, when the ex-post bank capital is negative, the closure rule applies: a bank is expropriated from shareholders and is transferred to the government. New capital $K' = D_u - D'$ is injected net of the positive value of the bank's capital, $E(x) > 0$. In both cases, the government agency does not incur bankruptcy costs ($\eta = 0$).

Note that the PCA actually implements a *contingent* enforcement of a capital requirement. When triggered, such requirement is more stringent than the ex-ante capital requirement for two reasons. First, the PCA implements a more restrictive rule on ex–ante book capital contingent on certain (low) realizations of ex-post capital. Second, the risk of expropriation of shareholders based on the realization of negative ex-post capital imposes a higher shadow cost on the current capital requirement. For these reasons, we consider the PCA as a proper regulation in itself, so that we can compare the impact of a contingent capital requirement plus a closure rule, as implemented by the PCA just defined, with (non–contingent) capital requirements.[16]

For banks subject to the PCA, the Bellman equation of the solution of the bank's program when the bank is solvent (i.e., $V_t$ is positive) is as in Equation (18). Under the PCA closure rule, the value of the net payoff to the government is defined by the recursive equation

$$G(x) = (1 - \Delta(x)) \left( \mathcal{T}(y') + \mathbb{E}[M(x, x')G(x')] \right) - \Delta(x) \left( K' - E(x) \right), \tag{28}$$

with $\Delta(x)$ denoting the indicator of the event $V < 0$ at $x$.

As before, the government is assumed to be able to finance any recapitalization shortfall of an individual bank with general tax proceeds. However, in the specific case $V \leq 0$ and the debt issuance, $B < 0$, is fully collateralized, bond holders, (old) depositors, and the government are paid in full.[17] In this case, the financing shortfall (net obligations minus available liquid funds) of the bank is $(1 + r_d)D - (1 + r_f)B + \mathcal{T}(y) - (D' + \pi(L)Z)$. A portion of loan portfolio is liquidated and the proceeds match (net of loan liquidation costs) the shortfall:

$$L - L' - m(L(1 - \delta) - L') = (1 + r_d)D - (1 + r_f)B + \mathcal{T}(y) - (D' + \pi(L)Z).$$

By solving this equation with respect to $L'$, a new level of loans, $L^*$, is determined, which is positive because the bank satisfies the collateral constraint. Clearly, in this reorganization procedure, an indirect cost is still incurred because of loan liquidation costs, $m(L(1 - \delta) - L^*)$.

[16] In practice, however, PCA–type policies apply to banks that are already subject to other regulations, such as capital requirements. In the sequel, we will also consider the impact of the PCA when implemented together with capital and liquidity requirements.

[17] This is because the collateral constraint in Equation (10) is

$$L + \pi(L)Z_d + (1 + r_f)B - (1 + r_d)D - \mathcal{T}(y^{\min}) \geq m(-L(1 - \delta)) - D_d,$$

the closure rule $V \leq 0$ gives

$$L + \pi(L)Z + (1 + r_f)B - (1 + r_d)D - \mathcal{T}(y) \leq 0,$$

and the left–hand side of the second inequality is higher than the corresponding side of the first inequality. Therefore, all stakeholders can be paid by liquidating the assets.

# IV. Regulations in a simplified version of the model

To illustrate in a simple way some trade–offs on bank optimal policies implied by capital and liquidity requirements, and to point out the restrictions implied by assuming banks are short–lived, we collapse our model to three periods. Now $t+1$ is the decision date, $t+2$ is the final date, and the bank initial conditions are determined at $t$.

We make the following simplifying assumptions. The discount factor is deterministic, so $M(x_t, x_{t+1}) = \beta$. There are no taxes, no adjustment costs, deposits are deterministic and constant ($D_t = D_{t+1} = D_{t+2} = D > 0$), $\delta = 0$, $\beta \leq (1+r_f)^{-1}$, and $r_d = r_f$. Furthermore, we assume a simple two–point credit shock distribution: $Z^H$ with probability $p \in (0,1)$, and $Z_d$ otherwise, where $Z_d$ is such that $Z_d = \frac{Z^L L_{t+1}}{\pi(L_{t+1})}$, with $Z^H > 0 \geq Z^L \geq -1$. Under these assumptions, the collateral constraint for $B_{t+1} < 0$, denoted with (C), the capital constraint, denoted with (K), and the liquidity constraint, denoted with (L), are:

$$B_{t+1} \geq \frac{r_f}{1+r_f} D - \frac{1+Z^L}{1+r_f} L_{t+1}, \tag{C}$$

$$B_{t+1} \geq D - (1-k) L_{t+1}, \tag{K}$$

$$B_{t+1} \geq \frac{\ell r_f}{1+r_f} D - \frac{Z^L}{1+r_f} L_{t+1}. \tag{L}$$

Recall that by Equation (13), the cash flow to shareholders is $U_t = W_t + L_t - B_{t+1} - L_{t+1}$ if $U_t > 0$, and $U_t(1+\lambda)$ if $U_t < 0$, as the bank issues new equity at a cost $\lambda \geq 0$.

The bank chooses $(L_{t+1}, B_{t+1})$ to maximize

$$\begin{aligned} e_t + \beta \mathbb{E}_t \left[ e_{t+1} \right] = {} & (W_t + L_t)(1+\lambda) - (1 + \lambda - \beta p (1+r_f)) B_{t+1} - (1+\lambda) L_{t+1} \\ & + \beta \left[ p \left( Z^H \pi(L_{t+1}) - (1+r_f) D + L_{t+1} \right) \right. \\ & \left. + (1-p) \max \left\{ 0, (1+Z^L) L_{t+1} + (1+r_f) B_{t+1} - (1+r_f) D \right\} \right]. \end{aligned} \tag{29}$$

Since $1 + \lambda > \beta p (1+r_f)$, it is optimal to maximize debt ($B_{t+1} < 0$), because profits are increasing in debt in the good state, while in the bad state losses are bounded to be non-negative by limited liability. This implies that at most one of the constraints (C), (K), and (L) will be binding.

The unregulated bank maximizes Equation (29) subject to constraint (C). Inserting (C) into Equation (29), the $\max\{\cdot\}$ term in the third line of Equation (29) vanishes. Therefore, the optimal loan level $L^c_{t+1}$ satisfies the (necessary and sufficient) first order condition

$$\beta p Z^H \pi'(L^c_{t+1}) = 1 + \lambda - \beta p - (1 + \lambda - \beta p(1 + r_f))\frac{1 + Z^L}{1 + r_f}. \tag{30}$$

Suppose now that the capital constraint (K) is tighter than (C), that is, (K) is binding. The third line of Equation (29) becomes $\max\{0, (1 + Z^L - (1 + r_f)(1 - k))L_{t+1}\}$.

The optimal loan investment when (K) is binding, defined by $L^k_{t+1}$, satisfies:

$$\beta p Z^H \pi'(L^k_{t+1}) = 1 + \lambda - \beta p - (1 + \lambda - \beta p(1 + r_f))(1 - k). \tag{31}$$

By comparing the right–hand sides of Equations (30) and (31), it is straightforward to verify that $L^k_{t+1} > L^c_{t+1}$ when $(1 + Z^L) < (1 + r_f)(1 - k)$, owing to the strict concavity of function $\pi$.

Observe that the inequality $(1 + Z^L) < (1 + r_f)(1 - k)$ holds for relatively low levels of $k$, but it is reversed for values of $k$ close to one. Thus, there exists a threshold value $\hat{k}$ such that $L^k_{t+1} < L^c_{t+1}$ for all $k > \hat{k}$. In other words, under a sufficiently mild capital constraint (or $k < \hat{k}$), lending is higher than in the unregulated case. Thus, when (K) is binding, depending on parameters, lending could be higher than in the unregulated case under mild capital requirements, even though borrowing is lower ($B^k_{t+1} > B^c_{t+1}$ holds when constraint (K) is more stringent than (C)). This is because the capital requirement lowers the return of holding cash relative to the expected return on loan investment. In sum, there may exist parameter configurations such that the relationship between loans and capital requirements is inverted U-shaped. Interestingly, this result may (but needs not) hold for any $\lambda \geq 0$.

Consider now the addition of a liquidity requirement to the capital requirement and suppose that the liquidity constraint (L) is tighter than (K) at the optimal choice $L^k_{t+1}$, that is, (L) is binding. Replacing (L) in Equation (29), the $\max\{\cdot\}$ term turns into $\max\{0, L_{t+1} + (r_f(\ell - 1) - 1)D\}$.

If at the optimal solution $L_{t+1} + (r_f(\ell - 1) - 1)D \leq 0$, then $L^\ell_{t+1}$ satisfies

$$p Z^H \pi'(L^\ell_{t+1}) = r_f - (1 - p)Z^L. \tag{32}$$

Otherwise, $L^{\ell}_{t+1}$ satisfies

$$pZ^H \pi'(L^{\ell}_{t+1}) = r_f - Z^L. \tag{33}$$

Comparing Equation (32) with Equation (30), it is easy to verify that the right hand side of Equation (30) is always strictly lower than that of Equation (32). By strict concavity of the revenue function, this implies that $L^{\ell}_{t+1} < L^{k}_{t+1}$: the liquidity constraint unambiguously reduces lending relative to the bank subject to a (binding) mild capital constraint. Comparing Equation (30) with Equation (33), the same result is obtained if $p$ is close to 1. Thus, there may exist parameter configurations such that the liquidity constraint reduces lending relative to the capital constraint.

Summing up, this simplified version of our model illustrates cases –depending on parameters– in which an inverted U-shaped relationship with the stringency of capital regulation may arise, and where a liquidity requirement may reduce lending. However, these conclusions may, or may not, hold under complex dynamic trade-offs arising from the fully dynamic version of our model. Contrary to the short finite–horizon bank just described, the full dynamics takes into account bank management of retained earnings and optimal portfolio allocations under the true shadow costs of regulatory restrictions. Importantly, in our model the default decision of a bank is endogenous, rather than being exogenously imposed.

# V. The impact of bank regulation

In this section we illustrate the results of the calibration and simulation of the model. Subsection A describes a set of benchmark parameters calibrated using selected statistics from U.S. banking data, some previous studies, and regulatory and tax parameters. Subsection B analyzes optimal bank policies along the business cycle, while Subsection C analyzes the steady state and its welfare properties.

## A. Calibration

Our calibration is based on three sets of parameters, summarized in Table I. The first set comprises parameters of the credit shock and deposits process. We estimated the VAR in

Equation (15) using U.S. yearly aggregate time series for the period 1983-2009 for the entire universe of banks included in the Federal Reserve Call Reports constructed by Corbae and D'Erasmo (2011). The shock process was proxied by the return on bank investments before taxes, given by the ratio of interest and non-interest revenues to total lagged assets. As can be seen in Table I, the shock process exhibits high persistence and the correlation with the process of (log) deposit is negative. Estimates of the autocorrelation process for (log) deposit produced estimates closed to unity, indicating the possibility that such process has a unit root. To guarantee convergence of the fixed point algorithm, we set this parameter equal to 0.98.

The second set of parameters is taken from previous research. The parameters of the stochastic process of the macroeconomic risk factor $u$, $\kappa_u$ and $\sigma_u$ are set equivalent, on an annual basis, to the quarterly values of 0.98 and 0.007 reported in Jones and Tuzel (2013). The parameters of the stochastic discount factor, $\gamma_1 = 3.22$ and $\gamma_2 = -15.30$, are also taken from Jones and Tuzel (2013). The annual discount factor $\beta$ is 0.95, equal to that used by Zhu (2008) and Cooley and Quadrini (2001). The rate, $r_f$, is set to 2.5% and the deposit rate, $r_d$, is set to 0. These values are consistent with the average effective cost of funds documented in Corbae and D'Erasmo (2011). With regard to corporate taxation, recall that the tax function is defined by the marginal tax rates, $\tau^+$ and $\tau^-$, for positive and negative income, respectively. Since we do not explicitly consider dividend and capital gain taxation for shareholders, or interest taxation for depositors and bond holders, the two marginal rates for corporate taxes need to be considered net of the effect of personal taxes. For this reason we choose $\tau^+ = 15\%$, which is close to the values determined by Graham (2000) for the marginal tax rate. The marginal tax rate for negative income is $\tau^- = 0$ to allow for convexity in the corporate tax schedule.

Furthermore, the proportional bankruptcy cost is $\eta = 0.10$, This is a value close to the (structural) estimate of 0.104 based on data for U.S. non–financial firms found by Hennessy and Whited (2007). Since this estimate is based on non–financial firms, it can be viewed as a lower bound for bankruptcy costs incurred in the financial sector. The annual percentage of reimbursed loan is 20%, so that the average maturity of outstanding loans is 4 years, in line with the assumption made by Van den Heuvel (2009). The underwriting cost for seasoned equity issuance is 6%, not far from the estimates provided by Altinkilic and Hansen (2000) and Hennessy and Whited (2005).

The revenue function from loan investment is $\pi(L) = L^{\alpha}$ as in Zhu (2008), and our base case value for $\alpha$ is set equal to 0.90, which is a value in line with that used in other studies. Lastly, we obtain $m^{+} = 0.04$ and $m^{-} = 0.05$ by matching two moments from empirical data. The first moment is the average ratio of bank credit over deposits, where bank credit includes loans and other financial investments. From our dataset, this ratio is 1.271. The second moment we match is bank's book leverage, defined as deposits plus other financing liabilities over loans and other financial investments. In the data, the average book leverage is 0.89. The corresponding unconditional moments from a Monte Carlo simulation of the model with the selected parameters are respectively 1.1153 and 0.9043. Hence, our calibration delivers per–unit loan liquidation costs higher than per–unit costs of loan extensions ($m^{-} > m^{+}$).

The third set of parameters is based on regulatory prescriptions. These are the ratio of capital to risk–weighted assets and the liquidity coverage ratio. The benchmark capital ratio $k$ is set equal to 4%, while the benchmark liquidity ratio is set equal to $\ell = 20\%$.

The relationships between the credit and liquidity shocks affecting the banking system, the systematic (macroeconomic) risk factor and the idiosyncratic factor are derived in Appendix B and illustrated in Figure 3. As expected, credit risk is positively correlated with the systematic risk factor, as credit quality and loan demand increase in an upturn, and decline in a downturn. On the other hand, liquidity risk, as captured by the dynamics of insured deposits, turns out to be mildly negatively correlated with the systematic risk factor. This pattern is consistent with an increase in savings in a downturn, as insured deposits are a component of savings, and with the reallocation of agents' portfolios towards safer assets. Lastly, the correlations between idiosyncratic risk and credit and liquidity risks are fairly small, slightly negative for the former, and almost null for the latter.

### B. Bank policies along the business cycle

The correlations between optimal bank policies and the systematic (macroeconomic) risk factor, viewed as a proxy measure of the business cycle, are summarized in Figure 4 and Tables II-IV. Five cases are considered: unregulated banks, banks subject to capital requirements only, subject to both capital and liquidity requirements, subject to the PCA only, and subject to the PCA and capital requirements. Specifically, Figure 4 depicts lending and debt policies for a given set of states centered around the steady state (unconditional) median values.[18] Tables II-IV are based on Monte Carlo simulation (the design of the simulation exercise is detailed in Appendix C). In these tables we report bank lending, capital ratios, and liquidity ratios for solvent banks sorted against quartiles of total risk (the sum of systematic and idiosyncratic shocks, $u+v$). In the top panel of each table the sorting of these variables is based on the whole sample. The influence of the business cycle is obtained by sorting variables conditional on the state of the economy being in a business cycle upturn (with $u = 0.0352$), or in a downturn (with $u = -0.0352$).

### B.1. Unregulated banks

As shown in Figure 4 and Table II, lending and debt are positively correlated with the systematic shock. Recall that such a shock translates into a positive credit shock, $Z$, and a negative liquidity shock, since deposits are negatively correlated with the systematic shock. Thus, a positive systematic shock prompts banks to increase lending, but also increases their need of financing when deposits become relatively scarce, inducing them to increase debt as well. Thus, a positive correlation between lending, debt, and the systematic shock is a feature of banks' optimal policies *independently* of bank regulations.

As shown in Table III, capital ratios are positively correlated with the systematic shock, owing to lower deposits in an upturn (deposits are a negative component of the ratio), while changes in loans and short–term debt almost offset each other.[19] By contrast, as shown in Table IV, liquidity ratios are negatively correlated with the systematic shock, because in a downturn debt declines and deposits increase. The opposite holds in an upturn.

[18] The analysis is centered at the steady states for both deposits ($D = 2$) and credit shock ($Z = 0.0717$), while choosing $B = 0$ to avoid the impact of current liquidity, and $L = 4.7$, which is close to the unconditional median of $L$ for several versions of the model. As a result, bank's capital is $K = 2.7$.

[19] Note that (book) capital ratios may be negative for some realizations of systematic and idiosyncratic shocks. Since there is no restriction on book capital and, as is standard in the literature, the (concave) loan function has the book value of loans as an input, in the presence of debt a bank can be operating as long as the market value of its capital is positive, even though its book capital can be negative.

### B.2. Capital regulation

Similarly to unregulated banks, banks under the base capital requirement exhibit a positive correlation between the systematic shock, lending, and debt (see Figure 4). However, banks invest more in loans and choose less debt than their unregulated counterparts for all realizations of the systematic shock. This implies that a bank subject to a mild capital requirement finds it optimal to satisfy it by increasing loans at a rate proportionally higher than the capital ratio coefficient (see Equation (25)), rather than by reducing lending. In essence, a mild capital requirement reduces the rate of return on short–term debt relative to the expected returns on loans, prompting a higher investment in loans. In turn, higher revenues generated by higher lending are employed in the building up of capital. Interestingly, in a downturn banks subject to a mild capital requirement reduce lending *less* than unregulated banks. This implies that risk–based capital regulation does not necessarily amplify the contraction of lending in a downturn.

These mechanisms are reflected in the evolution of the capital and liquidity ratios along the cycle. A mild capital requirement encourages banks to build–up capital buffers in upturns to reduce the risk of costly loan liquidations in downturns: this is witnessed by the lower positive correlation between capital ratios and the systematic shock relative to unregulated banks (see Table III). On the other hand, under a mild capital requirement, the negative correlation between liquidity ratios and the systematic shock is reduced relative to unregulated banks. This occurs because in a downturn banks use retained earnings and the financial resources freed by the reduction in lending to strengthen their liquidity position. In other words, a mild capital requirement contributes to strengthen banks' liquidity position in a downturn.

### B.3. Liquidity requirements

The addition of a liquidity requirement to a capital requirement changes bank optimal policies significantly. A shown in Figure 4 and Table II, banks' lending and debt shrink for every realization of the systematic shock, with the reduction in debt being the most dramatic. Thus, the liquidity requirement turns out to be far more restrictive than the capital requirement, forcing banks to reduce both debt and lending. Moreover, relative to the case of banks subject to capital regulation only, the positive correlation between lending and the systematic shock is drastically reduced.

The dominant tightness of the liquidity requirement is also reflected in the evolution of capital ratios along the business cycle. As shown in Table III, capital ratios become inflated, being pushed up by a relatively large net bond holding (the numerator of the ratio) and a lower investment in loans (the denominator of the ratio). Note that the mechanism driving this result is totally different from that induced by capital regulation: in that case, the capital ratio is pushed up by retained earnings generated by higher revenues from lending, while in this case, the capital ratio is mainly pushed up by a significant reduction of lending. Therefore, under a liquidity requirement the positive correlation between the capital ratio and the systematic shock *increases* significantly. This increased correlation is the result of banks being forced to reduce lending significantly in an upturn. In a downturn, however, capital ratios are not significantly different from those attained by banks subject only to capital regulation.

As shown in Table IV, liquidity ratios are significantly higher than the prescribed level ($\ell = 20\%$), since the (shadow) cost associated with the liquidity constraint forces banks to hold precautionary cash to avoid hitting that constraint. On the other hand, the negative correlation between the liquidity ratios and systematic shocks is *reduced*, as witnessed by a comparison of the values under an upturn and a downturn. As a result, in a downturn the increase in liquidity holdings does not lead to liquidity ratios significantly different from those attained by banks subject to capital regulation only.

### B.4. PCA

Under the PCA, the correlations between lending, debt, capital, and liquidity ratios are similar to those under capital regulation. However, as shown in Table III, the correlation of the capital ratio with the systematic shock is higher than that under capital regulation, primarily owing to capital ratios significantly lower in downturns. The lower capital ratios under the PCA in a downturn indicate that banks will implement a lower reduction in lending, while keeping open their options to either liquidate loans or issuing equity in the event the realization of current earnings is unfavorable. In essence, in a downturn the PCA appears to provide banks some flexibility which is unavailable under an (unconditional) ex-ante capital requirement. In the next section, we will show that the closure rule embedded in the PCA limits banks' incentives to abuse such flexibility.

### B.5. Summary

Our analysis of banks' optimal policies along the business cycle delivers two results relevant to the issue of whether bank regulation enhances the procyclicality of lending, as captured in our model by the correlation between lending and the systematic risk factor.

First, relative to unregulated banks, mild capital requirements and the PCA *reduce* the positive correlation between lending, debt, and capital ratios, as well as the negative correlation with liquidity ratios, relative to unregulated banks. Specifically, in an upturn banks increase lending at a rate consistent with the build–up of capital buffers through retained earnings generated by higher lending revenues, while in a downturn they reduce debt by building up liquidity buffers. Perhaps not surprisingly, these results differ from those obtained in static or semi–static models that do not allow banks to issue equity or debt and to manage retained earnings. For example, procyclicality is enhanced by capital requirements in the model of Repullo and Suarez (2013), who consider short–lived banks that do not manage retained earnings, and are not allowed to issue either debt or equity.

Second, the addition of liquidity requirements to capital requirements reduces the procyclicality of lending and debt, but also increases the procyclicality of capital ratios. Yet, capital buffers in downturns are not significantly different from those resulting from banks subject only to capital regulation. In essence, the reduction in lending procyclicality is skewed toward upturns, significantly hampering lending.

## C. Bank policies, efficiency and welfare in steady state

We now turn to the analysis of bank optimal policies and the metrics of efficiency and welfare in steady state obtained through Monte Carlo simulation of the numerical solution of the bank's optimal program (see Appendix C for details). Table V presents statistics of policies, assets and liabilities, welfare and efficiency metrics, and default frequency, when banks are not regulated, when they are subject to capital requirements only, and when they are subject to both capital and liquidity requirements. Table VI presents a comparison of the same statistics for banks under the PCA, and for banks subject to the PCA superimposed on capital and liquidity requirements. In Table VII we report the same statistics under perturbed parameters of underwriting costs, loan liquidation costs, and the degree of bank's maturity transformation.

### C.1. Capital requirements

Compared to the unregulated case, Table V shows that banks operating under the base case capital requirement ($k = 4\%$) lend more and have less debt. Remarkably, the steady state percent increase in lending relative to the unregulated case is a significant 15%. Since deposits are not a control variable and follow the same exogenous process of the unregulated case, the bank can fund these additional loans by reducing payouts and increasing retained earnings and equity issuance. Specifically, from Equations (12) and (13), given the choice of $L_{t+1}$ and $B_{t+1}$, more earnings are retained from $W_t$, or shares are issued (incurring underwriting costs) if $U_t$ is negative.

As a result of these optimal policies, banks also hold a higher capital ratio than that prescribed by regulation. This is because the positive shadow price of the capital constraint forces banks to manage their earnings and investments so as to maintain a capital buffer that minimizes the risk that the constraint is hit. In such an event, it would become too expensive to either liquidate loans or inject new equity capital to comply with the regulatory restriction. These findings reflect the well-known result that constraints may not be binding on equilibrium paths (see, e.g., Ayagari (1995)). Importantly, capital regulation results in a bank with a lower probability of default than in the unregulated case. Thus, a capital requirement is unambiguously successful in abating default risk under deposit insurance.

With regard to the efficiency and welfare metrics, a mild capital requirement results in a small decrease in banks' enterprise value (about 1%), which is offset by an increase in government value. The higher government value stems from higher tax receipts accruing from a larger taxable profit base, as well as from a lower probability of bank default, which reduces expected bailout costs. As a result, the welfare metrics is larger than that of the unregulated case.

However, the benefits of capital regulation can turn into welfare–reducing costs if such regulation becomes too stringent. An increase in the capital requirement (from $k = 4\%$ to $k = 12\%$) results in reductions of lending, debt, as well as in the efficiency and welfare metrics. Recall that banks satisfy a relatively mild capital requirement by an increase in loans financed with a combination of debt and retained earnings. But when the capital requirement becomes too stringent, such a strategy becomes too costly. In such a case, payouts need to be significantly reduced since it may become too expensive either to raise new equity capital owing to equity issuance costs, or to reduce investment owing to loan liquidation costs in unfavorable states of nature. Thus, banks are compelled to reduce both lending and debt. Thus, there exists an inverted U–shaped relationship between lending and welfare associated with capital requirements. This further suggests the existence of optimal levels of regulatory capital as a function of banks' characteristics. In other words, under a mild capital requirement, banks optimally choose to increase lending so as to generate higher revenues supporting the building up of capital; if capital requirements are too high, however, banks find it optimal to satisfy them by reducing lending. This latter reduction also impacts negatively on welfare, as it reduces both the enterprise and government values of bank activities.

### C.2. Liquidity requirements

When liquidity requirements are added to capital requirements, Table V shows that lending contracts dramatically relative to the bank subject only to the base capital requirement (by about 26%). The efficiency and welfare metrics declines significantly as well (by about 20%). As noted earlier, the liquidity requirement generates over–bloated banks' book capital ratios. Such high ratios may be viewed as an indication of safe but very inefficient banks.

Furthermore, an increase in the capital requirement (from $k = 4\%$ to $k = 12\%$) for the bank already subject to a liquidity requirement ($\ell = 20\%$), implies only small positive changes in lending and the efficiency and welfare metrics. Similarly, an increase in the liquidity requirement (from $\ell = 20\%$ to $\ell = 50\%$, with constant $k = 4\%$) implies relatively small changes in lending, efficiency and welfare. These results are due to the significant stringency of the liquidity requirement, which makes banks' optimal policies relatively insensitive to changes in capital requirements.

### C.3. PCA

As shown in Table VI, the PCA prompts banks to increase lending relative to the unregulated case, as in the case of banks subject to capital regulation only. Importantly, however, the PCA also achieves higher lending, capital, and higher levels of the efficiency and welfare relative to banks subject only to a capital requirement, even though default (in the form of bank closure) is more frequent. Notably, our calibration implies that immediate capital restoration when $0 < V < kL$ is triggered only 0.27% of the times, so banks rarely incur this additional burden. This is because the potential costs generated by the PCA to banks generate a shadow cost of violation large enough to prompt banks to choose policies that minimize the probability of these ex–post violations. In essence, the PCA *de facto* implements a contingent transfer of control of banks from the shareholders to the deposit insurance authority that allows shareholders to get the upside potential of investment, while giving the controlling authority a way to mitigate the welfare costs of liquidations and bailouts.

The superiority of the PCA over (non–contingent) capital and liquidity requirements stems in part from the resolution of the difficulties introduced by the absence of contingencies in deposit payments due to deposit insurance. In set–ups similar to ours, when demandable deposits are either fully contingent on the realization of the states, as in De Nicolò (1996), or contingencies are introduced allowing deposit runs, as in Allen and Gale (1998), an optimal banking allocation is achieved. Deposit insurance eliminates state contingency, and non–contingent capital and liquidity requirements do not replace the lack of contingencies of deposit payments. By contrast, the PCA is a state contingent intervention scheme that substitutes for the lack of contingencies in demandable deposits, with these contingencies being induced now by a regulator acting as a representative of depositor. Under deposit insurance, state contingencies are moved to states of the world in which there is still equity value remaining.

It is interesting to assess the impact of the PCA depending on whether a bank is hit by a systematic or idiosyncratic shock. In the sample generated by our simulation, we find that the PCA is never triggered for any realization of the idiosyncratic shock when the systematic shock is positive. Moreover, conditional on a negative systematic shock, when the idiosyncratic shock is positive, the PCA is triggered with a low frequency (0.08%), whereas it occurs more frequently when such a shock is negative (0.19%). Therefore, the PCA is triggered differently according to weather shocks are systematic or idiosyncratic: bank closures and restructuring occur more often in the case of an adverse systematic shock, whereas forced recapitalizations and restructuring of payouts are more frequent in the case of an adverse idiosyncratic shock.

Finally, when the PCA is introduced in addition to banks subject to capital requirements, banks are forced to hold a capital ratio higher than $k$ in all states, and this requirement is made more demanding by the PCA when $V$ falls short $kL$. Relative to the case of capital requirement only, the addition of the PCA results in small reductions in lending, efficiency and welfare metrics. Moreover, the marginal effect of PCA is negligible when applied to banks that are already subject to capital and liquidity requirements, owing primarily to the dominant stringency of liquidity requirements, which nullify the benefits of the PCA.

### C.4. The role of issuance costs, loan liquidation costs, and the degree of maturity transformation

Here we examine the impact of capital and liquidity requirements under different parameters of underwriting costs, loan liquidation costs, and the degree of bank's maturity transformation. In Table VII we consider: no issuance costs, as well as an increase of $\lambda$ from the benchmark value 0.06 to 0.2; an increase of loan liquidation costs, $m^{-}$, from 0.05 to 0.08; a reduction of $\delta$, the parameter gauging maturity transformation, from 20% to 10%, indicating a longer loan maturity (from 4 years to 9 years).

To what extent do underwriting costs contribute to determine the inverted U-shaped relationship between capital requirements and lending and welfare? On the one hand, with no issuance costs, Table VII shows that the inverted U–shaped relationship between lending, welfare, and capital requirements is strengthened. This is perhaps not surprising, since banks can cope with distress at a lower cost and find it optimal to increase lending even under higher levels of capital requirements. On the other hand, increasing issuance costs relative to the benchmark generates a decline in lending, enterprise, and social values, but this decline is relatively small. Note that these results hold for banks subject to capital requirements only, as well as for banks subject to both capital and liquidity requirements. Thus, the role of issuance costs in determining the costs of more stringent capital requirements appears relatively less important than the management of retained earnings.[20]

To what extent do different levels of loan liquidation costs change the impact of capital and liquidity requirements? When we increase the relevant parameter ($m^-$) in our simulation, we find that lending, enterprise, and social values do not change significantly relative to the base case. This result reveals again the key role of retaining earnings in supporting bank optimal choices: when facing higher loan liquidation costs, a bank would respond by increasing lending and retained earnings, with the latter increase aimed at minimizing the probability of incurring in large loan liquidation costs in the event of distress. However, the addition of liquidity requirements to capital requirements still results in lower lending and a worsening of the efficiency and welfare metrics, as in all previous simulations. Therefore, the benefits of mild capital requirements may be even more important when loan liquidation costs are high.[21]

Lastly, the role of liquidity requirements in hampering the maturity transformation function of banks is starkly illustrated by the case in which banks have a longer loan maturity. Under capital requirements only, banks with a larger maturity mismatch undertake a more intense maturity transformation, as witnessed by higher levels of lending relative to banks with a milder maturity mismatch. When liquidity requirements are added to capital requirements, however, the reduction in lending, efficiency and welfare is significantly greater than that witnessed banks whose loan maturity is shorter under capital requirements only. Again, liquidity requirements turn out to be the more detrimental to lending, efficiency, and welfare, the more intense is banks' transformation of short–term liabilities into longer–term assets.

[20] We run simulations treating equity issuance costs as undervaluation costs. In this case, while underpricing of newly issued equity affects current shareholders, it benefits new shareholders because they can buy a share in the bank's capital for a lower price. Therefore, these costs are not deadweight costs but just a wealth transfer from old to new shareholders, and are added to the welfare function. The qualitative results we obtain are essentially the same as those just presented.

[21] As a robustness check, we considered loan liquidation costs as fire sales costs, since these costs have been identified as one of the key sources of systemic risk in the financial crisis of 2007–09 (see, e.g., Kashyap, Brener, and Goodhart (2011)). The welfare criterion now includes these costs, since fire sales affect negatively current shareholders, but benefit investors who can buy assets at low fire sales prices. While all other statistics remain unchanged, the social values are higher than those obtained in Table V, but still hump–shaped.

### C.5. Summary

Mild capital requirements induce banks to *increase* lending, resulting in higher welfare relatively to unregulated banks, but these welfare gains are destroyed when requirements become too strict. This result is consistent with the finding of relatively high welfare costs associated with capital regulation by Van den Heuvel (2008). Thus, attaining the benefits of capital requirements is not a free lunch. The resulting inverted U-shaped relationship between lending, welfare, and the level of capital requirements is also robust to different levels of underwriting costs and loan liquidation costs.

When liquidity requirements of the type currently envisioned in regulatory proposals are added to capital requirements, they undo the benefits of mild capital requirements, since they prompt significant reductions in lending, efficiency, and welfare. In essence, liquidity requirements constraint banks' maturity transformation function, forcing them to under-invest in lending and over-invest in unproductive liquidity buffers.

Importantly, the PCA as a contingent resolution procedure dominates (unconditional) capital and liquidity requirements in terms of lending, efficiency and welfare, since it provides stronger incentives for banks to manage risk relative to requirements set ex-ante. This is accomplished by introducing contingencies based on observed equity, which effectively replace those contingencies in deposit payments that might improve bank efficiency and welfare, but that are ruled out by deposit insurance.

The above findings remain essentially unchanged under different configurations of key parameters. In sum, PCA-type policies may be best (and mild capital requirements second best) under a wide range of business strategies chosen by banks, as well as with regard to different configurations of credit and liquidity risks they may be exposed to. By contrast, high capital requirements and liquidity requirements place non–contingent restrictions on bank optimal policies that produce no gain in terms of lending, efficiency, and welfare.

## VI. Concluding remarks

This paper has formulated a dynamic model of banks under deposit insurance that are exposed to credit and liquidity risks arising from systematic and idiosyncratic shocks, undertake maturity transformation, invest in risky loans, issue secured debt, costly equity, and may face financial distress. In this environment, we assessed the impact of capital regulation, liquidity requirements, and the PCA on banks' optimal policies and value metrics of bank efficiency and welfare.

Along the business cycle, capital requirements do not appear to increase lending procyclicality, while the addition of liquidity requirements to capital requirements reduces lending procyclicality. In particular, capital requirements give banks incentives to accumulate liquidity buffers in a downturn. In steady state, we uncovered an inverted U–shaped relationship between bank lending, welfare, and regulatory capital ratios, suggesting the existence of optimal bank-specific levels of regulatory capital. By contrast, liquidity requirements significantly reduce lending as well as bank efficiency and welfare, as a result of the severe repression of the maturity transformation function of bank intermediation. Importantly, the implementation of contingent capital requirements with a bank closure rule embedded in the PCA dominates non–contingent capital and liquidity requirements in terms of lending, bank efficiency, and welfare.

Overall, these results support the argument by Admati, DeMarzo, Hellwig, and Pfleiderer (2011) that capital requirements can be a substitute for liquidity requirements. Yet, they do not support proposals of sharp increases in the stringency of capital requirements. Rather, the dominance of the PCA over non–contingent regulatory prescriptions supports the desirability of recent proposals to introduce contingencies in the regulation of capital structure, such as those advanced in the pioneering work by Flannery (2005), Flannery (2009), and more recently by Hart and Zingales (2011).

We should note that in our model bank risk choices arise from the dependence of the volatility of cash flows on the level of lending owing to the concavity of the loan revenue function. A risk shifting problem would just arise if equity holders decided to liquidate loan and distribute dividends. This would be a decision that would potentially shift NPV value from debt holders and depositors to equity holders. However, the collateral constraint prevents such a behavior. Thus, in our model a bank's choice of risk is not a choice of a loan revenue distribution.

Our set-up therefore differs from set–ups such as those analyzed by Blum (1999) and Calem and Rob (1999). In these set–ups a bank makes a choice of a loan revenue distribution. Incentives for risk shifting might be induced by capital requirements when banks are poorly capitalized and the costs of issuing equity are too high or prohibitive, leading banks to "gamble for resurrection." A similar finite–horizon set–up characterizes the model by Calomiris, Heider, and Horeova (2012), who show examples where liquidity requirements might be a substitute for capital requirements. Yet, these results are obtained under the assumption of an exogenously given choice of default in the final period, so that a positive bond investment (as a "liquid" component of capital) that could be used to avoid default to preserve the (continuation) value of a bank is ruled out, since there is no continuation value. In other words, default is exogenous. In Calem and Rob (1999) dynamic (infinite–horizon) model, bank risk–taking increases if capital requirements are sufficiently high. Yet, in their model equity issuance is ruled out and the size of a bank is fixed, so that banks cannot liquidate assets. Extending our model by allowing banks to choose a loan revenue distribution might be useful to assess the robustness of the foregoing conclusions. In any event, a contingent resolution policy such as the PCA would likely eradicate at the source the potential risk-shifting problems pointed out by these contributions.

Our quantitative results are in line with some empirical findings. As noted earlier, the result that optimal capital ratios are always higher than required ratios along equilibrium paths is a straightforward implication of bank optimal policies derived in a fully dynamic framework, and is consistent with the empirical evidence presented in Flannery and Kasturi (2008) and Flannery (2005).

The procyclicality of lending induced by capital requirements has been mainly examined in the context of macroeconomic models with either short–lived banks, or long–lived banks whose optimal policies have been proxied by ad–hoc capital adjustment rules (see Panetta and Angelini (2009) and Angelini, Enria, Neri, Panetta, and Quagliarello (2010) for reviews of this literature). Recent central banks' efforts to quantify the impact of increases in capital requirements and the introduction of liquidity requirements on banks' lending and the real economy have produced mixed results as well.[22] Kashyap, Stein, and Hanson (2010) review the empirical literature on U.S. banks, and assess the impact of increases in capital requirements through regression analyses, finding relatively large reductions in lending in the transition to a steady state with higher capital requirements, but small reductions of lending in steady state. Yet, many of the results of this empirical literature are obtained with reduced-form statistical models, rather than informed by a "structural" model of the type characterizing our approach.

Our calibration results suggest that implementing non-trivial increases in capital requirements and the introduction of liquidity requirements may be associated with significant reduction of lending, as well as with efficiency and welfare costs. The estimated magnitude of these costs appears significantly larger than that obtained with currently available reduced–form estimates. Designing and introducing resolution procedures with appropriate contingencies related to observed levels of capital, such as the PCA, might be a necessary step to move towards optimal bank regulation.

[22]Basel Committee on Banking Supervision (2010) evaluates the long–term economic impact of the proposed capital and liquidity reforms using a variety of models, including dynamic stochastic general equilibrium models. This study finds that the net economic benefits of these reforms, as measured as a reduction in the expected yearly output losses associated with a lower frequency of banking crises, are positive for a broad range of capital ratios, but become negative beyond a certain range. Yet, these quantitative assessments are surrounded by notable uncertainty. Moreover, measurement errors in the computation of net economic benefits also arise from the use of banking crises classifications which record government responses to crises rather than adverse shocks to the banking system (see Boyd, De Nicolò, and Loukoianova (2010)).

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# Appendix

## A. Properties of the unregulated bank program

Compactness of the feasible set of the bank can be shown as follows. Given the strict concavity of $\pi(L)$, there exists a level $L_u$ such that $\pi(L_u)Z_u - rL_u = 0$, where $r$ (which can be either $r_f$ or $r_d$) is the cost of capital of the marginal dollar raised either through deposits or short–term financing.[23] Thus, any investment $L > L_u$ would be unprofitable. This establishes an upper bound on the feasible set of $L$, given by $[0, L_u]$ for some $L_u$. With an upper bound on $L$, and because the stochastic process $D$ has compact support, the collateral constraint sets a lower bound $B_d$ (i.e., an upper bound on bond issuance). Specifically, this is obtained by putting $D_d$ in place of $D_t$ and $L_u$ in place of $L_t$ in Equation (10).

Lastly, an upper bound on $B$ can be obtained assuming that the proceeds from risk–free investments made by the bank are taxed at a higher rate than the personal tax rate and that floatation costs are positive. Specifically, assume that the current deposits $D$ are all invested in short–term bonds, $B$, with no investments in loans. To further increase the investment in bonds of one dollar, the bank must raise equity capital. A shareholder thus incurs a cost $1 + \lambda$, where $\lambda$ is the floatation cost. This additional dollar is invested at the rate $r_f$, so that at the end of the year, the proceeds of this investment that can be distributed are $(1 + r_f(1 - \tau^+))$. Alternatively, the shareholder can invest $1 + \lambda$ in a risk–free bond, obtaining $(1 + \lambda)(1 + r_f)$. Because $\tau^+ \geq 0$ and $\lambda \geq 0$, then $(1+\lambda)(1+r_f) \geq (1+r_f(1-\tau^+))$, there is no incentive of the bank to have a cash balance larger than $D$ as long as either $\lambda$ or $\tau^+$ are strictly positive. The foregoing argument is made for simplicity. If the shareholders are taxed on their investment proceeds at a rate $\tau_p$, they obtain $(1+\lambda)(1+r_f(1-\tau_p))$ from their investment in the risk–free asset. If $\tau_p \leq \tau^+$, then $(1 + \lambda)(1 + r_f(1 - \tau_p)) > (1 + r_f(1 - \tau^+))$, and the bank has no incentive to increase the investment in risk–free bonds beyond $D$. Moreover, if floatation costs associated with equity issuance are strictly increasing in the amount issued, no assumption about differential tax rates are needed to establish an upper bound on $B$. In conclusion, the feasible set of the bank can be assumed to be $[0, L_u] \times [B_d, B_u]$.

[23] Deposits and short–term bonds are the cheapest form of financing. If the same dollar were raised by issuing equity, the cost would be higher owing both to the higher cost of equity capital and to floatation costs. In this case the upper bound would be even lower.

Furthermore, standard arguments establish the existence of a unique value function $E(x) = E(L, B, D, u, v)$ that satisfies Equation (18) and is continuous in all its arguments. The existence and uniqueness of the value function $E$ follow from the Contraction Mapping Theorem (Theorem 3.2 in Stokey and Lucas (1989)). The continuity of $E$ follow from the continuity of $e$ and the Monotonicity of the Markov transition function of the process $(u, v)$.

## B. The dynamics of deposits and credit shocks

Introducing a more compact notation, the joint dynamics of the systematic and idiosyncratic risk is described by equation

$$s_t = H s_{t-1} + \zeta_t, \tag{34}$$

where

$$H = \begin{pmatrix} \kappa_u & 0 \\ 0 & \kappa_v \end{pmatrix}, \qquad \zeta_t = \begin{pmatrix} \sigma_u \varepsilon_t^u \\ \sigma_v \varepsilon_t \end{pmatrix}, \qquad \mathbb{E}_t\left[\zeta_{t+1} \cdot \zeta_{t+1}'\right] = \begin{pmatrix} \sigma_u^2 & 0 \\ 0 & \sigma_v^2 \end{pmatrix} = T$$

and $(\varepsilon_t^u, \varepsilon_t)$ are standard Normal variates with truncated support. The dynamics of $(Z_t, \log D_{t+1})$ in (15) is

$$X_t = \overline{X} + K X_{t-1} + \xi_t \tag{35}$$

where

$$X_t = \begin{pmatrix} Z_t \\ \log D_{t+1} \end{pmatrix}, \qquad \overline{X} = \begin{pmatrix} (1-\kappa_Z)\overline{Z} \\ (1-\kappa_D)\log \overline{D} \end{pmatrix}, \qquad K = \begin{pmatrix} \kappa_Z & 0 \\ 0 & \kappa_D \end{pmatrix}, \qquad \xi_t = \begin{pmatrix} \xi_t^Z \\ \xi_t^D \end{pmatrix},$$

and

$$\mathbb{E}_t\left[\xi_{t+1} \cdot \xi_{t+1}'\right] = \begin{pmatrix} \sigma_Z^2 & \rho\sigma_Z\sigma_D \\ \rho\sigma_Z\sigma_D & \sigma_D^2 \end{pmatrix} = \Sigma.$$

In the bank's model, to achieve the stochastic structure in Equation (35), we have introduced the transformation (14), $X = \mu + Ns$, where $\mu = (\mu_1, \mu_2)$, and $N$ is a matrix

$$N = \begin{pmatrix} \nu_{Z,u} & \nu_{Z,v} \\ \nu_{D,u} & \nu_{D,v} \end{pmatrix}$$

such that $\det N \neq 0$. While the bank's program will be solved (and simulated) in the state space $\mathcal{S}$, we will use the above affine transformation to calculate the values of the variables $Z$ and $D$.

The parameters of the stochastic discount factor are taken from Jones and Tuzel (2013). Therefore, we are left with eight unknown parameters describing the stochastic structure of the model: $\mu_1$, $\mu_2$, $\kappa_v$, $\sigma_v$, $\nu_{Z,u}$, $\nu_{D,u}$, $\nu_{Z,v}$, $\nu_{D,v}$. From empirical data we calculate seven moment conditions: $\overline{Z}$, $\log \overline{D}$, $\kappa_Z$, $\sigma_Z$, $\kappa_D$, $\sigma_D$, $\rho$, as reported in Table I. In what follows, we derive the equations that relate the unknown parameters to the moment conditions. From Equation (14), we have $s = N^{-1}(X - \mu)$. Replacing this in Equation (34), after some manipulations we have

$$X_t = \left(I - NHN^{-1}\right)\mu + NHN^{-1}X_{t-1} + N\zeta_t.$$

This dynamics is equal to the dynamics in Equation (35) if the following conditions are simultaneously true: $\mu = \left(I - NHN^{-1}\right)^{-1}\overline{X}$, $NHN^{-1} = K$, $NTN' = \Sigma$. Given the second equation, $\mu = (I - K)^{-1}\overline{X}$, so that the solution for $\mu = (\mu_1, \mu_2)$, is found in closed form relative to known moments conditions. The second and third conditions set seven equations (three from $NTN' = \Sigma$, given that the two matrices are symmetric, and four from $NHN^{-1} = K$) in the remaining six unknowns. We find a least square solution to this overidentified system of seven non–linear equations. Using the parameter values in Table I in the paper, with the additional parameters for the macroeconomic risk, $\kappa_u = 0.98$, $\sigma_u = 0.007$, the robust solution obtained with a global optimization routine is $\mu_1 = 0.0717$, $\mu_2 = 0.6931$, $\kappa_v = 0.901992$, $\sigma_v = 0.009548$, $\nu_{Z,u} = 1.660682$, $\nu_{D,u} = -2.988127$, $\nu_{Z,v} = -0.798126$, and $\nu_{D,v} = 0.044359$. Although the model is overidentified, the proposed calibration procedure does a good job at matching a number of moments of the process $X$, from Equation (15), with moments of $\hat{X} = \mu + Ns$, where $s$ is the process in Equation (34) with the above estimated parameters. This is can be seen in Table VIII.

To understand the relationship that this calibration generates between the variables $(u, v)$ and the variables $(Z, D)$, we plot the values of $Z$ and $\log D$ resulting from a numerical approximation of the dynamic of $(u, v)$ in Figure 3. The variable $\log D$ appears strongly counter–cyclical, and depends only marginally on the idiosyncratic component, whereas the credit shocks $Z$ are pro–cyclical and negatively correlated with idiosyncratic risk.

## C. Numerical solution and simulation of the valuation problem

Given the dynamics of $(u, v)$, we solve the program

$$E(x) = \max \left\{ 0, \max_{(L',B') \in \mathcal{A}(D)} \left\{ e(x, L', B') + \mathbb{E}\left[ M(x, x') E(x') \right] \right\} \right\},$$

where function $e(x, L, B)$, is defined in Equation (13), and $\mathcal{A}(D)$ is the case specific feasible set defined differently for the unregulated and the regulated case. The solution of the Bellman equation above is obtained numerically by a value iteration algorithm. The valuation model for bank's equity is a continuous–decision and infinite–horizon Markov Decision Processes. The solution method is based on successive approximations of the fixed point solution of the Bellman equation. Numerically, we apply this method to an approximate discrete state-space and discrete decision valuation operator.[24]

The variables $u$ and $v$ are discretized using the numerical approach proposed by Rouwenhorst (1995), as the stochastic process of systematic risk is quite persistent. The feasible interval for loans, $[0, L_u]$, and for the face value of bonds, $[B_d, B_u]$ (with $B_d < 0 < B_u$), is set so that they are never binding for the equity maximizing program. We discretize $[L_d, L_u]$, to obtain a grid of $n_L$ points

$$\widetilde{L} = \left\{ \widetilde{L}_j = L_u(1 - \delta)^j \mid j = 1, \ldots, n_L - 1 \right\} \cup \{ L_{n_L} = 0 \}$$

such that, if the bank chooses inaction, the loan's level is what remains after the portion $\delta L$ has been repaid. The interval $[B_d, B_u]$ is discretized into $n_B$ equally–spaced values, making up the set $\widetilde{B}$. To keep the notation simple, we also denote $x = (s, L, B)$ the generic element of the discretized state.

For the set of parameters in Table I, we use $L_u = 18$, $B_d = -7$ and $B_u = 3$. Given the properties of the quadrature scheme, we solve the model using only 5 points for $u$, 7 points for $v$. However, we need to allow for many more points when discretizing the control variables, so we choose $n_L = 29$, and $N_B = 34$. The tolerance for termination of the value function iteration is set at $10^{-5}$.

[24] See Rust (1996) or Burnside (1999) for a survey on numerical methods for continuous decision infinite horizon Markov Decision Processes.

Given the optimal solution, we can determine the optimal policy and the transition function $\varphi(x)$ in Equation (19) based on the arg-max of equity value at the discrete states $x$. The optimal policy is used to generate 50 simulated economies, each characterized by a specific path of the systematic shock, and comprising 2,000 independent paths/banks for the idiosyncratic shock, for 100 periods (years). In particular, given the simulated dynamics of the state variables $(u, v)$ by application of the recursive formula in Equation (15), we start from $Z(0) = \overline{Z}$ and $D(0) = D_d$. Then, setting a feasible initial choice $L(0) = 0$ and $B(0) = D_u$ (so that the initial bank capital is $D_u - D_d$), we apply the transition function $\varphi$ along each simulated path recursively. If a bank defaults at a given step, then the current depositors receive the full value of their claim, while the deposit insurance agency pays the bankruptcy cost. Afterwards, a seed capital $D_u - D_d$ is injected in the bank. Together with deposit $D_d$, the total amount $D_u$ is momentarily invested in bonds, $B = D_u$, while $L = 0$. Then the "new" bank follows on the same path by applying the optimal policy. To limit the dependence of our results on the initial conditions, we drop the first 50 steps.

| | | |
|---|---|---|
| $\kappa_Z$ | annual persistence of the credit shock | 0.88 |
| $\sigma_Z$ | annual conditional std. dev. of the credit shock | 0.0139 |
| $\overline{Z}$ | unconditional average of the credit shock | 0.0717 |
| $\kappa_D$ | annual persistence of the log of deposits | 0.98 |
| $\sigma_D$ | annual conditional std. dev. of the log of deposits | 0.0209 |
| $\overline{D}$ | unconditional average of deposits | \$2 |
| $\rho$ | correlation between log–deposit and credit shock | -0.85 |
| $\beta$ | time discount factor | 0.95 |
| $\gamma_1$ | constant price of risk parameter | 3.22 |
| $\gamma_2$ | time varying price of risk parameter | -15.30 |
| $r_f$ | annual rate on bonds | 2.5% |
| $r_d$ | annual rate on deposits | 0% |
| $\tau^+$ | corporate tax rate for positive earnings | 15% |
| $\tau^-$ | corporate tax rate for negative earnings | 0% |
| $\delta$ | annual percentage of reimbursed loan | 20% |
| $\eta$ | bankruptcy costs | 0.10 |
| $\lambda$ | flotation cost for equity | 0.06 |
| $\alpha$ | return to scale for loan investment | 0.90 |
| $m^+$ | unit price for loan investment | 0.04 |
| $m^-$ | unit price for loan fire sales | 0.05 |
| $k$ | percentage of loans for capital regulation | 4% |
| $\ell$ | liquidity coverage ratio | 20% |

Table I: **Base case model parameters**

| Unconditional | | | | |
|---|---|---|---|---|
| $e^{u+v}$ | 0.99 | 1.01 | 1.02 | 1.06 |
| Unregulated | 1.12 | 1.01 | 0.87 | 0.73 |
| Capital | 1.18 | 1.13 | 1.00 | 0.91 |
| Cap + Liq | 0.92 | 0.81 | 0.76 | 0.63 |
| PCA | 1.23 | 1.13 | 1.03 | 0.86 |
| PCA + Cap | 1.18 | 1.13 | 1.01 | 0.87 |
| **Upturn** | | | | |
| $e^{u+v}$ | 1.02 | 1.04 | 1.05 | 1.09 |
| Unregulated | 2.14 | 1.85 | 1.77 | 1.53 |
| Capital | 2.07 | 2.03 | 1.83 | 1.68 |
| Cap + Liq | 1.50 | 1.38 | 1.31 | 0.99 |
| PCA | 2.07 | 2.02 | 1.83 | 1.67 |
| PCA + Cap | 2.07 | 2.03 | 1.83 | 1.68 |
| **Downturn** | | | | |
| $e^{u+v}$ | 0.95 | 0.97 | 0.98 | 1.02 |
| Unregulated | -0.04 | -0.08 | -0.08 | -0.07 |
| Capital | 0.06 | -0.03 | -0.06 | -0.07 |
| Cap + Liq | 0.07 | -0.03 | -0.02 | -0.03 |
| PCA | 0.06 | -0.05 | -0.08 | -0.09 |
| PCA + Cap | 0.06 | -0.03 | -0.05 | -0.06 |

Table II: **Loan investment policy**. This table shows the simulated ratios $L^*/((1-\delta)L)$, where $L^*$ is the optimal solution for a solvent bank, sorted against quartiles of total risk, $e^{u+v}$, under the unregulated, the capital requirement, the capital and liquidity requirement cases, the PCA case, and the one with PCA and capital requirement. We report the values from the total sample (top panel), the values conditional on the economy being in an upturn, when the systematic risk $u = 0.0352$ (middle panel), or in a downturn, when $u = -0.0352$ (bottom panel). These results are based on the numerical solution of the valuation problem in Equation (18) and the simulation of the optimal solution, as described in Appendix C, based on the parameter values in Table I. The table presents the averages across the economies of the time series averages of the cross-sectional sortings.

| Unconditional | | | | |
|---|---|---|---|---|
| $e^{u+v}$ | 0.99 | 1.01 | 1.02 | 1.06 |
| Unregulated | -5.75 | -3.30 | -1.10 | -1.79 |
| Capital | 0.11 | 0.15 | 0.15 | 0.14 |
| Cap and Liq | 0.33 | 0.36 | 0.34 | 0.32 |
| PCA | 0.06 | 0.04 | 0.02 | 0.01 |
| PCA + Cap | 0.11 | 0.16 | 0.15 | 0.15 |
| **Upturn** | | | | |
| $e^{u+v}$ | 1.02 | 1.04 | 1.05 | 1.09 |
| Unregulated | 0.18 | 0.14 | 0.12 | 0.06 |
| Capital | 0.19 | 0.18 | 0.15 | 0.13 |
| Cap and Liq | 0.62 | 0.60 | 0.59 | 0.53 |
| PCA | 0.19 | 0.19 | 0.15 | 0.13 |
| PCA + Cap | 0.19 | 0.18 | 0.15 | 0.13 |
| **Downturn** | | | | |
| $e^{u+v}$ | 0.95 | 0.97 | 0.98 | 1.02 |
| Unregulated | -29.00 | -23.49 | -13.04 | -14.29 |
| Capital | 0.06 | 0.20 | 0.19 | 0.19 |
| Cap and Liq | 0.07 | 0.21 | 0.20 | 0.19 |
| PCA | -0.07 | -0.14 | -0.17 | -0.21 |
| PCA + Cap | 0.06 | 0.20 | 0.19 | 0.19 |

Table III: **Capital ratios**. This table shows the simulated capital ratios (i.e., bank capital over loans, or $K^*/L^* = (L^* + B^* - D')/L^*$, where $(L^*, B^*)$ is the optimal solution for a solvent bank and $D'$ is the new possible level of deposits, sorted against quartiles of total risk, $e^{u+v}$, under the unregulated, the capital requirement, the capital and liquidity requirement cases, the PCA case, and the one with PCA and capital requirement. We report the values from the total sample (top panel), the values conditional on the economy being in an upturn, when the systematic risk $u = 0.0352$ (middle panel), or in a downturn, when $u = -0.0352$ (bottom panel). These results are based on the numerical solution of the valuation problem in Equation (18) and the simulation of the optimal solution, as described in Appendix C, based on the parameter values in Table I. The table presents the averages across the economies of the time series averages of the cross-sectional sortings.

| Unconditional | | | | |
|---|---|---|---|---|
| $e^{u+v}$ | 0.99 | 1.01 | 1.02 | 1.06 |
| Unregulated | -535.32 | -270.79 | -182.18 | -129.10 |
| Capital | -534.41 | -270.23 | -181.53 | -128.61 |
| Cap and Liq | 25.10 | 8.99 | 9.05 | 6.22 |
| PCA | -535.18 | -270.67 | -181.97 | -128.52 |
| PCA + Cap | -534.41 | -270.22 | -181.51 | -128.22 |
| **Upturn** | | | | |
| $e^{u+v}$ | 1.02 | 1.04 | 1.05 | 1.09 |
| Unregulated | -26.32 | -24.45 | -23.35 | -21.82 |
| Capital | -26.56 | -25.43 | -24.38 | -23.01 |
| Cap and Liq | 0.91 | 0.75 | 0.91 | 0.70 |
| PCA | -26.77 | -25.54 | -24.45 | -22.86 |
| PCA + Cap | -26.56 | -25.42 | -24.36 | -22.88 |
| **Downturn** | | | | |
| $e^{u+v}$ | 0.95 | 0.97 | 0.98 | 1.02 |
| Unregulated | -0.26 | -0.12 | -0.05 | -0.01 |
| Capital | 2.59 | 3.01 | 3.20 | 3.34 |
| Cap and Liq | 3.04 | 3.40 | 3.42 | 3.52 |
| PCA | 2.53 | 2.98 | 3.15 | 3.31 |
| PCA + Cap | 2.58 | 3.01 | 3.21 | 3.39 |

Table IV: **Liquidity ratios**. This table shows the simulated liquidity ratios (i.e., end–of–period total cash available in the worst case scenario over the end–of–period net cash outflows due to a variation in deposits, or $(\delta L^* + \pi(L^*)Z_d - \mathcal{T}(y^{\min}) + B^*(1+r_f))/(D'(1+r_d) - D_d)$, where $(L^*, B^*)$ is the optimal solution for a solvent bank and $D'$ is the new possible level of deposits, sorted against quartiles of total risk, $e^{u+v}$, under the unregulated, the capital requirement, the capital and liquidity requirement cases, the PCA case, and the one with PCA and capital requirement. We report the values from the total sample (top panel), the values conditional on the economy being in an upturn, when the systematic risk $u = 0.0352$ (middle panel), or in a downturn, when $u = -0.0352$ (bottom panel). These results are based on the numerical solution of the valuation problem in Equation (18) and the simulation of the optimal solution, as described in Appendix C, based on the parameter values in Table I. The table presents the averages across the economies of the time series averages of the cross-sectional sortings.

| | *Unreg.* | *Capital* | | *Capital & Liquidity* | | |
|---|---|---|---|---|---|---|
| | | $k = 4\%$ | $k = 12\%$ | $k = 4\%$<br>$\ell = 20\%$ | $k = 12\%$<br>$\ell = 20\%$ | $k = 4\%$<br>$\ell = 50\%$ |
| Loans (book) | 4.41 | 5.08 | 4.96 | 3.71 | 3.75 | 3.71 |
| Net Bond Holdings (book) | -2.75 | -2.30 | -2.05 | 0.34 | 0.32 | 0.38 |
| Bank Capital (book) | -0.32 | 0.80 | 0.92 | 2.07 | 2.09 | 2.12 |
| Equity (mkt) | 6.97 | 7.32 | 7.36 | 7.65 | 7.66 | 7.69 |
| Deposits (mkt) | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 |
| Enterprise Value (mkt) | 11.70 | 11.61 | 11.40 | 9.29 | 9.33 | 9.29 |
| Government Value (mkt) | 0.82 | 0.97 | 0.97 | 0.90 | 0.90 | 0.91 |
| Social value (mkt) | 12.52 | 12.58 | 12.37 | 10.19 | 10.23 | 10.19 |
| Default (%) | 1.30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Table V: **The impact of bank regulation**. The table presents different dimensions of the bank, based either on book or market values. The columns represent different choices of parameters: the "Unregulated" case is obtained with the parameters in Table I. The case with capital constraint ("Capital") has either $k = 4\%$ or $k = 12\%$. The case with both capital and liquidity restrictions ("Capital & Liquidity") is obtained for the base case parameters ($k = 4\%$ and $\ell = 20\%$), and two alternative combinations, with $k = 12\%$ and $\ell = 20\%$, and with $k = 4\%$ and $\ell = 50\%$, respectively. These results are based on the numerical solution of the valuation problem in Equation (18) and the simulation of the optimal solution, as described in Appendix C, based on the parameter values in Table I. The table presents the averages across the simulated economies of the time series averages of the cross–sectional averages (computed on non–defaulted instances) of the different metrics.

| | *Unreg.* | *PCA* | *Cap.* | *PCA+ Cap.* | *Cap.&Liq.* | *PCA+ Cap.&Liq.* |
|---|---|---|---|---|---|---|
| Loan (book) | 4.41 | 5.12 | 5.08 | 5.03 | 3.71 | 3.72 |
| Net Bond Holdings (book) | -2.75 | -2.38 | -2.30 | -2.25 | 0.34 | 0.34 |
| Bank Capital (book) | -0.32 | 0.77 | 0.80 | 0.80 | 2.07 | 2.07 |
| Equity (mkt) | 6.97 | 7.46 | 7.32 | 7.30 | 7.65 | 7.65 |
| Deposits (mkt) | 1.89 | 1.88 | 1.89 | 1.89 | 1.89 | 1.89 |
| Enterprise Value (mkt) | 11.70 | 11.81 | 11.61 | 11.53 | 9.29 | 9.30 |
| Government Value (mkt) | 0.82 | 0.97 | 0.97 | 0.98 | 0.90 | 0.91 |
| Social value (mkt) | 12.52 | 12.78 | 12.58 | 12.50 | 10.19 | 10.20 |
| Default (%) | 1.30 | 3.71 | 0.00 | 0.00 | 0.00 | 0.00 |
| PCA frequency (%) | – | 0.27 | – | 0.02 | – | 0 |

Table VI: **Prompt Corrective Action**. The table presents different dimensions of the bank, based either on book or market values. The columns offer a comparison among the three cases (unregulated bank, capital requirement, and capital plus liquidity requirement) without and with the Prompt Corrective Action (PCA). These results are based on the numerical solution of the valuation problem in Equation (18) and the simulation of the optimal solution, as described in Appendix C, based on the parameter values in Table I. The table presents the averages across the simulated economies of the time series averages of the cross–sectional averages (computed on non–defaulted instances) of the different metrics.

| | *Unreg.* | base | $\lambda = 0$ | $\lambda = .2$ | $m^- = .08$ | $\delta = .1$ |
|---|---|---|---|---|---|---|
| | | | | *Capital* | | |
| Loans (book) | 4.41 | 5.08 | 5.13 | 4.98 | 5.09 | 7.38 |
| Net Bond Holdings (book) | -2.75 | -2.30 | -2.32 | -2.24 | -2.31 | -2.98 |
| Bank Capital (book) | -0.32 | 0.80 | 0.83 | 0.75 | 0.80 | 2.41 |
| Equity (mkt) | 6.97 | 7.32 | 7.35 | 7.22 | 7.29 | 10.30 |
| Deposits (mkt) | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 |
| Enterprise Value (mkt) | 11.70 | 11.61 | 11.65 | 11.44 | 11.57 | 15.26 |
| Government Value (mkt) | 0.82 | 0.97 | 0.99 | 0.95 | 0.97 | 1.37 |
| Social value (mkt) | 12.52 | 12.58 | 12.64 | 12.40 | 12.55 | 16.63 |
| | | | | *Capital & liquidity* | | |
| Loans (book) | 4.41 | 3.71 | 3.85 | 3.57 | 3.69 | 5.64 |
| Net Bond Holdings (book) | -2.75 | 0.34 | 0.31 | 0.38 | 0.35 | 0.64 |
| Bank Capital (book) | -0.32 | 2.07 | 2.18 | 1.97 | 2.06 | 4.30 |
| Equity (mkt) | 6.97 | 7.65 | 7.80 | 7.49 | 7.64 | 10.86 |
| Deposits (mkt) | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 |
| Enterprise Value (mkt) | 11.70 | 9.29 | 9.47 | 9.09 | 9.27 | 12.20 |
| Government Value (mkt) | 0.82 | 0.90 | 0.93 | 0.87 | 0.90 | 1.34 |
| Social value (mkt) | 12.52 | 10.19 | 10.40 | 9.96 | 10.17 | 13.54 |

Table VII: **The role of equity issuance costs, adjustment costs and maturity transformation**. The table shows two panels: on top the case of a bank with capital requirement, and at the bottom the case of a bank subject to both capital and liquidity restrictions. The table presents different dimensions of the bank, based either on book or market values. The columns represent different choices of parameters: the column denoted "base" is the base case, with the parameters in Table I. The others are obtained by changing only the parameter used to denominate the column (e.g., in "$\lambda = 0$" all the parameters are at the base case value, but $\lambda$, which is set to zero). These results are based on the numerical solution of the valuation problem in Equation (18) and the simulation of the optimal solution, as described in Appendix C, based on the parameter values in Table I. The table presents the averages across the simulated economies of the time series averages of the cross–sectional averages (computed on non–defaulted instances) of the different metrics.

| | *Original* | $X$ | $\hat{X}$ |
|---|---|---|---|
| $\mathbb{E}[Z]$ | 0.0717 | 0.0718 | 0.0723 |
| $\sigma[Z]$ | 0.0293 | 0.0236 | 0.0429 |
| $\mathbb{E}[\log D]$ | 0.6931 | 0.6926 | 0.6920 |
| $\sigma[\log D]$ | 0.0669 | 0.0471 | 0.0701 |
| $(1-\kappa_Z)\overline{Z}$ | 0.0086 | 0.0129 | 0.0039 |
| $\kappa_Z$ | 0.8800 | 0.8184 | 0.9357 |
| $\sigma_Z$ | 0.0139 | 0.0135 | 0.0150 |
| $(1-\kappa_D)\log\overline{D}$ | 0.0347 | 0.0733 | 0.0374 |
| $\kappa_D$ | 0.9500 | 0.8940 | 0.9450 |
| $\sigma_D$ | 0.0209 | 0.0210 | 0.0226 |
| $\rho$ | -0.8500 | -0.7514 | -0.8503 |

Table VIII: **Calibration**. Results from the calibration procedure described in Appendix B. Column "*Original*" reports the value of the moments from Table I. In Column "$X$" there is the mean value of the moments estimated from a simulation of $X$. Column "$\hat{X}$" reports the mean value of the moments estimated from a simulation of $s$ in Equation (34), and than using at each step of the simulated sample the transformation in Equation (14) to obtain $\hat{X}$.

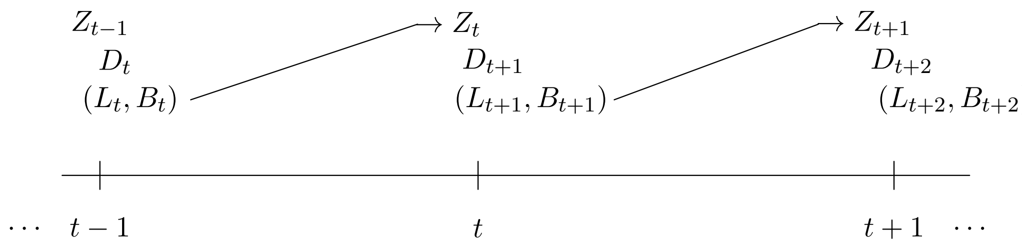


Figure 1: **Bank's dynamic**. Evolution of the state variables (credit shock, $Z$, and deposits, $D$) and of the bank's control variables (cash and liquid investments, $B$, and loans, $L$) assuming the bank is solvent at each date.

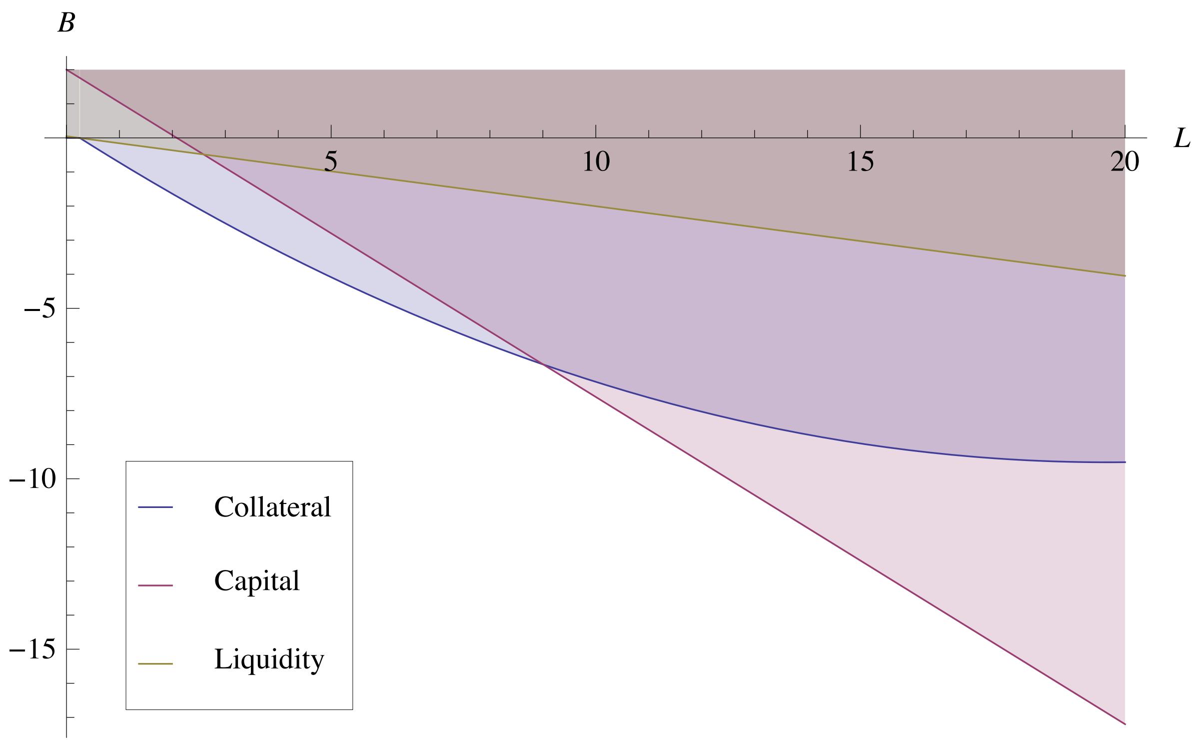


Figure 2: **Comparison of constraints**. This figure presents the three feasible regions of $(L, B)$ defined by the collateral constraint, $\Gamma(D)$ in Equation (11), the capital requirement, $\Theta(D)$ from Equation (25), and by the liquidity requirement, $\Lambda(D)$ from Equation (27). The plot is based on the parameter values in Table I, for a current $D = 2$.

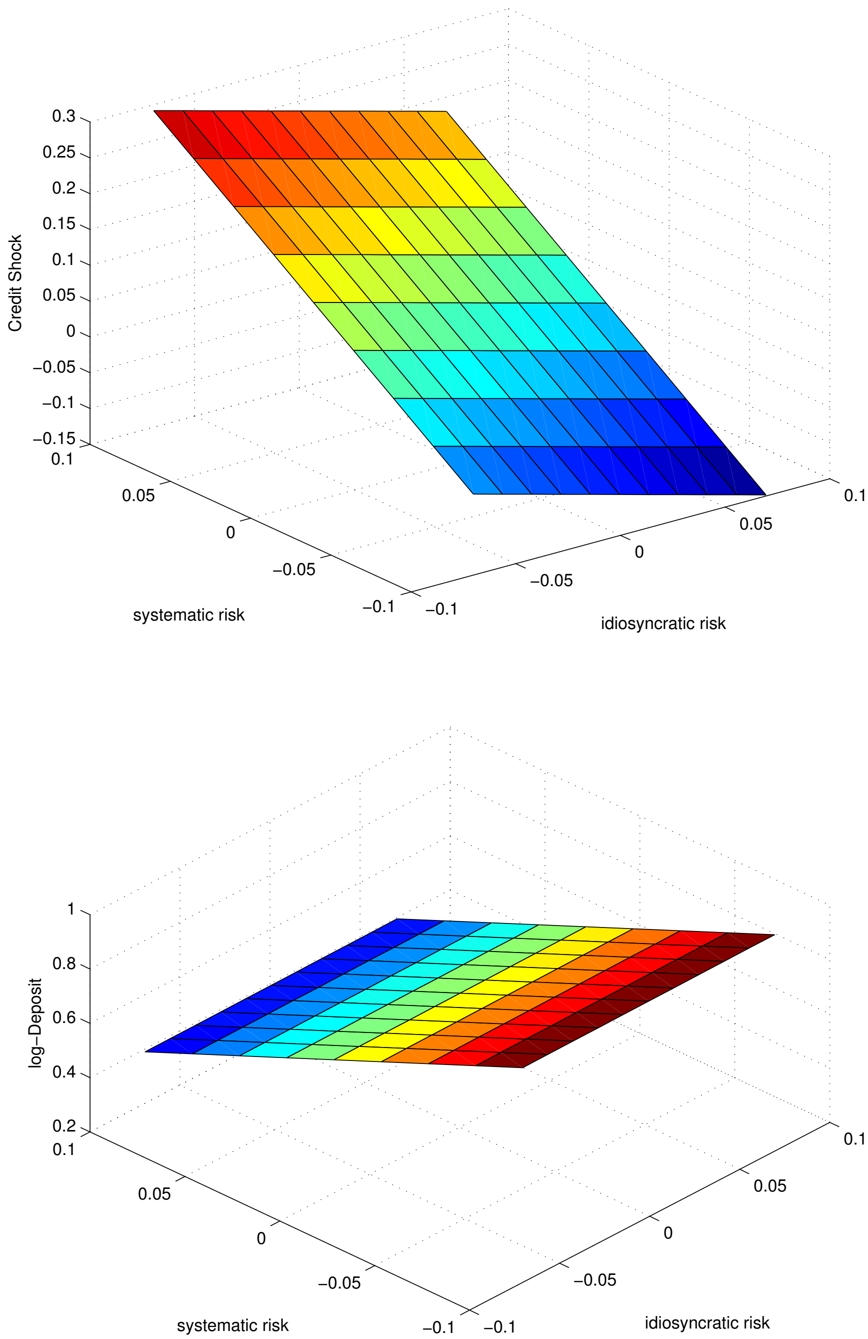


Figure 3: **Credit shock and deposit against systematic and idiosyncratic risk**. The values are from the numerical solution of the model using 9 points for $u$, 11 points for $v$, based on the estimated parameter values in Appendix B.

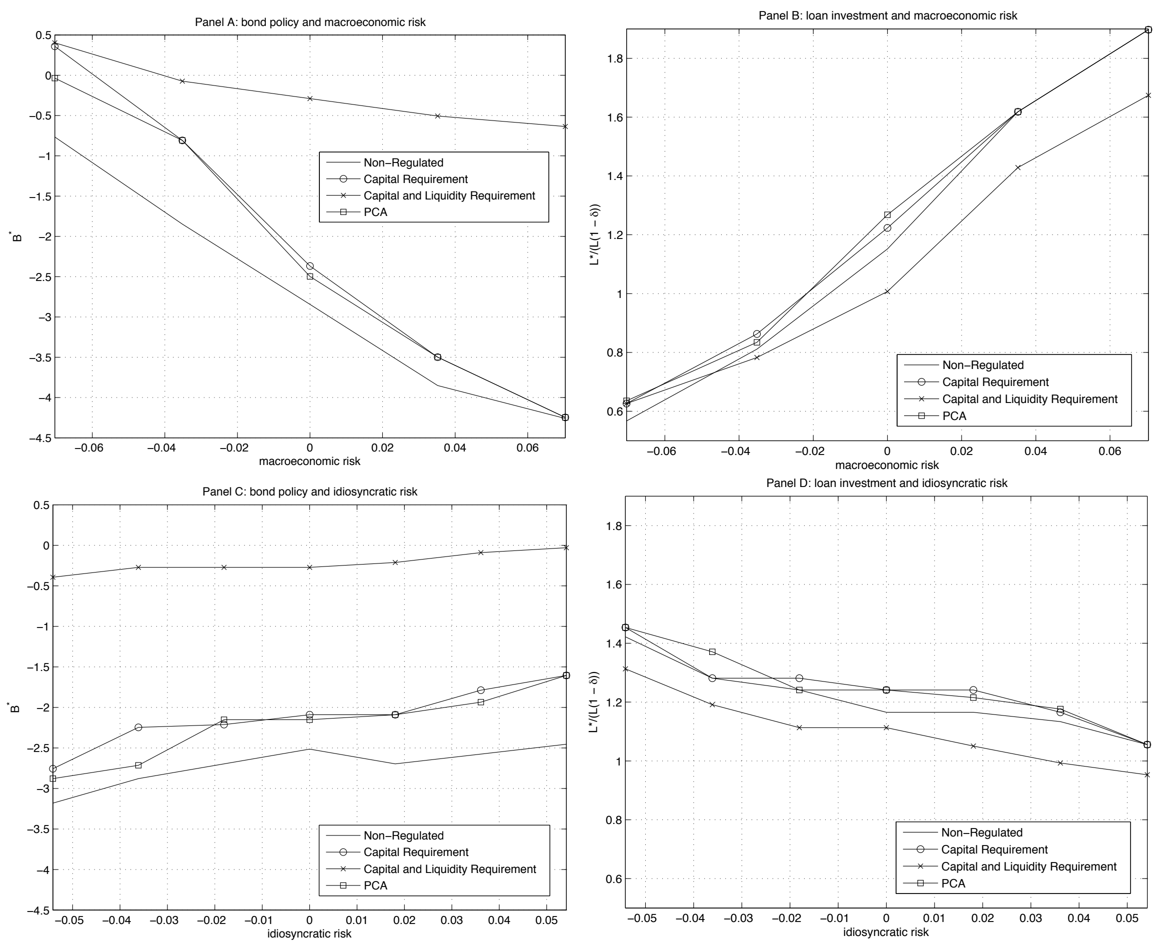


Figure 4: **Bank's policy**. This figure illustrates the impact of regulatory restrictions on the bank's policy related to loan investment represented by the ratio $L^*/L(1-\delta)$ and to short–term investment and financing with bonds, $B^*$, for the non–regulated case, for the cases with capital constraint, with both capital and liquidity constraints altogether, and the PCA case. These values are plotted against the macroeconomic risk factor, $u$, in the upper panels and the idiosyncratic risk factor, $v$, in the remaining panels, and are obtained assuming that the bank is currently at the steady state (so that the credit shock is 0.0717, and the deposits from the previous date are $D = 2$, respectively), while $B = 0$, and $L = 4.7$ so that the current bank capital (right before making the decision) is $K = 2.7$. The investment and financing policies are given by averaging out idiosyncratic risk when plotted against $u$ and averaged across the systematic risk when plotted against $v$. These results are based on the numerical solution of the valuation problem in (18), as described in Appendix C, based on the parameter values in Table I.

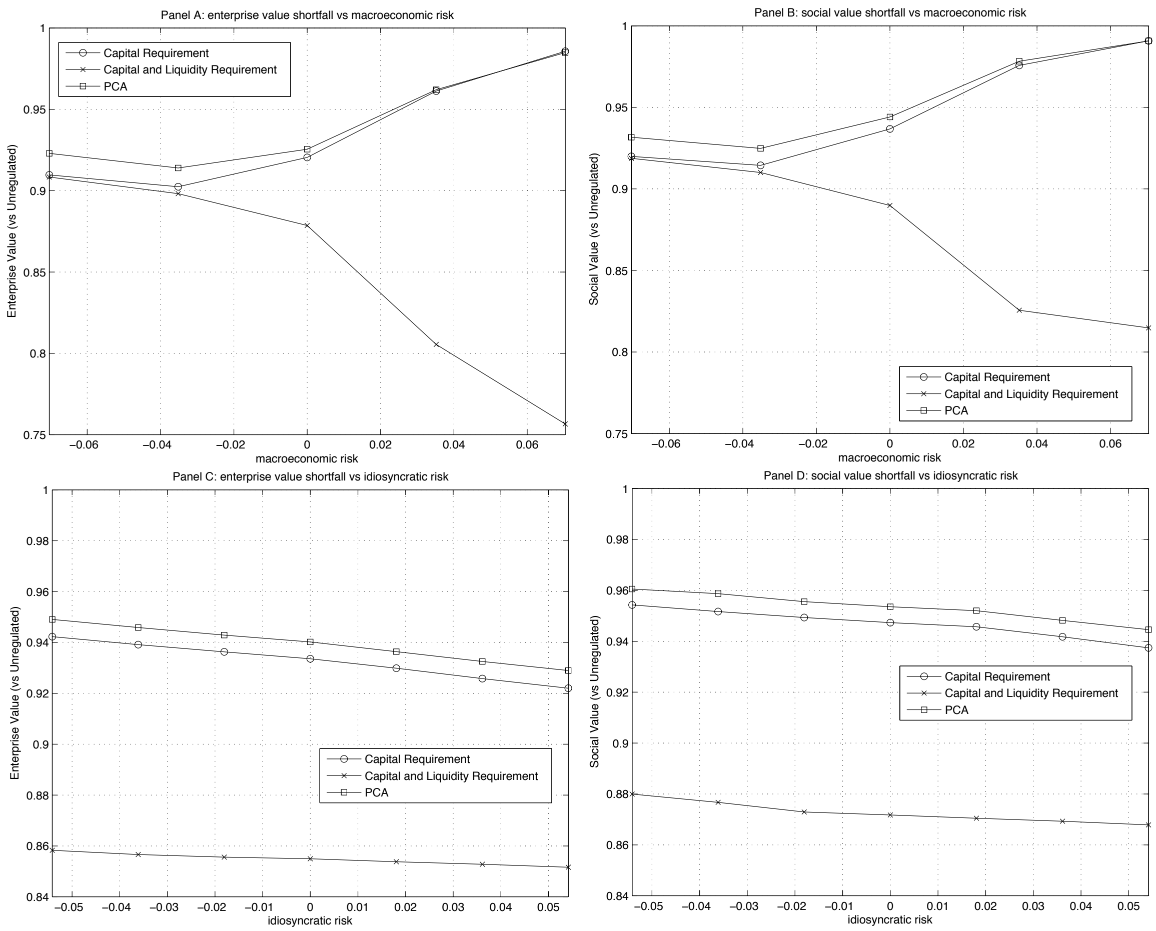


Figure 5: **Value loss associated with regulatory restrictions**. This figure illustrates the impact of regulatory restrictions by comparing the enterprise value (i.e., value of deposits plus market value of equity net of cash balance, or plus short–term debt), and the social value (i.e., book value of deposits plus market value of equity plus the value to the government, plus the present value of issuance costs) of the bank, for the case with capital constraint, with both capital and liquidity constraints, and the PCA case, as a proportion of the value from the non–regulated case. These values are plotted against the systematic shock, $u$, in the upper panels and the idiosyncratic shock, $v$, in the lower panels, and are obtained assuming that the bank is currently at the steady state (so that the credit shock is 0.0717, and the deposits from the previous date are $D = 2$, respectively), while $B = 0$, and $L = 4.7$ so that the current bank capital (right before making the decision) is $K = 2.7$. The value ratios are given by averaging out idiosyncratic risk when plotted against $u$ and averaged across systematic risk when plotted against $v$. These results are based on the numerical solution of the valuation problem in (18), as described in Appendix C, based on the parameter values in Table I.