Stationarity of Ibbotson Associates Equity Risk Premiums

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Equity Risk premium data from Ibbotson and Sinquefeld (1982), which have been annually updated by Ibbotson Associates and are currently updated by Morningstar (e.g., Morningstar, 2013) which acquired Ibbotson Associates in 2006, are commonly used by financial practitioners to develop estimates of the cost of capital. "The equity risk premium (ERP) (often interchangeably referred to as the market risk premium) is defined as the extra return (over the expected yield on risk-free securities) that investors expect to receive from an investment in a diversified portfolio of common stocks."[1] Historic equity risk premiums based on the Ibbotson Associates data are cited in leading business valuation texts, such as Pratt and Niculita (2008), as the starting point for estimating the cost of equity for use in discounted cash flow approaches to business valuation. Historic equity risk premiums are also used as a starting point in deriving risk adjusted discounts for use in commercial litigation.

Morningstar (2013) provides the argument by Ibbotson Associates that the average equity risk premium from 1926, the earliest time period covered by the Ibbotson Associates, to present provides the best forecast of the future equity risk premium. Morningstar (2013) argues that over the period from 1926 to 2011, the annual arithmetic equity risk premium exhibits virtually zero first order serial correlation, which means that the equity risk premium in one year cannot be used to forecast the equity risk premium in the following year. As a result, the annual equity risk premium has behaved in a random fashion over time and the mean is the best forecast of expected values of the equity risk premium. In other words, the annual equity risk premium is stationary as it has a constant mean.

The inference that the equity risk premium is stationary has not been completely without controversy. Finnerty and Leistikow (1993 and 1994) conducted tests of the stationarity of equity risk premiums over the period from 1926 to 1989, finding that equity risk premiums had

[1] Pratt & Grabowski (2008), p. 89.

declined over time through comparison of the 1926-1957 and 1958-1989 time periods. Ibbotson and Lummer (1994) countered that these declines were not statistically significant. Responding, Finnerty and Leistikow (1994) point out that Ibbotson and Lummer's exhibits displaying mean annual risk premiums from 1926 to 1993 suggest a downward trending equity risk premium. Because securities markets have experienced considerable turmoil since 1993, especially in 2008, the time is right for a second look at the statistical properties of equity risk premiums.

This paper provides tests of the stationarity of the equity risk premium using monthly returns for 1926-2012 from Morningstar (2013). The Augmented Dickey-Fuller (Dickey and Fuller, 1981, and Said and Dickey, 1984) and the Kwiatkowski, Phillips, Schmidt and Shin (Kwiatkowski, et al., 1992) tests are used to test for stationarity.

However, before analyzing equity risk premiums, it is worth detouring and understanding what an equity risk premium from the standpoint of financial theory. In particular, what is the relationship of equity risk premiums measured with historic data to the equity risk premium discussed in financial theory.

Equity Risk Premium

As noted above, the equity risk premium is the difference between the return on a well-diversified portfolio of stocks and a riskless asset. Financial theory assumes that investment decisions and the valuation of assets are based on what Fernandez (2013) refers to as the "expected equity risk premium". The expected equity risk premium is equal to the return that an investor expects to receive in the stock market above the return on risk free investments. The use of an average equity risk premium for some historic period to estimate the expected equity risk premium implies that investors expect history to guide the future. Shiller (2000) and Booth (1999) both argue that there is no support for the proposition that future changes in the equity risk premium can be predicted by current values. As Fernandez (2013) point out in his survey of the literature, historic equity risk premiums are constructed with survivor bias. Because observed returns are conditional on survivorship of the stocks included in the stock index over time (Brown, Goetzmann and Ross, 1995), historic equity risk premiums overstate expected equity risk premiums.

Asset pricing models, such as the Capital Asset Pricing Model, are used to determine a “required equity risk premium”, which following the terminology of Fernandez (2013), is equal to the incremental return above the rate on risk free assets that investors require in order to be compensated for the risk of holding a diversified stock portfolio. This is a key concept since it is used in financial theory to describe the required rate of return for a company or the required rate of return required for capital budgeting purposes. As Fernandez (2013) points out, a drop in the required rate on equity will not be reflected in the historic equity risk premium because a reduction in the required rate of return on equity for a company will increase the stock price and result in higher historic equity risk premiums.

Mehra and Prescott (1985) coined the term “the equity risk puzzle” to describe the problem of reconciling a model of the pricing of assets by representative consumers with data on the historic equity risk premium. As Fernandez (2013) notes in his literature review, researchers are still trying to disentangle the equity risk puzzle.

The difficulty of using historic equity risk premiums, such as data from Morningstar (2013), is that historic equity risk premiums are ex-post returns. However, investment decisions and valuations require the use of ex-ante measures of the equity risk premium.

Data and Methodology

The data used in this paper was obtained from Morningstar (2013). The return series in Morningstar (2013) are available in both monthly and annual periods. We used monthly return series to construct measures of the equity risk premium, which is a historic equity risk premium measure. The data covered the period from January 1926 through December 2012. We conducted tests on the monthly arithmetic risk premium:

$r_{mt} - r_{ft}$

where: r_{mt} is the monthly return on large company stocks in month t and

r_{ft} is the return on risk free assets in month t.

Arithmetic return series were constructed using the following alternative measures of the risk free assets:

- Total return on Treasury Bills;
- Income return on intermediate term Treasury bonds; and
- Income return on long term Treasury bonds.

Note that this paper follows Morningstar (2013) in using the income returns of intermediate and long term Treasuries instead of total return. Total return is made up of (Morningstar, 2013):

- **Income return**, the portion of the total return that results from coupon payments;
- **Capital appreciation return,** the portion of the total return that results from the price change of the instrument; and
- **Reinvestment return,** which is the return on a given period's investment that results from reinvestment in the same asset class in a subsequent period.

Of these components of total return, only income return can be viewed as a proxy for a risk free investment. The capital appreciation return results from the change in the price of a bond over some time period. Bond prices are generally reacting to changes in yields to maturity, which are not anticipated.[2] The reinvestment return is subject to reinvestment risk that results from the likelihood that when reinvestment of future coupon payments occurs, the reinvestment will not occur at the interests that were prevailing when the bond was initially purchased.

We also follow Morningstar (2013) in using the total return on Treasury bills. The rationale is based on standard financial theory that suggests that the reinvestment risk and fluctuations in Treasury bill prices associated with unanticipated changes in yields are minimal, as compared to intermediate and long term Treasuries.

In addition to conducting tests of stationarity on arithmetic monthly equity risk premiums, we conducted tests on geometric monthly equity risk premiums where the geometric monthly equity risk premium is defined as:

$(r_{mt} - r_{ft})/(1+ r_{ft})$

Table 1 displays the sample means and standard deviations for each of the monthly arithmetic and geometric equity risk premium measures. There is substantial volatility for each of these series relative to their sample means. The means for each series range from 0.5 percent to 0.6 percent while the standard deviation for each series is approximately 5.8 percent.

We define an economic time series as stationary if after an exogenous shock is applied to the series, it reverts to its mean. As a practical matter, most economic time series are not stationary because they grow over time (Kennedy, 2008).

The Augmented Dickey-Fuller (ADF) test involves the assumption that a particular time series y follows an AR(p) process. The ADF test regression is then:

$$\Delta y_t = \alpha y_{t-1} + x_t'\delta + \beta_1 \Delta y_{t-1} + \beta_2 \Delta y_{t-2} + \cdots + \beta_p \Delta y_{t-p} + \gamma_t \qquad (1)$$

[2] See Morningstar (2013) p. 55.

where x_t are exogenous variables.

Under ADF, a t-test is conducted of the null hypothesis that $\alpha=0$, the time series y has a unit root and is nonstationary versus the alternate hypothesis that $\alpha< 0$, the time series is stationary. In our specifications of the ADF test equation, we used a constant as our exogenous variable. This follows the test strategy suggested by Elder and Kenney (2001) when analyzing an interest rate series, which we have applied to analyzing a return series. Rather than arbitrarily choosing the length of the lag p, we have used the Modified Akaike's Information Criterion to select p with the maximum number of lags set at 12.[3]

The Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test is an alternative test of whether the equity risk premium is stationary. The KPSS statistic is based on the residuals of a regression of y_t on the exogenous variables x_t . Again, we used a constant as our exogenous variable. In the case of the KPSS test, the null hypothesis is that the time series is stationary and the alternative hypothesis is that the time series is nonstationary.

Results

Figures 1-6 display graphs of each of the annual equity risk premium measures over time. Visual inspection of these graphs does not reveal any obvious pattern of the mean shifting over time.

Table 2 summarizes the results of the ADF tests on each of the annual equity risk premium measures. As shown in Table 2, the null hypothesis of a unit root was strongly rejected for each of the equity premium measures. The results of these tests were not affected when the equity premium was calculated as a geometric return rather than an arithmetic return.

[3] This follows the strategy of Brooks (2008) of setting the maximum number of lags at 12 when working with monthly financial data. In all of our ADR tests on formulation of the equity risk premiums, the chosen value of p was 5.

Table 3 displays the results of the KPSS tests conducted on each of the annual equity risk premiums. As shown in Table 3, for each of the annual equity risk premium measures, the null hypothesis of stationarity could not be rejected at the five or ten percent significance level. The statistical inference was again not affected when the equity premium was computed as a geometric return.

Conclusions

Based upon the analysis of the full sample of monthly arithmetic and geometric equity risk premiums from January 1926 to December 2012, each of the measures of the equity risk premium examined are stationary based upon the ADF and KPSS tests. This suggests that historical equity risk premiums are stationary.

However, for the purpose of business valuations and the derivation of risk-adjusted discount rates, ex-ante measures of equity risk premiums are needed in order to determine the expected or required rates of return on equity. Ex-post measures of the equity risk premium, while stationary from a time series standpoint, are not necessarily consistent with ex-ante measures.

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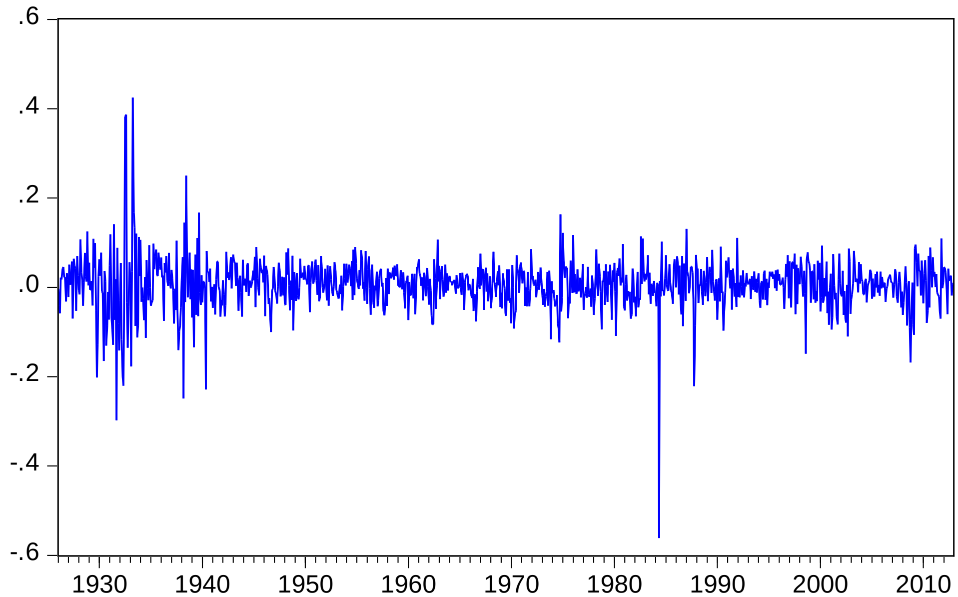
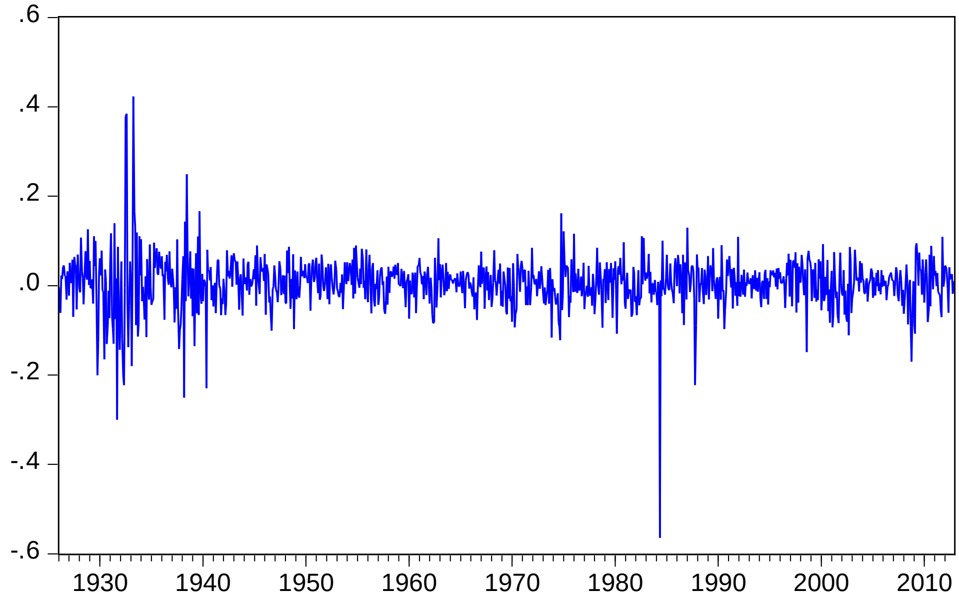
Table 1. **Sample Statistics for Monthly and Geometric Equity Risk Premiums**

	Monthly Arithmetic Equity Risk Measure Using:		
	Treasury Bill Total Return	**Intermediate Term Treasury Bond Income Return**	**Long Term Treasury Bond Income Return**
Mean	0.0059	0.0050	0.0045
Standard Deviation	0.0579	0.0579	0.0579
	Geometric Equity Risk Measure Using:		
	Treasury Bill Total Return	**Intermediate Term Treasury Bond Income Return**	**Long Term Treasury Bond Income Return**
Mean	0.0059	0.0049	0.0045
Standard Deviation	0.0578	0.0577	0.0576

<u>Figure 1</u>. Monthly Arithmetic Equity Risk Premiums Calculated Using Total Return on Treasury Bills



<u>Figure 2</u>. Monthly Arithmetic Equity Risk Premiums Calculated Using Income Returns on Intermediate Term Treasury Bonds



<u>Figure 3</u>. Monthly Arithmetic Equity Risk Premiums Calculated Using Income Return on Long Term Treasury Bonds

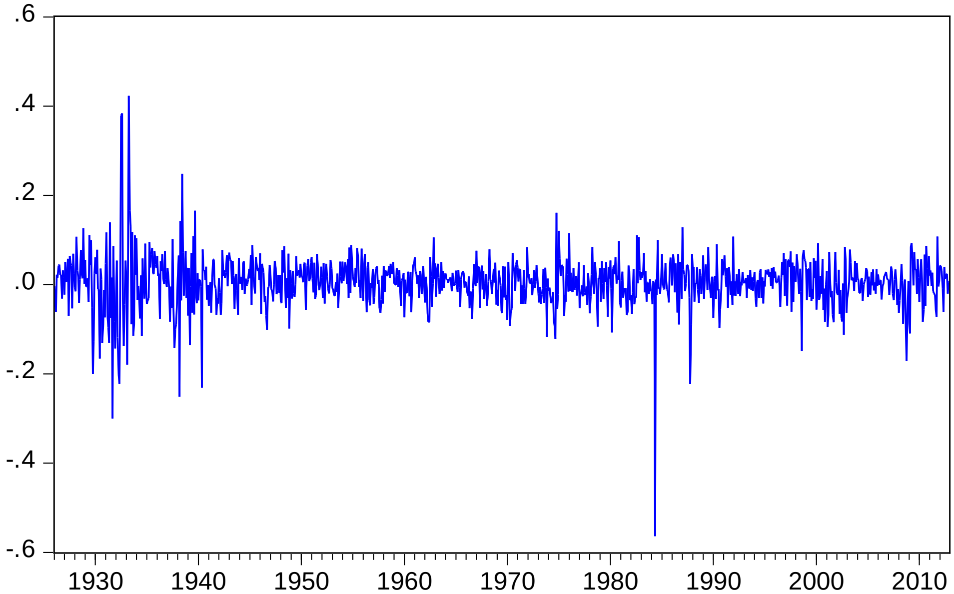
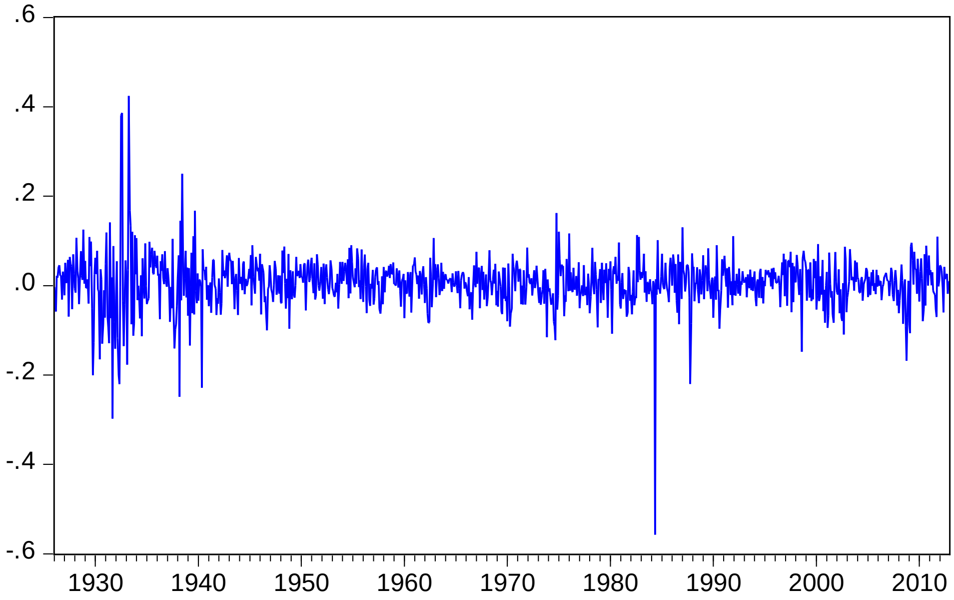
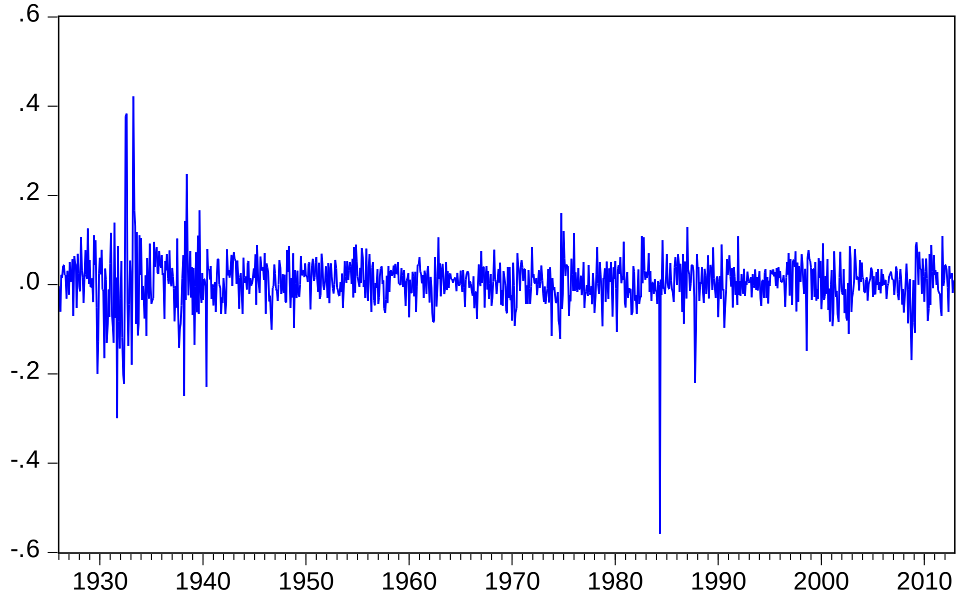


Figure 4. Monthly Geometric Equity Risk Premiums Calculated Using Total Return on Treasury Bills



<u>Figure 5.</u> Monthly Geometric Equity Risk Premiums Calculated Using Income Returns on Intermediate Term Treasury Bonds



<u>Figure 6.</u> Monthly Geometric Equity Risk Premiums Calculated Using Income Returns on Long Term Treasury Bonds

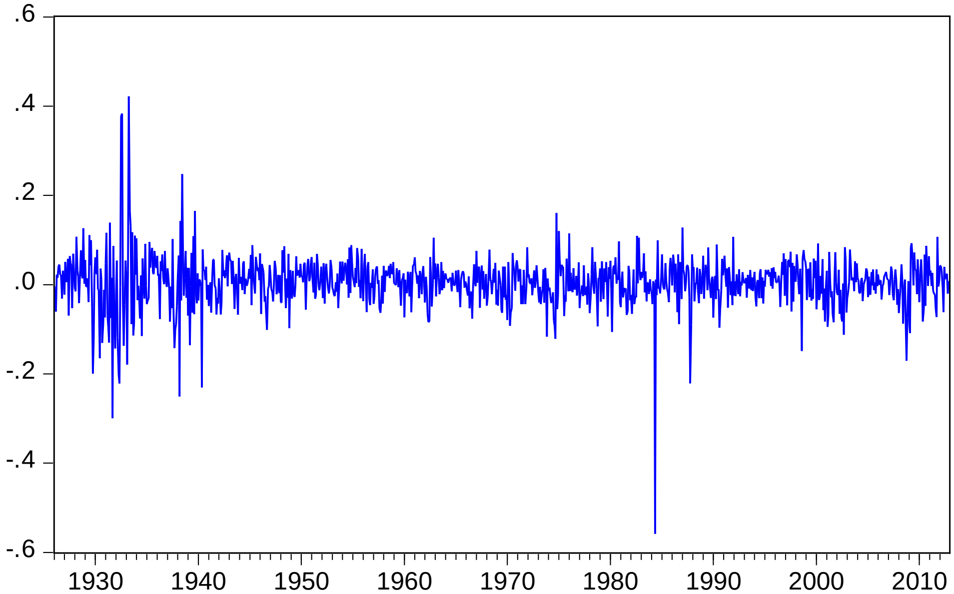


Table 2. Augmented Dickey Fuller Tests of Equity Risk Premiums

	Monthly Arithmetic Equity Risk Measure Using:		
	Treasury Bill Total Return	**Intermediate Term Treasury Bond Income Return**	**Long Term Treasury Bond Income Return**
Augmented Dickey Fuller Lagged Coefficient	- 0.9826	- 0.9829	- 0.9844
Standard Error	0.0732	0.0733	0.0733
T-Statistic	-13.4206	-13.4165	-13.4304
Significance Level	0.0000	0.0000	0.0000
	Geometric Equity Risk Measure Using:		
	Treasury Bill Total Return	**Intermediate Term Treasury Bond Income Return**	**Long Term Treasury Bond Income Return**
Augmented Dickey Fuller Lagged Coefficient	- 0.9827	- 0.9828	- 0.9845
Standard Error	0.0732	0.0733	0.0733
T-Statistic	-13.4237	-13.4180	-13.4331
Significance Level	0.0000	0.0000	0.0000

Table 3. Kwiatkowski, Phillips, Schmidt and Shin (KPSS) Tests of Equity Risk Premiums

	Monthly Arithmetic Equity Risk Measure Using:		
	Treasury Bill Total Return	**Intermediate Term Treasury Bond Income Return**	**Long Term Treasury Bond Income Return**
KPSS Test Statistic	0.14161	0.14987	0.16089
Critical value for rejecting null hypothesis of stationarity at 10% significance level	0.34700	0.34700	0.34700
	Monthly Geometric Equity Risk Measure Using:		
	Treasury Bill Total Return	**Intermediate Term Treasury Bond Income Return**	**Long Term Treasury Bond Income Return**
KPSS Test Statistic	0.14205	0.15066	0.16168
Critical value for rejecting null hypothesis of stationarity at 10% significance level	0.34700	0.34700	0.34700