How Much Do Official Price Indexes Tell Us About Inflation?

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Abstract

Official price indexes, such as the CPI, are imperfect indicators of inflation calculated using *ad hoc* price formulae different from the theoretically well-founded inflation indexes favored by economists. This paper provides the first estimate of how accurately the CPI informs us about "true" inflation. We use the largest price and quantity dataset ever employed in economics to build a Törnqvist inflation index for Japan between 1989 and 2010. Our comparison of this true inflation index with the CPI indicates that the CPI bias is not constant but depends on the level of inflation. We show the informativeness of the CPI rises with inflation. When measured inflation is low (less than 2.4% per year) the CPI is a poor predictor of true inflation even over 12-month periods. Outside this range, the CPI is a much better measure of inflation. We find that the U.S. PCE Deflator methodology is superior to the Japanese CPI methodology but still exhibits substantial measurement error and biases rendering it a problematic predictor of inflation in low inflation regimes as well.

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1 Introduction

We have long known that the price indexes constructed by statistical agencies, such as the Consumer Price Index (CPI) and the Personal Consumption Expenditure (PCE) deflator, measure inflation with error. This error arises for two reasons. First, formula biases or errors appear because statistical agencies do not use the price aggregation formula dictated by theory. Second, imperfect sampling means that official price indexes are inherently stochastic. A theoretical macroeconomics literature starting with Svensson and Woodford [2003] and Aoki [2003] has noted that these stochastic measurement errors imply that one cannot assume that true inflation equals the CPI less some bias term. In general, the relationship is more complex, but what is it? This paper provides the first answer to this question by analyzing the largest dataset ever utilized in economics: 5 billion Japanese price and quantity observations collected over a 23 year period. The results are disturbing. We show that when the Japanese CPI measures inflation as low (below 2.4 percent in our baseline estimates) there is little relation between measured inflation and actual inflation. Outside of this range, measured inflation understates actual inflation changes. In other words, one can infer inflation changes from CPI changes when the CPI is high, but not when the CPI close to zero. We also show that if Japan were to shift to a methodology akin to the U.S. PCE deflator, the non-linearity would be reduced but not eliminated. This non-linear relationship between measured and actual inflation has important implications for the conduct of monetary policy in low inflation regimes.

The basic intuition is straightforward. There is little disagreement that a superlative price index, such as the Törnqvist, has the best theoretical properties of any price index we have discovered [Diewert and Nakamura, 1993]. However, it is not possible to construct this index given the limited data resources of most statistical agencies. Thus, price indexes in all countries are constructed using other functional forms. We demonstrate that the measurement error that results from using the wrong formula is not diminished by averaging over larger sets of observations and is not constant over time. The empirical macroeconomics literature has documented that the volatility of inflation rises with the level of inflation. We study the volatility of both true inflation and the CPI measurement errors with surprising results. While we confirm that the volatility of true inflation rises rapidly with its level, we find that the variance of the CPI measurement errors does not.

Thus, when inflation is high, most of the observed movement in the CPI is due to actual inflation movements, but when the inflation is low, much of the movement in the CPI is noise. Just as a bathroom scale is suitable for determining changes in a person's weight but not those of a mouse, so the CPI is more accurate when inflation is high than when it is low.

To understand the math underlying this result, think of the CPI as having two components: a signal of true inflation and a measurement error, or noise. If the measurement errors are uncorrelated with inflation rates, it is easy to show that the CPI inflation will be more informative about actual inflation if the signal-to-noise ratio is high. If this ratio is high, the variance of true inflation must be a large component of CPI variation, and one should expect that true inflation moves nearly one to one with the CPI. If the variance of true inflation is small relative the movements in the CPI, then the signal-to-noise ratio will, likewise be low, and it is safe to largely ignore CPI movements when trying to determine inflation changes.

One can formalize this intuition mathematically by realizing that our best estimate of true inflation movements equal CPI movements multiplied by a coefficient that equals the variance of true inflation divided by the variance of the CPI. Since the variance of the CPI rises with both the variance of true inflation and the variance of the error, this coefficient will tend to be less than one. If the variance of true inflation is high relative to the variance of the CPI error (a high signal-to-noise ratio), this coefficient will be close to one, and true inflation will move almost one-to-one with the CPI. However, if the signal-to-noise ratio is low, CPI movements will not be informative about true inflation movements and one should not expect true inflation to move much as the CPI moves.

While this result is a simple application of econometric theory, economics can inform our understanding of when we should expect the signal-to-noise ratio of the CPI to be high and when we should expect it to be low. Starting with Okun [1971], many studies (e.g., Friedman [1977], Taylor [1981], Ball et al. [1988], Ball and Cecchetti [1990], Kiley [2000]) have found that countries with high inflation rates tend to have high inflation volatility.[1] Critically, these studies imply that the signal in the CPI (i.e., the variance of true inflation) should be rising as inflation rises. The rising inflation signal in the CPI implies that as long as CPI noise does not rise too quickly with inflation, it must be the case that there is a much tighter relationship between the CPI and inflation in high inflation regimes than in low ones.

This has profound implications for how one should think about inflation and monetary policy. First, it means that one cannot write true inflation as a linear function of the CPI. In particular, true inflation cannot be assumed to equal the CPI less a constant bias term. These non-linearities matter enormously in our data. For example, in our baseline estimates based on Japanese data that we will discuss later, we find that when the annual Japanese CPI registers an inflation rate of 0 percent, true inflation is -0.8 percent—but when CPI inflation is 2 percent the bias rises to -1.8 percent. In other words, a 2 percent CPI inflation target is actually a price stability target, when using annual data. Since the methodology underlying the Japanese CPI is more prevalent internationally than that used in the U.S. PCE deflator, this potentially has broad implications. Our results even have implications for the U.S. PCE deflator. When we replicate the PCE deflator methodology using Japanese data, we find the PCE deflator methodology generates substantial upward biases: the PCE deflator methodology overstates inflation by about a percentage point when reported inflation is 2

[1] Friedman [1977] argues that "a burst of inflation produces strong pressure to counter it. Policy goes from one direction to the other, encouraging wide variation in the actual and anticipated rate of inflation." A similar idea is proposed by Ball [1992], who argues that when inflation is low, there is a consensus that it should be kept low; however, when inflation is high, there is disagreement about the importance of reducing it, leading to high variability of inflation. Taylor [1981] suggests that accommodative monetary policies may lead to high inflation and greater variability in response to supply shocks. Cukierman and Meltzer argue that an exogenous increase in the variance of monetary control errors gives a central bank a stronger incentive to create surprise inflation, leading to a high inflation and high volatility of inflation. Ball et al. [1988] show, using a menu cost model, that high trend inflation reduces nominal price rigidity and thus steepens the short-run Phillips curve. As a result, shocks to aggregate demand have smaller effects on output but larger effects on inflation. Gagnon [2009] confirms that the micro price data is consistent with this story by showing that the frequency of price adjustment increases with the level of inflation.

percent and overstates inflation changes by 30 percent when inflation is below 2.2 percent.[2]

A second important implication concerns how central banks and other economic agents should respond to CPI inflation movements. Again, the non-linearity arising from the fact that the signal-to-noise ratio in the Japanese CPI is much higher in high inflation regimes matters. We estimate that a one percentage point rise in the CPI should raise one's assessment of true inflation by only half a percentage point when inflation is less than 2 percent, but this same increase should raise one's assessment by 2 percent when inflation is high. In other words, failure to take into account the change in an official price index's signal-to-noise ratio is likely to mean that central banks pay far too much attention to official price indexes when they are low and not enough attention when they are high.

In order to demonstrate these points, our paper makes use of the Nikkei Point of Sale database.[3] Nikkei collects daily scanner purchase data from hundreds of large and small stores covering hundreds of thousands of barcodes spread across Japan in the grocery sector. These data are typically sold by Nikkei to customers interested in accurately measuring the sales of individual barcodes. Our data covers the period from 1988 to 2010 and contains 4.82 billion price *and* quantity observations. The Japanese CPI, like the CPIs of most countries, contains no quantity data and a tiny fraction of the number of price observations. Crucially, the Nikkei data come as close as one could come empirically to observing the universe of grocery price and quantity observations in the Japanese economy. Moreover, the long time span means that we observe Japanese periods of substantial inflation and deflation. The fact that we have both price and quantity data enables us to construct the theoretically well-founded Törnqvist index of "true" inflation for a set of items covering 17 percent of Japanese consumer expenditure. By following Abe and Tonogi's [2010] concordance matching the items in the Nikkei POS with those of the Japanese CPI, we are able to observe both a true inflation measure and the CPI value.

[2]The variability of the bias has not been the focus of most prior work in this area (see the excellent surveys by Hausman [2003], Lebow and Rudd [2003], and Reinsdorf and Triplett [2009]).

[3]This is an expanded version of the dataset used in Abe and Tonogi [2010].

These data let us make a number of important empirical findings. The first is the volatility in the CPI error. Our estimates of this error based on grocery data are likely to be substantially lower than for the rest of the CPI because it is much easier to measure the prices of the goods sold in a grocery store than major CPI components like housing, transport and communication, and recreation. Nonetheless, we find that inflation mismeasurement is large. Our results indicate that over 12-month periods, the grocery CPI sometimes overstated inflation relative to a Törnqvist by as much as four percentage points while at other times it understated inflation by as much as 2 percentage points. The fact that the standard deviation in the bias is 0.9 percent means that much of the fluctuation in observed inflation during this time period was due to CPI errors. It is the surprising magnitude of these errors that drive our basic finding that the CPI is not very informative when the variance of inflation is low.

It is not possible to precisely decompose the reasons for all of the differences between the CPI and the Törnqvist because the two indexes differ in a myriad of ways. Some of the important differences include the use of unweighted arithmetic (or sometimes geometric) averages rather than sales-weighted geometric averages at the lower-level, upper level weights that differ from those of a Törnqvist because they are computed from different sources than the lower level data and at different times, and finally, samples of prices instead of the universe of prices. In sum, official price indexes contain two classes of errors: "sampling errors" arising from using a subset of the price data and "formula errors" arising from using the wrong weights and formulas to aggregate the price information. An important difference between these two types of errors is that formula errors are not diminished by building price indexes with more data. For example, if the correct formula required a set of numbers to be multiplied together, but instead the statistical agency added them, the result would be different in general, and this difference would not be diminished by working with larger sets of numbers.

In order to better understand the sources of this noise, we show how one can express

CPI errors relative to a Törnqvist index, so that one can decompose the variance of the CPI into movements in true inflation, upper-level weighting errors, and mismeasurement of lower-level item indexes. True inflation can also be decomposed into fluctuations due to aggregate inflation movements and fluctuations arising from relative price movements (which will matter if some goods have nontrivial sales shares).

Since we can directly measure the CPI variance and its components, we can precisely explain the non-linear relationship between the CPI and true inflation. We first show that while there is some tendency for the variance of CPI noise to rise as inflation rises, the rise is small compared to the tremendous increase in inflation volatility that occurs as inflation rises. In other words, while Cecchetti [1997] and Shapiro and Wilcox [1996] were right to point the tendency for CPI noise to rise with inflation because lower-level substitution bias rises with inflation, this effect is very small compared to the rise in inflation volatility.

Because we observe the underlying individual price and quantity data, we are able to be precise about how the non-linear relationship between the two series is generated. We decompose inflation volatility into two components: one related to movements in components of inflation that are common across products and another related to the relative price dispersion across products. Consistent with previous work, we find that relative price dispersion rises with inflation (c.f. Vining and Elwertowski [1976], Parks [1978], Fischer [1981] and Stockton [1988]). The variance in relative price shocks more than doubles as inflation rises past 2.4 percent, but this dispersion explains only a small component of the increase in aggregate price volatility. Most of this increase is explained by the variance in the aggregate or common component of inflation, which rises by a massive 320 percent. The non-linear relationship between inflation and the CPI is primarily driven by the fact that CPI measurement errors are much less sensitive to inflation than either component of aggregate price volatility: the variance of upper- and lower-level measurement errors only increase by 35–60 percent as we move from low inflation to high inflation regimes. The critical factors driving the non-linearity are the higher inflation variance in high inflation regimes and the relative insensitivity of CPI measurement errors to inflation movements.

The richness of our data enables us to delve deeper into the sources of these errors by performing a number of informative decompositions. First our results suggest that upper-level weighting errors—which in the U.S. account for most of the difference between chained and unchained indexes—matter for the average bias but are relatively unimportant in understanding movements in measurement error. Second, we find that one major source of formula error comes from using arithmetic, rather than geometric, averages of prices. This suggests that the move to geometric averaging of prices not only reduced the upward bias in inflation indexes but also reduced the noise in these indexes. Third, we are able to show that geometric averaging is not a complete solution in itself. In our simulations using Japanese data, we show that the PCE deflator methodology still produces significant biases and measurement error relative to the Törnqvist due to the choice of simple geometric averaging rather than weighted averaging. Interestingly, sampling errors seem to be relatively unimportant in understanding why our price indexes are flawed.

The structure of the paper is as follows. Section 2 describes our data and provides a preview of the CPI measurement issues, biases and data challenges. Section 3 describes the econometric strategies we follow in decomposing the relationship between true and measured inflation. Section 4 presents our main empirical findings, and Section 5 concludes.

2 Data

Our paper makes use of two principal datasets. The first is detailed Japanese CPI data. We obtained the price indexes and CPI weights from various issues of the Annual Report on the Consumer Price Index produced by the Statistics Bureau (SBJ) at the Ministry of Internal Affairs and Communications.[4] We then used these data to build the CPI for grocery items following the same formulas used by the SBJ. In all of our work, the inflation rate we work

[4] Although the official name of the bureau is the "Statististics Bureau," we will follow the Japanese website and abbreviate it with SBJ, which stands for "Statistics Bureau of Japan." Data are available at the SBJ website (http://www.stat.go.jp/english/data/cpi/). Issues prior to 2001 are only available in paper form.

with is the inflation rate in a given month relative to the same month in the previous year. This lets us avoid seasonality problems.

The SBJ produces a CPI index conforming to the International Labor Organization standard. The ILO methodology is the basic approach used in all advanced countries.[5] Thus, many of the strengths and weaknesses of the Japanese CPI are common to most advanced countries. Nevertheless, it is worth describing how the index is constructed. Price quotes for each good are collected each month during the span of the same week in stores in hundreds of locations across Japan. Indeed the geographic representation of the Japanese CPI is superior to that of the U.S. CPI. Like the U.S. CPI, there is highly imperfect information available on what weights to use at the lower level. The SBJ defines product type specifications for each of the item categories and only collects prices for products fitting these specifications.[6] For example, the price quotes for the "Butter" item include only the prices of products that are 200 grams and packed in a paper container, excluding unsalted varieties [Imai et al., 2012]. The SBJ then calculates inflation for each item as a Dutot index; that is, the ratio of the mean price quotes collected for an item in one period relative to another. Once these item indexes are constructed, the SBJ aggregates the data to form an overall CPI using weights from its consumer expenditure survey. The upper-level item weights are based on item expenditure shares in a base year that is updated every five years.[7]

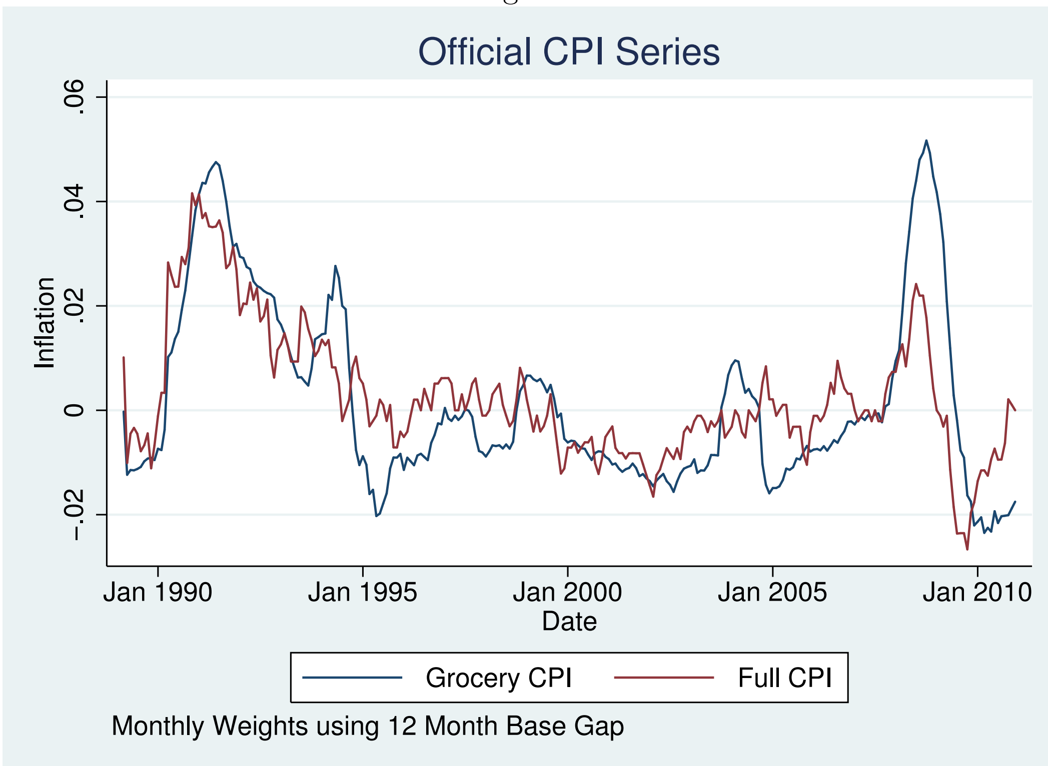
The items in the grocery elements of the CPI account for 17 percent of the full CPI. Grocery items are extremely standardized items that are easy to price, so it is far easier to measure price changes for this sample of the CPI than it is for most of the remainder. For example, it is easier to measure the the 30-day price change of a 300 ml can of Coca Cola sold in a particular store than the 30-day price change of other major expenditure items like imputed rent or recreational services. One should therefore anticipate that our paper

[5]See http://www.stat.go.jp/english/data/cpi/1585.htm#B2

[6]See Imai et al. [2012] for a more detailed description of this "purposive" sampling rule and a comparison of its performance relative to the probability sampling used by the BLS.

[7]There are a range of differences between how the BLS calculates the U.S. CPI and the Japanese methodology described above. We explore how these technical differences potentially reduce the variance of the CPI errors that drive our main results in Section 4.4 below.

Figure 1



understates the magnitude of CPI measurement error.

An obvious concern with our focus on grocery items is that this sample of goods might not be representative. While it is true that inflation measurement errors are likely to be much smaller in our sample of grocery items than for the whole CPI, the grocery CPI tends to track the overall Japanese CPI quite closely. Figure 1 plots both the Grocery CPI and the full CPI, and it is immediately apparent that the two are fairly closely correlated ($\rho = 0.8$) despite the fact that grocery items make up less 20 percent of the CPI. Our sample of products seems to capture most of the variation in the official CPI including the overall trend in inflation.

Our second dataset is the Nikkei Point of Sale (Nikkei POS) dataset. The Nikkei data differs from other barcode datasets in frequency, scope, and length. Nikkei collects data at a daily frequency. We observe the number of units of a barcoded good purchased in a store on a day and the store's sales revenue for that barcode on that day. In a typical month we observe prices for about a quarter of a million different grocery products, each identified by a barcode. Nikkei also has impressive scope relative to other datasets. Where other datasets focus on one store or chain, the Nikkei POS database includes observations taken from hundreds of grocery and convenience stores spread across Japan each day. This is done so that Nikkei can provide customers with an accurate picture of how goods are sold in general and across different markets. Finally, the truly amazing feature of the Nikkei POS data is its time dimension. Our data makes use of 23 years of these data collected from 1988 through 2010, all together comprising close to 5 billion observations.

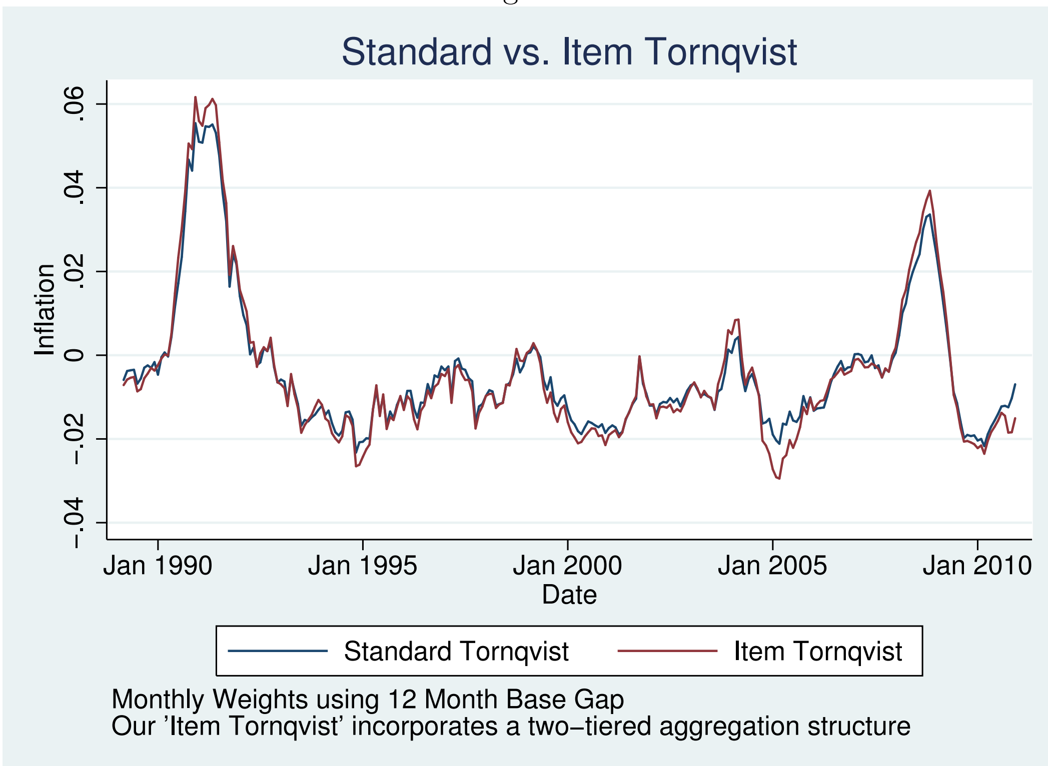
The detail in the Nikkei data allows us to construct a price index under fewer assumptions than the official CPI. We measure "true" inflation using a Törnqvist index. As a superlative index, the Törnqvist is a second order approximation to *any* twice-differentiable homothetic expenditure function, and is as close as we can come to computing an exact inflation index without actually specifying preferences.[8] Since our data permits us to compare the prices of identical goods purchased in the the same store using the correct weights for that store, we are able to exactly produce this index.[9]

It is difficult to formally compare the properties of the two-tiered CPI with those of the Törnqvist, which does not have a two-tier system. For example, it is not impossible to formally assess the importance of measurement error at the CPI's lower level because the Törnqvist has no lower level. In order to make progress on understanding what causes CPI errors, we utilize a Törnqvist index that has a two-tiered structure. We call this two-tiered index the "item Törnqvist" because the lower tier indexes are calculated at the "item" level to correspond with the Japanese CPI and then aggregated using Törnqvist weights. Fortunately, the item Törnqvist produces inflation rates that are almost identical to those of the standard Törnqvist as is shown in Figure 2. Thus, we can be confident that virtually all of the difference between Törnqvist and the CPI can be understood in terms of the

[8]The Törnqvist measures inflation in common goods prices. Assumptions on preferences are required in order to account for the welfare effects of entry and exit of goods (see, for example, Broda and Weinstein [2010]).

[9]See Appendix B for details on the Törnqvist index calculation.

Figure 2



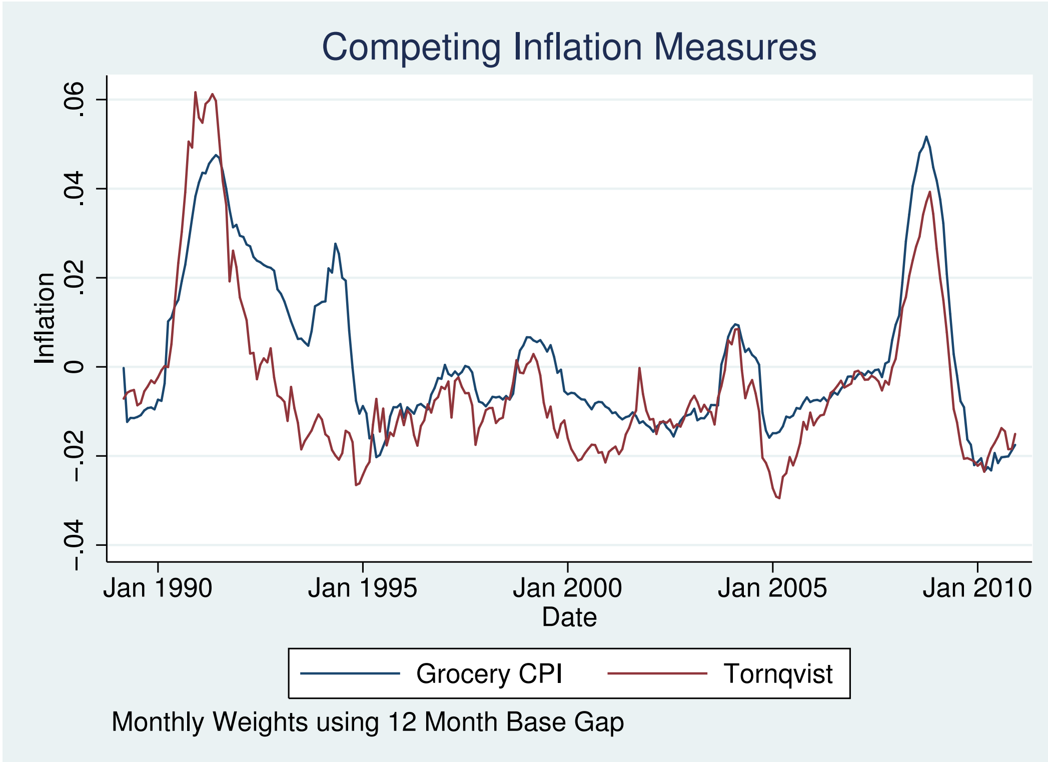
decomposition between an item Törnqvist and the CPI and not as differences between the item Törnqvist and the standard Törnqvist.[10]In the interest of simplicity, we will therefore ignore the difference between the item Törnqvist and the Törnqvist indexes of inflation and refer simply to to the "item Törnqvist" as the "Törnqvist" index throughout.

2.1 Measurement Error in the Official CPI

We calculate the measurement error in the official CPI by comparing it with the Törnqvist index. Figure 3 presents the grocery component of the official CPI and the Törnqvist index calculated using the Nikkei data. As one can see in the figure, there are substantial differences between the CPI and the Törnqvist index. Figure 3 suggests that there is a very clear upward

[10]Diewert [1978] shows that a superlative index, such as the Törnqvist, is approximately consistent in aggregation (that is, the value of an index calculated in two stages approximately coincides with the value of an index calculated in a single stage) and that this approximation is closer for chained indexes, like those constructed here using changes in prices and quantities between successive periods, rather than fixed base indexes. It is not surprising, therefore, that all of our results are qualitatively identical whether we use the standard or the item Törnqvist.

Figure 3



bias in the official index. While the Törnqvist index became negative in 1993, the official index did not register deflation until 1995. However, the correlation between the CPI and the Törnqvist is much less tight than what one might expect for two indexes measuring the same component of inflation. The correlation between the two indexes is 0.87, which reflects the fact that there are some periods in with the two indexes differ quite substantially. For example, in 1994 the CPI was registering an inflation rate that was at one point 4.7 percentage points above that of Törnqvist, in 1991 CPI *understated* inflation by almost 2 percentage points, and between 1995-1999, the bias was close to zero. Thus, Figure 3 indicates that one should be very wary of assuming that one can infer inflation by looking at an official index and subtracting off a constant bias term.

In Table 1, we study the bias in the CPI, defined as the difference between the CPI and the Törnqvist. As we can see in the first column, this difference averages to 0.63 percent per year. The Törnqvist index suggests that Japanese grocery goods entered a period of sustained deflation in 1993. If we focus our attention on that period we see that the official bias in the CPI was actually larger, at 0.76 percent per year.

Table 1
Index Biases

	$\pi^{\text{CPI}} - \pi^{T}$
Annualized Total Bias	0.625
Standard Deviation of Bias	0.961
Annualized Total Bias (Post-93)	0.762
Standard Deviation of Bias (Post-93)	0.763

This bias implies a radically different impression of what has happened to the price level in Japan (at least for grocery goods). As one can see in Figure 4 the official price index suggests that prices only fell by 4.2 percent between 1993 and 2010, suggesting that deflation was quite mild. The superlative index, on the other hand, indicates that deflation averaged .91 percent per year, implying a far more substantial 14.5 percent drop in the price level over the whole period.

What is most striking in Table 1 is the magnitude of the standard deviation of the bias: 0.96 percentage points. This means that while, on average, the CPI inflation rate is biased upwards by 0.6 percentage points per year, one can only say with 95 percent confidence that this bias lies between -1.3 and 2.5 percentage points. In other words, if the official inflation rate is one percent per year and aggregate CPI errors are the same as those for grocery items, one can only infer that the true is inflation rate is between -1.5and 2.3 percentage points. Thus, a one percent measured inflation rate would not be sufficient information for a central bank to know if the economy is in inflation or deflation. The magnitude and standard deviation of the bias in the grocery component of the CPI is particularly striking since other components of the CPI are subject to much larger measurement errors, potentially yielding an even larger and more noisy bias in the overall CPI.[11]

[11] Of course, averaging over more price series might improve the picture for the overall CPI—a point we will address later—but the magnitude of the errors is still worrying.

Figure 4

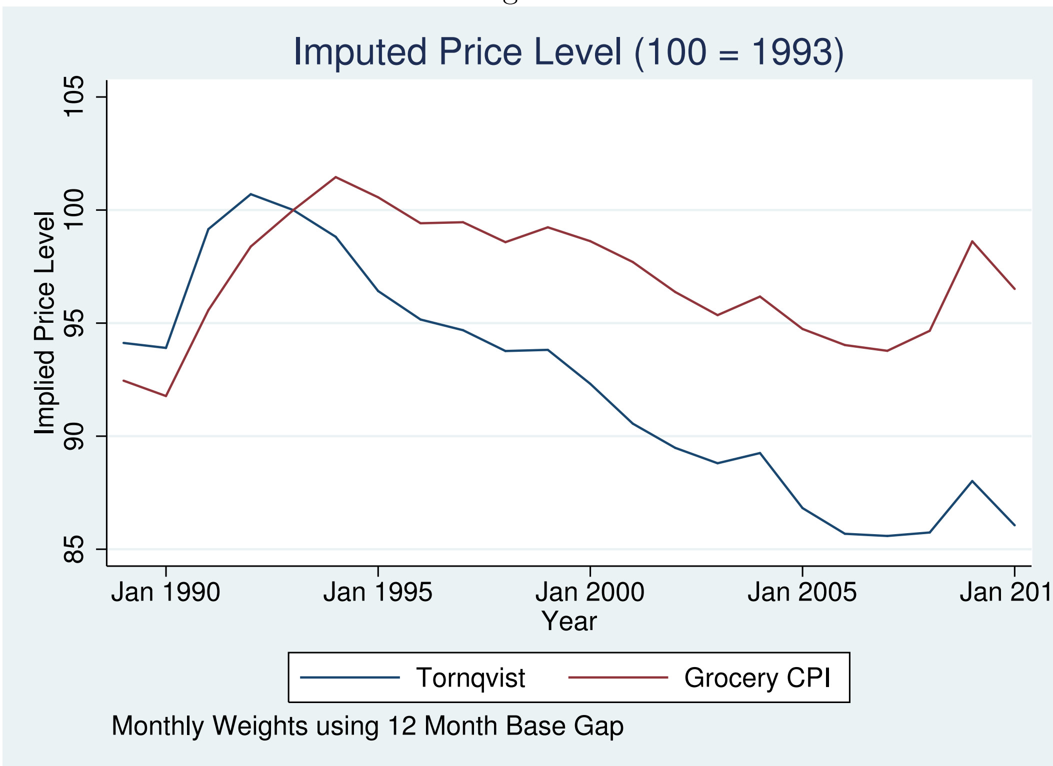
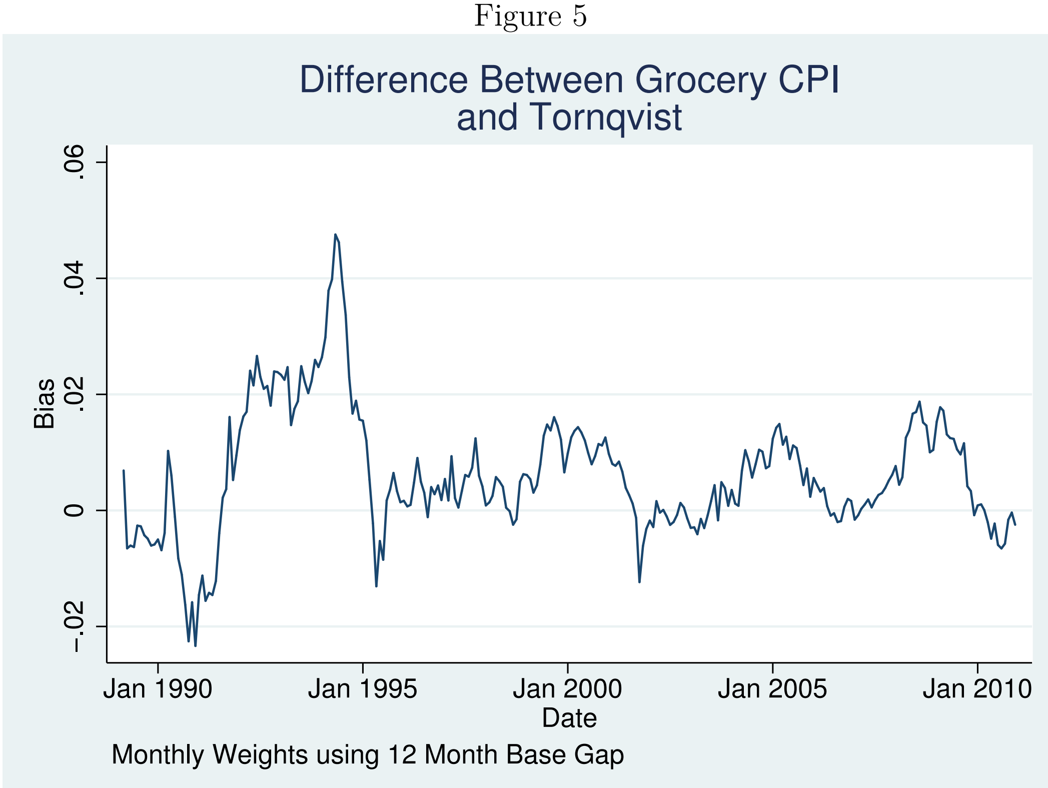


Figure 5



We can see the magnitude of the noise in Figure 5. The figure plots the difference between the CPI and the Törnqvist index over the full time period. There are two important findings revealed by the figure. First, as we saw in Table 1, the CPI bias is not constant. While the bias is positive on average, the difference between the CPI and the Törnqvist fluctuates. Second, the magnitude of the measurement error is quite large in comparison with the underlying Japanese inflation rate. While one can see that the bias is clearly positive on average, there appears to be no clear relationship between the magnitude of the bias and the underling inflation rate. For example, the bias was not exceptionally high or low in either of the two periods that Japan experience inflation peaks, 1990-1 and 2008-9. The plot suggests that it is not implausible to think of this noise as classical measurement error.

The results in Figure 5 and Table 1 have extremely important implications for our understanding of inflation. As basic econometrics tells us, the fact that there is substantial measurement error in the CPI means those who assume that a one percent movement in the CPI corresponds to a one percent movement in inflation will systematically overstate true inflation movements. We turn to formalizing this intuition in the next section.

3 Econometric Theory

3.1 Inferring True Inflation from Measured Inflation

The basic question we are trying to ask is how do we infer true inflation from the measured values of inflation. We think about this problem mathematically by first assuming that measured inflation—the CPI, which we denote π_t^{CPI}—is equal to true inflation, which we denote π_t^T, plus some measurement error, ϕ_t:

$$\pi_t^{CPI} = \pi_t^T + \phi_t \tag{1}$$

where π_t^T and ϕ_t are both unobserved. Equation 1 can be thought of as the standard equation used in the CPI bias literature.[12] However, most users of the CPI are are interested in a related but different question: what is the expected value of true inflation given measured inflation, or $E\left[\pi_t^T|\pi_t^{CPI}\right] = \alpha' + \beta'\pi_t^{CPI}$? Since π_t^T and π_t^{CPI} are both univariate and they follow a linear relationship, the minimum mean square estimate of π_t^T is its conditional expectation, which we can express as:

$$E\left(\pi_t^T|\pi_t^{CPI}\right) = E\left(\pi_t^T\right) + \frac{Cov\left(\pi_t^T, \pi_t^{CPI}\right)}{Var\left(\pi_t^{CPI}\right)}\left[\pi_t^{CPI} - E\left(\pi_t^{CPI}\right)\right] \tag{2}$$

indicating that the extent to which an unexpected change in the CPI is a signal of an unexpected change in true inflation is governed by the ratio of the covariance of inflation and the CPI to the variance of the CPI.

The amount of information on true inflation, π_t^T, that is contained in observed inflation, π_t^{CPI}, therefore depends crucially on the covariance of the two series relative to the variance of the observed inflation series. It is useful to rewrite equation 2 in terms of a regression coefficient, β, that would be obtained from regressing π_t^T on π_t^{CPI}:

$$E\left(\pi_t^T|\pi_t^{CPI}\right) = \left[E\left(\pi_t^T\right) - \beta E\left(\pi_t^{CPI}\right)\right] + \beta\pi_t^{CPI}$$

where:

$$\beta \equiv \frac{Cov\left(\pi_t^T, \pi_t^{CPI}\right)}{Var\left(\pi_t^{CPI}\right)} = \frac{Var\left(\pi_t^T\right) + Cov\left(\pi_t^T, \phi_t\right)}{Var\left(\pi_t^T\right) + Var\left(\phi_t\right) + 2Cov\left(\pi_t^T, \phi_t\right)} \tag{3}$$

where the second equality follows from equation (1).

Equation 3 forms the foundation of everything that follows. In particular, the notion that a given movement in the CPI produces a movement in actual inflation of the same

[12] One can think of this formally by realizing that the price measurement literature has focused on obtaining a link between true and measured inflation, i.e. estimating $\pi_t^{CPI} = \pi_t^T + \alpha + \epsilon_t$, where π^{CPI} is the CPI inflation rate, π_t^T is the true inflation rate, α is the bias, and ϵ_t is a mean-zero error term. This approach is equivalent to equation (1) with $\phi_t = \alpha + \epsilon_t$.

magnitude, i.e. $\beta = 1$, relies on the assumption that there is no variation in the measurement error of CPI. If there is *any* volatility measurement error in the CPI, true inflation cannot be expressed as equal to measured inflation less some constant term. In other words, the fact that the bias term in Figure 5 is not a horizontal line, provides *prima facie* evidence that the common practice of assuming true inflation equals measured inflation less some bias term is wrong.

The noisy indicators literature has examined cases where β is constant. We will next show that one should not expect this. In order to do this, it will be useful to build some theory from first principles on what we should expect the relationship to be.

3.2 Decomposing "True" Inflation

We need to make a few assumptions about the underlying error structure causing CPI mismeasurement in order to understand why β in equation 3 is unlikely to equal one or be constant. We begin by assuming that inflation, π_t^T, is given by a Törnqvist index:

$$\ln\left(1 + \pi_t^T\right) = \sum_{i=1}^{n} w_{it}\pi_{it} \tag{4}$$

where $w_{it} = \frac{1}{2}\left(s_{it} + s_{it-1}\right)$, s_{it} is the expenditure share of good i in period t, and $1 + \pi_{it}$ is the relative price of good i in period t relative to its price in period $t - 1$, and $\sum_{i=1}^{n} w_{it} = 1$. The price change of every good i from period t to period $t-1$ can be decomposed as follows:

$$\pi_{it} = \mu_t + \nu_{it}, \tag{5}$$

where μ_t is the aggregate component of inflation and ν_{it} is an idiosyncratic component of that good's price change. This enables us to rewrite the Törnqvist inflation index as

$$\pi_t^T \;\; = \;\; \mu_t + \sum_{i=1}^{n} w_{it}\nu_{it}. \tag{6}$$

3.3 The Variance of "Measured" Inflation

As we have argued earlier, statistical agencies cannot compute the Törnqvist index in equation (4) because the data requirements are too high. We can think of official inflation rates as weighted averages of price changes in which the weights and formulas contain errors. We can therefore write a general equation for the CPI, π_t^{CPI} in terms of the true underlying price indexes as:

$$\pi_t^{CPI} = \sum_{i=1}^{n} \left(w_{it} + \epsilon_{it}\right) \left(\pi_{it} + \delta_{it}\right), \tag{7}$$

where we use ϵ_{it} to denote the error in the upper-level weight for good i at time t and δ_{it} to denote the lower-level error in the measured inflation rate for the good. In this formulation, if the the upper- and lower-level errors were eliminated, the CPI inflation rate would collapse to that of the Törnqvist.

In order to make the analysis that follows tractable, we need to assume that these errors are random independent draws from distributions with zero means. In particular, we assume (i.e. $E\left(\epsilon_{it}\right) = E\left(\delta_{it}\right) = 0$, and $Cov\left(\epsilon_{it}, \epsilon_{jt}\right) = Cov\left(\delta_{it}, \delta_{jt}\right) = Cov\left(\nu_{it}, \nu_{jt}\right) = 0$, $\mu_t \perp \epsilon_{it}, \delta_{it}$). As we show in Appendix A, these assumptions are sufficient to ensure that the error term in equation 3 is independent of the level of inflation, a condition that we will examine in the empirical section. These assumptions enable us to derive the variance of the the CPI in terms of the signal, given by the variance of true inflation $\left(Var\left(\pi_t^{CPI}\right)\right)$, and the variances of the various errors given below:

$$\begin{aligned} Var(\pi_t^{CPI}) \;\; &= \;\; Var(\pi_t^T) + \sigma_{\delta_t}^2 \left(\sum_{i=1}^{n} s_{i,t-1}^2 + \frac{\gamma^2}{4}\left(n-1\right)\sigma_{\nu_t}^2\right) \\ &\quad + n\sigma_{\epsilon_t}^2 \left[\sigma_{\mu_t}^2 + \sigma_{\nu_t}^2 + \sigma_{\delta_t}^2\right], \end{aligned} \tag{8}$$

where the algebra used to derive this equation is relegated to Appendix A.

Equation 8 is key to understanding the problems of formula bias in understanding the variance of inflation measures. As we argued before, if $w_{it} \approx 1/n \approx s_{i,t-1}$ then in the limit, as the number of products goes to infinity, the law of large numbers will cause the variance in the first term to approach the variance in underlying inflation. Similarly, the law of large numbers will also cause the second term to also approach zero because any idiosyncratic error in calculating lower-level price quote will not affect the average. However, the last term in equation (8) will not in general approach zero as the number of items in the index approaches infinity. The reason more data does not improve the accuracy of the CPI stems from the fact that CPI contains a formula bias, which is contained in the ϵ_t's. These errors do not disappear as one averages across more sectors because they are correlated with the inflationary errors and the δ_t's. In other words, one cannot correct for the fact that the CPI uses the wrong formula to measure inflation by simply using more data: averaging over more sectors will not solve the problem of formula bias.

Our next task is to compute correlation between π_t^{CPI} and π_t^T. In Appendix A we show that under reasonable assumptions about the independence of the various idiosyncratic shocks we can write:

$$Cov\left(\pi_t^T, \pi_t^{CPI}\right) = Var\left(\pi_t^T\right) \tag{9}$$

The key assumption behind this result is that the CPI measurement errors, ϵ_{it} and δ_{it}, are independent draws from a distribution and, therefore, uncorrelated with the components of true inflation, μ_t and ν_{it}. While equation 9 does not need to be true for all possible price shocks, we will show in the empirical section that it is an extremely good approximation of reality. This result has important implications for our understanding of 2. In particular, if we maintain the assumptions necessary to derive equation 9, we can rewrite the expression for β derived in equation 3 in terms of the fundamental microeconomic price shocks as,

$$\beta = \frac{Var\left(\pi_t^T\right)}{Var\left(\pi_t^T\right) + Var\left(\phi_t\right)} \tag{10}$$

$$= \frac{Var\left(\pi_t^T\right)}{Var\left(\pi_t^T\right) + \underbrace{\sigma_{\delta_t}^2\left(\sum_{i=1}^{n} s_{i,t-1}^2 + \frac{\gamma^2}{4}\left(n-1\right)\sigma_{\nu_t}^2\right) + n\sigma_{\epsilon_t}^2\left[\sigma_{\mu_t}^2 + \sigma_{\nu_t}^2 + \sigma_{\delta_t}^2\right]}_{\text{“CPI Noise”}\geq 0}} \leq 1 \tag{11}$$

where $\sigma_{\mu_t}^2$ is the variance in common price shocks inflation, and $\sigma_{\nu_t}^2$ is the variance in idiosyncratic price shocks..

Equation 11 is the critical equation for everything that follows. A first clear implication of this equation is that *a rise in CPI inflation of a given percentage should always be associated with an equal or smaller percentage rise in actual inflation.* A second implication is that with non-constant measurement errors (*i.e.*, $\sigma_{\delta_t}^2 > 0$ or $\sigma_{\epsilon_t}^2 > 0$) changes in the CPI always overestimate actual inflation changes.

Our ability to decompose this bias into the underlying components enables us to make much stronger statements about the relationship between real inflation and measured inflation. Ball et al. [1988], among others, have argued that the variance of inflation should rise as inflation rises. However, they were only able to make this claim using U.S. CPI data, and thus, it is not clear that this rise in variance is due to actual inflation becoming more volatile, i.e., a rise in $\sigma_{\mu_t}^2$, or just an increase in the volatility of idiosyncratic price movements, $\sigma_{\nu_t}^2$. The former might occur if monetary shocks become more volatile when inflation rises, and the latter would arise if price dispersion rises with inflation. One of the interesting elements of our setup is that we can not only examine if $Cov\left(Var\left(\pi_t^T\right), \pi_t^T\right) > 0$, but we can also understand why it is true.

A second, and more important, feature of our approach is that we also have a direct measure of the “CPI Noise” term in equation 11. If $Cov\left(Var\left(\pi_t^T\right), \pi_t^T\right) > 0$ and this term is approximately constant, then this implies that there will be a non-linear relationship between CPI inflation changes and actual inflation changes. When the CPI is low, the signal to noise ratio will be low because most movements in the CPI will be driven by noise, and hence one should not adjust expectations of true inflation from movements in the CPI. But when inflation is high and $Var\left(\pi_t^T\right)$ is large, movements in our estimate of actual inflation much more with movements in the CPI. In order to compute the exact magnitudes we need to first compute the elements of equation 11.

4 Results

We divide our results into several sections. First, we document that there is a strong non-linear relationship between true inflation and the CPI. Second, we investigate the microstructure of this non-linearity with an aim of understanding what elements of measured inflation explain our findings. Third, we conduct a series of robustness checks to examine whether the results stem from the methodologies employed by statistical agencies or whether our results simply stem from using a different dataset to measure prices. Finally, we consider some potential fixes to the measurement of inflation.

4.1 Inferring Inflation

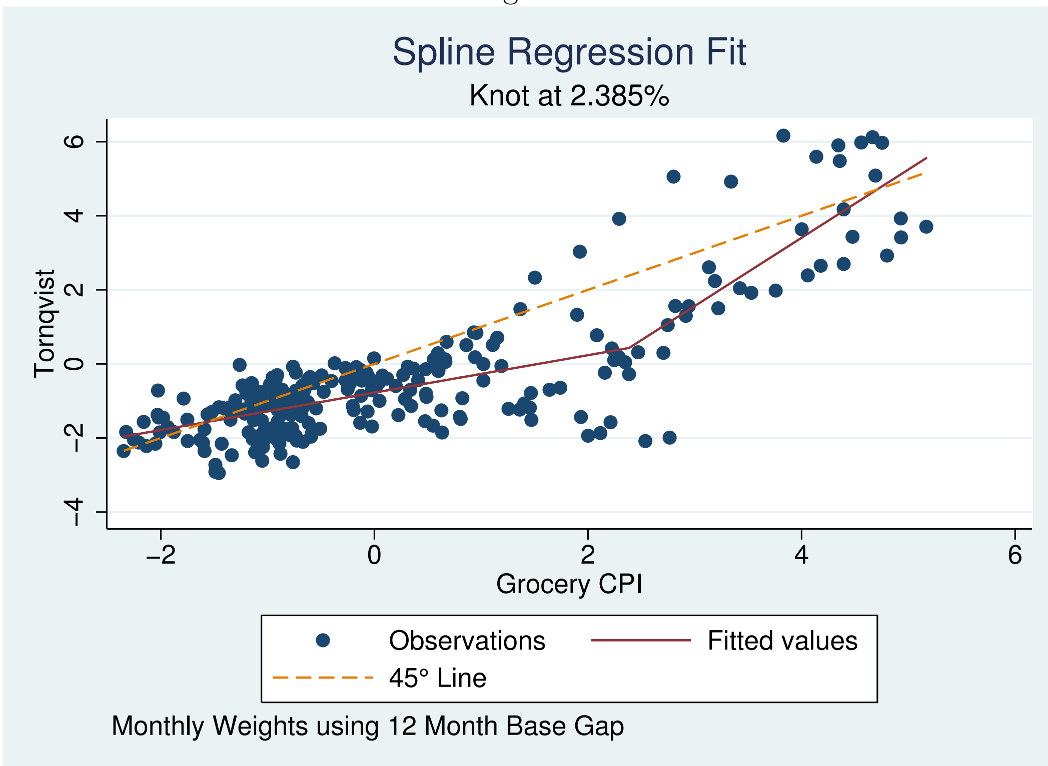
We now turn to studying the relationship between the official CPI and the Törnqvist index addressing a series of questions motivated by the theoretical framework outlined above:

1. Is the signal-to-noise coefficient (β) equal to one?

2. Is the relationship between true inflation and measured inflation non-linear, resulting in a poor relationship between measured inflation when it is low and a stronger one when it is high?

Fortunately, both of these questions can be qualitatively assessed by looking at the data. We start by plotting π_t^T against π_t^{CPI} in Figure 6. Each point in the plot represents a 12-

Figure 6



month inflation rate taken from our sample. There is a very strong positive relationship between true inflation and CPI inflation when the inflation rate exceeds 2 percent, but there is a much weaker connection between CPI inflation and actual inflation when the CPI is registering rates of inflation below 2 percent per year.

Regression evidence confirms that our eyes are not deceiving us. In Table 2 we present a number of regressions of of π_t^T against π_t^{CPI}. In the first column we perform a simple OLS regression and obtain a coefficient of 0.83. The fact that this coefficient is less than one suggests that CPI measurement error could be causing an attenuation bias. However, simple inspection of Figure 6 indicates that this bias is not stable. In column 2 we regress π_t^T against π_t^{CPI} and its square. The data indicate that the attenuation bias is much larger when CPI inflation is low relative to when it is higher. In column 3, we run a spline regression with a knot placed at 1 percent measured inflation. The interesting feature of this regression is that that for inflation rates below 1 percent, there is a much weaker relationship

Table 2
Grocery CPI vs. Tornqvist Inflation Regressions (Lag-11 Newey-West Standard Errors)

	Tornqvist	Tornqvist	Tornqvist	Tornqvist
Grocery CPI	0.832***	0.553***		
	(0.148)	(0.151)		
Grocery CPI^2		0.119**		
		(0.0494)		
Grocery CPI ($\leq$ 1%)			0.398**	
			(0.189)	
Grocery CPI ($>$ 1%)			1.257***	
			(0.241)	
Grocery CPI ($\leq$ 2.385%)				0.505***
				(0.165)
Grocery CPI ($>$ 2.385%)				1.843***
				(0.446)
Constant	-0.584***	-0.908***	-0.894***	-0.775***
	(0.173)	(0.224)	(0.216)	(0.183)
Observations	262	262	262	262
Adjusted R^2	0.668	0.717	0.713	0.739

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

between measured and actual inflation. If we chose an endogenous knot, the data place it at 2.4 percent, indicating that there is a substantially weaker relationship between measured inflation and true inflation below this point.[13]

The results presented in the last column suggest that the relationship between measured and actual inflation is highly non-linear. Consider, for example, how to interpret a move in the Japanese CPI from -1 to 2 percent. According to the regression evidence in the last column in Table 2, an inflation rate of -1 percent per year—as the Japanese CPI averaged for many years—corresponds to a true inflation rate of -1.3. In other words the bias in inflation is very small when inflation is moderately negative. However, if CPI inflation were to rise by three percentage points to 2 percent, the negative bias would rise in magnitude dramatically, reaching 1.8 percentage points. Our point estimate suggests that two percent measured inflation rate is only a 0.2 percent inflation rate. *In other words the current Bank of Japan target inflation rate of 2 percent is extremely close to the correct rate to achieve price stability!* Moreover, the fact that the bias point estimate varies considerably with the underlying inflation rate means that one should exercise caution in comparing estimates of the bias computed using samples composed of observations using different inflation rates.

A second striking feature of the non-linearity that we observe is that further increases in CPI inflation imply much sharper rises in true inflation. For example, our estimation implies that an increase in inflation from 2 percent to 5 percent would correspond to an increase in true inflation from 0.24 percent to 3.4 percent. Since the upward bias largely disappears at higher inflation rates, central banks should pay much more attention to inflationary changes when inflation is high than when it is close to zero. A central bank that deems a movement in CPI inflation from 0 to 2 percent as the same as a movement from 2 to 4 percent is liable to dramatically overreact to inflation when it is low and and underact when it is high.

[13] It is possible the relationship between measured inflation and true inflation depends on the absolute magnitude of inflation, rather than its level. Suppose, for example, that inflation volatility is increasing in the magnitude of inflation, rather than its level, that is, $Cov\left(Var\left(\pi_t^T\right), |\pi_t^T|\right) > 0$ rather than $Cov\left(Var\left(\pi_t^T\right), \pi_t^T\right) > 0$. We considered a specification with two knots symmetrically placed around 0, but the data did not support this parameterization instead selecting a model with a single knot placed at 2.4 percent. This may reflect the fact that we do not experience any highly deflationary periods.

4.2 The Microstructure of Inflation Measurement Errors

The analysis above gives visual and econometric evidence for a non-linear relationship between the CPI and actual inflation, but we still have not identified the micro-foundations of this relationship. Our theory depends on the CPI measurement error being classical. Theoretically, one can construct cases in which this is not true, but its veracity is an empirical question. In order to assess whether $Cov\left(\pi_t^T, \phi_t\right) = 0$, we regressed ϕ_t on π_t^T and could not statistically reject that there was no relationship between the measurement error and the level of inflation. Since we cannot statistically reject the hypothesis that the inflation measurement error is classical, we assume it is so and proceed with our decomposition.

Table 3
Summary Statistics (Means): Item Tornqvist

	All	Ratio (1% Knot)	Ratio (2% Knot)	Ratio (Endogenous Knot)
$\beta(\pi^{\mathrm{CPI}}, \pi^T)$	0.833	2.56	3.42	3.65
$\sigma^2_{\pi^T}$	3.18e-04	10.41	8.01	5.68
$\sigma^2_{\nu_t}$	3.49	2.46	2.55	2.34
$\sigma^2_{\hat{\mu}_t}$	0.79	7.45	5.77	4.16
$\sigma^2_{\varepsilon_t}$	0.32	1.14	1.19	1.35
$\sigma^2_{\delta_t}$	2.83	1.57	1.60	1.62
Herf_t	0.019	1.05	1.05	1.05
CPI Noise	0.20	2.36	2.49	2.56
$\#\mathrm{Items}_t$	135.2	0.97	0.98	0.99
γ_t	-0.68			
Observations	262	0.32	0.21	0.15

Note: Entries for $\sigma^2_{\nu_{i,t}}$, $\sigma^2_{\hat{\mu}_t}$, $\sigma^2_{\varepsilon_{i,t}}$, $\sigma^2_{\delta_{i,t}}$, and the CPI Noise are divided by the entry for $\sigma^2_{\pi^T}$. The "Ratio" columns give the ratio of a variable in periods where π^{CPI} is greater than the knot value to that variable's value calculated from observations in those months when π^{CPI} is less than the knot. The "Endogenous Knot" was determined to be 2.385%.

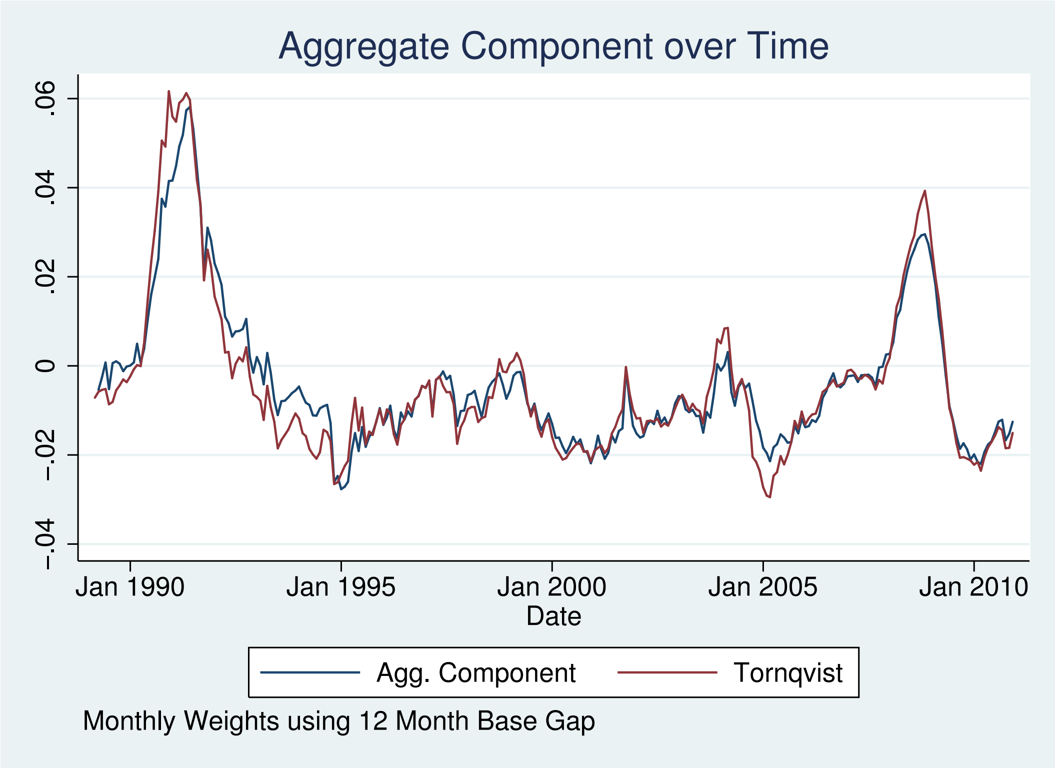
Equation 11 provides the formula for decomposing β into the underlying variances. Fortunately, our dataset is rich enough that we can directly measure all of the variance components that are likely to create the biases. In Table 3 we compute $\hat{\beta}$ along with all of its components to examine why it varies with the level of inflation. In the first column of the table, we present the results from the full sample with the first three elements of the column presenting our regression coefficient from the first column of Table 2 along with its decomposition as given in equation 11.

A critical feature of equation 2 is that it tells us how to decompose movements in the link between CPI inflation and actual based on movements in the underlying forces driving movements in variance of actual inflation $\sigma^2_{\pi^T_t}$ and movements in the CPI noise term. Let's begin by focusing on the determinants of inflation volatility.

We know from equation 5 that we can decompose actual inflation of an item, π^T_{it}, into two components: the aggregate component of inflation, μ_t, and its idiosyncratic component, ν_{it}. By simply running this OLS regression we can identify each element of this decomposition. In Figure 7, we plot the inflation rate implied by the aggregate component of inflation, μ_t, against that of true inflation, π^T_t. Not surprisingly, these two series track each other extremely closely, which is what one would expect if aggregate inflation were driven largely by shocks common to all sectors rather than to shocks to a few sectors.

In the next cells of the first column of Table 3, we express each of the component variances as a share of $\sigma^2_{\pi^T}$, so that one can get some sense of how important each factor is relative to the variance of CPI inflation. The fact that the variance of the aggregate component of inflation, $\sigma^2_{\mu_t}$, is 80 percent as large as $\sigma^2_{\pi^T}$ implies that actual inflation movements are an important determinant of CPI inflation movements as one might suspect. It is a little harder to interpret the importance of idiosyncratic inflation volatility because this term only enters into the formula for actual or CPI inflation volatility after being multiplied by another term, but the fact that we saw in Figure 7 that π^T_{it} tracks μ_t so closely means that these idiosyncratic terms cannot be an important determinant of actual inflation. The next two cells show the importance of error terms that only affect the variance of measured (CPI) inflation but not actual inflation. As one can see from the table, the variance of the lower-level errors, $\sigma^2_{\delta_t}$, are much larger than the upper-level ones, $\sigma^2_{\epsilon_t}$, although here, too, it is difficult to assess the importance of each error because they affect the CPI noise interactively.

Figure 7



In order to make some progress on this issue, we can think of performing a number of counterfactual exercises in which we assume that a statistical agency could eliminate different types of errors and see what this does to the variance of the aggregate measurement error. The move to producing chained CPI's can be thought of as attempts to eliminate the upper-level weighting problems captured by $\sigma^2_{\epsilon_t}$. Equation 11 lets us compute how much the variance in the CPI noise would fall if we eliminated these errors by setting $\sigma^2_{\epsilon_t} = 0$. Our results suggest that eliminating upper-level weighting from the current weighting structure a perfect weighting structure would only reduce the variance in CPI noise by 49 percent. However, if we were to eliminate lower-level measurement errors, *i.e.* setting $\sigma^2_{\delta_t} = 0$, this would reduce the variance in CPI noise by 71 percent.[14] It is clear that at least half of the problem in the CPI is that there are substantial formula biases and other measurement errors at the lower level. The importance of lower-level errors in driving overall CPI measurement

[14]The two numbers do not sum to 100 percent because the $n\sigma^2_{\epsilon_t}\sigma^2_{\delta_t}$ term can be eliminated by setting either $\sigma^2_{\epsilon_t}$ or $\sigma^2_{\delta_t}$ to zero.

error probably stems from the fact that the inability to weight goods by sales and the choice of the price aggregation method at the lower level is not rigorously based on theory.

In addition to decomposing the sources of the errors, we also can back out the combined impact of each of these terms by using equation 11 and our parameter estimates to solve for the variance in the CPI noise. This number is presented in the row labeled "CPI Noise" in Table 3 and indicates that this noise is about 20 percent as big as the variance of actual inflation implying a signal-to-noise ratio of about 5.[15]

The following three columns of Table 3 present what happens to each parameter as we divide the data into different bins. The first two columns corresponds to results of assuming that there are knots at inflation rates of one percent and two percent, and the last column corresponds to the endogenous knot we identified in Table 2. For example, we see in the first row that if we estimated β using data in which the CPI exceeded 2.4 percent (the value of our endogenous knot) we would obtain a value that is 3.65 times larger than if we estimated β using data in which measured inflation is less than 2.4 percent. The lower rows provide information how changes in the key variances that determine β in equation 11 vary above and below each knot. The second row of the table shows that the variance of true inflation, $\sigma^2_{\pi^T}$, rises by a factor of almost 5.7 to 10.4 as we move from a low inflation regime to a high inflation regime. This is the standard result that variance of inflation rises with inflation.

However, we are able to drill deeper into this result to understand its cause.As one can see from the third and fourth rows of Table 3 the rise in the variance of true inflation is driven by two forces. First and foremost, we see that the variance of the aggregate component of inflation, $\sigma^2_{\mu_t}$, rises by a factor of 4 in the endogenous knot specification, which is consistent with the idea that the variance of aggregate shocks rise as inflation rates rise. Second, we see that variance of idiosyncratic price shocks, $\sigma^2_{\nu_t}$, doubles, which suggests that rising price dispersion—a feature of many staggered pricing models—also increases with inflation.

[15] As a check on our results we can also solve for the translog demand parameter, γ, which comes out equal to -0.68, implying that firm with a 20 percent market share would have a 29 percent markup. Our procedure is surely not the most efficient means of estimating this demand parameter; we report the number simply to demonstrate that our approach does not imply an implausible elasticity.

While the data also indicate that the variance of CPI noise rises with inflation, the increase is much smaller: as we move from low to high inflation regimes, CPI noise only rises half as much as the variance of true inflation. This increase comes from a number of sources. The rise in idiosyncratic price shocks, $\sigma^2_{\nu_t}$, interacts with the substantial rise in lower level measurement errors, $\sigma^2_{\delta_t}$, reinforcing the level of measurement error. Similarly, upper level weighting errors, $\sigma^2_{\epsilon_t}$, also rise, further boosting measurement error. However, despite these effects, the CPI's signal-to-noise ratio almost doubles in the high inflation regime, which explains why the CPI is a much more reliable predictor of inflation when inflation is high.

4.3 Robustness Check: Are the results due to dataset differences?

An obvious concern about our results is that we are working with two different datasets—the Nikkei POS and the CPI—and that our results may be due to the fact that the data collection methods differ. In order to test whether this is driving our results we need to replicate the Japanese CPI using the Nikkei POS data. This is not a trivial exercise as the purposive sampling method used by SBJ uses a non-random selection of goods. Fortunately, Imai et al. [2012] used the CPI price quote descriptions to identify the barcodes that match these specifications in the Nikkei data and then replicated the Japanese CPI methodology on this sub-sample of the Nikkei data for 2000–10.

Table 4 reports the results of replicating the regressions in Table 2 on the Imai et al. [2012] set of barcodes believed to be used in the Japanese CPI. A striking feature of these results is how similar the two sets of results are. One cannot statistically reject that the coefficients in in Table 4 are the same as their counterparts in in Table 2. These results indicate that the reason for the non-linearity is not based on differences in the underlying price data in the Nikkei and CPI samples, but must emanate from the different methodologies used to construct the price indexes.

Table 4
Replicated CPI vs. Tornqvist Inflation Regressions (Lag-11 Newey-West Standard Errors)

	Tornqvist	Tornqvist	Tornqvist	Tornqvist
Replicated CPI	0.671***	0.608***		
	(0.0947)	(0.0612)		
Replicated CPI^2		0.0655***		
		(0.0209)		
Replicated CPI ($\leq$ 1%)			0.434***	
			(0.0972)	
Replicated CPI ($>$ 1%)			1.075***	
			(0.0939)	
Replicated CPI ($\leq$ 1.507%)				0.464***
				(0.0895)
Replicated CPI ($>$ 1.507%)				1.181***
				(0.124)
Constant	-0.622***	-0.849***	-0.852***	-0.804***
	(0.150)	(0.189)	(0.176)	(0.164)
Observations	113	113	113	113
Adjusted R^2	0.748	0.794	0.810	0.812

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Index begins in 2000

4.4 Are other price index methodologies superior?

There have been a number of analyses that have suggested that one way to improve the CPI is to move from Dutot index of prices (which is comprised of a simple average of prices) to a Jevons index (which uses a geometric average of prices) at the lower level. Both methods are acceptable according to ILO standards, and the ILO reports that slightly more countries use the Dutot method than the Jevons.[16] For example, Belgium, Germany, and the U.K. also use Dutot indexes. However, many countries, including the U.S., switched to the Jevons methodology in the 1990s because it better controls for consumer substitution. It therefore is reasonable to ask whether a methodology akin to that used by the U.S. in the construction of the PCE deflator eliminates the non-linearity. In order to assess the importance of arithmetic versus geometric averaging, we replicated the methodology used by the BLS in the selection of the price quotes, and then constructed a price index based on the PCE deflator that consisted of geometric averaging at the lower level and a Törnqvist index at the upper level.[17] The results from this exercise are presented in Table 5. If we compare our results based on US methodology with those using Japanese methodology in Tables 2 or 4, we see that using geometric averaging improves our measure of inflation both in terms of raising the R^2 and in terms of reducing the level of the bias. We can see this most clearly in Figure 8, which plots the movements of the bias in the grocery CPI ($\pi_t^{CPI} - \pi_t^T$) against its level (π_t^{CPI}) in green, and the movement of the bias in our replication of the PCE deflator ($\pi_t^{PCE} - \pi_t^T$) against its level (π_t^{PCE}) in blue. As one can see from the plots, both indexes tend to overstate

[16] See `http://www.ilo.org/public/english/bureau/stat/download/cpi/survey.pdf`.

[17] In order to do this we replicated both the sample size and the sampling procedure used by the BLS. The BLS collects prices for 85,000 products each month. These products are classified into 305 entry level items (ELIs), of which 62 are food as opposed to 135 items in Japan. We chose the number of store-barcodes according to the following formula:

$$\frac{\text{62 Food ELIs}}{\text{305 Total ELIs}} \times \frac{\text{85,000 price quotes}}{\text{135 Japan CPI items}} = \text{128 store-barcodes per Japan CPI item}$$

The BLS chooses products for inclusion on a rotating basis. The BLS methodology replaces 1/16th of their sample of price quotes each quarter, and we replicated this in our data. We did this by resampling 8 store-barcodes in each of our item categories each quarter. Selection probability was a store-barcode's share of the previous quarter's sales within its CPI item. Sampling weights were based on the sales of each barcode in the two years prior to each base year.

inflation. However, the extent to which the Japanese CPI overstates inflation increases much more dramatically than the upward bias in the replicated PCE deflator. The upward bias in the Japanese CPI is largest at 1.5 percentage points when measured inflation approaches 2 percent and decreases thereafter, while the upward bias in the replicated PCE deflator increases consistently but slowly with the level of measured inflation never reaching more than one percentage point. Furthermore, although our replicated PCE deflator indicates that a substantial share of movement in the PCE is noise at low inflation rates, our replicated PCE deflator does appear more accurate than the Japanese CPI. A rise of measured inflation from 0 percent to 2 percent is associated with a 1.4 percent increase in inflation if when using the PCE deflator methodology but only 1 percent when using the Japanese CPI methodology.

Table 5
Replicated PCE vs. Tornqvist Inflation Regressions (Lag-11 Newey-West Standard Errors)

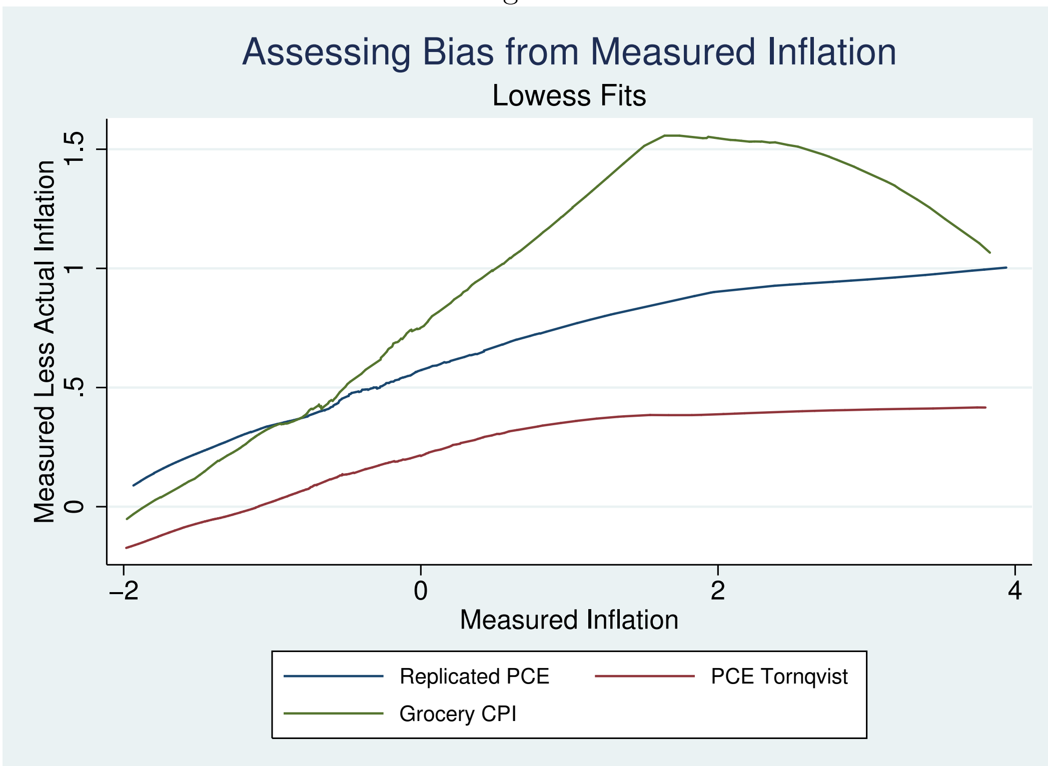
	Tornqvist	Tornqvist	Tornqvist	Tornqvist
Replicated PCE	0.925***	0.789***		
	(0.0791)	(0.0480)		
Replicated PCE^2		0.0635***		
		(0.0121)		
Replicated PCE ($\leq$ 1%)			0.668***	
			(0.0623)	
Replicated PCE (> 1%)			1.202***	
			(0.0964)	
Replicated PCE ($\leq$ 2.251%)				0.725***
				(0.0663)
Replicated PCE (> 2.251%)				1.454***
				(0.0812)
Constant	-0.433***	-0.645***	-0.657***	-0.591***
	(0.0877)	(0.0950)	(0.0992)	(0.0910)
Observations	240	240	240	240
Adjusted R^2	0.898	0.923	0.921	0.925

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Index starts in January 1991, the first month we have 2 full calendar years' worth of data.

Figure 8



One obvious question is how important are sampling errors versus formula errors in the generation of measurement errors in price indexes. We can easily address this question in the case of the PCE deflator methodology by maintaining the BLS sampling technique but switching the price aggregation method from a simple geometric average (the Jevons index) to the Törnqvist. By constructing this "PCE Törnqvist" index, we eliminate all of the formula bias in the price estimate but keep errors due to sampling only a subset of prices. Figure 8 and Table 6 report the results of this exercise. Our results suggest that eliminating the formula bias substantially improves the accuracy relative to the purposive sampling approach. Figure 8 shows that moving from the PCE deflator formula to a Törnqvist index yields a much better predictor of the level of true inflation: the bias drops to around a third of the bias in the replicated PCE deflator. The relationship between the Törnqvist index with the replicated BLS sample is still non-linear, but the kink is at a lower level of inflation than the kink in either the replicated PCE deflator or the official grocery CPI. Moreover, the bias becomes fairly stable when measured inflation is above 1 percent. In fact, the final column in Table 6 shows that the PCE deflator Törnqvist index moves almost one-to-one with true inflation when inflation exceeds 0.6 percent per year. In other words, if the BLS used the correct aggregation formula, actual and measured inflation would move one-to-one at most levels of inflation even if the sampling methodology were unchanged. This establishes that it is the use of the wrong formula and not the sampling techniques that is the principal culprit generating measurement errors in the PCE deflator methodology.

Table 6
Tornqvist PCE (All Stores) vs. Tornqvist (All Stores) Inflation Regressions (Lag-11 Newey-West Standard Errors)

	Tornqvist	Tornqvist	Tornqvist	Tornqvist
Tornqvist PCE	0.896***	0.818***		
	(0.0465)	(0.0206)		
Tornqvist PCE^2		0.0420***		
		(0.00401)		
Tornqvist PCE ($\leq 1\%$)			0.710***	
			(0.0341)	
Tornqvist PCE ($> 1\%$)			1.120***	
			(0.0306)	
Tornqvist PCE ($\leq .611\%$)				0.695***
				(0.0317)
Tornqvist PCE ($> .611\%$)				1.087***
				(0.0306)
Constant	-0.0610	-0.261***	-0.286***	-0.311***
	(0.0614)	(0.0495)	(0.0557)	(0.0531)
Observations	262	262	262	262
Adjusted R^2	0.957	0.975	0.974	0.974

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

4.5 Applicability of our results for the broader CPI

Another way of seeing that formula errors are driving our results more than sampling errors can be garnered from realizing that the CPI errors are not mitigated much by building indexes that use more categories of goods. We can obtain some sense of how much working with data for a larger set of items might matter by bootstrapping the underlying errors in the CPI and seeing how the variance of the CPI falls with as we average over more product categories. Any CPI measure of aggregate prices can be thought of as a weighted average over a set of K individual item indexes. Thus we can write:

$$\pi_t^{CPI_K} = \sum_{i \in K} w_{i,t}^{CPI_K} \pi_{i,t}^{CPI}, \tag{12}$$

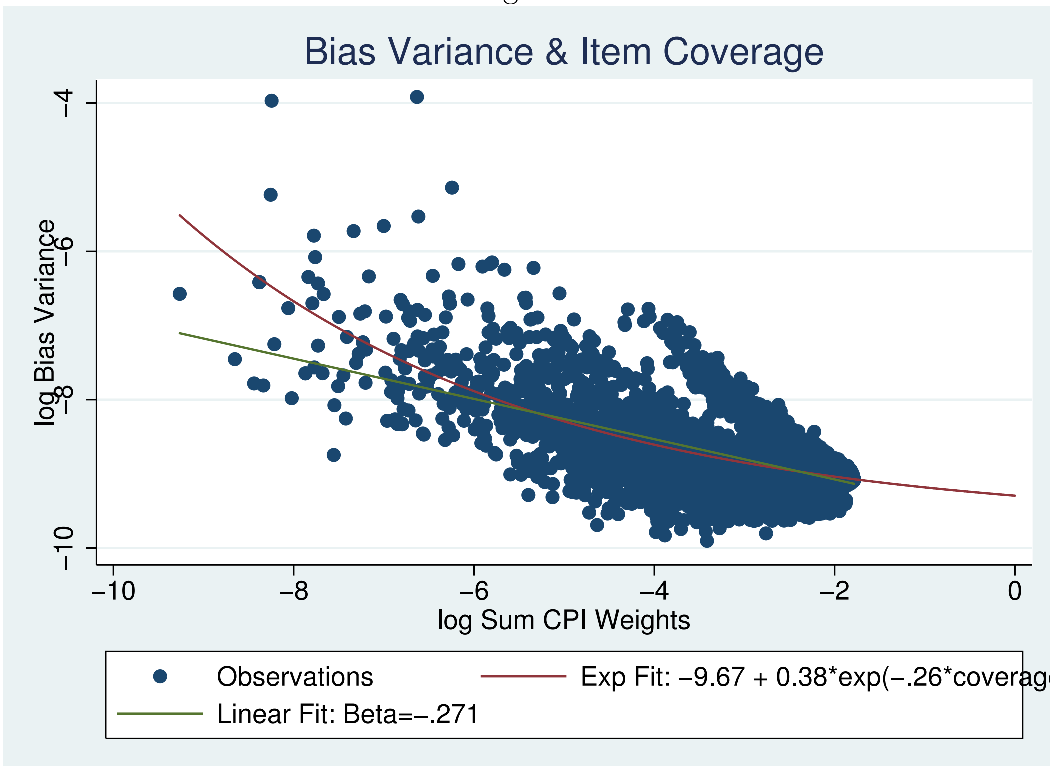
where $\pi_t^{CPI_K}$ is the CPI computed over set of K items, $w_{i,t}^{K}$ is the weight of item i in a CPI computed over set of K items, and $\pi_{i,t}^{CPI}$ is the CPI item index. Similarly, one can define a Törnqvist index over the same set of items:

$$\pi_t^{T_K} = \prod_{i \in K} \left(\pi_{i,t}^{T}\right)^{w_{i,t}^{T_K}} - 1, \tag{13}$$

where $\pi_t^{T_K}$ is a Törnqvist index computed over the K items, $w_{i,t}^{T_K}$ is the Törnqvist weight of item i in a Törnqvist index computed over the K items, and $\pi_{i,t}^{T}$ is the Törnqvist item index. We now can define the bias in the K item index as $\phi_t^K \equiv \pi_t^{CPI_K}$-$\pi_t^{T_K}$, and its variance across all time periods as $\sigma^2_{\phi_K}$. Similarly, we can define share of of the K items in the *full* CPI as $W_K = \frac{0.168}{T} \sum_{t \in T, i \in K} w_{i,t}^{CPI_K}$, where T is the number of time periods, and 0.168 is the share of the total CPI expenditure categories in our data. For each value of K from one to 178, we can draw fifty samples with replacement and record the values of $\sigma^2_{\phi_K}$ and W_K.

Figure 9 plots the result of this exercise. Not surprisingly, we see that the bias variance declines with sample size, but what is striking is that the impact of increasing the sample size has a rapidly diminishing effect on reducing the variance. This is exactly what one might have expected given that some of the variance of the error is due to formula errors

Figure 9



and not due to simply having a larger sample. Indeed if we fit a Weibull function to this distribution, we estimate that using the full sample of all CPI products would only reduce the variance by 30 percent relative to our sample, and this result, of course, assumes that the sampling and formula errors in the non-grocery components of the CPI are not higher than those within the grocery sector. Thus, it appears that the mere fact that the CPI is computed over more items is unlikely to dramatically alter the fact that formula biases generate significant inflation measurement errors that cannot be averaged away.

4.6 Can time averaging reduce non-linearities?

Comprehensive overhaul of a country's price indexes is not easy, so one question that remains is whether there are any quick fixes that could mitigate this problem. One such approach is to use time averaging and compute inflation over longer periods: perhaps a 24-, 36-, or 48-month inflation rate will be more accurate than a 12-month one. Averaging might help eliminate errors if the covariance of inflation in consecutive years was positive. The intuition behind this insight comes from the fact that if true inflation rates are positively correlated but the noise is not, then the noise will tend to cancel as we average over longer time periods, but the true inflation signal will amplify. This is essentially the same intuition about why a sample average produces a lower variance estimate of a mean than a single observation.[18]

We can see the impact of time averaging in Table 7. Since most of the non-linear relationship between CPI inflation and actual inflation can be captured by the quadratic functional form (because there are not many periods of severe deflation in Japan), we will focus our results on this specification. As one can see, computing inflation over 24, 36, or 48 month periods does not tend to change the coefficients much, although the standard errors rise, presumably because longer differences require us to throw out more data.

Table 7
Grocery CPI vs. Item Tornqvist Inflation Regressions (Newey-West Standard Errors)

Base Gap	12	24	36	48
Grocery CPI	0.553***	0.571**	0.621**	0.633***
	(0.151)	(0.221)	(0.241)	(0.198)
Grocery CPI^2	0.119**	0.164**	0.153*	0.110
	(0.0494)	(0.0753)	(0.0906)	(0.101)
Constant	-0.908***	-0.876***	-0.823***	-0.785***
	(0.224)	(0.263)	(0.225)	(0.190)
Implied Price Stability Target	1.285	1.152	1.052	1.049
Adjusted R^2	0.717	0.727	0.732	0.773

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

[18]Temporal aggregation reduces the variance of random variables with non-negative serial correlation and the rate of decay is increasing with the extent to which the variable is serially correlated (see, for example, Wei [1990]). Let $\Pi^T_{t,m} = \frac{1}{m}\sum_{t=1}^{m}\pi^T_{t-m+1}$ and $\Phi^T_{t,m} = \frac{1}{m}\sum_{t=1}^{m}\phi_{t-m+1}$ be true inflation and the CPI bias aggregated over m periods and $\beta^m = Var(\Pi^T_{t,m})/[Var(\Pi^T_{t,m}) + Var(\Phi_{t,m})]$ is the conditional expectation coefficient on expected annualized inflation implied by equation 11. In the extreme case, in which we assume that true inflation is positively serially correlated but the noise is not, then the variance of true inflation, $Var(\Pi^T_{t,m})$, will decay faster with the order of temporal aggregation, m, than the variance of the measurement error, $Var(\Phi_{t,m})$ and, therefore, that the conditional expectation coefficient, β_m, increases towards one as m increases.

The results in this section provide two important takeaways for understanding the implications of our technical results when using CPI data. First, the fact that formula biases are not dramatically reduced when moving from the analyses of subsamples of the CPI to full samples means that CPI index errors are likely to be a problem for inflation measurement in low inflation regimes. Second, one possible solution to these problems is to measure inflation using longer time averages.

5 Conclusion

This paper shows that the relationship between the economic concepts of inflation and the inflation indexes generated by statistical agencies is nonlinear. In particular, changes in the CPI and PCE deflator overstate changes in true inflation when inflation is low, and are only accurate measures when inflation is high. This result stems from the fact that much of the movement in the CPI (or our replicated PCE deflator) in low inflation regimes arises from formula errors in the computation of price indexes. Since the variance of true inflation is high in high inflation regimes, the signal-to-noise ratio of the CPI rises with inflation making it more accurate in inflationary periods.

Moreover, our estimates suggest that the the biases of official inflation indexes are not constant. Inflationary biases peak when official inflation rates are at two percent per year, but approach zero at higher inflation rates. Thus, there is no single bias number for the CPI. Moreover, we are also able to show that even a chained official index, like the PCE deflator, has an upward bias relative to a Törnqvist index. This bias is largely due to the usage of unweighted geometric averages of price quotes at the lower level.

Our results have a number of implications for policy. First, given the biases inherent in the Japanese CPI, an inflation target of 2 percent approximately corresponds to price stability. Similarly, our results suggest that price stability in the U.S. would correspond to a PCE deflator inflation rate of 1 percent. These result also suggest that the switch to geometric averaging at the lower level of the U.S. CPI and chaining at the upper level have not eliminated the upward bias in the PCE deflator or chained-CPI. This implies that indexing benefits to a chained index may provide real income gains to recipients.

Second, our results also imply that central banks would do well to pay less attention to official inflation measures in low inflation regimes. Similarly, economists using official price indexes to measure inflation should be cognizant of the fact that these measures do not always move one to one with actual inflation.

References

Naohito Abe and Akiyuki Tonogi. Micro and macro price dynamics in daily data. *Journal of Monetary Economics*, 57(6):716–728, 2010.

Kosuke Aoki. On the optimal monetary policy response to noisy indicators. *Journal of Monetary Economics*, 50(3):501–523, 2003.

Laurence Ball. Why does high inflation raise inflation uncertainty? *Journal of Monetary Economics*, 29(3):371–388, 1992.

Laurence Ball and Stephen G. Cecchetti. Inflation and uncertainty at short and long horizons. *Brookings Papers on Economic Activity*, 1990(1):215–254, 1990.

Laurence Ball, N. Gregory Mankiw, and David Romer. The new Keynesian economics and the output-inflation trade-off. *Brookings Papers on Economic Activity*, 1988(1):1–82, 1988.

Christian Broda and David E. Weinstein. Product creation and destruction: Evidence and price implications. *American Economic Review*, 100:691–723, 2010.

Stephen G. Cecchetti. Measuring short-run inflation for central bankers. *Review of the Federal Reserve Bank of St. Louis*, 79(3):143–156, 1997.

Alex Cukierman and Allan H. Meltzer. A theory of ambiguity, credibility, and inflation under discretion and asymmetric information. *Econometrica*, 54(5):1099–1128.

W Erwin Diewert. Superlative index numbers and consistency in aggregation. *Econometrica*, pages 883–900, 1978.

W. Erwin Diewert and Alice O. Nakamura. *Essays on Index Number Theory: Volume 1*. North-Holland, 1993.

Stanley Fischer. Relative shocks, relative price variability, and inflation. *Brookings Papers on Economic Activity*, 1981(2):381–441, 1981.

Milton Friedman. Nobel lecture: Inflation and unemployment. *The Journal of Political Economy*, pages 451–472, 1977.

Etienne Gagnon. Price setting during low and high inflation: Evidence from mexico. *The Quarterly Journal of Economics*, 124(3):1221–1263, 2009.

Jerry Hausman. Sources of bias and solutions to bias in the consumer price index. *the Journal of Economic Perspectives*, 17(1):23–44, 2003.

Satoshi Imai, Chihiro Shimizu, and Tsutomu Watanabe. How fast are prices in japan falling? Discussion papers 12075, Research Institute of Economy, Trade and Industry (RIETI), 2012.

Michael T. Kiley. Endogenous price stickiness and business cycle persistence. *Journal of Money, Credit and Banking*, pages 28–53, 2000.

David E. Lebow and Jeremy B. Rudd. Measurement error in the consumer price index: Where do we stand? *Journal of Economic Literature*, 41(1):159–201, 2003.

Arthur M. Okun. The mirage of steady inflation. *Brookings Papers on Economic Activity*, 1971(2):485–498, 1971.

Richard W. Parks. Inflation and relative price variability. *The Journal of Political Economy*, pages 79–95, 1978.

Marshall Reinsdorf and Jack E. Triplett. A review of reviews: Ninety years of professional thinking about the consumer price index. In *Price Index Concepts and Measurement*, pages 17–83. University of Chicago Press, 2009.

Matthew D. Shapiro and David W. Wilcox. Mismeasurement in the consumer price index: An evaluation. In *NBER Macroeconomics Annual 1996, Volume 11*, pages 93–154. MIT Press, 1996.

David J. Stockton. Relative price dispersion, aggregate price movement, and the natural rate of unemployment. *Economic Inquiry*, 26(1):1–22, 1988.

Lars E.O. Svensson and Michael Woodford. Indicator variables for optimal policy. *Journal of Monetary Economics*, 50(3):691–720, 2003.

John B. Taylor. On the relation between the variability of inflation and the average inflation rate. In *Carnegie-Rochester Conference Series on Public Policy*, volume 15, pages 57–86. Elsevier, 1981.

Daniel R. Vining and Thomas C. Elwertowski. The relationship between relative prices and the general price level. *American Economic Review*, 66(4):699–708, 1976.

W.W.S. Wei. *Time series analysis: univariate and multivariate methods*. Addison-Wesley, 1990.

Appendix A Derivations

Variance of the CPI: The CPI is related to the Törnqvist index by the following equation

$$\pi_t^{CPI} := \sum_i (w_{it} + \epsilon_{it}) (\pi_{it} + \delta_{it}) = \pi_t^T + \sum_i w_{it}\delta_{it} + \sum_i \epsilon_{it}\pi_{it} + \sum_i \epsilon_{it}\delta_{it}. \tag{14}$$

By expanding (14), we obtain

$$\begin{aligned}
V\left(\pi_t^{CPI}\right) &= V\left(\pi_t^T\right) + V\left(\sum_i w_{it}\delta_{it}\right) + V\left(\sum_i \epsilon_{it}\pi_{it}\right) + V\left(\sum_i \epsilon_{it}\delta_{it}\right) \\
&+ 2Cov\left(\pi_t^T, \sum_i w_{it}\delta_{it}\right) + 2Cov\left(\pi_t^T, \sum_i \epsilon_{it}\pi_{it}\right) + 2Cov\left(\pi_t^T, \sum_i \epsilon_{it}\delta_{it}\right) \\
&+ 2Cov\left(\sum_i w_{it}\delta_{it}, \sum_i \epsilon_{it}\pi_{it}\right) + 2Cov\left(\sum_i w_{it}\delta_{it}, \sum_i \epsilon_{it}\delta_{it}\right) + 2Cov\left(\sum_i \epsilon_{it}\pi_{it}, \sum_i \epsilon_{it}\delta_{it}\right).
\end{aligned}$$

Notice that all the covariance terms are zero because the error terms are independent. For example, consider the first covariance term. We know that the expected value of the weighted average of the error terms is zero because

$$E\left[\sum_i w_{it}\delta_{it}\right] = \sum_i E\left[w_{it}\delta_{it}\right] = \sum_i E\left[w_{it}\right] E\left[\delta_{it}\right] = 0$$

which gives

$$\begin{aligned}
nCov\left(\pi_t^T, \sum_i w_{it}\delta_{it}\right) &= E\left[\left(\sum_i w_{it}\pi_{it}\right)\left(\sum_i w_{it}\delta_{it}\right)\right] = E\left[\sum_{i,j} w_{it}w_{jt}\pi_{it}\delta_{jt}\right] \\
&= \sum_{i,j} E\left[w_{it}w_{jt}\pi_{it}\right] E\left[\delta_{jt}\right] = 0.
\end{aligned}$$

A similar argument holds for all other covariance terms. Therefore,

$$V\left(\pi_t^{CPI}\right) = V\left(\pi_t^T\right) + V\left(\sum_i w_{it}\delta_{it}\right) + V\left(\sum_i \epsilon_{it}\pi_{it}\right) + V\left(\sum_i \epsilon_{it}\delta_{it}\right). \tag{15}$$

We can write the second term of (15) in terms of the underlying variances of the errors through a bit of algebra. First recall $E\left[\sum_i w_{it}\delta_{it}\right] = \sum_i E\left[w_{it}\delta_{it}\right] = \sum_i E\left[w_{it}\right] E\left[\delta_{it}\right] = 0.$

Now we can write,

$$
\begin{aligned}
V\left(\sum_i w_{it}\delta_{it}\right) &= E\left[\left\{\frac{1}{n}\sum_{i,j}\left(s_{it-1}+\frac{\gamma}{2}\left(\nu_{it}-\nu_{jt}\right)\right)\delta_{it}\right\}^2\right] \\
&= \frac{1}{n^2}\sum_{i,j,k}E\left[\left(s_{it-1}+\frac{\gamma}{2}\left(\nu_{it}-\nu_{jt}\right)\right)\left(s_{it-1}+\frac{\gamma}{2}\left(\nu_{it}-\nu_{kt}\right)\right)\delta_{it}^2\right] \quad (\because \delta_{it}\perp\delta_{jt} \text{ if } i\neq j) \\
&= \frac{\sigma_{\delta_t}^2}{n^2}\sum_{i,j,k}E\left[\left(s_{it-1}^2+\frac{\gamma^2}{4}\left(\nu_{it}-\nu_{jt}\right)\left(\nu_{it}-\nu_{kt}\right)\right)\right] \quad (\because E\left[\nu_{it}-\nu_{kt}\right]=0\forall i,k) \\
&= \frac{\sigma_{\delta_t}^2}{n^2}\left(\sum_{i,j,k}s_{it-1}^2+\frac{\gamma^2}{4}\left(\sum_{i,j,k}\underbrace{E\left[\nu_{it}^2\right]}_{=\sigma_{\nu_t}^2}-\sum_{i,j,k}\underbrace{E\left[\nu_{it}\nu_{kt}\right]}_{=\sigma_{\nu_t}^2 1_{\{i=k\}}}-\sum_{i,j,k}\underbrace{E\left[\nu_{jt}\nu_{it}\right]}_{=\sigma_{\nu_t}^2 1_{\{j=i\}}}+\sum_{i,j,k}\underbrace{E\left[\nu_{jt}\nu_{kt}\right]}_{=\sigma_{\nu_t}^2 1_{\{j=k\}}}\right)\right) \\
&= \frac{\sigma_{\delta_t}^2}{n^2}\left(\sum_i s_{it-1}^2\sum_{j,k}1+\frac{\gamma^2}{4}\sigma_{\nu_t}^2\left(\sum_{i,j,k}1-\sum_{i,j,k}1_{\{i=k\}}-\sum_{i,j,k}1_{\{j=i\}}+\sum_{i,j,k}1_{\{j=k\}}\right)\right) \\
&= \frac{\sigma_{\delta_t}^2}{n^2}\left(n^2\sum_i s_{it-1}^2+\frac{\gamma^2}{4}\sigma_{\nu_t}^2\left(n^3-n^2-n^2+n^2\right)\right) \\
&= \sigma_{\delta_t}^2\left(\sum_i s_{it-1}^2+\frac{\gamma^2}{4}\left(n-1\right)\sigma_{\nu_t}^2\right).
\end{aligned}
$$

We can rewrite the third term of (15) in terms of the underlying variances by first remembering that $E\left[\sum_i \epsilon_{it}\pi_{it}\right]=\sum_i E\left[\epsilon_{it}\right]E\left[\pi_{it}\right]=0$. The independence of ϵ_{it} guarantees that the cross terms disappear, and we get

$$
\begin{aligned}
V\left(\sum_i \epsilon_{it}\pi_{it}\right) &= E\left[\left(\sum_i \epsilon_{it}\pi_{it}\right)^2\right]=\sum_i E\left[\left(\epsilon_{it}\pi_{it}\right)^2\right]=\sum_i E\left[\epsilon_{it}^2\right]E\left[\pi_{it}^2\right] \\
&= n\sigma_{\epsilon_t}^2\left(\sigma_{\mu_t}^2+\sigma_{\nu_t}^2\right).
\end{aligned}
$$

Finally, since the error terms are iid when $i\neq j$ and $E\left[\epsilon_{it}\right]=E\left[\delta_{it}\right]=0$, the fourth term of (15) can be rewritten as

$$
V\left(\sum_i \epsilon_{it}\delta_{it}\right)=n\sigma_{\epsilon_t}^2\sigma_{\delta_t}^2.
$$

In summary,

$$\begin{aligned} V\left(\pi_t^{CPI}\right) &= V\left(\pi_t^T\right) + \sigma_{\delta_t}^2 \left(\sum_i s_{it-1}^2 + \frac{\gamma^2}{4}(n-1)\,\sigma_{\nu_t}^2\right) + n\sigma_{\epsilon_t}^2\left(\sigma_{\mu_t}^2 + \sigma_{\nu_t}^2\right) + n\sigma_{\epsilon_t}^2\sigma_{\delta_t}^2 \\ &= V\left(\pi_t^T\right) + \sigma_{\delta_t}^2 \left(\sum_i s_{it-1}^2 + \frac{\gamma^2}{4}(n-1)\,\sigma_{\nu_t}^2\right) + n\sigma_{\epsilon_t}^2\left(\sigma_{\mu_t}^2 + \sigma_{\nu_t}^2 + \sigma_{\delta_t}^2\right) \end{aligned}$$

Covariance of the CPI and true inflation: Note $Cov\left(\pi_t^T, \sum_i w_{it}\delta_{it}\right) = Cov\left(\pi_t^T, \sum_i \epsilon_{it}\pi_{it}\right) = Cov\left(\pi_t^T, \sum_i \epsilon_{it}\delta_{it}\right) = 0$. Hence,

$$\begin{aligned} Cov\left(\pi_t^{CPI}, \pi_t^T\right) &= V\left(\pi_t^T\right) + Cov\left(\pi_t^T, \sum_i w_{it}\delta_{it}\right) + Cov\left(\pi_t^T, \sum_i \epsilon_{it}\pi_{it}\right) + Cov\left(\pi_t^T, \sum_i \epsilon_{it}\delta_{it}\right) \\ &= V\left(\pi_t^T\right). \end{aligned}$$

Appendix A.1 Decomposing CPI Measurement Error

We can use the upper-level components of the item Törnqvist to study the measurement error in the Japanese CPI through the lens of the theoretical framework presented in Section 3 above. In this section, we derived the relationship between the aggregate measurement error in the Japanese CPI and the measurement errors in the underlying item-level components of this index (Equation 8). We can now define these errors in the weight and item inflation index components empirically as

$$\varepsilon_{i,t}^{IT} = w_{i,t}^{CPI} - w_{i,t}^T \tag{16}$$

$$\delta_{i,t}^{IT} = \pi_{i,t}^{CPI} - \pi_{i,t}^T \tag{17}$$

The relationship between these errors and the aggregate measurement error defined in Equation 8 also depended on the aggregate and idiosyncratic components of the true item level inflation. We extract these components by running the following unweighted regression: $\pi_{i,t}^T = \mu_t + \nu_{i,t}$.

In Section 3, we additionally formally characterized the measurement error in the Japanese

CPI. We found that the conditional expectation of true inflation based on observed inflation depends crucially on the variances of the inflation components (μ_t, ν_{it}) and errors (ϵ_t, δ_{it}) listed above. To characterize the inference problem empirically, we estimate these variances. We define the variance of the aggregate component of inflation, μ_t as

$$\hat{\sigma}^2_{\mu_t} = \frac{1}{N}\sum_{t=1}^{N}\left(\hat{\mu}_t - \bar{\hat{\mu}}\right)^2, \text{ where } \bar{\hat{\mu}} = \frac{1}{N}\sum_{t=1}^{N}\hat{\mu}_t \tag{18}$$

and where N is the number of observations (months) in the sample. We then calculate the variances of ε, δ, ν within each item as

$$\hat{\sigma}^2_{x_{i,t}} = \frac{1}{N}\sum_{t=1}^{N}\left(\hat{x}_{i,t} - \bar{\hat{x}}_i\right)^2, \text{ where } \bar{\hat{x}}_i = \frac{1}{N}\sum_{t=1}^{N}\hat{x}_{i,t} \tag{19}$$

where $x = \varepsilon, \delta, \nu$ specifies the variable under consideration. We aggregate these variances across items using Törnqvist item weights:

$$\hat{\sigma}^2_{x_t} = \sum_{i=1}^{I} w^T_{i,t}\hat{\sigma}^2_{x_{i,t}}$$

We use the values of each of these estimated variances to solve for the implied value of γ_t by equating

$$\frac{Cov\left(\pi^T_t, \pi^{CPI}_t\right)}{Var\left(\pi^{CPI}_t\right)} = \frac{Var\left(\pi^T_t\right)}{Var\left(\pi^T_t\right) + \sigma^2_{\delta_t}\sum_{i=1}^{n} s^2_{it-1} + n\left[\sigma^2_{\epsilon_t}\sigma^2_{\mu_t} + \sigma^2_{\epsilon_t}\sigma^2_{\nu_t} + \sigma^2_{\epsilon_t}\sigma^2_{\delta_t} + \frac{\gamma^2_t}{4}\left(\sigma^2_{\delta_t}\sigma^2_{\nu_t}\right)\right]} \tag{20}$$

using variables' period t values then solving for γ_t.

Appendix B Index Calculation

Our price and quantity data are at a daily frequency, so we sum over quantity and expenditure to find their monthly values. We then back out the monthly price by dividing. For all days

D_t in the month t, in each store s and JAN code j, we define

$$p_{j,s,t} = \frac{\sum_{d \in D_t} p_{j,s,d} q_{j,s,d}}{\sum_{d \in D_t} q_{j,s,d}} \tag{21}$$

We aggregate the daily item purchase and price data to a monthly frequency denoting the sales-weighted average price charged by store s for JAN code j in month t as $p_{j,s,t}$ and the corresponding sales quantity $q_{j,s,t}$. In order to do the decompositions indicated in equation 4, we need to work with Törnqvist index that is composed of Törnqvist item indexes. Each "item" level index is a Törnqvist index of the twelve-month within-store JAN code price changes aggregated across all JAN codes that belong to an "official" item category. The item-level Törnqvist index for item i in month t, $\pi^T_{i,t}$, is defined as follows:

$$1 + \pi^T_{i,t} = \prod_{j \in J_{i,s,[t-12,t]}} \prod_{s \in S_{[t-12,t]}} \left(\frac{p_{j,s,t}}{p_{j,s,t-12}} \right)^{w^T_{j,s,t}}$$

where $J_{i,s,[t-12,t]}$ is the set of item i JAN codes sold in store s in months $t-12$ and t; $S_{[t-12,t]}$ is the set of stores in the sample for months $t-12$ and t; and $w^T_{j,s,t}$ is item Törnqvist weight for JAN code j in store s, and month t given by

$$w^T_{j,s,t} = \frac{s_{j,i,s,t} + s_{j,i,s,t-12}}{2} \quad \text{for} \quad s_{j,i,s,t} = \frac{p_{j,s,t} q_{j,s,t}}{\sum_{j \in J_{i,s,[t-12,t]}} \sum_{s \in S_{[t-12,t]}} p_{j,s,t} q_{j,s,t}}$$

The item Törnqvist index aggregates these indexes across items $i = 1, ..., I$, using a weighted geometric average:

$$1 + \pi^T_t = \prod_{i=1}^{I} \left(1 + \pi^T_{i,t} \right)^{w^T_{i,t}} \tag{22}$$

with Törnqvist item weights, $w_{i,t}^{T}$ given by

$$w_{i,t}^{T}=\frac{s_{i,t}+s_{i,t-12}}{2}\quad\text{for}\quad s_{i,t}=\frac{\sum_{j\in J_{i,s,[t-12,t]}}\sum_{s\in S_{[t-12,t]}}p_{j,s,t}q_{j,s,t}}{\sum_{i=1}^{I}\sum_{j\in J_{i,s,[t-12,t]}}\sum_{s\in S_{[t-12,t]}}p_{j,s,t}q_{j,s,t}}.$$

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[19] In the theory section, we make the approximation that $\ln\left(1+\pi_t^T\right)=\pi_t^T$, but equation 22 does not make this approximation. This has virtually no impact on the results.