Energy Supply Interruption, Climate Change, and Water Conservation

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Abstract

In some areas, agriculture that depends on irrigation from groundwater dominates both peak period energy use and consumption of water. Energy is a key input for pumping water from aquifers. This linkage means that public policies and contract terms designed for either factor may affect the use of the other factor. Even though the link between energy and water in groundwater-fed irrigation has been recognized, no rigorous economic study of interactions between these factor markets has appeared in the economics literature. We look in particular at the effects on groundwater use of energy supply interruptions. We analyze the intra-seasonal irrigation decisions of individual agricultural producers facing stochastic energy supply interruption and rainfall using stochastic dynamic programming. We find that agricultural producers should increase the amount of water applied per irrigation opportunity to hedge against the risk of future energy outages. Further, numerical analysis calibrated to intensive irrigation in Nebraska, USA, where groundwater use is regulated, shows that random energy supply interruption could increase the total amount of water consumption despite reduced opportunities for irrigation. This finding indicates that energy supply interruptions could have adverse effects on groundwater use, potentially complicating the management of water resources. We also find that changes in the distribution of rainfall, as may accompany climate change, exacerbate the effects of energy supply interruptions on total groundwater consumption.

JEL codes: Q51, Q15

1 Introduction

This study examines relationships between climate, water use, and energy use at the groundwater-fed irrigated agriculture. Irrigation is used worldwide to buffer agricultural production against climate variability and accounts for more than half of all freshwater consumption worldwide (Naylor, 1996; Shah et al., 2003). Groundwater is a major source of water for irrigation; this practice has contributed substantially to aquifer depletion worldwide (Malik, 2002; Siebert et al., 2010; Scanlon et al., 2012; Wada et al., 2012). Pumping water out of aquifers uses energy. Energy use for groundwater-fed irrigation varies over space and time because crop water demands and weather vary over space and time. Thus, there is a strong climate-energy-water nexus (Scott, 2011, 2013). These linkages mean that public policies and contract terms designed for either factor - energy or water - may affect the use of the other factor. Energy supply capacity and water resources are both constrained in many areas so understanding the interactions between these markets is important.

The need to manage peak energy loads has led many power suppliers to introduce offer interruptible energy supply contracts. Conversely, in the developing world, energy supply is often unreliable due to problems with infrastructure or management (Malik, 2002). In both cases, energy supplies are stochastic. This stochasticity may influence the water use choices of irrigators with implications for water resource management. These relationships may also be affected by changes in rainfall and temperature patterns.

There are three main research questions considered in this paper. First, how would profit-maximizing farmers adjust their irrigation strategy under energy supply interruption as opposed to the case of no control? Second, does energy supply interruption increase or decrease total groundwater consumption? Finally, how does climate variation affect groundwater consumption under energy supply interruption?

We develop and solve a stochastic dynamic programming problem in which producers decide when and how heavily to irrigate throughout a crop growth season. The model uses a probabilistic interruptible energy supply and stochastic rainfall. Analytical optimality conditions show that increasing the likelihood of energy supply interruption has an ambiguous impact on the total amount of irrigation. However, numerical examples calibrated to conditions in the High Plains, USA, suggest that total firm-level water demand is likely to increase as a consequence of energy supply interruption

This paper is laid out as follows. Section 2 discusses the phenomenon of energy supply interruption. Section 3 reviews pertinent literature. Sections 4-6 present a formal decision model, analytical results, and numerical examples, respectively. Section 7 discusses policy implications and extensions.

2 Energy Supply Interruption

Many agricultural producers rely on public power provision for groundwater extraction from aquifers. However, in some regions, that public energy supply is interruptible. Therefore, producers may not be able to irrigate when they desire if energy supply is interrupted, intentionally or not.

In developing countries, energy supply interruption occurs often because of the inability to supply energy reliably due to multiple factors including poor facility maintenance (Dethier et al., 2011; Steinbuks and Foster, 2010). In India, one of the largest groundwater users in the world, groundwater-fed irrigation can use so much energy that supply cannot keep up with demand, causing frequent energy outages (Malik, 2002). In Punjab, India, in fact, 85% of farmers are unsatisfied with the reliability of energy supply (Perveen et al., 2012).

In developed countries, energy supply reliability is not generally a concern. However, public energy suppliers have introduced a number of demand-side management (DSM) measures to manage energy peak loads (Eto, 1996). Direct irrigation load control, which involves cuts in energy supply, is one form of DSM. Therefore, producers may be faced with energy supply interruptions meant to curtail peak energy loads. In the U.S., such programs are implemented in regions with heavy groundwater-fed irrigation such as Nebraska, Kansas, and

Idaho. Direct irrigation load control programs usually offer farmers a choice of contracts with varying levels of restrictions on the ability to irrigate, accompanied by some form of financial incentive to compensate for the increased risk in irrigation scheduling (Table 1). Energy suppliers set financial incentives generously to encourage producers to choose a contract with energy supply interruption that gives suppliers more control over the peak energy load. Consequently, in developed countries, many producers *opt* into energy supply interruption, even though the total hours with no energy supply can be significant (Table 2). In developed as well as developing countries, then, agricultural producers need to take into account the possibility of energy supply interruption when forming their irrigation strategies.

3 Literature Review

There are three main strands of literature relevant to this study. We first review the literature on energy supply reliability in developing countries and then energy DSM for developed countries. Finally, we look at the literature on the intraseasonal irrigation scheduling problem, which also forms the framework for our model.

3.1 Energy Supply Reliability

In many developing countries, energy supply interruption is common; the economics literature therefore includes numerous studies on the negative economic impact of energy outages (e.g. Steinbuks and Foster, 2010; Adenikinju, 2003; Gulyani, 1999). However, analysis of their impact on groundwater use is scant. Malik (2002) reports that agricultural producers in India, to cope with energy supply interruption, install their own electricity generators to stabilize their energy supply. Those who cannot afford such an investment extract more water when energy supply is available than they would if the energy supply were reliable (Malik, 2002). Shah et al. (2008) report that the combination of energy supply hour rationing and more reliable supply as part of the *Jyotigram* scheme led to a reduction in water consumption in Gujarat, India. However, all the existing studies provide only anecdotal evidence of the linkage between energy supply reliability and groundwater use. No rigorous economic analysis has been conducted to date.

3.2 Energy Demand-Side Management

One common DSM is time-of-use (TOU) rates, which differentiate marginal energy prices depending on time of the day. The numerous empirical studies on the impacts of energy DSM programs are primarily focused on the impact of TOU rates on residential energy consumption behavior (e.g.Aigner and Leamer, 1984; Caves et al., 1984; Howrey and Varian, 1984; Park and Acton, 1984; Hausman and Trimble, 1984). In economics, to the author's knowledge, Train and Toyama (1989) is the only study that looks at the impact of an energy DSM program in the agricultural sector. This study found empirically that energy TOU rates shift consumers' electricity use from on-peak hours to off-peak hours, contributing to a reduction in the peak load. Moreover, they find that total electricity use increased as the decrease in the price during the off-peak period was more than enough to make up for the increased price during the on-peak periods. However, they did not make the connection that the increased energy use implies greater water use. Outside of economics, Scott (2013) provides some evidence of increases in water consumption after the introduction of energy TOU rates in Mexico, though the study lacks rigorous statistical examination. In fact, there have been no rigorous economic studies, theoretical or empirical, on the implications of energy supply interruption on groundwater-fed irrigation.

3.3 Optimal Intra-Seasonal Irrigation Scheduling

The dynamic optimization problem considered in this study falls into the class of intra-seasonal water allocation (irrigation scheduling) problems. The two main strands of literature on these problems differ in their objectives. One strand pays almost no attention to economic factors such as the price of a crop or water, but only to physical aspects that maximize the crop production level (e.g. Hajilal et al., 1998). The other incorporates economic factors, focusing on profit rather than yields (e.g. Gowing and Ejieji, 2001; Paul et al., 2000; Dudley et al., 1971; Rhenals and Bras, 1981; Yaron and Dinar, 1989; McGuckin et al., 1987). Most economic studies of irrigation scheduling use a numerical stochastic dynamic programming approach since it naturally fits into a sequential irrigation decision problem with uncertainty in climatic conditions.

While a large number of studies have been published on the subject, none of them has considered the optimal irrigation strategy under electricity (water) supply interruption. Moreover, the primary objective of this study is fundamentally different from that of previous studies in that our ultimate goal is to determine the impact of energy supply interruption on the amount of irrigation, while the previous studies focused almost exclusively on profit maximizing in the irrigation strategy itself. The only exception, to the best of our knowledge, is Shani et al. (2004), where water use under profit maximization and yield maximization is contrasted. Moreover, as Shani et al. (2004) point out, most of the previous studies are somewhat ad hoc in that their objective is to find the optimal irrigation strategy for a particular case. Shani et al. (2004) use optimal control theory to find a generalizable solution to the optimal irrigation scheduling problem. They find that a turnpike strategy is optimal, in which a fixed level of soil moisture is reached as soon as possible and maintained throughout the season. They do not, however, consider cases where climate variables are random or energy supply interruption.

4 The Model

4.1 Single Period Payoff

We model crop growth as a function of soil moisture level, following previous studies on irrigation scheduling[1]. Payoff for each period is written as follows: $Y(S_t) - P_w \cdot x_t$. The

[1] While nutrients like phosphorous also determine crop growth, this study focuses on soil moisture level.

first term, $Y(S_t)$, represents incremental crop growth as a function of soil moisture level, S_t, at time t. Crop growth is assumed to be increasing with soil moisture level at a decreasing rate, $Y'(\cdot) > 0$ and $Y''(\cdot) < 0$. The crop-water function will be kept unspecified throughout the theoretical investigation[2]. As a consequence, the optimal solution concept will be quite general. Pw is the unit pumping cost of water relative to the price of the crop (numeraire) and X_t is the amount of irrigation[3]. Therefore, single period payoff is crop growth (equivalent to revenue) less pumping cost.

It is well known that the water requirement of a crop differs depending on its growing stage. In this study, however, we work with a fixed crop-water function over time because it will not change the economic insights into adaptation to random power supply interruption.

4.2 State Equation

Soil moisture level decays over time due to transpiration, evaporation, and deep percolation. Irrigation and rainfall supplement the soil moisture. This process is represented mathematically as $S_{t+1} = \alpha(\beta S_t + \theta \cdot X_t + R_t)$. The proportion of the soil moisture that remains after crop consumption (transpiration) is represented by $\beta \in [0, 1]$, which varies by type of crop. The proportion of soil moisture remaining after percolation and evaporation is represented by $\alpha \in [0, 1]$. The rate of percolation varies depending on the soil type. For example, percolation is greater with sandy soil than with silty soil. Not all water applied reaches the soil, however, due to evaporation and surface runoff. The proportion of the applied water that goes into the soil, irrigation efficiency, is represented by $\theta \in [0, 1]$. Irrigation efficiency varies depending on irrigation technology: Irrigation efficiency for center pivot irrigation is about 90% as opposed to 60% for flood irrigation. In addition to irrigation, stochastic rainfall,

[2]An analytical solution for optimal irrigation is possible for a very limited class of crop-water functions and under the assumption of non-random temperature and rainfall. While an analytical solution could make comparative statics easier to derive, these assumptions severely limit the applicability of the derived economic insights.

[3]Unit pumping cost can differ across farmers even with the same price for electricity because of differences in pumping efficiency.

represented by R_t, also adds to the soil moisture level. The distribution of rainfall is kept unspecified throughout an irrigation season, and agricultural producers are allowed to act on their subjective beliefs about rainfall events.

4.3 Agricultural Producer's Problem

The agricultural producer's problem is to maximize the expected value of profit given the state equation and random energy supply interruption. This problem is written mathematically as follows:

$$
\begin{aligned}
& \max_{x_t} \ \mathbb{E}\left[\sum_{t=1}^{T} Y(S_t) - P_w \cdot x_t\right] \\
\text{s.t} \quad & S_{t+1} = \alpha(\beta S_t + \theta \cdot x_t + R_t) \\
& x_t = 0 \quad \text{with probability} \quad \rho \in [0,1]
\end{aligned}
\tag{1}
$$

The final crop yield at the end of the production season is assumed to be the summation of incremental crop growth in each period over the production season. The additivity of crop yield is commonly used in the optimal irrigation scheduling literature (e.g. Bras and Cordova, 1981; Cai et al., 2011)[4].

The unique feature of the model is that the control variable is set at 0 for probability ρ. The probability of energy supply interruption is assumed to be fixed over the irrigation season even though it may be predicted by observing future and past climatic conditions. Nonetheless, this assumption does not interfere with deriving economic insights, and detailed updating of subjective belief should be left to a numerical analysis. The model does not have a intra-seasonal discount factor because an irrigation season is short enough that any discount should be negligible. Finite time is appropriate considering that the end period is clearly

[4]While some agronomy literature claims that additive models are unrealistic in that they do not allow for the existence of the wilting point, it is only because the additive model is written in a very restricted way in the literature. It is indeed possible to allow for the wilting point by letting $Y(S_t)$ take negative values that are large in magnitude at the lower range of soil moisture level.

defined. In this study, no limit is placed on the well yield: we assume it to be high enough that producers can apply as much water as they desire for a given time period. This assumption certainly would not hold for some wells. Agricultural producers with low well yield may not be able to irrigate as much as they desire, thus potentially limiting their capability to adapt to random energy outages. For example, in northwest Texas, the well yield of some wells were so low that in 2011 their owners could not meet crop water requirements even without energy supply interruption, resulting in severe production loss. On the other hand, most of the wells for irrigation in Nebraska still have quite a high yield (see Appendix for further detail).

Along with these assumptions, there are several minor technical assumptions that are used in our mathematical proofs. It is assumed that $\lim_{S \to 0} Y'(S) < C$ for some positive constant C (assumption 1). This assumption simply states that the marginal production of soil moisture is bounded and does not approach infinitity as the soil moisture level moves infinitesimally close to 0. The optimization problem is not well defined without this assumption, t. It is also assumed that $S_t > \alpha\beta S_t + \alpha E[R_t]$, ${}^{\forall}S_t \in R_+$ and ${}^{\forall}t$ (assumption 2). This assumption states that given the current soil moisture S_t, the expected amount of rainfall is small enough that the expected value of soil moisture in the next period is smaller than S_t if no water is applied at the current period. This assumption does not limit the practical relevance of our model because irrigation would hardly be necessary in the first place if the assumption were not satisfied.

5 Solutions and Results

In this section, we establish the first order condition of the optimal irrigation schedule problem with random energy supply interruption, find the value functions, and then derive comparative statics.

5.1 Bellman Equations

Let $V_t(S,0)$ and $V_t(S,1)$ denote the value function at t given soil moisture level S when energy is and is not available, respectively. The Bellman equations of the dynamic optimization problem for time t are then as follows:

$$\begin{aligned} V_t(S,1) =& Max_{x\in R_+}\ \ Y(S) - P_w x + \rho E\Big[V_{t+1}\Big(\alpha(\beta S + \theta x + R_t), 0\Big)\Big] \\ &+ (1-\rho)E\Big[V_{t+1}\Big(\alpha(\beta S + \theta x + R_t), 1\Big)\Big] \end{aligned} \tag{2}$$

$$\begin{aligned} V_t(S,0) =& Max_{x\in \{0\}}\ \ Y(S) - P_w x + \rho E\Big[V_{t+1}\Big(\alpha(\beta S + \theta x + R_t), 0\Big)\Big] \\ &+ (1-\rho)E\Big[V_{t+1}\Big(\alpha(\beta S + \theta x + R_t), 1\Big)\Big] \\ =& Y(S) + \rho E\Big[V_{t+1}\Big(\alpha(\beta S + R_t), 0\Big)\Big] + E\Big[(1-\rho)V_{t+1}\Big(\alpha(\beta S + R_t), 1\Big)\Big] \end{aligned} \tag{3}$$

where expectation is taken over R_t. When finding the optimal irrigation amount, producers need to consider how irrigation influences the stream of future profits as well as the immediate return. Here, the immediate return is incremental crop revenue, $Y(S)$, less pumping cost, $P_w x$. Future profits are uncertain because of the stochasticity of energy supply and rainfall. Producers transition to the state with or without energy supply in the next period, with probability ρ and $1-\rho$, respectively. The expected maximum profits from $t+1$ onwards, conditional on irrigation at t, are represented by $\rho E\Big[V_{t+1}\Big(\alpha(\beta S + \theta x + R_t), 0\Big)\Big]$ and $(1-\rho)E\Big[V_{t+1}\Big(\alpha(\beta S + \theta x + R_t), 1\Big)\Big]$, for the states with or without energy supply interruption, respectively. Thus, the summation of the two terms represents the expected future profit. When energy is not available in the current period, irrigation is necessarily zero, and thus the Bellman equation reduces to equation 3 when energy is not available in the current period.

5.2 Optimal Irrigation Strategy

Using backward induction, value functions and optimal irrigation strategies can be found sequentially from the end period to the first period (See Appendix for the proof). Along with solving the problem, the following proposition can be established that helps explain the economic meaning of the first order condition.

***Proposition 1*:** Optimal irrigation strategy has the following form for all t:

$$x_t^* = \frac{\sigma_t}{\theta} - \frac{\beta}{\theta} S_t \tag{4}$$

where σ_t is a constant that depends on all the parameters and the distributions of all the rainfall events from time t on, but does not depend on S_t.

This proposition has several important implications. When the above irrigation strategy is followed, the soil moisture level in the next period before deep percolation and evaporation takes place is σ_t if there is no rain. Therefore, σ_t can be interpreted as the minimum amount of soil moisture level guaranteed irrespective of the amount of rainfall in that period before deep percolation and evaporation loss. That is, the profit maximizing agricultural producer will irrigate just as much as necessary to fill the gap between the current and the target soil moisture (σ_t) levels, independent of the current soil moisture level.

The expected value of the soil moisture level given the current soil moisture level S_t is the following;

$$E[S_{t+1}] = E[\alpha(\beta S_t + \theta x_t^* + R_t)] = E[\alpha(\sigma_t + R_t)]$$

Since σ_t is independent of S_t, $E[S_{t+1}]$ is also independent of S_t. This further means that if the distribution of rainfall events is stationary, agricultural producers will try to keep the same level of expected soil moisture whenever they can irrigate, except for the last few periods. In the last few periods, the expected value of irrigation decreases because the soil moisture is assumed to have no value after harvest. When the rainfall variable is not stationary, the soil moisture target will vary over time. However, the expected soil moisture level will remain independent of the current soil moisture level. This leads to the following corollaries:

***Corollary 1*:** Irrigation and rainfall in the current periods have no impact on future soil moisture levels for all periods after energy supply interruption ends, except the next period.

The key insight from proposition 1 is that whenever irrigation is possible, the soil moisture level will be targeted at a fixed level that is independent of the starting soil moisture level. Therefore, irrigation and rainfall in the current period will have an effect on future soil moisture only as long as energy supply is continuously on. For example, any additional irrigation at t will have no effect on the soil moisture level at $t+2$ if irrigation is allowed at $t+1$. This corollary is important in understanding the first order condition. An analogous corollary holds for rainfall.

5.3 First Order Condition

In order to understand the first order condition, it is instructive to first look at the case with no energy supply interruption ($\rho = 0$). The first order condition at $T-k$ ($k = 1, 2, \ldots, T-1$) is the following:

$$Pw = \theta\alpha E\Big[Y'\Big(\alpha(\beta S_{T-k} + \theta x + R_{T-k})\Big)\Big] + \frac{\alpha\beta Pw}{\theta} \tag{5}$$

The left-hand side is the marginal cost of pumping. The marginal benefit of water represented on the right-hand side consists of two parts. The first term represents the marginal expected revenue due to crop growth in the next period from adding one more unit of water in the current period. The second term represents the cost savings in the next period from adding one more unit of water in the current period. The cost savings occur because an additional unit of irrigation leads to a higher soil moisture level in the next period, which in turn reduces the optimal amount of irrigation required in the next period. Agricultural producers need to be concerned about the impact of irrigation on soil moisture only in the next period, but not in future periods, which is an immediate consequence of Corollary 1.

The essential structure of the first order condition remains the same for the case with non-zero probability of energy supply interruption ($\rho > 0$). The first order condition at $T-k$ ($k = 1, 2, \dots, T-1$) is (see Appendix for derivation):

$$\begin{aligned} Pw =& \theta \sum_{i=1}^{k} \alpha^i \rho^{i-1} \beta^{i-1} E\Big[Y'\Big(\alpha^i \beta^{i-1}(\beta S_{T-k} + \theta x) + \sum_{j=1}^{i} \alpha^j \beta^{j-1} R_{T-k-j+i}\Big)\Big] \\ &+ (1-\rho)\alpha\beta \cdot Pw \left[\sum_{j=1}^{k-1} (\alpha\beta\rho)^{j-1}\right] \end{aligned} \tag{6}$$

The first term on the right-hand side of the first order condition is the summation of expected marginal crop growth (or, equivalently, revenue, because the crop is the numeraire). Each term is conditioned on *continued* power outage, adjusted for the portion of irrigation remaining in the soil moisture. For example, denoting the current period t, the second term in the summation represents marginal crop growth at $t+2$ if energy is not available at $t+1$. The third term is the expected marginal crop growth that may be realized at $t+3$ if energy was not available in the previous two consecutive periods. Following from proposition 1 and its corollary, agricultural producers need to consider the impact of irrigation only for the periods that have experienced consecutive energy outages. For example, agricultural producers do not have to account for how irrigation at time t would affect crop growth at $t+2$ if energy is available at $t+1$.

Analogously, each term in the second summation term represents the expected marginal cost savings conditioned on *sustained* power outage, adjusted for the portion of irrigation remaining in the soil moisture. Each term is multiplied by $1-\rho$ because cost savings are realized only when farmers can irrigate, which occurs with probability $1-\rho$. For example, the second term in the summation is the expected cost savings that are realized two periods later if farmers cannot irrigate in the next period, but can irrigate two periods later.

The first order condition encapsulates the economic intuition: agricultural producers need to take into account now the impact of irrigation on the soil moisture levels in future periods. This compares to the case of no energy supply interruption when producers do not need to account for the impact of current irrigation on future soil moisture because they can irrigate in any period.

5.4 Comparative Statics

5.4.1 Energy supply interruption

***Proposition 2*:** Random energy supply interruption always leads to an increase in the optimal amount of irrigation per irrigation opportunity (see Appendix for the proof):

$$\frac{\partial X_t^*}{\partial \rho} > 0$$

This finding is consistent with our expectation and quite intuitive: producers increase irrigation to hedge against the risk of not being able to irrigate in future periods. An immediate consequence of this proposition is that the optimal target soil moisture level goes up as the probability of energy supply interruption increases.

From the groundwater conservation perspective, the total amount of irrigation is more important than the amount of irrigation per irrigation opportunity, which does not necessarily lead to an increase in the total amount of irrigation because farmers have fewer opportunities to irrigate. The question is whether the increase in irrigation per irrigation opportunity is large enough to make up for the loss of irrigation opportunities.

Unfortunately, the sign of the impact cannot be determined definitively, so the effect on overall water use is an empirical question[5]. Nonetheless, it is possible to identify conditions that are more likely to lead to an increase in the total amount of irrigation: 1) the crop-growth function has a very steep slope (high marginal production of water) at the lower range of soil moisture and 2) the slope of the crop-growth function flattens very fast after

[5]The mathematical expression for the derivative of total irrigation with respect to the probability of energy supply interruption is convoluted and not presented here. Please see the Appendix for more.

the steep slope. In other words, a large loss in crop growth will be incurred when even a small amount of water is not supplied due to an energy outage. In order to avoid a potentially large loss, producers need to irrigate more when they can. A steeply declining slope of the crop-water function (rapidly declining marginal production of water) means that the optimal soil moisture level would be very close to the high slope portion of the crop-water function if it were not for energy supply interruption.

5.4.2 Rainfall

An important difference between the first order conditions with and without energy supply interruption is the role of rainfall events. For the case with no energy supply interruption, agricultural producers need to consider only the distribution of rainfall in the same period, as can be seen in equation 5. This leads to the following lemmas:

***Lemma 1*:** When energy supply interruption is not present, a change in the distribution of rainfall at t will influence decision making only in period t.

On the other hand, when the probability of energy supply interruption is nonzero, all rainfall events from the current period until the end period enter the decision making process because farmers need to take into account the expected future soil moisture levels due to potential energy outages. This leads to the following lemma:

***Lemma 2*:** A change in the distribution of rainfall in period s will influence decision making in all periods up to and including period s if energy supply interruption is possible.

Lemmas 1 and 2 immediately lead to the following proposition:

***Proposition 3*:** Information about the distribution of future rainfall events is more valuable when energy supply interruption is possible than otherwise.

These results hold for any kind of change in the distribution of rainfall in general. Now we consider specifically the implications of two aspects of the distribution of rainfall: expected rainfall ($E[R]$) and variance (assuming a mean-preserving spread).

First, it can be shown that the marginal impact of the expected rainfall at t on the optimal amount of irrigation at t is negative and of the same magnitude with or without energy supply interruption (see Appendix D):

$$\frac{\partial x_{t^*}}{\partial \gamma_t}\Big|_{\rho=0} = \frac{\partial x_{t^*}}{\partial \gamma_t}\Big|_{\rho>0} = -\frac{1}{\theta} < 0 \tag{7}$$

This implies that with or without energy supply interruption, the expected total groundwater consumption will decrease (increase) when expected rainfall increases (decreases). Second, an increase in the expected value of a rainfall event in a future period $s(> t)$ will reduce the optimal amount of irrigation when energy supply interruption is present, but has no effect otherwise (See Appendix D):

$$\frac{\partial x_{t^*}}{\partial \gamma_s}\Big|_{\rho=0} = 0 \quad \text{and} \quad \frac{\partial x_{t^*}}{\partial \gamma_s}\Big|_{\rho>0} < 0 \tag{8}$$

The insensitivity of optimal irrigation to a reduction in the expected value of future rainfall events is a direct consequence of Lemma 1. Combining the two findings results in the following proposition:

***Proposition 4*:** The optimal irrigation per irrigation opportunity is more elastic to the expected value of future rainfall events with energy supply interruption than without it.

$$\sum_{s=t}^{T} \frac{\partial x_{t^*}}{\partial \gamma_s}\Big|_{\rho=0} > \sum_{s=t}^{T} \frac{\partial x_{t^*}}{\partial \gamma_s}\Big|_{\rho>0} \tag{9}$$

For example, if the expected amount of rainfall at t and $t+1$ decreases, the increase in the optimal amount of irrigation at t is greater when energy supply interruption is present.

This, however, does not mean the expected *total* groundwater consumption experiences a larger increase in response to a decrease in the expected amount of rainfall events when there is energy supply interruption than otherwise. In fact, this cannot be signed definitively and remains an empirical question. This is analogous to the impact of energy supply interruption on the total amount of groundwater consumption, discussed above. When the probability of energy supply interruption is high, producers have fewer opportunities to irrigate.

Now we discuss how a change in uncertainty about future rainfall events influences the optimal irrigation strategy. Quite intuitively, it can be shown that a mean-preserving spread of the distribution of a rainfall event in future periods will lead to an increase in the amount of optimal irrigation per irrigation opportunity with or without energy supply interruption (see Appendix E). With a slight simplification of mathematical notation,

$$\left.\frac{\partial x_{t^*}}{\partial Var(R_t)}\right|_{\rho\geq 0} > 0 \tag{10}$$

This is because a mean-preserving spread will increase the risk of crop yield reduction and leads to an increase in total groundwater consumption, with or without energy supply interruption.

Another interesting question is whether the impact of a change in the variance of rainfall is greater when energy supply is intermittent. First, with Lemmas 1 and 2, it is straightforward to show that a mean-preserving spread of rainfall events in a future period $s(>t)$ increases the optimal amount of irrigation at t when energy supply interruption is present, but has no effect otherwise. However, one cannot determine whether the impact of a mean-preserving spread of rainfall in the current period t is greater with energy supply interruption than otherwise (see Appendix E).

As with the expected value of rainfall, it is not possible to determine analytically whether an increase in uncertainty about rainfall events leads to greater *total* groundwater consumption with energy supply interruption than otherwise. This is ultimately determined by the nature of the crop-water function and is an empirical question.

6 Numerical Examples

The theoretical analysis suggests that the impact of energy supply interruption on total irrigation, the variable of most interest, is ambiguous. We also found that it is impossible to determine whether changes in the distribution of rainfall have a greater impacts on total irrigation with interruptible energy supply than otherwise. We turn to see the signs of these effects and their magnitude in a realistic parameter space. We first describe the calibration method, discusses the numerical simulation results, and derive policy implications.

6.1 Calibration

The calibration requires, first, a physical model that relates soil moisture level to crop growth. We employ the AquaCrop model calibrated for corn production in the High Plains, USA by Foster (2013). AquaCrop is an intra-seasonal irrigation scheduling model developed by the Food and Agriculture Organization (Steduto et al., 2009; Raes et al., 2009). Soil moisture is the major determinant of crop growth, as in our study. Aquacrop has been widely used by researchers and calibrated to many agricultural regions with a wide variety of climatic conditions and crop types (e.g. Farahani et al., 2009; Araya et al., 2010; Stricevic et al., 2011; García-Vila and Fereres, 2012). The crop-water function and state equation in this study are then calibrated to the outcomes of the calibrated AquaCrop model so the resulting total irrigation falls within historically observed levels in the region. The calibrated crop-water function is shown in Figure 1.

6.2 Results

The dynamic problem is solved numerically for the rainfall pattern of 2012, the driest year in the region for the past 30 years. Analysis with the high rainfall pattern of 1992 follows for comparison.

Figure 2 shows the contour map of profit per acre with respect to pumping cost and the probability of energy supply interruption for the rainfall pattern of 2012. The profits are calculated based on a corn price of \$5 per bushel and production costs, excluding pumping, of \$3 per bushel. As can be seen in the figure, profitability is highly resilient to energy supply interruption - the contours are nearly horizontal. This is primarily because the cost of pumping is extremely low relative to the usually high corn price. Under these conditions, a relatively small financial incentive can induce farmers to adopt interruptible energy contracts. As an illustration, the profits associated with the energy contracts summarized in Table 1 are represented by red circles in the figure. Agricultural producers are likely to choose interruptible contracts, given the steep differential in profits.

Figure 3 shows the contour map of total irrigation per acre with respect to the pumping cost and the probability of energy supply interruption for 2012. It shows that energy supply interruption could indeed increase the amount of irrigation, *ceteris paribus.* For example, for a pumping cost of \$2/acre-inch, the total amount of irrigation that maximizes profits is greater by about 1.5 inches when the probability of energy outage is 0.35 than when power is not interruptible. If the energy price is lowered as a compensation for energy supply interruptibility as in load control programs, for example to \$1.5/acre-inch, optimal total irrigation would increase an additional 0.5 acre-inches.

The magnitude of the supply interruption effect for the 2012 drought is in the 5% to 10% range. Drought would also reduce surface runoff and percolation. Added groundwater withdrawals would amplify the drought-induced water resource stresses.

Figure 4 shows the contour map of total irrigation per acre with respect to the pumping cost and the probability of energy outages for the year of 1992. The total amount of irrigation is much less. Energy supply interruption still increases the total amount of irrigation, but to a much smaller degree than in 2012. That is, if climate change reduces the amount of rainfall in an irrigation season, groundwater irrigation may see a greater increase if agricultural producers are faced with power supply interruption, or vice versa.

6.3 Policy Implications

Energy supply interruption is a common element of direct load control program for peak energy load management. In agricultural areas dependent on irrigation from groundwater, reducing irrigation is critical to shave peak loads. While this clearly means that instantaneous water use would decrease, total water use in irrigation would not necessarily be reducedincrease application in off-peak periods could compensate partially or more than fully for water not applied during power interruptions. Thus, it policies and contracts applied to energy use may have consequences for water use.

In a realistic parameter space, energy supply interruption could lead to an increase in total groundwater consumption as a risk management strategy, *ceteris paribus*. Reduced unit prices of energy associated with interruptible supply contracts could further increase the total amount of irrigation. Therefore, both supply interruption and price effects could add pressure to extract groundwater. One could mitigate the price effect by financially compensating agricultural producers with a lump sum instead of lowering the marginal energy price. However, the supply interruption effect would persist.

For the cases analyzed here, profit is highly resilient to energy supply interruption. Agricultural producers would choose interruptible contracts even if the price compensation for energy were smaller than the levels actually observed. The energy price for interruptible contracts could be increased without affecting agricultural producers' choice of energy contract, thereby reducing the price effect on water us without diminishing energy suppliers' ability to control peak energy load.

When the supply interruption effect on groundwater consumption is positive, however, there is no incentive-compatible energy contract structure that would lead to lower water consumption for interruptible contracts. The unit energy price for interruptible contracts would need to be higher than that for non-interruptible contracts in order to induce lower groundwater consumption. However, in this case, agricultural producers would not opt into an interruptible contract since it would be financially disadvantageous. In order to maintain the groundwater consumption level after introduction of a load control, the unit energy price for interruptible contracts must be higher than the original unit energy price. Moreover, the new unit price for non-interruptible contract must be set even higher than that for interruptible contracts. This may be faced with strong oppositions from producers.

Although institutional contexts are very different in the developing world, energy supply reliability is an important issue for agriculture in many countries. The results in this paper about the interaction between energy supply interruption and groundwater consumption can shed some lights on its implications in the developing world as well. Our results suggest that improving energy supply reliability could lead to a reduction in total groundwater consumption. In India, for example, agricultural producers tend to be vehemently opposed to any form of increase in energy-related costs in most regions. Indeed, agricultural producers are a powerful lobby and have stalled many legislative moves to increase the price of energy (Malik, 2002; Shah et al., 2008). Therefore, even though energy pricing is widely discussed in the academic literature as a means of enhancing groundwater conservation, the feasibility of such a measure may be questionable in some regions. To make matters worse, there are so many wells that metering can be prohibitively expensive, which could make it impossible to charge agricultural producers on the basis of volume extracted. Given this background, it is particularly important to note that stabilizing energy supply could decrease the amount of groundwater use, even in the absence of the price effect. Farmers would certainly welcome an improvement in energy reliability, and the inability to meter wells has no impact on the positive effect of a stable energy supply.

In some regions, as in West Bengal, India (Mukherji et al., 2009), there is a volumetric charge for energy. In those regions, agricultural producers may accept an increase in the unit energy price if the consistency of energy supply is improved. Indeed, a recent survey conducted by the Columbia Water Center (CWC) confirmed that agricultural producers are willing to accept such a trade-off (Perveen et al., 2012). This combination of an increased unit energy price and improved energy supply reliability could provide a double dividend, with both the price effect and energy supply effect moving in favor of groundwater conservation.

In some regions, ongoing climate change is expected to create a lower amount of precipitation. It has been analytically shown that this change will lead to greater groundwater consumption. This means that energy suppliers may need to increase the frequency of energy supply interruption to gain greater control of energy peak loads, which in turn could lead to greater total groundwater consumption. Ongoing climate change, in addition to its direct impacts, may thus indirectly influence groundwater conservation via energy suppliers' adaptation.

It has been shown that the impact of energy supply interruption differs depending on the distribution of rainfall. Introducing energy supply interruption (improving energy supply reliability) can be more harmful (beneficial) to groundwater conservation in regions where rainfall is scant and variable. If climate change advances such that rainfall decreases and becomes less evenly distributed over a production season, the amount of irrigation is likely to increase, with or without energy supply interruption. However, the magnitude of the increase can be even greater if agricultural producers are faced with energy supply interruption. That is, energy supply interruption could exacerbate the impact of climate change on groundwater resources.

7 Conclusions

The ever-increasing and inextricably linked demand for energy and water warrants informed joint energy-water management. Energy supply interruption is common in some regions where the economy relies heavily on groundwater-fed irrigation. Nonetheless, the impact of energy supply interruption on groundwater consumption has not been studied rigorously in the economics literature. This study develops economic insights into agricultural producers' adaptation to energy supply interruption and its potential consequences on groundwater conservation. We modeled intraseasonal irrigation decisions by individual agricultural producers with random energy supply interruption and stochastic rainfall and analyzed them both analytically and numerically by means of a stochastic dynamic programming approach.

It has been shown that agricultural producers increase the amount of irrigation per irrigation opportunity in response to interruptibility in energy supply. This makes good economic sense because this strategy allows agricultural producers to hedge against the risk of not being able to irrigate in future periods. The impact of energy supply interruption on expected total groundwater consumption, however, is ambiguous and thus an empirical question. Numerical examples showed that energy supply interruption could lead to an increase in total groundwater consumption in a realistic parameter space. This led to differing policy implications for developed and developing countries, where energy supply interruptions occur for different reasons. However, in both contexts it is clear that energy and groundwater management should not be considered independently.

Rainfall is one of the most significant determinants of irrigation. A decrease in the expected value of and a mean-preserving spread of rainfall always leads to a higher optimal irrigation amount, with or without energy supply interruption, leading to an increase in total groundwater conservation. Energy supply interruption interacts with climate in an interesting way to affect groundwater consumption behavior. When there is no energy supply interruption, only the distribution of rainfall events in the current period influences producers' irrigation decisions. On the other hand, the distribution of rainfall events in all the future periods affects irrigation decisions in the current period with a declining degree of influence. Therefore, information about the distribution of future rainfall events is more valuable when there is energy supply interruption than otherwise. Moreover, numerical results suggest that it is possible that the distribution of rainfall events has a greater impact on total groundwater consumption with energy supply interruption changes than without.

There are multiple interesting extensions for future work. First, econometric analysis using energy and water consumption data under energy supply interruption will prove useful to gaining further insights into the sign and magnitude of the comparative statics results derived analytically in this study. Econometric analysis on observed data has an advantage over numerical analysis in that no a priori assumptions or parametrization of the model is necessary. Second, aside from the irrigation load control program, another popular way to manage peak energy load is time-of-use rates. Since under this program agricultural producers can irrigate whenever they want but at a higher price during on-peak hours, they are faced with quite a different incentive structure than under irrigation load control. It would be interesting to examine the implications of time-of-use rates on groundwater irrigation behavior and then compare them with those of irrigation load control. Finally, it may be interesting to examine whether providing future weather information would reduce groundwater consumption when energy supply interruption is possible.

Table 1: **Price structure of contracts in the Custer Public Power District**

Power Control Type	Unit Price in 2007 (per kwh)	Unit Price in 2008 (per kwh)
No Control	$0.0875	$0.1100
1 day Control	$0.0490	$0.0565
2 day Control	$0.0470	$0.0500
3 day Control	$0.0450	$0.0470
Full Control	$0.0430	$0.0460

Note: This table shows, as an example, the electricity price by energy contract type for the Custer Public Power District (PPD) in Nebraska. The unit price of electricity for no power control is about twice as high as that for interruptible contracts. Other public power districts lower the unit cost of horsepower of a water pump, which is a sunk cost once a irrigation season starts.

Table 2: **Control Hours in the Custer Public Power District**

Power Contract Type	Interrupted Hours in 2002 (hours)	Interrupted Hours in 2008 (hou
No Interruption	0	0
1 day Interruption	55	20
2 day Interruption	110	40
3 day Interruption	170	60
Daily Interruption	320	100

Note: Irrigation load control takes place primarily in July and August when the demand for electricity is the greatest. As an example, Table 2 shows the total hours of load control implemented by the Custer PPD.

Figure 1: Daily Crop-Water Function for Corn

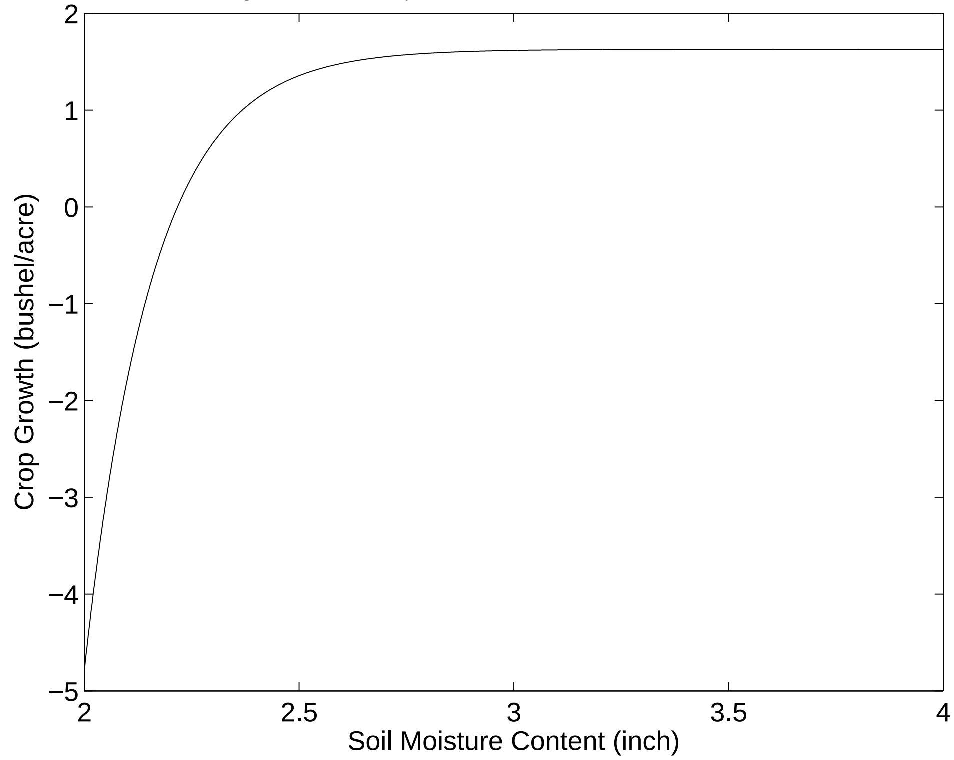


Figure 2: Contour Map of Expected Profit ($/acre) for 2012 (Dry Year)

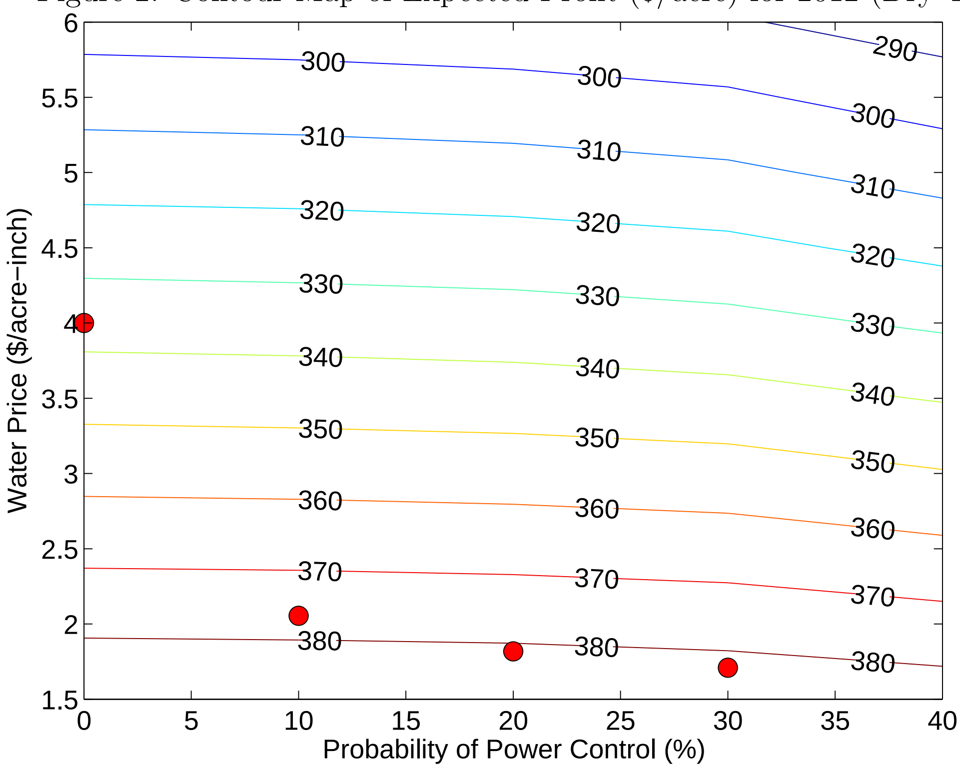


Figure 3: Contour Map of Expected Irrigation (inch) for 2012 (Dry Year)

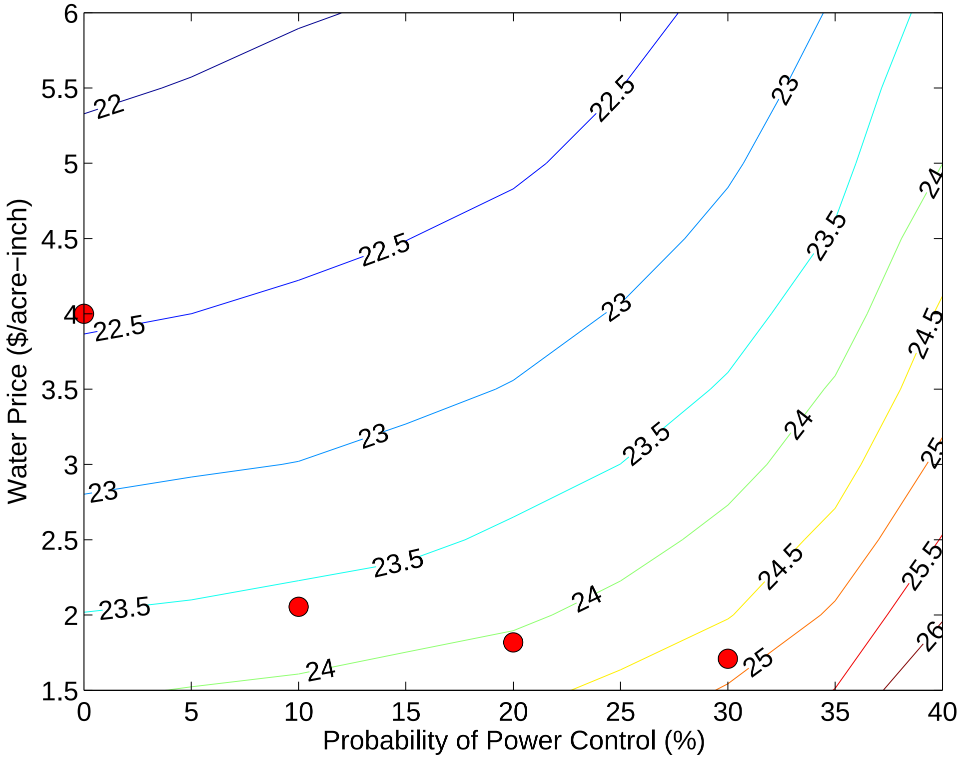
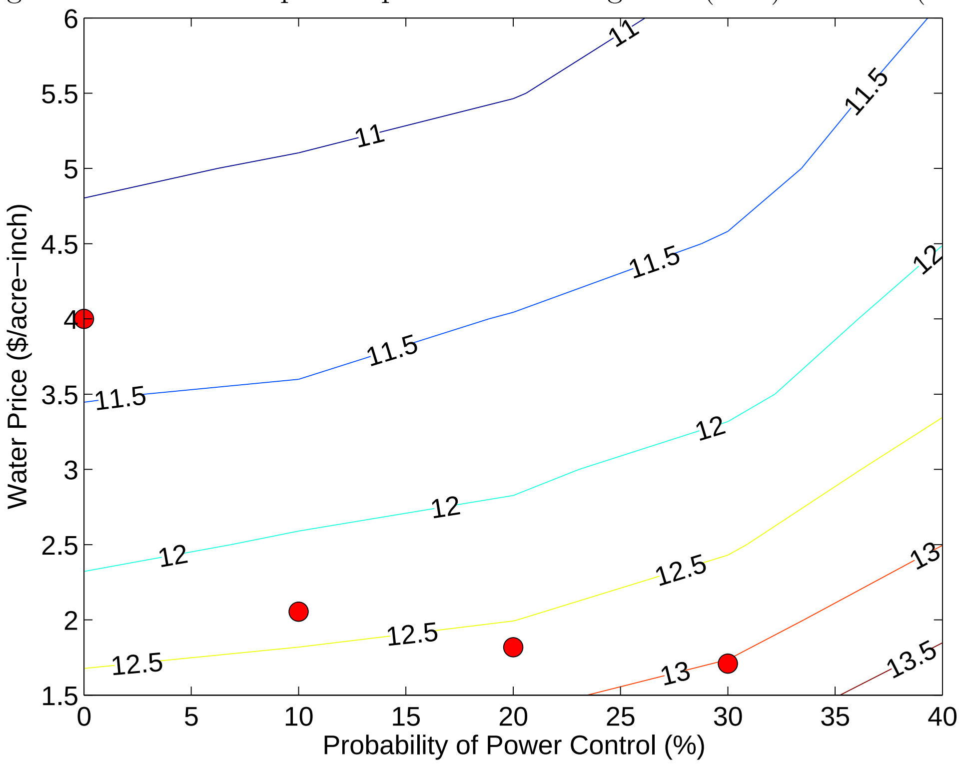


Figure 4: Contour Map of Expected Total Irrigation (inch) for 1992 (Wet Year))



Appendices

A Value Functions, First Order Conditions, and Proof of Proposition 1

Derivation of the value functions, first order conditions, and proof of proposition 1 come simultaneously. We do so by means of mathematical induction starting from $k = 1$ for x^*_{T-k} ($k = 2, \ldots, T-1$). At the last period ($t = T$), the optimal irrigation amount $x^*_T = 0$ because irrigation at time T does not contribute to crop growth at time T. Therefore, $x^*_T = 0$ irrespective of the power control status. Given x^*_T, the value functions $V_T(S_T, 0)$ and $V_T(S_T, 1)$ are written as follows,

$$V_T(S_T, 0) = V_T(S_T, 1) = Y(S_T) \tag{11}$$

(i) $k = 1$

Now, given $V_T(S_T, 0)$ and $V_T(S_T, 1)$,

$$\begin{aligned} V_{T-1}(S_{T-1}, 0) =& Y(S_{T-1}) + \rho E_{T-1}\Big[V_T\Big(\alpha(\beta S_{T-1} + R_{T-1}), 0\Big)\Big] \\ &+ (1-\rho)E_{T-1}\Big[V_T\Big(\alpha(\beta S_{T-1} + R_{T-1}), 1\Big)\Big] \\ =& Y(S_{T-1}) + E_{T-1}\Big[Y\Big(\alpha(\beta S_{T-1} + R_{T-1})\Big)\Big] \end{aligned} \tag{12}$$

where the expectation is taken over R_{T-1}. The optimal irrigation problem given S_{T-1} can be formulated as follows,

$$\begin{aligned} \text{Max}\{x\} \;\; & Y(S_{T-1}) - Pw \cdot x + \rho E_{T-1}\Big[V_T\Big(\alpha(\beta S_{T-1} + \theta x + R_{T-1}), 0\Big)\Big] \\ &+ (1-\rho)E_{T-1}\Big[V_T\Big(\alpha(\beta S_{T-1} + \theta x + R_{T-1}), 1\Big)\Big] \end{aligned} \tag{13}$$

First order condition of the problem is as follows,

$$
\begin{aligned}
Pw =& \frac{\partial}{\partial x}\Big\{\rho E_{T-1}\Big[V_T\Big(\alpha(\beta S_{T-1}+\theta x+R_{T-1}),0\Big)\Big]\Big\}\\
&+\frac{\partial}{\partial x}\Big\{(1-\rho)E_{T-1}\Big[V_T(\alpha(\beta S_{T-1}+\theta x+R_{T-1}),1\Big)\Big]\Big\}\\
\Rightarrow Pw =& \alpha\theta\rho E_{T-1}\Big[Y'\Big(\alpha(\beta S_{T-1}+\theta x+R_{T-1})\Big)\Big]\\
&+(1-\rho)\alpha\theta E_{T-1}\Big[Y'\Big(\alpha(\beta S_{T-1}+\theta x+R_{T-1})\Big)\Big]\\
\Rightarrow Pw =& \alpha\theta E_{T-1}\Big[Y'\Big(\alpha(\beta S_{T-1}+\theta x+R_{T-1})\Big)\Big] \qquad (14)
\end{aligned}
$$

By dropping the subscript for R for the sake of conciseness, the expectation part of the right hand side of equation (14) can be written as a function of $y=\beta S_{T-1}+\theta x$ as follows,

$$
G_1(y)=E\Big[Y'\Big(\alpha(y+R)\Big)\Big]=\int_0^\infty Y'\Big(\alpha(y+R)\Big)\cdot f(R)\,dR \qquad (15)
$$

By Assumption 1,

$$
G_1'(y) =\alpha\int_0^\infty Y''\Big(\alpha(y+R)\Big)\cdot f(R)\,dR<0
$$

Thus, $G_1(y)$ is invertible and one can solve equation (14) in terms of y,

$$
\begin{aligned}
y =& G_1^{-1}(\frac{P_w}{\theta}) \qquad (16)\\
\Rightarrow x =& \frac{\sigma_1}{\theta}-\frac{\beta}{\theta}S_{T-1} \qquad (17)
\end{aligned}
$$

where σ_1 depends on all the parameters and the distribution of R_{T-1}, but is independent of S_{T-1}. Therefore, the optimal irrigation strategy is linear in S_{T-1}.

Now, given x^*_{T-1},

$$\begin{aligned}
V_{T-1}(S_{T-1},1) =& Y(S_{T-1}) - Pwx^*_{T-1} + \rho E\Big[V_T\Big(\alpha(\beta S_{T-1} + \theta x^*_{T-1} + R_{T-1}), 0\Big)\Big] \\
&+ (1-\rho)E\Big[V_T(\alpha\Big(\beta S_{T-1} + \theta x^*_{T-1} + R_{T-1}), 1\Big)\Big] \\
V_{T-1}(S_{T-1},1) =& Y(S_{T-1}) + \frac{\beta Pw}{\theta} \cdot S_{T-1} - \frac{\sigma_1 Pw}{\theta} + \rho E\Big[V_T\Big(\alpha(\sigma_1 + R_{T-1}), 0\Big)\Big] \\
&+ (1-\rho)E\Big[V_T\Big(\alpha(\sigma_1 + R_{T-1}), 1\Big)\Big] \\
=& Y(S_{T-1}) + \frac{\beta Pw}{\theta} \cdot S_{T-1} + B_1
\end{aligned} \tag{18}$$

where $B_1 = \rho E\Big[V_T\Big(\alpha(\sigma_1 + R_{T-1}), 0\Big)\Big] + (1-\rho)E\Big[V_T\Big(\alpha(\sigma_1 + R_{T-1}), 1\Big)\Big] - \frac{\sigma_1 Pw}{\theta}$, which is independent of S_{T-1}.

(i) $k = 2$

Given $V_{T-1}(S_{T-1}, 0)$ and $V_{T-1}(S_{T-1}, 1)$, we can solve the case for $T-2$. First,

$$\begin{aligned}
V_{T-2}(S_{T-2},0) =& Y(S_{T-2}) + \rho E_{T-2}\Big[V_{T-1}\Big(\alpha(\beta S_{T-2} + R_{T-2}), 0\Big)\Big] \\
&+ (1-\rho)E_{T-2}\Big[V_{T-1}\Big(\alpha(\beta S_{T-2} + R_{T-2}), 1\Big)\Big] \\
=& Y(S_{T-2}) + \rho E_{T-2}\Big[Y\Big(\alpha(\beta S_{T-2} + R_{T-2})\Big)\Big] \\
&+ \rho E_{T-2}E_{T-1}\Big[Y\Big(\alpha^2\beta(\beta S_{T-2} + R_{T-2}) + \alpha R_{T-1}\Big)\Big] \\
&+ (1-\rho)E_{T-2}\Big[Y\Big(\alpha(\beta S_{T-2} + R_{T-2})\Big)\Big] \\
&+ \frac{(1-\rho)\beta Pw}{\theta}\Big(\alpha\beta S_{T-2} + \alpha E[R_{T-2}]\Big) + (1-\rho)B_1 \\
=& Y(S_{T-2}) + E_{T-2}\Big[Y\Big(\alpha(\beta S_{T-2} + R_{T-2})\Big)\Big] \\
&+ \rho E_{T-2}E_{T-1}\Big[Y(\alpha^2\beta(S_{T-2} + R_{T-2}) + \alpha R_{T-1})\Big] \\
&+ \frac{(1-\rho)\alpha\beta^2 Pw}{\theta} S_{T-2} + A_2
\end{aligned} \tag{19}$$

where $A_2 = (1-\rho)B_1 + \frac{(1-\rho)\beta Pw}{\theta} \cdot E[R_{T-2}]$, which is independent of S_{T-2}.

The optimal irrigation problem given S_{T-2} can be formulated as follows,

$$\begin{aligned} \text{Max}\{x\}\ \ & Y(S_{T-2}) - Pw\cdot x + \rho E_{T-2}\Big[V_{T-1}\Big(\alpha(\beta S_{T-2}+\theta x + R_{T-2}), 0\Big)\Big] \\ & + (1-\rho)E_{T-2}\Big[V_{T-1}\Big(\alpha(\beta S_{T-2}+\theta x + R_{T-2}), 1\Big)\Big] \end{aligned} \tag{20}$$

First order condition of the problem is as follows,

$$\begin{aligned} Pw =& \theta\rho\cdot\frac{\partial}{\partial x}\Big\{E_{T-2}\Big[V_{T-1}\Big(\alpha(\beta S_{T-2}+\theta x + R_{T-2}), 0\Big)\Big]\Big\} \\ & + (1-\rho)\theta\cdot\frac{\partial}{\partial x}\Big\{E_{T-2}\Big[V_{T-1}\Big(\alpha(\beta S_{T-2}+\theta x + R_{T-2}), 1\Big)\Big]\Big\} \\ \Rightarrow Pw =& \alpha\theta\rho E_{T-2}\Big[Y'\Big(\alpha(\beta S_{T-2}+\theta x + R_{T-2})\Big)\Big] \\ & + \alpha^2\beta\theta\rho E_{T-2}E_{T-1}\Big[Y'\Big(\alpha^2\beta(\beta S_{T-2}+\theta x + R_{T-2}) + \alpha R_{T-1}\Big)\Big] \\ & + \alpha\theta(1-\rho)E_{T-2}\Big[Y'\Big(\alpha(\beta S_{T-2}+\theta x + R_{T-2})\Big)\Big] + \theta(1-\rho)\frac{\alpha\beta Pw}{\theta} \\ \Rightarrow \gamma_2 =& \alpha\theta E_{T-2}\Big[Y'\Big(\alpha(\beta S_{T-2}+\theta x + R_{T-2})\Big)\Big] \\ & + \alpha^2\beta\theta\rho E_{T-2}E_{T-1}\Big[Y'\Big(\alpha^2\beta(\beta S_{T-2}+\theta x + R_{T-2}) + \alpha R_{T-1}\Big)\Big] \end{aligned} \tag{21}$$

where $\gamma_2 = (1-\alpha\beta(1-\rho))Pw$, a constant. Analogous with the previous argument made for $k=1$, the right hand side is monotonic decreasing with respect to $\beta S_{T-2}+\theta x$. Therefore, there exists a function $G_2(\cdot)$ such that $\gamma_2 = G_2(\beta S_{T-2}+\theta x)$ and $G_2'(\cdot) < 0$. Thus,

$$\begin{aligned} G_2^{-1}(\gamma_2) &= \beta S_{T-2} + \theta x \\ \Rightarrow x &= \frac{\sigma_2}{\theta} - \frac{\beta}{\theta}S_{T-2} \end{aligned} \tag{22}$$

where $\sigma_2 = G_2^{-1}(\gamma_2)$. Now, given x^*_{T-2}

$$\begin{aligned} V_{T-2}(S_{T-2},1) =& Y(S_{T-2}) + \frac{\beta P w}{\theta}\cdot S_{T-2} - \frac{\sigma_2 P w}{\theta} + \rho E_{T-2}\Big[V_{T-1}\Big(\alpha(\sigma_2 + R_{T-2}),0\Big)\Big] \\ &+(1-\rho)E_{T-2}\Big[V_{T-1}\Big(\alpha(\sigma_2 + R_{T-2}),1\Big)\Big] \\ =& Y(S_{T-2}) + \frac{\beta P w}{\theta}\cdot S_{T-2} + B_2 \end{aligned} \tag{23}$$

where $B_2 = \rho E_{T-2}\Big[V_{T-1}\Big(\alpha(\sigma_2 + R_{T-2}),0\Big)\Big] + (1-\rho)E_{T-2}\Big[V_{T-1}\Big(\alpha(\sigma_2+R_{T-2}),1\Big)\Big] - \frac{\sigma_2 P w}{\theta}$, which is independent of S_{T-2}.

(ii) $k = n$ **and** $k = n+1$

Now suppose our claim hold when $k = n$ $(n \geq 3)$ and we can write x^*_{T-n}, $V_{T-n}(S_{T-n},0)$, and $V_{T-n}(S_{T-n},1)$ as follows for some constants σ_n, A_n, and B_n,

$$X^*_{T-n} = \sigma_n - \frac{\beta}{\theta}S_{T-n} \tag{24}$$

$$V_{T-n}(S_{T-n},1) = Y(S_{T-n}) + \frac{\beta P w}{\theta}\cdot S_{T-n} + B_n \tag{25}$$

$$\begin{aligned} V_{T-n}(S_{T-n},0) =& Y(S_{T-n}) + E\Big[Y\Big(\alpha(\beta S_{T-n} + R_{T-n})\Big)\Big] + \\ & \rho E\Big[Y\Big(\alpha^2\beta(\beta S_{T-n} + R_{T-n}) + \alpha R_{T-n+1}\Big)\Big] + \cdots + \\ & \rho^{n-1}E\Big[Y\Big(\alpha^n\beta^{n-1}(\beta S_{T-n} + R_{T-n}) + \alpha^{n-1}\beta^{n-2}R_{T-n+1} \\ & \quad + \cdots + \alpha^2\beta R_{T-2} + \alpha R_{T-1}\Big)\Big] + \\ & + \left[\sum_{j=2}^{n}(\alpha\beta\rho)^{j-2}\right]\cdot(1-\rho)\frac{\alpha\beta^2 P w}{\theta}\cdot S_{T-n} + A_n \end{aligned} \tag{26}$$

Now, given $V_{T-n}(S_{T-n},0)$ and $V_{T-n}(S_{T-n},1)$, $V_{T-n-1}(S_{T-n-1},0)$ is as follows,

$$\begin{aligned}
V_{T-n-1}(S_{T-n-1},0) =& Y(S_{T-n-1}) + \rho E\Big[V_{T-n}\Big(\alpha(\beta S_{T-n-1}+R_{T-n-1}),0\Big)\Big] \\
&+ (1-\rho)E\Big[V_{T-n}\Big(\alpha(\beta S_{T-n-1}+R_{T-n-1}),1\Big)\Big] \\
=& Y(S_{T-n-1}) + \rho E\Big[Y\Big(\alpha(\beta S_{T-n-1}+R_{T-n-1})\Big)\Big] + \\
&\rho E\Big[Y\Big(\alpha^2\beta(\beta S_{T-n-1}+R_{T-n-1})+\alpha R_{T-n})\Big)\Big] + \cdots + \\
&\rho^n E\Big[Y\Big(\alpha^{n+1}\beta^n(\beta S_{T-n-1}+R_{T-n-1})+\alpha^n\beta^{n-1}R_{T-n} \\
&\qquad + \cdots + \alpha^2\beta R_{T-2}+\alpha R_{T-1}\Big)\Big] + \\
&\left[\sum_{j=2}^{n}(\alpha\beta\rho)^{j-2}\right]\cdot(1-\rho)\frac{\alpha\beta^2 Pw}{\theta}\cdot\rho E[\alpha(\beta S_{T-n-1}+R_{T-n-1})] + \\
&(1-\rho)E\Big[Y\Big(\alpha(\beta S_{T-n-1}+R_{T-n-1})\Big)\Big] + \\
&(1-\rho)\frac{\beta Pw}{\theta}E[\alpha(\beta S_{T-n-1}+R_{T-n-1})] + \rho A_n + (1-\rho)B_n \\
=& Y(S_{T-n-1}) + \rho E\Big[Y\Big(\alpha(\beta S_{T-n-1}+R_{T-n-1})\Big)\Big] + \\
&\rho^2 E\Big[Y\Big(\alpha^2\beta(\beta S_{T-n-1}+R_{T-n-1})+\alpha R_{T-n})\Big)\Big] + \cdots + \\
&\rho^n E\Big[Y\Big(\alpha^{n+1}\beta^n(\beta S_{T-n-1}+R_{T-n-1})+\alpha^n\beta^{n-1}R_{T-n} \\
&\qquad + \cdots + \alpha^2\beta R_{T-2}+\alpha R_{T-1}\Big)\Big] + \\
&\left[\sum_{j=2}^{n+1}(\alpha\beta\rho)^{j-2}\right]\cdot(1-\rho)\frac{\alpha\beta^2 Pw}{\theta}\cdot S_{T-n-1} + A_{n+1}
\end{aligned} \tag{27}$$

where $A_{n+1} = \left[\sum_{j=2}^{n+1}(\alpha\beta\rho)^{j-2}\right]\cdot(1-\rho)\frac{\alpha\beta^2 Pw}{\theta}\rho E[R_{T-n-1}] + \rho A_n + (1-\rho)B_n$, which is independent of S_{T-n-1}. Therefore, our claim for the form of $V_{T-n-1}(S_{T-n-1},0)$ holds for $n+1$. Now, given $V_{T-n}(S_{T-n},0)$ and $V_{T-n}(S_{T-n},1)$, the optimal irrigation problem given

S_{T-n-1} can be formulated as follows,

$$\begin{aligned}
\text{Max}\{x\} \;\; & Y(S_{T-n-1}) - Pw \cdot x + \rho E\Big[V_{T-n}\Big(\alpha(\beta S_{T-n-1} + \theta x + R_{T-n-1}), 0\Big)\Big] \\
& + (1-\rho) E\Big[V_{T-n}\Big(\alpha(\beta S_{T-n-1} + \theta x + R_{T-n-1}), 1\Big)\Big]
\end{aligned} \tag{28}$$

The first order condition is as follows,

$$\begin{aligned}
Pw =& \rho \cdot \frac{\partial}{\partial x}\Big\{E_{T-n-1}\Big[V_{T-n}\Big(\alpha(\beta S_{T-n-1} + \theta x + R_{T-n-1}), 0\Big)\Big]\Big\} \\
& + (1-\rho) \cdot \frac{\partial}{\partial x}\Big\{E_{T-n-1}\Big[V_{T-n}\Big(\alpha(\beta S_{T-n-1} + \theta x + R_{T-n-1}), 1\Big)\Big]\Big\} \\
\Rightarrow Pw =& \theta\alpha\rho E\Big[Y'\Big(\alpha(\beta S_{T-n-1} + \theta x + R_{T-n-1})\Big)\Big] + \\
& \theta\alpha^2\beta\rho E\Big[Y'\Big(\alpha^2\beta \cdot (\beta S_{T-n-1} + \theta x + R_{T-n-1}) + \alpha R_{T-n}\Big)\Big] + \cdots + \\
& \theta\alpha^{n+1}\beta^n\rho^n E\Big[Y'\Big(\alpha^{n+1}\beta^n(\beta S_{T-n-1} + \theta x + R_{T-n-1}) + \alpha^n\beta^{n-1}R_{T-n} \\
& \qquad + \cdots + \alpha^2\beta R_{T-2} + \alpha R_{T-1}\Big)\Big] + \\
& \theta\rho\left[\sum_{j=2}^{n}(\alpha\beta\rho)^{j-2}\right] \cdot (1-\rho)\frac{\alpha\beta^2 Pw}{\theta} \cdot \alpha + \\
& \theta\alpha(1-\rho)E\Big[Y'\Big(\alpha(\beta S_{T-n-1} + R_{T-n-1})\Big)\Big] + \theta(1-\rho)\frac{\alpha\beta Pw}{\theta} \\
\Rightarrow Pw =& \theta\alpha E\Big[Y'\Big(\alpha(\beta S_{T-n-1} + \theta x + R_{T-n-1})\Big)\Big] + \\
& \theta\alpha^2\beta\rho E\Big[Y'\Big(\alpha^2\beta \cdot (\beta S_{T-n-1} + \theta x + R_{T-n-1}) + \alpha R_{T-n}\Big)\Big] + \cdots + \\
& \theta\alpha^{n+1}\beta^n\rho^n E\Big[Y'\Big(\alpha^{n+1}\beta^n(\beta S_{T-n-1} + \theta x + R_{T-n-1}) + \alpha^n\beta^{n-1}R_{T-n} \\
& \qquad + \cdots + \alpha^2\beta R_{T-2} + \alpha R_{T-1}\Big)\Big] + \\
& \theta\left[\sum_{j=2}^{n+1}(\alpha\beta\rho)^{j-2}\right] \cdot (1-\rho)\frac{\alpha\beta Pw}{\theta} \\
=& \theta\sum_{i=1}^{k}\alpha^i\rho^{i-1}\beta^{i-1}E\Big[Y'\Big(\alpha^i\beta^{i-1}(\beta S_{T-k} + \theta x) + \sum_{j=1}^{i}\alpha^j\beta^{j-1}R_{T-k-j+i}\Big)\Big] \\
& + (1-\rho)\alpha\beta \cdot Pw\left[\sum_{j=1}^{k-1}(\alpha\beta\rho)^{j-1}\right]
\end{aligned} \tag{29}$$

Note that each component of the right hand side is monotonic decreasing in $\beta S_{T-n-1} + \theta x$ except the last term, which is a constant. Thus, there exists a function $G_{n+1}(\cdot)$ with $G'_{n+1}(\cdot) < 0$ such that,

$$Pw\left\{1 - \left[\sum_{j=2}^{n+1}(\alpha\beta\rho)^{j-2}\right]\cdot(1-\rho)\alpha\beta\right\} = G_{n+1}(\beta S_{T-n-1} + \theta x) \tag{30}$$

Since $G_{n+1}(\cdot)$ is invertible,

$$\begin{aligned}\beta S_{T-n-1} + \theta x =& G_{n+1}^{-1}\left(Pw\left\{1 - \left[\sum_{j=2}^{n+1}(\alpha\beta\rho)^{j-2}\right]\cdot(1-\rho)\alpha\beta\right\}\right)\\ \Rightarrow x =& \frac{\sigma_{n+1}}{\theta} - \frac{\beta}{\theta}S_{T-n-1}\end{aligned} \tag{31}$$

where $\sigma_{n+1} = G_{n+1}^{-1}\left(Pw\left\{1 - \left[\sum_{j=2}^{n+1}(\alpha\beta\rho)^{j-2}\right]\cdot(1-\rho)\alpha\beta\right\}\right)$. Therefore, the optimal irrigation is linear in S_{T-n-1} as claimed.

Finally, $V_{T-n-1}(S_{T-n-1}, 0)$ can be written as follows,

$$\begin{aligned}V_{T-n-1}(S_{T-n-1}, 1) =& Y(S_{T-n-1}) - \frac{\sigma_{n+1}Pw}{\theta} + \frac{\beta Pw}{\theta}\cdot S_{T-n-1} +\\ & \rho E\Big[V_{T-n}\Big(\alpha(\sigma_{n+1} + R_{T-n-1}), 0\Big)\Big] +\\ & (1-\rho)E\Big[V_{T-n}\Big(\alpha(\sigma_{n+1} + R_{T-n-1}), 1\Big)\Big]\\ =& Y(S_{T-n-1}) + \frac{\beta Pw}{\theta}\cdot S_{T-n-1} + B_{n+1}\end{aligned} \tag{32}$$

where $B_{n+1} = \rho E\Big[V_{T-n}\Big(\alpha(\sigma_{n+1} + R_{T-n-1}), 0\Big)\Big] + (1-\rho)E\Big[V_{T-n}\Big(\alpha(\sigma_{n+1} + R_{T-n-1}), 1\Big)\Big] - \frac{\sigma_{n+1}Pw}{\theta}$, which is independent of S_{T-n-1}. Therefore, our claim for the form of $V_{T-n-1}(S_{T-n-1}, 0)$ holds too.

B Proof of Proposition 2

Let $\hat{S}_{T-k+n}$ denote the soil moisture level at time $T-k+n$ when x^*_{T-k} is applied at $T-k$ and power control is continuously on for all the periods until $T-k+n$[6]. Substituting in the optimal amount of irrigation x^*, the first order condition at the equilibrium (for sufficiently large k) writes as follows,

$$\begin{aligned} Pw(\frac{1-\alpha\beta}{1-\alpha\beta\rho}) =& \theta E[\alpha Y'(\hat{S}_{T-k+1})] + \theta\rho E[\alpha^2\beta Y'(\hat{S}_{T-k+2})] + \cdots + \\ & \theta\rho^{k-1}E[\alpha^k\beta^{k-1}Y'(\hat{S}_T)] \end{aligned} \tag{33}$$

Now, differentiating both sides with respect to ρ,

$$\begin{aligned} Pw\alpha\beta\frac{1-\alpha\beta}{(1-\alpha\beta\rho)^2} =& \theta^2\Bigg\{\alpha^2 E[Y''(\hat{S}_{T-k+1})] + \rho(\alpha^2\beta)^2 E[Y''(\hat{S}_{T-k+2})] \\ & + \cdots + \rho^{k-1}(\alpha^k\beta^{k-1})^2 E[Y''(\hat{S}_T)]\Bigg\}\frac{\partial x^*}{\partial\rho} \\ & + \theta\Bigg\{\alpha^2\beta E[Y'(\hat{S}_{T-k+2})] + 2\alpha^3\beta^2\rho E[Y'(\hat{S}_{T-k+3})] \\ & + \cdots + (k-1)\alpha^k\beta^{k-1}\rho^{k-2}E[Y'(\hat{S}_T)]\Bigg\} \end{aligned} \tag{34}$$

Now, we denote the two components of the right hand side of equation (34) as follows,

$$\begin{aligned} A =& \theta^2\Bigg\{\alpha^2 E[Y''(\hat{S}_{T-k+1})] + \rho(\alpha^2\beta)^2 E[Y''(\hat{S}_{T-k+2})] \\ & + \cdots + \rho^{k-1}(\alpha^k\beta^{k-1})^2 E[Y''(\hat{S}_T)]\Bigg\}\frac{\partial x^*}{\partial\rho} \end{aligned}$$

[6]For example, $\hat{S}_{T-k+1} = \alpha(\sigma + R_{T-k})$, $\hat{S}_{T-k+2} = \alpha^2\beta(\sigma + R_{T-k}) + \alpha R_{T-k+1}$, and $\hat{S}_{T-k+3} = \alpha^3\beta^2(\sigma + R_{T-k}) + \alpha^2\beta R_{T-k+1} + \alpha\beta R_{T-k+2}$

$$
\begin{aligned}
B =&\theta\Bigg\{\alpha^2\beta E[Y'(\hat{S}_{T-k+2})] + 2\alpha^3\beta^2\rho E[Y'(\hat{S}_{T-k+3})] \\
&+ \cdots + (k-1)\alpha^k\beta^{k-1}\rho^{k-2}E[Y'(\hat{S}_T)]\Bigg\}
\end{aligned}
$$

Then, equation (34) writes as follows,

$$
\begin{aligned}
Pw\alpha\beta\frac{1-\alpha\beta}{(1-\alpha\beta\rho)}\cdot\frac{\alpha\beta}{1-\alpha\beta\rho} &= A + B \\
Pw\alpha\beta\frac{1-\alpha\beta}{(1-\alpha\beta\rho)}\cdot\alpha\beta &= (1-\alpha\beta\rho)A + (1-\alpha\beta\rho)B \qquad (35)
\end{aligned}
$$

By substituting equation (33) into the left hand side of the above equation, it writes as follows,

$$
\begin{aligned}
(\alpha\beta\rho - 1)A =& -\theta\Bigg\{\alpha^2\beta E[Y'(\hat{S}_{T-k+1})] + \alpha^3\beta^2\rho E[Y'(\hat{S}_{T-k+2})] \\
&+ \cdots + \alpha^k\beta^{k-1}\rho^{k-2}E[Y'(\hat{S}_{T-k+k})]\Bigg\} + (1-\alpha\beta\rho)B \\
=& -\theta\Bigg\{\alpha^2\beta E[Y'(\hat{S}_{T-k+1})] + \alpha^3\beta^2\rho E[Y'(\hat{S}_{T-k+2})] \\
&+ \cdots + \alpha^k\beta^{k-1}\rho^{k-2}E[Y'(\hat{S}_{T-k+k})]\Bigg\} \\
&+ \theta\Bigg\{\alpha^2\beta E[Y'(\hat{S}_{T-k+2})] + 2\alpha^3\beta^2\rho E[Y'(\hat{S}_{T-k+3})] \\
&+ \cdots + (k-1)\alpha^k\beta^{k-1}\rho^{k-2}E[Y'(\hat{S}_T)]\Bigg\} \\
&- \alpha\beta\rho\theta\Bigg\{\alpha^2\beta E[Y'(\hat{S}_{T-k+2})] + 2\alpha^3\beta^2\rho E[Y'(\hat{S}_{T-k+3})] \\
&+ \cdots + (k-1)\alpha^k\beta^{k-1}\rho^{k-2}E[Y'(\hat{S}_T)]\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
(\alpha\beta\rho - 1)A = & -\theta\Bigg\{\alpha^2\beta E[Y'(\hat{S}_{T-k+1})] + 2\alpha^3\beta^2\rho E[Y'(\hat{S}_{T-k+2})] \\
& \qquad + \cdots + (k-1)\alpha^k\beta^{k-1}\rho^{k-2}E[Y'(\hat{S}_{T-1})] + k\alpha^{k+1}\beta^k\rho^{k-1}E[Y'(\hat{S}_T)]\Bigg\} \\
& + \theta\Bigg\{\alpha^2\beta E[Y'(\hat{S}_{T-k+2})] + 2\alpha^3\beta^2\rho E[Y'(\hat{S}_{T-k+3})] \\
& \qquad + \cdots + (k-1)\alpha^k\beta^{k-1}\rho^{k-2}E[Y'(\hat{S}_T)]\Bigg\} \\
= & \theta\Bigg\{\alpha^2\beta\Big(E[Y'(\hat{S}_{T-k+2})] - E[Y'(\hat{S}_{T-k+1})]\Big) \\
& + 2\alpha^3\beta^2\rho\Big(E[Y'(\hat{S}_{T-k+3})] - E[Y'(\hat{S}_{T-k+2})]\Big) \\
& + \cdots + (k-1)\alpha^k\beta^{k-1}\rho^{k-2}\Big(E[Y'(\hat{S}_T)] - E[Y'(\hat{S}_{T-1})]\Big)\Bigg\} \\
& - k\alpha^{k+1}\beta^k\rho^{k-1}E[Y'(\hat{S}_T)]
\end{aligned}
\tag{36}
$$

Note that for $E_{T-k+m+1}[Y'(\hat{S}_{T-k+m})]$ in the above expression, expectation is taken over $R_{T-k}, R_{T-k+1}, \ldots, R_{T-k+m}$. Therefore, by the law of iterated expectation, for any m $(= 1, 2, \ldots, k)$,

$$
\begin{aligned}
& E[Y'(\hat{S}_{T-k+m+1})] - E[Y'(\hat{S}_{T-k+m})] \\
= & E_{T-k} \cdots E_{T-k+m}\Bigg[E_{T-k+m+1}\Big[Y'(\hat{S}_{T-k+m+1}) - Y'(\hat{S}_{T-k+m})\Big| R_{T-k}, \ldots, R_{T-k+m}\Big]\Bigg] \\
= & E_{T-k} \cdots E_{T-k+m}\Bigg[E_{T-k+m+1}\Big[Y'(\hat{S}_{T-k+m+1})\Big| R_{T-k}, \ldots, R_{T-k+m}\Big] - Y'(\hat{S}_{T-k+m})\Bigg]
\end{aligned}
$$

Now, by assumption 1, $E[\hat{S}_{T-k+m+1}|R_{T-k}, \ldots, R_{T-k+m}] = \alpha\beta\hat{S}_{T-k+m} + \alpha E[R_{T-k+m}] < \hat{S}_{T-k+m}$ and thus, $Y'(E[\hat{S}_{T-k+m+1}|R_{T-k}, \ldots, R_{T-k+m}]) > Y'(\hat{S}_{T-k+m})$. By Jensen's inequality, $E_{T-k+m+1}[Y'(\hat{S}_{T-k+m+1})|R_{T-k}, \ldots, R_{T-k+m}] > Y'(E[\hat{S}_{T-k+m+1}|R_{T-k}, \ldots, R_{T-k+m}])$. Therefore, $E_{T-k+m+1}[Y'(\hat{S}_{T-k+m+1})|R_{T-k}, \ldots, R_{T-k+m}] > Y'(\hat{S}_{T-k+m})$. Therefore, every single component of the first term of the right hand side of the above equation is positive. Finally, by assumption 2, as k goes sufficiently large, the last term goes infinitesimally close to 0.

Now, $\alpha\beta\rho - 1$ in the left hand side of equation 36 is always negative. Furthermore, the coefficient on $\partial x^*/\partial\rho$ in A is always negative. Therefore, $\partial x^*/\partial\rho > 0$.

C Expected Total Irrigation

The ex ante expected value of irrigation per irrigation opportunity at t conditional on no energy supply interruption is as follows:

$$E[x_t^*] = \frac{\sigma_t}{\theta} - \frac{\beta}{\theta}E[S_t] \tag{37}$$

Thus, the expected value of irrigation, which will be denoted as $E[IR_t]$ is,

$$E[IR_t] = (1-\rho)E[x_t^*] = (1-\rho)(\frac{\sigma_t}{\theta} - \frac{\beta}{\theta}E[S_t]) \tag{38}$$

Differentiating both sides with respect to ρ,

$$\frac{\partial E[IR_t]}{\partial \rho} = \frac{1}{\theta}\Big\{(1-\rho)(\frac{\partial \sigma_t}{\partial \rho} - \beta \cdot \frac{\partial E[S_t]}{\partial \rho}) - (\sigma_t - \beta E[S_t])\Big\}$$

We know that $\dfrac{\partial \sigma_t}{\partial \rho} > 0$ (see Appendix B) and also that $\sigma_t > 0$. Now, $E[S_t]$, the ex ante (before irrigation season starts) expected soil moisture level at t is mathematically represented as follows:

$$\begin{aligned} E[S_t] = (1-\rho)\alpha \sum_{i=1}^{t-1} \rho^{i-1}\Big\{(\alpha\beta)^{i-1}\sigma_{t-i} + \sum_{j=1}^{i}(\alpha\beta)^{j-1}E[R_{t-j}]\Big\} \\ + \rho^{t-1}\Big\{(\alpha\beta)^{t-1}S_1 + \alpha\sum_{j=1}^{t-1}(\alpha\beta)^{j-1}E[R_{t-j}]\Big\} > 0 \end{aligned} \tag{39}$$

where S_1 is the starting soil moisture level. Differentiating with respect to ρ,

$$\begin{aligned}\frac{\partial E[S_t]}{\partial \rho} =& -\alpha \sum_{i=1}^{t-1} \rho^{i-1} \Big\{ (\alpha\beta)^{i-1} \sigma_{t-i} + \sum_{j=1}^{i} (\alpha\beta)^{j-1} E[R_{t-j}] \Big\} \\ & + (1-\rho)\alpha \Big[\sum_{i=1}^{t-1} (i-1)\rho^{i-2} \Big\{ (\alpha\beta)^{i-1} \sigma_{t-i} + \sum_{j=1}^{i} (\alpha\beta)^{j-1} E[R_{t-j}] \Big\} \Big] \\ & + (1-\rho)\alpha \Big[\sum_{i=1}^{t-1} \rho^{i-1} (\alpha\beta)^{i-1} \frac{\partial \sigma_{t-i}}{\partial \rho} \Big] + (t-1)\rho^{t-2} \Big\{ (\alpha\beta)^{t-1} S_1 + \alpha \sum_{j=1}^{t-1} (\alpha\beta)^{j-1} E[R_{t-j}] \Big\}\end{aligned} \tag{40}$$

The sign of $\frac{\partial E[S_t]}{\partial \rho}$ is ambiguous. The sign of $\frac{\partial E[IR_t]}{\partial \rho}$ is ambiguous as well.

D Comparative Statics on Expected Value of Rainfall

First, it is shown that a reduction in the expected value of rainfall in the future leads to the higher optimal irrigation. In general, any random variable can be decomposed into a fixed part and a random part. Now, define γ_t and ε_t such that $R_t = \gamma_t + \varepsilon_t$ and $\gamma_t = E[R_t]$ ($E[\varepsilon_t] = 0$). Using this notation, the first order condition at $T-k$ can be written as follows:

$$\begin{aligned}Pw =& \theta \sum_{i=1}^{k} \alpha^i \rho^{i-1} \beta^{i-1} E\Big[Y' \Big(\alpha^i \beta^{i-1} \sigma_{T-k} + \sum_{j=1}^{i} \alpha^j \beta^{j-1} (\gamma_{T-k-j+i} + \varepsilon_{T-k-j+i}) \Big) \Big] \\ & + (1-\rho)\alpha\beta \cdot Pw \left[\sum_{j=1}^{k-1} (\alpha\beta\rho)^{j-1} \right]\end{aligned} \tag{41}$$

Differentiating both sides with respect to γ_{T-m} ($1 \le m \le k$),

$$\begin{aligned}0 =& \sum_{i=k-m+1}^{k} \alpha^i \rho^{i-1} \beta^{i-1} E\Big[Y'' \Big(\alpha^i \beta^{i-1} \sigma_{T-k} + \sum_{j=1}^{i} \alpha^j \beta^{j-1} (\gamma_{T-k-j+i} + \varepsilon_{T-k-j+i}) \Big) \Big] \\ & \cdot (\alpha^i \beta^{i-1} \cdot \frac{\partial \sigma_{T-k}}{\partial \gamma_{T-m}} + \alpha^{i-k+m} \beta^{i-1-k+m})\end{aligned} \tag{42}$$

Now, denoting $\alpha^i \rho^{i-1} \beta^{i-1} E\Big[Y''\Big(\alpha^i \beta^{i-1} \sigma_{T-k} + \sum_{j=1}^{i} \alpha^j \beta^{j-1} (\gamma_{T-k-j+i} + \varepsilon_{T-k-j+i})\Big)\Big]$ as η_i,

$$\frac{\partial \sigma_{T-k}}{\partial \gamma_{T-m}} = -\frac{\sum_{i=k-m+1}^{k} \eta_i \alpha^i \beta^{i-1}}{\sum_{i=k-m+1}^{k} \eta_i \alpha^{i-k+m} \beta^{i-1-k+m}} < 0 \tag{43}$$

Therefore, reduction in the mean of rainfall in any of the current and future periods will reduce the optimal amount of irrigation.

Now, it is shown that proposition 3 holds. When $\rho = 0$, $\frac{\partial \sigma_{T-k}}{\partial \gamma_m} = 0$ for $m = 1, 2, \ldots, k-1$. This immediately indicates that

$$\frac{\partial \sigma_{T-k}}{\partial \gamma_{T-m}}\Big|_{\rho=0} > \frac{\partial \sigma_{T-k}}{\partial \gamma_{T-m}}\Big|_{\rho>0} \tag{44}$$

However, when $m = k$,

$$\frac{\partial \sigma_{T-k}}{\partial \gamma_{T-m}}\Big|_{\rho>0} = -\frac{\sum_{i=1}^{k} \eta_i \alpha^i \beta^{i-1}}{\sum_{i=1}^{k} \eta_i \alpha^i \beta^{i-1}} = -1 = -\frac{\eta_1 \alpha}{\eta_1 \alpha} = \frac{\partial \sigma_{T-k}}{\partial \gamma_m}\Big|_{\rho=0} \tag{45}$$

This means that the change in the expected value of the rainfall that follows immediately after the current irrigation period has the impact of the same magnitude on the optimal irrigation.

E Uncertainty in Rainfall

First we prove that mean preserving spread of rainfall will unambiguously lead to an increase in the amount of optimal irrigation per irrigation opportunity. The first order condition at

$T - k$ is as follows,

$$\begin{aligned} Pw =& \theta \sum_{i=1}^{k} \alpha^i \rho^{i-1} \beta^{i-1} E\Big[Y'\Big(\alpha^i \beta^{i-1} \sigma_{T-k} + \sum_{j=1}^{i} \alpha^j \beta^{j-1} (R_{T-k-j+i})\Big)\Big] \\ &+ (1-\rho)\alpha\beta \cdot Pw \left[\sum_{j=1}^{k-1} (\alpha\beta\rho)^{j-1}\right] \end{aligned} \tag{46}$$

Now, consider two distribution functions of R_{T-m} $(1 \leq m \leq k)$, $F_{T-m}(R)$ and $G_{T-m}(R)$, where $G_{T-m}(R)$ is a mean preserving spread of $F_{T-m}(R)$. It is a well known result that $F_{T-m}(R)$ second-order stochastically dominates $G_{T-m}(R)$: that is, $\int_{-\infty}^{\infty} u(R) dF_{T-m}(R) > \int_{-\infty}^{\infty} u(R) dG_{T-m}(R)$ when $u(R)$ is concave. Here, the function of interest, $Y'(\cdot)$, is convex in R and $G_{T-m}(R)$ second-order stochastically dominates $F_{T-m}(R)$, instead. Therefore, a mean preserving spread of the distribution of R_{T-m} results in a rightward shift of the right-hand side of the first order condition. Moreover, the left-hand side of the first order condition remains unchanged. As a consequence, it increases the optimal amount of irrigation per irrigation opportunity. Therefore, irrespective of the existence of power control, an increase in uncertainty about rainfall will increase the optimal amount of irrigation per irrigation opportunity.

Now, Lemmas 1 and 2 immediately suggest that the impact of a mean preserving spread of the distribution of R_{T-m} on x_{T-k} $(m < k)$ is greater when energy supply interruption is present. When $m = k$, it is much more complicated. We know that when there is no power control $(\rho = 0)$, the optimal target soil moisture level, σ_t, satisfies the following first order condition:

$$\begin{aligned} Pw(1 - \alpha\beta^2) =& \theta\alpha E\Big[Y'\Big(\alpha(\sigma_{T-k} + R_{T-k})\Big)\Big] \\ =& \theta\alpha \int_{R_+} Y'\Big(\alpha\sigma_{T-k} + \alpha r\Big) f(r) dr \end{aligned} \tag{47}$$

Consider a infinitesimally small perturbation $\varepsilon\phi(r)$ to the distribution function of R_{T-k},

$F(r)$, so the resulting distribution function $G(r) = F(r) + \varepsilon\phi(r)$ is a mean preserving spread of $F(r)$. Now the functional derivative of the integrand (denoted as J) in the above equation is as follows:

$$\frac{\delta J}{\delta F} = \alpha Y''\Big(\alpha\sigma_{T-k}(F) + \alpha r\Big) \cdot f(r) \cdot \frac{\partial \sigma_{T-k}(F)}{\partial F} - \alpha Y''\Big(\alpha\sigma_{T-k}(F) + \alpha r\Big) \tag{48}$$

Thus, the functional differential is,

$$\begin{aligned} \delta J = \int_{R_+} \frac{\delta J}{\delta F}\phi(r)dr =& \alpha \cdot \frac{\partial \sigma_{T-k}(F)}{\partial F} \int_{R_+} Y''\Big(\alpha\sigma_{T-k}(F) + \alpha r\Big) \cdot f(r) \cdot \phi(r)dr \\ &- \alpha \int_{R_+} Y''\Big(\alpha\sigma_{T-k}(F) + \alpha r\Big)\phi(r)dr \end{aligned} \tag{49}$$

Since the functional differential of the left-hand side is 0, δJ must be 0 as well. Therefore,

$$\begin{aligned} \frac{\partial \sigma_{T-k}(F)}{\partial F}\Big|_{\rho=0} =& \frac{\int_{R_+} Y''\Big(\alpha\sigma_{T-k}(F) + \alpha r\Big)\phi(r)dr}{\int_{R_+} Y''\Big(\alpha\sigma_{T-k}(F) + \alpha r\Big) \cdot f(r) \cdot \phi(r)dr} \\ >& 0 \qquad \text{(by construction of } \phi(r)) \end{aligned} \tag{50}$$

Now consider the marginal impact of the same perturbation when there is energy supply interruption. The i th term of the first summation of the first order condition is the following:

$$J_i = \alpha^i \rho^{i-1} \beta^{i-1} E\Big[Y'\Big(\alpha^i\beta^{i-1}\sigma_{T-k} + \sum_{j=1}^{i} \alpha^j\beta^{j-1}(R_{T-k-j+i})\Big)\Big] \tag{51}$$

where the expectation is taken over R_s for $s = T-k, T-k+1, \ldots, T-k+i-1$. Now, denoting the integrals of $Y'(\cdot)$ over all R_s except $s = T-k$ as $E_-[Y'(\cdot)]$, J_i can be written as follows:

$$J_i = \alpha^i \rho^{i-1} \beta^{i-1} \int_{R_+} E_-\Big[Y'\Big(\alpha^i\beta^{i-1}\sigma_{T-k} + \alpha^i\beta^{i-1}r + \sum_{j=1}^{i-1} \alpha^j\beta^{j-1}(R_{T-k-j+i})\Big)\Big] f(r)dr \tag{52}$$

Analogously with J, the functional differential of J_i with respect to $F(r)$ is the following:

$$\begin{aligned}\delta J_i =&\alpha^{2i}\rho^{i-1}\beta^{2(i-1)}\cdot\frac{\partial\sigma_{T-k}(F)}{\partial F}\int_{R_+}E_-\Big[Y''\Big(S_i+\alpha^i\beta^{i-1}r\Big)\Big]f(r)\phi(r)dr\\&-\alpha^{2i}\rho^{i-1}\beta^{2(i-1)}\cdot\int_{R_+}E_-\Big[Y''\Big((S_i+\alpha^i\beta^{i-1}r\Big)\Big]\phi(r)dr\end{aligned} \tag{53}$$

where $S_i=\alpha^i\beta^{i-1}\sigma_{T-k}(F)+\sum_{j=1}^{i-1}\alpha^j\beta^{j-1}(R_{T-k-j+i})$. Since the functional differential of the left-hand side is 0, the functional differential of the right-hand side must be 0. This condition leads to the following:

$$\begin{aligned}&\sum_{i=1}^{k}\delta J_i=0\\\Rightarrow\frac{\partial\sigma_{T-k}(F)}{\partial F}\Big|_{\rho>0}&=\frac{\sum_{i=1}^{k}\alpha^{2i}\rho^{i-1}\beta^{2(i-1)}\int_{R_+}E_-\Big[Y''\Big(S_i+\alpha^i\beta^{i-1}r\Big)\Big]\phi(r)dr}{\sum_{i=1}^{k}\alpha^{2i}\rho^{i-1}\beta^{2(i-1)}\int_{R_+}E_-\Big[Y''\Big(S_i+\alpha^i\beta^{i-1}r\Big)\Big]f(r)\phi(r)dr}\end{aligned} \tag{54}$$

The sign of $\frac{\partial\sigma_{T-k}(F)}{\partial F}\Big|_{\rho>0}-\frac{\partial\sigma_{T-k}(F)}{\partial F}\Big|_{\rho=0}$ depends crucially on the curvature of the crop-water function at which it is evaluated for each i the term, and cannot be signed definitively.

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