Information Acquisition, Resource Allocation and Managerial Incentives

Oguzhan Ozbas and Heikki Rantakari
University of Southern California
Marshall School of Business

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Abstract

A manager's incentives to acquire information about different investment alternatives and then to choose how to allocate resources among them are jointly influenced by his compensation contract and the level of resources allocated to him. We show that the optimal compensation contract induces investment allocations that are more aggressive than the first-best allocation conditional on the precision of information, while the optimal level of resources may be set above or below the first-best level, depending on whether the desired level of investment is increasing or decreasing in the precision of information. Both distortions are used to motivate further information acquisition by the manager. Finally, we show how the choice of the level of resources can be delegated to the manager without any loss in efficiency through appropriately linking managerial compensation to the level of resources requested.

1 Introduction

Much research in finance and economics examines how firms can effectively use organizational systems and processes to deal with various frictions within the firm. A voluminous principal-agent literature following Holmstrom (1979) studies how firms can design compensation systems to address motivational problems, encourage information acquisition and improve biased decisions of agent managers. Another strand of literature following Harris, Kriebel and Raviv (1982), Antle and Eppen (1985) and Harris and Raviv (1996,1998) studies how firms can design capital budgeting processes to mitigate informational problems and minimize efficiency losses in resource allocation.

While both literatures separately shed light on two organizational design problems that significantly affect firm performance (compensation systems and capital allocation processes), little work simultaneously considers them. Our paper combines elements of both literatures to begin building a comprehensive picture of how the two problems may interact. First and foremost, our model has general normative value as it applies to any manager in charge of multiple projects or investments, e.g., CEOs, division managers, etc. From a positive perspective, our model generates equilibrium behavior by managers that is systematically biased from the firm's ex post perspective but is ex ante desirable and arises from standard preferences through the structure of the optimal compensation contract.

We consider a simple agency problem where a manager needs to first acquire information regarding the quality of different investment alternatives and then allocate the available resources among them. To manage the agency problem, the firm has two design parameters at its disposal. First, the firm determines the level of resources available to the manager for investment. Second, the firm decides on the compensation structure of the manager. The manager is risk-neutral and cares only about his expected compensation and there are thus no behavioral biases relative to the firm, except to the extent that such behavior is induced in equilibrium by the optimal compensation structure of the manager. If there were no additional constraints on the problem, the solution would be simple. The firm would simply sell the investment opportunities to the manager, who then becomes a full residual claimant and makes first-best choices. To avoid this trivial solution, we assume that the manager is wealth-constrained and protected by limited liability, so that his compensation can never fall below zero.

The results that follow from the model are two-fold, separating the roles of the level of resources made available to the manager and how those resources are allocated between competing tasks. First, we show that when the manager's desired total investment is decreasing in the quality of information, then restricting the manager's access to capital below the first-best level induces the manager to make more careful decisions on how to allocate the funds and this increases his motivation to learn more about the investment alternatives. The converse, of course, is also true. When the desired level of investment is increasing in the quality of information, the manager acquires more information when provided access to a resource budget above the first-best level. In short, the optimal *level* of resources allocated to the manager may be above or below the first-best level, depending on both how the expected return to the tasks is influenced by the resources invested and how the manager is compensated based on performance, which determine whether resources and information are subsitutes or complements from the manager's perspective.

Second, we derive the optimal compensation contract, which will determine how the manager will *allocate* the resources available to him. With limited liability, we show that the optimal contract always induces the manager to exhibit over-aggressive investment behavior, in the sense that given the quality of information acquired by the manager, he chooses a resource allocation that is more aggressive than the allocation that would maximize the expected return given the quality of his information. In other words, the manager invests too much in the projects that appear more attractive and too little in projects that appear less attractive. The reason for this distortion is the same as for the distortion in the level of resources. By inducing more aggressive investment behavior, the firm is able to increase the value of information to the manager by making mistakes more costly and thus achieve more information acquisition at the same expected monetary cost.

Finally, we consider, instead of centrally determining the level of resources available to the manager, delegating this choice to the manager. Here, we reach the natural conclusion that by linking managerial compensation appropriately to the level of resources requested, we can delegate this decision without any loss of efficiency. However, if we naively linked managerial compensation directly to the true cost of resources, then the manager will generically either over-invest or under-invest from the firm's perspective. The reason for this result is also simple. When choosing how much to invest, the manager wants to maximize his expected compensation, while the firm would like to maximize expected value. Because the optimal compensation contract biases the manager away from value maximization, his perceived return to investment will also be affected. Thus, empire-building behavior may also arise simply as the by-product of efficiently motivating information acquisition.

Our result on the equilibrium aggressiveness of the resource allocation is related to works on risk-seeking behavior induced by convex compensation structures. The main modeling difference is that the manager is not gambling by turning a single risk dial but by making resource allocation decisions with greater upside potential and in favor of ex ante more attractive projects. In addition, we show that this result continues to hold at the optimal contract, which consists of at most two bonuses for exceeding given performance targets. It is thus not the convexity of the contract that induces the behavior, but the location of the bonuses. Both equilibrium biases could be eliminated by particular bonus structures, but that is suboptimal from the perspective of effort incentives. The main message of our analysis is thus that capital allocation processes cannot be fully understood without at the same time considering compensation systems, because managerial compensation is one of the key determinants behind how managers will actually value and use the capital allocated.

2 Related literature

Our paper is related to the literature on capital budgeting that has followed the contributions of Harris, Kriebel and Raviv (1982), Antle and Eppen (1985) and Harris and Raviv (1996,1998), where some of the more recent contributions include Berkovitch and Israel (2004) and Marino and Matsusaka (2005).[1] The main difference is that while much of the literature has focused on the problem of information revelation given particular managerial preferences, we focus on the motivational effects of capital budgeting and derive explicitly the optimal compensation contract, which endogenously determines the extent to which the manager's preferences will differ from the firm's.

The key building blocks of the model are (i) the ex ante information acquisition step by the manager, (ii) the resource allocation decision following the information acquisition stage, and (iii) the joint role of the compensation contract and the initial resource allocation in motivating the manager. Since the individual components have been studied before, it is instructive to relate the overall framework of our paper to papers in the literature and highlight our contribution.

First, there is a small but growing literature that has examined the dual agency problem where an agent first needs to acquire information regarding the value of a risky investment alternative and then choose whether to take the risky or risk-free alternative. Contributions examining this tradeoff include Lambert (1986), Holmstrom and Ricart I Costa (1986), Levitt and Snynder (1997) and Inderst and Klein (2007). The key similarity with this literature is that the agents are not inherently biased but may differ in their risk preferences or become biased due to the compensation contract that is offered to them. The key difference is that these papers deal with the binary choice between a risky and a risk-free alternative, whereas we consider a continuous allocation problem between two equally risky alternatives, which results in two fundamental differences. First, because the alternatives are equally risky, there is no truth-telling constraint that needs to be satisfied for the manager to reveal which alternative is better (while being at the heart of the other models).

[1] A generalized variant of the basic idea of Berkovitch and Israel (2004) can be found in Armstrong and Vickers (2010).

Second, the assumption of binary choice in the earlier works leaves the role of resources largely unexamined, whereas the use of resources is at the heart of our model, deriving predictions for both the level and allocation of resources as a strategic tool.

To illustrate this difference, consider Levitt and Snyder (1997) (or Inderst and Klein (2007)), which are most similar to ours. They show that the manager will, in equilibrium, end up choosing the risky alternative more frequently than optimal, because inducing the manager to not to go ahead is costly due to the truth-telling constraint and limited liability. One interpretation for this result is overinvestment (if the risk-free alternative is taken to be no investment), and the other is over-aggressiveness (choosing the risky alternative too frequently). In contrast, in our setting, because the level of resources is endogenous and explicitly modeled, we show that the equilibrium may exhibit either underinvestment or overinvestment in levels, but has over-aggressiveness as a general feature of the solution, in terms of excessive skewing of the final resource allocation between the alternatives (as opposed to choosing a riskier alternative).[2]

Other papers related to motivating information acquisition are Lewis and Sappington (1997), Szalay (2005,2009) and Bernardo, Cai and Luo (2009). All four papers consider variants of a mechanism design problem where, in addition to inducing the revelation of private information, the menu needs to motivate the acquisition of information in the first place. In Lewis and Sappington (1997), the motivation is achieved by increasing the difference between the compensation contracts, composed of a fixed reward and a linear cost-sharing component, in Szalay (2005), the motivation is achieved by ruling out some intermediate decisions, in Szalay (2009) by increasing the distortions in the quantity schedule, and in Bernardo, Cai and Luo (2009) by increasing disortions in the compensation contract, as in Lewis and Sappington, and also in the level of resources allocated to a project. The key similarity to these papers is the idea of increasing the sensitivity of the agent's pay to information, whether it is distortions in the capital allocation schedule, allowed decisions, compensation, or the quantity schedule, analogous to our over-aggressiveness result. The main differences are three-fold. First, these papers assume commitment by the principal to the allocation, while we illustrate how that behavior can be induced through the compensation contract of the agent.[3] Second, by focusing on a single task, all these models contain the truth-telling constraint of the agent, whereas our setting isolates the purely motivational consequences of capital use. Third, and relatedly, the two-task framework we consider is able to make the distinction between the differential effect of the *level* of resources used,

[2] See also Chen and Jiang (2004), who show that when a manager has empire-building preferences, then biasing the project acceptance rule against investment will increase his incentives to acquire information when the information generated is observable.

[3] Appendix B derives the corresponding mechanism design solution under the assumption of limited liability.

which is not a consideration in the above papers, and how those resources are used.

Some papers considering the general effort consequences of capital allocations are Paik and Sen (1995), Bernardo, Cai and Luo (2001) and Han, Hirshleifer and Persons (2009). Paik and Sen (1995) consider how the capital allocation menu is distorted if there is a direct complementarity or substitutability between capital and effort and the agent is endowed with private information regarding the production technology. Bernardo, Cai and Luo (2001) consider a model where the principal hires an agent with empire-building preferences to both reveal the quality of an investment opportunity and then exert effort to implement it, with assumed complementarity between effort and capital allocation. The key difference, in addition to qualitative predictions, is that we do not assume any ex ante relationship between the value of effort and the level of resources. Indeed, the key observation is that depending on the particular setting, the value of effort may be either increasing or decreasing in the level of resources available to the agent, which is exactly what may make the level of resources made available to the agent to be either above or below the first-best level. Finally, Han, Hirshleifer and Persons (2009) also examine the interaction between incentives and capital allocation but they do not consider optimal incentives. They show that it can be optimal to restrict the managers' access to capital when they are participating in a promotion tournament, because the value of being first and obtaining a promotion can lead to excessive risk-seeking in the absence of capital restrictions.

Finally, our work is related to the large literature on internal capital markets in that one of the stages in our model is a resource allocation decision between two investment opportunities. The difference is that the literature on internal capital markets deals with the case where the opportunities belong to separate strategic agents and the analysis focuses on the conflicting interests between the two recipients of funding to compete for that funding. In our setting, a single agent is responsible for both and that eliminates the truth-telling constraint in the present setting. Future extensions of our framework to account for the possibility of multiple agents appears promising since as with the capital budgeting literature, the literature on internal capital markets has paid only limited attention to the interaction between managerial compensation and the use of internal capital markets, instead assuming empire-building preferences. Two exceptions are Friebel and Raith (2010), who consider a resource allocation problem where two managers first attempt to generate good quality projects and then make claims regarding their need for resources and Bernardo, Cai and Luo (2004), which extends Bernardo, Cai and Luo (2001) to account for competing divisions.

3 Model

The model consists of a manager (say, a division head) and a principal. The manager is responsible for two projects or tasks and needs to allocate resources between them. One of the projects is more attractive in that the project has a greater expected marginal productivity than the other project for any given level of investment x. The manager, however, does not initially know enough to rank the projects.

At a personal cost $C(q)$, the manager can acquire information to rank the two projects correctly with probability $q \geq 1/2$. Having formed his interim belief regarding which of the projects is more productive, the manager allocates x_h to the more attractive project and x_l to the less attractive project, subject to a resource budget constraint $x_h + x_l \leq I$. We assume that the qualities of the projects are perfectly negatively correlated, so that one of the projects is always better than the other. Then, the gross profits generated by the resource allocation are given by

$$\pi = \pi_H\left(x_i\right) + \pi_L\left(x_j\right) + \varepsilon,$$

where $\pi_j\left(x\right)$ denotes the expected cash flows from a project of type $j \in \{H, L\}$ given the resource level x, with H denoting the more productive task and L denoting the less productive task, and $\varepsilon \sim G(\varepsilon)$ with $E\left(\varepsilon\right) = 0$ is a stochastic shock to the cash flow that is outside the manager's control. For the expected cash flows, we make the standard assumptions that $\frac{\partial \pi_j(x)}{\partial x} > 0$, $\frac{\partial^2 \pi_j(x)}{\partial x^2} < 0$, so that the expected cash flows are concave in the level of investment and that $\frac{\partial \pi_H(x)}{\partial x} > \frac{\partial \pi_L(x)}{\partial x}$ $\forall x$, so that the marginal return to a dollar invested in the more productive task is always higher than in the less productive task and there is no ambiguity in terms of which project is more attractive. Finally, given that the manager identifies the right ordering with probability q, we can write the expected cash flow as

$$E\left[\pi | x_h, x_l\right] = q\left(\pi_H\left(x_h\right) + \pi_L\left(x_l\right)\right) + \left(1-q\right)\left(\pi_H\left(x_l\right) + \pi_L\left(x_h\right)\right).$$

The agency problem arises from the fact that the manager does not directly care about either the gross cash flow or the cost of resources. Instead, the manager cares only about his compensation. To motivate the manager to perfom this dual task of first acquiring information q and then choosing the resource allocation $\left(x_h, x_l\right)$, the firm has two control instruments at its disposal. First, the firm offers the manager a wage contract $w\left(\pi\right)$, which ties the manager's compensation to the realized cash flow. The only restrictions we place on the wage contract are (i) limited liability, so that $w\left(\pi\right) \geq 0$ $\forall \pi$ and (ii) monotonicity, so that $w'(\pi) \geq 0$. Second, the firm determines ex ante the level of resources available to the manager, I, where the cost of resources to the organization is r.[4] We can thus write the design problem as

$$\max_{w(\pi),I} E\left[\pi - w(\pi)|x_h, x_l\right] - rI$$

$$\text{s.t. } \{q, (x_h, x_l)\} \in \arg\max E\left[w(\pi)|x_h, x_l\right] - C(q)$$
$$x_h + x_l \leq I.$$
$$w(\pi), w'(\pi) \geq 0$$

That is, the organization will choose the compensation contract $w(\pi)$ and the level of resources I to maximize its cash flow net of managerial compensation and the cost of resources, subject to the quality of information q and the resource allocation (x_h, x_l), as chosen by the manager to maximize his expected surplus $E\left[w(\pi)|x_h, x_l\right] - C(q)$.

A summary of timing and assumptions: Before proceeding with the analysis, we will briefly summarize and discuss the timing and the assumptions underlying the model. First, the game unfolds as follows. The principal offers a contract $(I, w(\pi))$ to the manager, which the manager either accepts or rejects. Second, the manager chooses the level of information acquisition q to create his posterior. Third, the manager chooses the resource allocation $(x_h(q), x_l(q))$ and the payoffs are realized. Both the distribution of the shocks $G(\varepsilon)$ and the functional form of the expected cash flow $\pi_H(x_i), \pi_L(x_j)$ are common knowledge.

For the underlying assumptions, we assume that the investment levels are purely mean-shifting for the cash-flow distribution. This assumption simplifies the analysis considerably without affecting the basic logic of the analysis.[5] Second, the model assumes that the resource allocation decision is made by the manager. This assumption stands in contrast to the more common assumption where the principal commits to a menu of resource allocations. We have made this assumption because, in practice, the manager may have considerable influence in how the resources are allocated across the tasks under his control, and even circumvent any direct demands by the principal because of the difficulties in verifying the true use of resources. For example, a manager may have a marketing budget but then discretion in how to allocate that budget across different media and which advertising agencies to use, or a manager in charge of a particular research project may have a budget for the whole project, but then have flexibility in how to allocate those resources across the sub-components of the project. Or more immediately, a fund manager chooses how

[4] In the extensions, we discuss how the choice of I could be delegated to the manager without any loss of efficiency through a contract $w(\pi, I)$ that ties managerial compensation to the level of resources invested.

[5] An earlier version of the paper considered general return distributions, and available from the authors on request.

to allocate the available funds across the different investments. In short, we believe that such allocation problems are relatively common for many managerial positions. But this assumption is also not essential to the basic logic of the analysis. In Appendix B, we solve the mechanism design version of the problem, showing how the basic logic of the analysis remains the same, although full efficiency may now be possible in some cases.

Third, the model assumes that the quality of the projects is perfectly negatively correlated. This is another simplifying assumption that allows us to decouple the effects of (i) the level of resources allocated to the manager and (ii) how aggressively those resources are allocated between the competing alternatives as the desired total level of investment is known ex ante, and also eliminates the much-analyzed screening problem in the interim stage. The setting is also equivalent to the manager being in charge of an infinite number of small tasks, each of which can be either high or low productivity with equal probability and iid draws, and where q is then the fraction of tasks correctly identified as high or low productivity.

Fourth, we assume that only the total performance of the manager is verifiable. That is, the manager cannot be compensated on the individual performance of the different investments or sub-tasks because arbitrariness in setting internal transfer prices all too often makes individual assessment of different investments difficult. The reasonableness of this assumption depends on the particular application as it depends on the fungibility of performance. For example, it may be very difficult to disentangle the impact of different marketing campaigns on the final sales of a product, while it is easier to identify the returns to individual investments of a fund manager, although even that may not be straightforward. This assumption, however, is not crucial for the analysis as the basic logic of the results continues to hold even if individual performance can be measured without any additional manipulation. Finally, we assume that the manager is protected by limited liability and is ex ante budget constrained. In the absence of these constraints, the firm could replicate the first-best solution by selling the tasks to the manager.

4 First-Best

Before considering the solution to the problem where the principal needs to rely on the manager for both the resource allocation and information acquisition decisions, it is instructive to consider what the solution would be if the principal could perform both tasks herself. Her maximization problem is

$$\max_{x_h, x_l, q} \int_0^\infty \pi \left(q f_H \left(\pi | x_h, x_l \right) + (1-q) f_L \left(\pi | x_h, x_l \right) \right) d\pi - r \left(x_h + x_l \right) - C(q),$$

where $f_H(\pi|x_h, x_l)$ and $f_L(\pi|x_h, x_l)$ are the probability distribution functions for the realized cash flow when the allocation is correct (H) and when it is wrong (L). The solution is given directly by the first-order conditions, as summarized by the following proposition:

Proposition 1 ***First-best solution:***

(i) Investment levels solve

$$q\left(\frac{\partial \pi_H}{\partial x_h}\right) + (1-q)\left(\frac{\partial \pi_L}{\partial x_h}\right) = r = q\left(\frac{\partial \pi_L}{\partial x_l}\right) + (1-q)\left(\frac{\partial \pi_H}{\partial x_l}\right).$$

(ii) The quality of information solves

$$\left(\pi_H\left(x_h\right) - \pi_H\left(x_l\right)\right) - \left(\pi_L\left(x_h\right) - \pi_L\left(x_l\right)\right) = C'(q).$$

(iii) Information and the aggressiveness of the investment are complements: $\frac{\partial x_h^{FB}}{\partial q} > 0$ *and* $\frac{\partial x_l^{FB}}{\partial q} < 0$.

(iv) Information and the total level of investment can be either complements or substitutes: $\frac{\partial\left(x_h^{FB} + x_l^{FB}\right)}{\partial q} \gtreqless 0$.

The first-order conditions summarized in (i) and (ii) contain the basic relationships among the investment levels and the quality of information, which provide the intuition for parts (iii) and (iv). First, $x_h > x_l$ whenever $q > \frac{1}{2}$ because the expected marginal productivity of the more productive task is higher for all levels of investment. Second, $\frac{\partial x_h}{\partial q} > 0$ and $\frac{\partial x_l}{\partial q} < 0$, so that more precise information allows the organization to adopt a more aggressive investment policy. The reason is that the more confident the organization is regarding having identified the better task, the less it needs to worry about low returns to x_h if it accidentally allocates it to the low-productivity task and similarly the less it needs to worry about potentially high returns to x_l. The converse of this logic, which will play an important role below when considering the motivational effects of the resource allocation, is that the investment strategy influences the value of information. In particular, the aggressiveness of the allocation and the quality of information are strategic complements, because an increase in aggressiveness makes mistakes more costly.

Third, while the investment policy becomes more aggressive with information, its impact on the level of overall investment is ambiguous, so that $\frac{\partial\left(x_h^{FB}+x_l^{FB}\right)}{\partial q} \gtreqless 0$. This is the second key element of the model, because it implies that the quality of information and the level of resources can be either strategic complements or substitutes. The intuition for this result is as follows. While the investment strategy is more aggressive, we don't know whether the increase in x_h is larger or smaller than the decrease in x_l. As an example, consider a firm investing in TV and social media advertising. If $q = \frac{1}{2}$, the firm decides to invest $\$5M$ in both campaigns. Now, suppose that the firm acquires information so that $q > \frac{1}{2}$. In response, the firm decides to invest $\$7M$ on a TV campaign and $\$2M$ on social media. The total expense is now $\$9M$ and the additional information has allowed the firm to economize on its total capital expenditure because the funds can be targeted better. But alternatively, the firm may increase its TV campaign to $\$8M$ while decreasing the social media campaign to $\$3M$, in which case the additional information leads the organization to invest more overall because it can be confident that the funds are invested appropriately.

The relationship between total investment and information depends then on the shape of the expected returns. To examine this relationship more, suppose that the expected returns are given by $\pi_i(x) = \theta_i \pi(x)$, so that the productivity is multiplicative. Then, we can reach the following proposition:

Proposition 2 ***The relationship between the precision of information and the desired investment level:***

The desired level of investment is increasing in q if $\frac{(\pi'(x_l))^2}{|\pi''(x_l)|} < \frac{(\pi'(x_h))^2}{|\pi''(x_h)|}$ and vice versa

Proof. See Appendix A.1 ■

Intuitively, the total level of investment is increasing in the precision of information if x_h is more sensitive to improvements in the precision of information than x_l, which implies that the rate at which the marginal return decreases cannot be too high. Using this condition, we can easily categorize expected return functions in terms of whether they lead to complementarity, substitutatibility or independence between the precision of information and the level of investment, as follows:

Corollary 3 ***The relationship between the precision of information and the desired level of investment for particular expected return functions:***

(i) Suppose $\pi(x) = \alpha x^{\frac{1}{g}}$, *with* $g > 1$. *Then,* $\frac{\partial\left(x_h^{FB}+x_l^{FB}\right)}{\partial q} > 0$.

(ii) Suppose $\pi(x) = \ln(1+\alpha x)$. *Then,* $\frac{\partial\left(x_h^{FB}+x_l^{FB}\right)}{\partial q} = 0$.

(iii) Suppose $\pi(x) = \alpha\left(1 - e^{-\beta x}\right)$. *Then,* $\frac{\partial\left(x_h^{FB}+x_l^{FB}\right)}{\partial q} < 0$.

The basic observation is thus as follows. While simple intuition suggests that the value of information is increasing in the resources available for investment because more resources implies greater potential total returns, the value of information is determined by the value of investing the marginal dollar correctly, which will depend on the properties of the expected returns. It is easy to construct examples of returns that will imply either complementarity or substitutability between the level of investment and the precision of information.

5 Analysis

Having considered the case where the firm is able to do both tasks, we can now consider the case where the manager is responsible for both the capital allocation (x_h, x_l) and information acquisition (q) and the resulting distortions.[6] The design problem for the principal is now

$$\max_{I,w(\pi)} \int_\theta (\pi - w(\pi))\left(qf_H(\pi|x_h,x_l) + (1-q) f_L(\pi|x_h,x_l)\right) d\pi - rI$$

$$\begin{aligned}
\text{s.t.} \qquad & q^M \in \arg\max_q \int_\theta w(\pi)\left(qf_H(\pi|x_h,x_l) + (1-q) f_L(\pi|x_h,x_l)\right) d\pi - C(q) \\
& x_i^M \in \arg\max_{x_i} \int_\theta w(\pi)\left(qf_H(\pi|x_h,x_l) + (1-q) f_L(\pi|x_h,x_l)\right) d\pi, \qquad i \in \{h,l\} \\
& \int_\theta w(\pi)\left(qf_H(\pi|x_h,x_l) + (1-q) f_L(\pi|x_h,x_l)\right) d\pi - C(q) \geq 0 \\
& w(\pi), w'(\pi) \geq 0 \\
& x_h^M + x_l^M \leq I.
\end{aligned}$$

The constraints that the principal must respect are thus as follows. First, the level of information acquisition q and the capital allocation (x_h, x_l) are chosen by the manager to maximize his expected compensation. Second, the manager must be willing to accept

[6] The case where the manager is responsible for information acquisition alone is discussed in Appendix B.

the offered contract (participation constraint). Third, the compensation contract must be weakly increasing in performance and satisfy the limited liability constraint. Finally, the manager's allocation must satisfy the resource constraint set by the principal.

To examine this problem, we will consider it in three steps. First, we will highlight the general motivational effect of the level or resources made available to the manager, which operates for all $w(\pi)$. Second, we will consider the structure of the optimal compensation contract $w(\pi)$ and its implications for both managerial behavior and the optimal level of resources. Third, we will consider how the same solution can be implemented through allowing the manager to choose I but linking his compensation on the resource allocation requested.

For the analysis, we assume that the MLRP holds so that $\frac{f_H(\pi|x_h,x_l)}{f_L(\pi|x_h,x_l)}$ is increasing in π and $\frac{f_H(\pi|x_h,x_l)}{f_L(\pi|x_h,x_l)} \to \infty$ as $\pi \to \infty$. Also, we will be using the first-order conditions for the manager's problem to characterize the solution. The manager's problem is globally concave in the precision of information q, but this need not hold for the allocation $\left(x_h^M, x_l^M\right)$. Further, since the manager's payoff is supermodular in q and the aggressiveness of the allocation, there may be multiple solutions to the manager's overall problem. However, our characterization applies to any potential solution, and thus our basic conclusions are not dependent on the uniqueness of the solution. Relatedly, we use a simple replication argument to characterize the shape of the optimal contract given the target (q, x_h, x_l) instead of the standard Lagrangian approach. Also, given the limited liability constraint, the participation constraint will not be binding and we will drop it from the rest of the analysis.[7] If the participation constraint was binding, then the cost of information acquisition would be fully covered by the compensation contract and both the level and the allocation of resources would be again set at their first-best levels.

5.1 Motivational effects of the level of resources available

To examine the motivational effects of the level of resources allocated to the manager, we can write the manager's problem as

$$\max_{x_h,x_l,q} \int_0^\infty w(\pi)\left(q f_H(\pi|x_h,x_l) + (1-q) f_L(\pi|x_h,x_l)\right) d\pi - \lambda\left(x_h + x_l - I\right) - C(q),$$

where λ is the Lagrange multiplier for the constraint $x_h + x_l \le I$. The analysis of the maximization problem establishes the following proposition:

[7] For any positive compensation, the manager could earn positive rents by simply investing randomly, so the participation constraint cannot bind.

Proposition 4 ***The motivational effect of the level of resources:***

The effort level of the manager is decreasing in the level of resources if and only if the shadow value of capital (λ) is decreasing the in the quality of information: $\frac{dq}{dI} < 0$ iff $\frac{d\lambda}{dq} < 0$.

Proof. See Appendix A.2 ■

This is the first key observation of the paper, highlighting the motivational role of the level of resources allocated to the manager: the effort of the manager is increasing in the level of resources if effort and resources are complements and vice versa. Intuitively, if resources and effort are substitutes, then by artificially withholding some resources away from the manager, we can increase the value of effort and thus induce the manager to work harder. The reason is that such withholding increases the marginal value of investing the last dollar correctly by forcing the manager to be more careful in his investments. Conversely, if resources and effort are complements, then we can motivate the manager by giving him access to more than the first-best level of resources. Further, whether the two are substitutes or complements can be determined by looking at how the precision of information influences the shadow value of capital.[8]

The novel element of this result is how this mechanism arises through the response of the allocation choices (x_h, x_l) to both the quality of information q and the total resources available I, and whereby the strategic manipulation of I influences the value of information through the investment choices. Further, the effect can go in either direction, depending on the structure of compensation $w(\pi)$ and the distribution of returns. This basic observation thus qualifies the common approach of viewing capital and information as complements, whereby the more resources the organization has available, the more valuable information is to the organization because returns to information scale with the level of investment. What the present paper highlights is that resources and information can also be substitutes: by having more precise information, the organization can actually save on the level of overall resources because it is able to target the funds available more precisely. And when this is the case, then restricting the manager's access to capital functions as a motivational tool over and above any direct compensation to the manager.

The final observation before considering the optimal compensation contracts is the relationship between propositions 2 and 4. If the compensation of the manager was linear,

[8] We are dealing with the Lagrange multiplier because the level of resources allocated to the manager is given in the present setting. Section 5.3 discusses the case where the manager is allowed to choose I, where the equivalent statement is then that $\frac{\partial q^M}{\partial r} > 0$ iff $\frac{\partial\left(x_h^M + x_l^M\right)}{\partial q} < 0$ or the total investment by the manager is decreasing in the precision of information.

then the sign would follow proposition 2 and corollary 3. But for other contracts, this will no longer be the case. The reason is that, unlike the expected return, the likelihood of meeting any particular performance level, $\overline{\pi}$, is not additively separable in the contributions of x_h and x_l, and how the manager responds to information will no longer be the same as how the firm would respond to that information. For example, suppose that the manager is paid a single bonus B for meeting a threshold performance $\overline{\pi}$, and suppose that $\pi_i(x) = \theta_i \ln(1+x)$, in which case we saw that the optimal level of investment is independent of q. In contrast, increased level of resources can either motivate or demotivate the manager. For example, setting $\overline{\pi}$ such that $f_H(\overline{\pi}) = f_L(\overline{\pi})$ creates complementarity between the precision of information and the level of resources.[9]

5.2 Optimal compensation contracts

Having illustrated the motivational effect of the level of resources allocated to the manager, the second step is to consider the structure of the optimal compensation contract. The optimal compensation contract is of interest because it will have clear implications for the behavior of the manager. Further, such behavior could not be induced by a simple linear contract commonly assumed in theoretical work. The properties of the optimal compensation contract are as follows:

Proposition 5 ***Optimal compensation contracts:***

(i) The optimal compensation contract consists of at most two bonuses $\underline{B}$ and $\overline{B}$, paid for exceeding performance thresholds $\underline{\pi}$ and $\overline{\pi}$, respectively

(ii) A sufficient condition for a single bonus B, paid for exceeding performance threshold π, to be optimal is that for all $\overline{\pi} > \pi > \underline{\pi}$,

$$\frac{f_L(\overline{\pi})(g(\overline{\pi})-g(\pi))}{f_L(\underline{\pi})(g(\underline{\pi})-g(\pi))} > \frac{\underline{\underline{B}}}{\overline{B}} = \frac{[\Delta F_L(\overline{\pi},\pi)(1-F_H(\pi))-\Delta F_H(\overline{\pi},\pi)(1-F_L(\pi))]}{[\Delta F_L(\pi,\underline{\pi})(1-F_H(\underline{\pi}))-\Delta F_H(\pi,\underline{\pi})(1-F_L(\underline{\pi}))]},$$

where $g(\pi)$ is the likelihood ratio at π and $\Delta F_i(\overline{\pi},\pi) = F_i(\overline{\pi}) - F_i(\pi)$.

(iii) The optimal compensation contract induces the manager to invest more aggressively than the first-best allocation, conditional on the information available to him: $\frac{x_h^M(q)}{x_h^M(q)+x_l^M(q)} > \frac{x_h^{FB}(q)}{x_h^{FB}(q)+x_l^{FB}(q)}$.

[9] Even if having $f_H(\overline{\pi}) = f_L(\overline{\pi})$ implies that the manager would choose resource allocations (x_h, x_l) that match those desired by the principal.

Proof. See Appendix A.3 ■

The optimal compensation contract thus consists of at most two bonuses, paid for exceeding particular performance thresholds. The logic behind the result is as follows. First, note that the principal needs to manage two agency problems. The first is the level of effort exerted by the manager to acquire information and the second is how the resources are allocated. To solve the first problem alone, MLRP implies that a single bonus, with $\pi, B \to \infty$ would be optimal. The reason is that with MLRP, extreme events are most informative of the manager's choice and thus the best way to motivate effort. The second problem is the resource allocation problem, for which there are multiple potential solutions. For example, any linear contract would achieve the right allocation choice. But there also exists a contract that consists of a single bonus that is able to achieve this, where we set the performance threshold $\widehat{\pi}$ to be such that the relative marginal contribution of x_h and x_l to meeting the performance threshold would exactly match their relative marginal contribution to expected cash flow. Not surprisingly, this is achieved when the likelihood ratio $g(\widehat{\pi})$ equals one. And since each of the agency problems (effort and allocation choices) could be solved with a single bonus, it comes as no surprise that the joint problem can be solved with a contract that consists of two bonuses.

Of more interest is the result that a single bonus can dominate two bonuses. While the general condition has no simple interpretation, it relates to the convexity of the likelihood ratio. The logic is as follows. Both the motivational effect of the contract and the equilibrium resource allocation are driven by a linear combination of the two bonuses. But the motivational effect is driven by the linear combination of the difference in the likelihood of meeting the threshold when the resource allocation is right or wrong, $F_L(\pi) - F_H(\pi)$, while the aggressiveness of the equilibrium resource allocation is driven by the linear combination of the likelihood ratios $g(\pi)$ associated with the two performance thresholds. Then, if the likelihood ratio is convex, then it is generally the case that a two-bonus contract that is able to achieve the same level of effort is going to generate a bigger distortion in the equilibrium allocations, making a contract consisting of a single bonus preferred.

The second and main result relates to the investment behavior induced by the optimal contract, which states that the investment behavior of the manager will be more aggressive than the first-best investment behavior, given the precision of information. The intuition for this result is simple. Because the value of information is increasing in the aggressiveness of the resource allocation, inducing more aggressive behavior by the manager following the acquisition of information through the compensation contract will induce the manager to acquire more information and thus allow the firm to economize on the monetary cost of information acquisition. In addition, such aggressiveness will make the realized cash flow more informative of whether the manager made the right choice, thus further reducing the expected monetary compensation that needs to be paid to the manager. To summarize, while there exists a compensation contract that is able to induce $\left(x_h^{FB}(q), x_l^{FB}(q)\right)$, it is optimal for the firm to distort the contract to induce more aggressive investment behavior as it allows it to reduce the expected monetary compensation both because the manager will be induced to acquire more information and the realized cash flow becomes more informative of the correctness of the allocation.

5.2.1 The interaction between the compensation contract and the level of resources

From the analysis above, it is clear that both the level of resources and the compensation contract are valuable tools for guiding the behavior of the manager, and the two will naturally also interact. First, suppose that the current compensation contract is chosen optimally. Then, if $\frac{dq}{dI} < 0$, the principal can induce further information acquisition and thus economize on the compensation contract by restricting the manager's access to resources below the first-best level, and vice versa, per proposition 4. Further, the sign of $\frac{dq}{dI}$ can depend on the compensation contract itself, as discussed above. In particular, inducing more aggressive behavior may lead the manager to view resources and information as complements, even if they are substitutes from the perspective of the principal. On the other hand, if the compensation contract cannot be chosen optimally, then the level of resources may function as an additional lever for influencing the agressiveness of the manager. Finally, there is an additional distortion caused by the positive level of managerial compensation, whereby the simple fact that the manager will receive rents will reduce the value of investment to the principal. Thus, the principal will mechanically want to invest less than the first-best level due to the lower perceived expected return.

5.3 Delegating the choice of I to the manager

Above, we assumed that the manager was allocated a given level of resources, I. However, in the present setting it is straightforward to delegate this choice to the manager without any loss in efficiency. The logic is as follows. While we clearly cannot charge the manager for the amount of resources he will use because of limited liability, what we can do is to condition the compensation contract of the manager on the total level of investment. In particular, let the bonus threshold be $\pi_i = \widetilde{\pi}_i + \widetilde{r}I$, where $\widetilde{r}$ will then be the sensitivity of the bonus thresholds to the level of resources used by the manager. The manager will then choose (x_h, x_l) that will solve

$$\max_{x_h, x_l} \sum_i B_i \left(q \left(1 - F_H \left(\pi_i | x_h, x_l \right) \right) + (1-q) \left(1 - F_L \left(\pi_i | x_h, x_l \right) \right) \right),$$

which then gives us the first-order conditions of

$$qy \frac{\partial \pi_H}{\partial x_h} + (1-q) \frac{\partial \pi_L}{\partial x_h} = \left[qy + (1-q) \right] \widetilde{r} = qy \frac{\partial \pi_L}{\partial x_l} + (1-q) \frac{\partial \pi_H}{\partial x_l},$$

where $y = \frac{\left(\underline{B} f_H(\underline{\pi}) + \overline{B} f_H(\overline{\pi}) \right)}{\left(\underline{B} f_L(\underline{\pi}) + \overline{B} f_L(\overline{\pi}) \right)}$ controls the aggressiveness of the resource allocation. As in the case of a fixed level of resources, I, the principal can use the structure of the compensation contract to manage the aggressiveness of the allocation by influencing y, while being able to induce any desired level of investment by altering $\widetilde{r}$. This cost of resources, $\widetilde{r}$, resembles an internal rate of return in the sense that it measures how the manager will evaluate the use of resources when making his investment decisions. It is then straightforward to relate managerial desire for resources to the precision of information, as follows:

Proposition 6 ***Managerial desire for resources***

The manager will want to invest more than the first-best level of resources if given access to resources at cost r if and only if his desired level of investment is increasing in the quality of information: $I^M > I^{FB}$ *if and only if* $\frac{\partial I^M}{\partial q} > 0$.

Proof. See Appendix A.4 ■

The intuition behind this result is as follows. From above, we know that the optimal contract will induce the manager to behave as if he is right more frequently than he truly is. But this bias will then naturally influence also the value that he places on having access to additional resources. If his desire for capital is decreasing in the quality of his information, then he will naturally also desire less resources at any given cost, while if his desire for capital is increasing in the quality of information, then he will desire more capital at any given cost.

As a final observation, it is worth noting that the proposition relates the investment behavior of the manager only to the first-best level of investment, absent any agency problems. Above, we have seen how the principal may want to distort the level of resources away from the first-best level to induce better information acquisition incentives, in which case $\widetilde{r}$ needs to be further adjusted to account for these effects. In particular, when $\frac{\partial I^M}{\partial q} > 0$, the additional resources have a motivational benefit to the manager (per proposition 4), so that the principal actually prefers the manager to invest more than the first-best level, other things constant.

6 An example

To illustrate the results discussed above, we will conclude by considering three numerical solution to the problem. First, the shocks are normally distributed with mean zero and variance σ^2. We can numerically confirm that the condition of proposition 5(ii) is satisfied and thus the optimal contract consists of a single bonus. Second, we take the precision of information to be $q = \frac{1}{2} + p$, where p is the effort exerted by the manager at personal cost $C(p) = -\mu((ap) + \ln(1-(ap)))$, where $a \geq 2$ determines the maximal precision of information with $\overline{q} = \frac{1}{2} + \frac{1}{a}$. We use this cost function simply because it is a simple function that guarantees that $\lim_{q \to 1/2} C'(q) = 0$ and $\lim_{q \to \overline{q}} C'(q) = \infty$. An important aspect to note is that for positive levels of information acquisition to take place, the cost function must be sufficiently convex. The reason is that the returns to information are convex as well, with the aggressiveness of the allocation increasing in q. Alternatively, very little information is practically worthless because the equilibrium resource allocation will barely respond to that information. Finally, the expected revenue to each task is given by $\pi_i(x) = \theta_i \pi(x)$, with $\theta_i \in \{\theta_L, \theta_H\}$. For the first two examples, we use $\pi(x) = (1 - e^{-x})$. Recall from corollary 3 that this case is associated with a negative relationship between the first-best level of investment and the precision of information. For the third example, we use $\pi(x) = \sqrt{x}$, for which we know that the first-best level of investment is positively related to the precision of information.

6.1 Equilibrium compensation structure and investment levels

The solution to the principal's problem in all three cases is illustrated in figure 1. For all three cases, the first panel plots the equilibrium compensation contract and the resulting precision of equilibrium information.[10] Recalling from above that the basic tradeoff in the design of the compensation contract is to motivate information acquisition and then to use that information appropriately, the logic is simple. As information becomes costlier, motivating its acquisition becomes increasingly important, and thus the principal optimally increases the threshold and the bonus that accompanies that threshold to induce more aggressive behavior and thus higher value of information. However, once the threshold becomes sufficiently high, then the distortion in the allocation may become too large relative to the benefits of additional information acquisition. This occurs in example (B). Then,

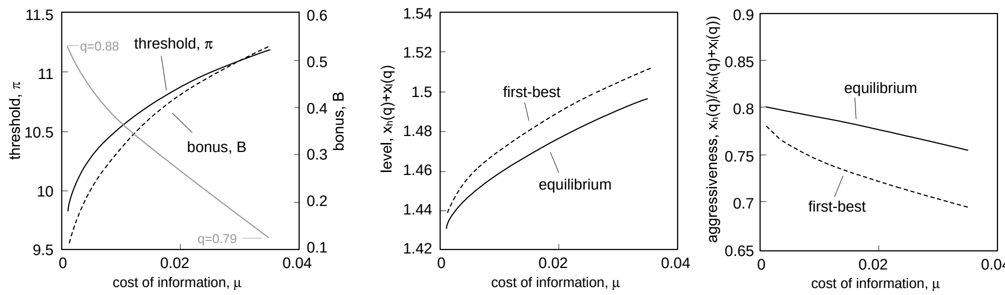
[10] The parameters are, for example (A), $\theta_H = 15$, $\theta_L = 5$, $\sigma^2 = 3$, $r = 4.5$ and $a = 2.5 \rightarrow \overline{q} = 0.9$, example (B) is identical to (A) except $\sigma^2 = 10$. For example (C), to create similar incentives to acquire information, we have $\theta_H = 6$, $\theta_L = 2$, $\sigma^2 = 3, r = 1.2$ and $a = 2.5$.

the principal may begin to lower the threshold and the bonus, essentially giving up on motivating further information acquisition and instead focusing on making better use of the information already available.[11]

The consequences of the compensation contract and the underlying production technology for the optimal level and the induced use of resources are then illustrated in panels (ii) and (iii). Consider first panels (iii), which capture the aggressiveness of the resource use. As the equilibrium information becomes less precise, we know from proposition 1 that the first-best aggressiveness decreases. Similarly, we can see that the equilibrum aggressiveness is also generally decreasing, but the relative distortion is increasing. The reason is that while the reduction in the precision of information directly reduces the aggressiveness of the manager as well, this is countered by the increase in the equilibrium performance threshold, which makes the manager more aggressive. In example (B), the latter effect is sufficiently strong so that the equilibrium aggressiveness is actually increasing for low-enough costs. In short, as information becomes more costly, the principal generally responds by increasing the performance threshold, which increases information acquisition by increasing the aggressiveness of the resource allocation, conditional on the precision of information.

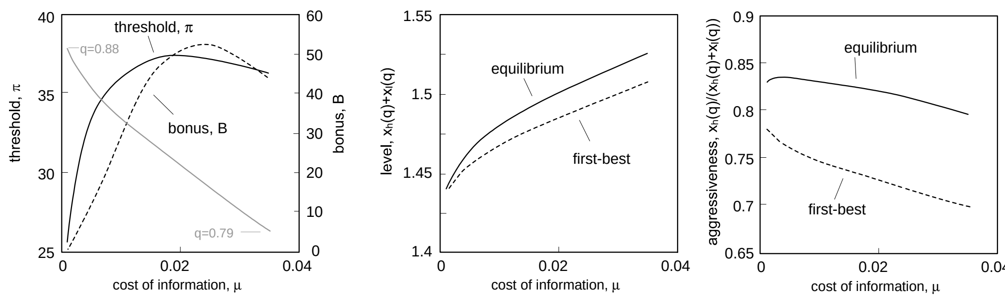
These two aspects are robust features of the solution, independent of the underlying parameters. The element that is sensitive to the underlying environment is how the level of resources is distorted from the first-best level, as discussed above, and illustrated in panels (ii). Example (A) gives a first illustration. Given that $\pi(x) = 1 - e^{-x}$, we know that the first-best level of information is decreasing in the precision of information. Similarly, the compensation contract is such that the manager also views the two as substitutes, implying that the manager is offered a level of resources that is below the first-best level. But how the manager views the relationship between resources and information depends on his compensation contract. In example (B), we use the same parameterization as in (A), except that we have increased the noise in the cash flow distribution. This change makes any threshold less informative of the performance, to which the principal responds optimally by increasing the performance threshold. But the increase in the performance threshold makes the manager to view information and resources as complements, and thus the principal now optimally offers the manager a resource level that is above the first-best level. Finally, in example (C), $\pi(x) = \sqrt{x}$, which implies that the first-best level of investment and information are positively related. Further, because the manager also views them as complements, the equilibrium level of investment is again above the first-best level.

[11] Note that the examples stop at the cost of information where the equilibrium $q \approx 0.8$. The reason for this is that for slightly higher costs, it becomes optimal to motivate no information acquisition, despite the fact that $\lim_{p \to 0} C'(p) = 0$. The reason is that the value of information is created by the response in the resource allocation. For sufficiently low precisions, the value is not enough to cover the cost.



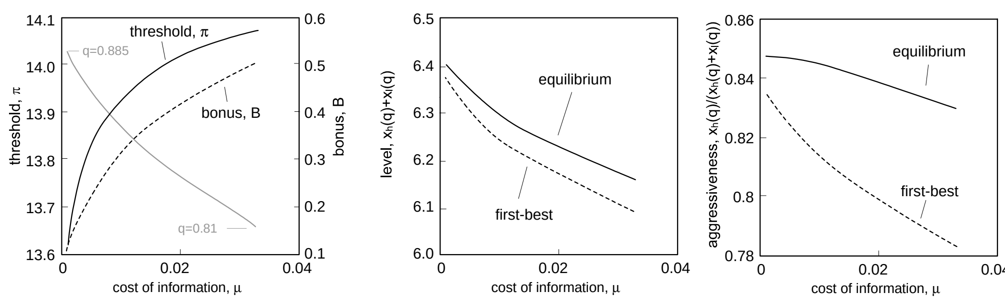
(i) optimal compensation and induced q *(ii) resource level* *(iii) aggressiveness of the allocation*

(A) Example 1: $\pi_i(x_j)=\theta_i(1-e^{-x_j})$, low variance



(i) optimal compensation and induced q *(ii) resource level* *(iii) aggressiveness of the allocation*

(B) Example 2: $\pi_i(x_j)=\theta_i(1-e^{-x_j})$, high variance



(i) optimal compensation and induced q *(ii) resource level* *(iii) aggressiveness of the allocation*

(C) Example 3: $\pi_i(x_j)=\theta_i x_j^{1/2}$

Figure 1: An illustration of the equilibrium compensation structure and the resource use.

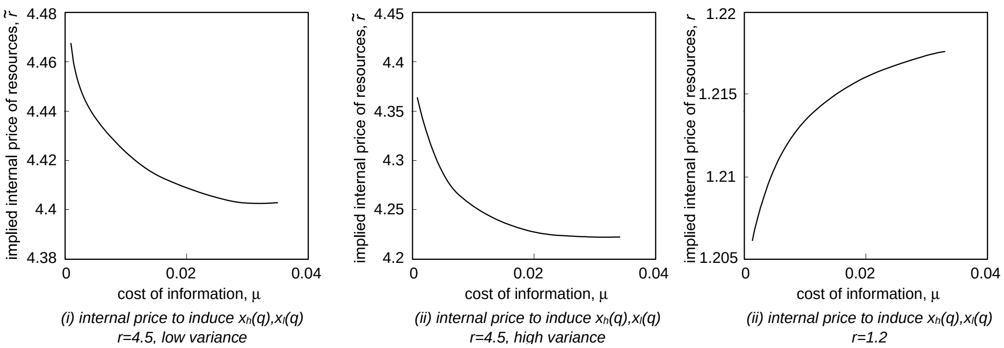


Figure 2: An illustration of the internal price of resources.

6.2 Implied internal price of resources

The remaining illustration relates to the ability to delegate the choice of the level of resources to the manager. From above, we know that we can do this by tying the manager's compensation to the amount of resources requested, with the threshold given by $\widetilde{\pi} = \underline{\pi} + \widetilde{r}I$, where $\widetilde{r}$ is then the imputed price charged to the manager. In other words, $\widetilde{r}$ is the rate at which the manager's bonus threshold increases as he chooses to invest an additional dollar of resources. From the manager's first-order condition, we know that his chosen level of resources solves

$$x_i : -\frac{\left[\left(q\frac{\partial F_H(\widetilde{\pi}|x_h,x_l)}{\partial x_i}+(1-q)\frac{\partial F_L(\widetilde{\pi}|x_h,x_l)}{\partial x_i}\right)\right]}{[qf_H(\widetilde{\pi}|x_h,x_l)+(1-q)f_L(\widetilde{\pi}|x_h,x_l)]} = \widetilde{r},$$

and since above we already solved for the target (x_h, x_l) and optimal $\widetilde{\pi}$, we can simply plug in the values to solve for the imputed $\widetilde{r}$ that will induce these choices. This solution is illustrated in figure 2, which plots the internal price of resources to the three examples of the previous subsection. The key message is that there is no simple relationship between $\widetilde{r}$, used to evaluate the managerial use of resources, and the desired level of investment.

In example (A), the desired level of investment is below the first-best level and the manager views resources and information as substitutes so that he also wants to invest below the first-best level. However, this underinvestment is too much from the principal's perspective and thus the optimal $\widetilde{r}$ is below the first-best level to encourage some additional investment. In example (B), the desired level of investment is above the first-best level. Even if the manager also wants to over-invest, he is offered an internal price of resources that is even lower than the true cost to encourage sufficient over-investment. Finally, in example (C), the desired level of investment is above the first-best level, but the manager would want to invest even more, so that the internal price of resources needs to be above the true cost, despite the fact that over-investment occurs in equilibrium. In short, the basic message of the analysis is that there is no direct relationship between the internal price of resources and over- or under-investment because how the manager evaluates resources depends on both the underlying production technology and the compensation contract.

7 Empirical Implications

While it is unlikely that a resource allocation problem illustrated here is the only problem that any firm attempts to solve with its managers, the analysis does suggest some general themes that should be borne out by the data if such a task is at least prominent enough to be of concern in the design of contracts. The two main theoretical predictions of the model are that (i) motivating managers to acquire information requires inducing them (via compensation contracts) to be overly aggressive in their resource allocation decisions and (ii) the level of resources that is available for investment can be either a substitute or a complement for information acquisition. Naturally, the analysis also highlights that one of the key determinants of managerial behavior is how managers are paid, so that any empirical work that ignores the compensation structure may lead to misleading conclusions. In particular, the analysis raises the question of whether equilibrium behavior is induced by the compensation contract because it is desired by the principal or because it is due to an underlying preference misalignment, which the principal can try to attenuate through the compensation contract.

Below, we illustrate these points and discuss how our theoretical predictions relate to two prominent debates in the corporate finance literature: the assumption that managers are empire builders and the question of whether managers are overconfident. As such, the logic of the analysis would be most applicable to division-level managers, but the scarcity of data on their compensation structure may make the analysis challenging. But it seems possible to consider these behavioral elements at the firm-level as well, taking the CEO as the manager.

Empire building versus the optimal level of resources: It is commonly assumed that managers desire more resources than optimal because they derive private benefits from that. In contrast, the present model suggests that it may be optimal to give managers excessive resources because that functions as a motivational tool. A key difference is that it may also be optimal to give managers less resources because that may provide motivation as well. To our knowledge, this represents an unexplored rationale for explaining significant heterogeneity in investment rates.

Categorizing firms by whether providing more or less resources to the manager is motivationally beneficial depends on the shape of the return distribution. The broad conclusion, however, is that projects that have less upside (highly concave returns) support limited access to resources while projects that have more upside (less concave returns) support excessive access to resources. If younger and more innovative industries have more upside than older and less innovative industries as commonly assumed, the empirical prediction is that the extent of over-investment is positively related to the innovativeness of an industry or the inverse of its age. This prediction is also supported by the observation that uncertainty in cash flow can lead the firm to choose a compensation structure that induces complementarity between information and resources from the perspective of the manager, even if they are substitutes in terms of expected cash flow (example (B) above), to the extent that such uncertainty is also higher in younger and more innovative industries.

The broader observation is that even if managers derive direct private benefits from the size of their operations, the equilibrium use of resources can be managed by linking managers' compensation to the level of resources used. Thus, any equilibrium use of resources that exceeds the first-best level should be beneficial to the firm, except in the case of entrenched managers who may be able to design their own compensation, and in addition, some distortion prevents managers from simply paying themselves more, which in turn leads them to instead use resources as the source of compensation (Scharfstein and Stein, 2000).

Overconfidence versus the optimal aggressiveness of the allocation decision: Managerial overconfidence has recently received increasing attention in the literature (see, e.g., Malmendier and Tate, 2005). One implication of overconfidence is that managers will be overly aggressive in their resource allocation decisions, which is observationally equivalent to the prediction here that such behavior may be induced by compensation contracts to motivate information acquisition. As with empire building, our claim is not that managers are never inherently overconfident, but that the equilibrium behavior may be desirable from the firm's perspective and therefore optimally induced. Empirical work that ignores the compensation structure (or obtains biased estimates of overconfidence if implicit contracts are important) may lead to misleading conclusions.

The important caution above also applies to a recent stream of research that compares the investment behavior of seemingly similar firms, which may in fact be compensating their managers differently if they face different opportunities. This can help explain puzzling differences in investment behavior across firms operating in the same industry (see, e.g., Sheen, 2011, for a comparison of the capacity expansion decisions of public and private producers of commodity chemicals, Gilje and Taillard, 2012, for a comparison of the well drilling decisions of public and private natural gas producers). Last but not least, the analysis suggests that so-called managerial styles (see, e.g., Bertrand and Schoar, 2003, and Fee, Hadlock, and Pierce, 2013) may be optimally induced. The general lesson is that one should not leave out compensation contracts when studying firm behavior.

8 Extensions

There are a number of natural extensions to the framework, where the complete analysis are left for future research but briefly discussed here to highlight some of the basic ideas.

Resource allocation decision as a strategic variable: The analysis assumed that the resource allocation decision was inalienable from the manager responsible for the division. Alternatively, it may be possible that the decision can be used as a strategic design variable, where it is either retained by the headquarters or delegated to the division manager.[12] In the present setting, delegating authority to the manager is unlikely to be optimal, because by retaining control the principal could customize the compensation contract to motivate information acquisition alone, which, without additional constraints, can achieve first-best.

If, however, there are other constraints on the design of incentives, then also strategic delegation of the resource allocation decision may become optimal. The reason is that any over-responsiveness to the information acquired by the manager through the existing compensation contract implies under-responsiveness by the headquarters. As a result, under the *same* compensation contract, the manager will be more motivated to acquire information when delegated the resource allocation decision because he will respond more to that information. The headquarters is then facing a tradeoff familiar from Aghion and Tirole (1997): by delegating authority to the manager, the headquarters suffers a loss of control in the sense that the manager will choose resource allocations that are more aggressive than desired ex post by the principal, but at the same time and, indeed, because of it, the manager will be motivated to acquire more precise information on which to base his investment decisions.

Separate performance measures: The analysis assumed that only joint performance on the two tasks was measurable. The only result afforded by this assumption was the characterization of the optimal contract. Even with separately measured tasks, the principal will want to economize on the wage bill needed to induce a given level of effort, for which overly aggressive resource allocation as induced by the compensation contract and either over- or undersupply of initial resources are two answers. The only thing afforded by having two separate performance measures is that the excessive responsiveness may be induced in a more cost-efficient way than with a single performance measure.

Variable overall productivity: The analysis also assumed that the productivity of the tasks was perfectly negatively correlated. This assumption allowed us to cleanly separate

[12] Or, the principal dictates the resource allocation but the manager can undo some of that, potentially at a cost.

the effects of the desired level of resources, which was known ex ante, and the aggressiveness of how those resources were used. From above, it is clear that allowing the desired level of resources to be uncertain does not change the value of overly aggressive allocation of those resources when different productivities are suggested by the information acquired, but how the role of the level of resources is changed is unknown and left for future research.

Multiple agents: Finally, the analysis considered only a single manager making an allocation choice between two alternatives. An interesting avenue for future research is how to optimally group tasks among different managers, where the key tradeoff appears to be that a single manager in charge of two projects will have no conflict in revealing which alternative is better, while informational rents need to be paid to induce a manager in charge of a single project to truthfully reveal when that project is truly bad. However, competition for resources between two managers may provide an additional motivational tool.

9 Conclusion

We analyze a simple model of resource allocation where a manager first needs to acquire information regarding the relative productivity of different investment alternatives and then allocate the resources available to him among the alternatives. The firm has two design parameters at its disposal: the structure of the compensation contract offered to the manager and the level of resources allocated to the manager. The main results from the analysis are two-fold. First, with respect to the level of resources, we identify an important motivational role whereby restricting the manager's access to resources below the first-best level could either sharpen or lessen the manager's incentive to acquire information. If more precise information decreases the manager's desired level of investment, then by restricting the manager's access to resources below the first-best level, the firm can induce the manager to acquire more information. Conversely, if more precise information increases the manager's desired level of investment, then by increasing the manager's access to resources above the first-best level, the firm can induce the manager to acquire more information.

Second, with respect to the optimal structure of compensation, we show how the optimal compensation contract induces the manager to be overly aggressive, in the sense that he allocates resources in a fashion that is more aggressive than what would maximize the expected cash flow. The intuition for this result is that by inducing more aggressive investment behavior by the manager, the firm could increase the value of information to him and thus economize on the expected monetary cost inducing any given level of information acquisition.

Third, we make the simple observation that we can delegate the choice of how much to invest overall simply by tying the compensation of the manager appropriately to the level of resources invested. However, if we linked the compensation only to the true cost of resources, the investment choices of the manager will generally be biased. The reason for this result is that because the manager invests to maximize his expected compensation, and the optimal compensation contract biases the manager away from maximizing expected value, the value that the manager places on the resources invested will generally be different from their true value. In particular, the manager may exhibit empire-building preferences, in the sense that he would prefer to invest more resources than the first-best level if given access to resources at their true cost. But at the same time, the firm may want the manager to invest more than the first-best level of resources, so that the imputed charge on capital that affects the compensation of the manager may be above or below the true cost of resources independent of whether the manager wants to over- or underinvest given the true cost of resources. This lack of direct relationship between the internal rate of return imposed on managers and the actual use of capital also highlights that having a higher IRR does not necessarily mean suboptimally low levels of investment if part of the purpose is to counter behavior that is caused by a compensation contract that is inducing biased evaluation of investment alternatives.

Of these two biases, it is the empire-building preferences that have received the most attention in the literature, with most of the capital budgeting literature exogenously assuming such preferences and the proceeding with the analysis to understand how the allocation should be distorted to induce truth-telling. Our analysis can be thus viewed as a model that is able to generate such preferences endogenously, providing some justification for such an assumption, but at the same time highlights that such equilibrium behavior can be optimal once we take into account the motivational effects of access to resources, and how the equilibrium use of resources can be managed by linking managerial compensation to the level of investment. The analysis thus suggests that it is dangerous to analyse compensation and capital budgeting processes in isolation, because one of the main drivers how managers will behave and evaluate different investment alternatives is how they are compensated in equilibrium.

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A Proofs and derivations

A.1 Proof of proposition 2

Suppose that the returns are given by $\theta_i \pi(x)$, where $\theta_i \in \{\theta_H, \theta_L\}$ and $\theta_H > \theta_L$. Then, the first-order conditions for the optimal level of investment become

$$(q\theta_H + (1-q)\theta_L)\,\pi'(x_h) = r = (q\theta_L + (1-q)\theta_H)\,\pi'(x_l),$$

which then allows us to write

$$\frac{\partial x_h}{\partial q} = -\frac{(\theta_H - \theta_L)\pi'(x_h)}{(q\theta_H + (1-q)\theta_L)\pi''(x_h)} \qquad \textit{and} \qquad \frac{\partial x_l}{\partial q} = \frac{(\theta_H - \theta_L)\pi'(x_l)}{(q\theta_L + (1-q)\theta_H)\pi''(x_l)}.$$

Next, note that at the optimal level of investment, we have $(q\theta_H + (1-q)\theta_L) = \frac{r}{\pi'(x_h)}$ and similarly for x_l, which allows us to write the sum of the changes as

$$\frac{\partial x_h}{\partial q} + \frac{\partial x_l}{\partial q} = -\frac{(\theta_H - \theta_L)(\pi'(x_h))^2}{r\pi''(x_h)} + \frac{(\theta_H - \theta_L)(\pi'(x_l))^2}{r\pi''(x_l)} = \frac{(\theta_H - \theta_L)}{r}\left(\frac{(\pi'(x_h))^2}{|\pi''(x_h)|} - \frac{(\pi'(x_l))^2}{|\pi''(x_l)|}\right),$$

so that the sign is determined by $\left(\frac{(\pi'(x_h))^2}{|\pi''(x_h)|} - \frac{(\pi'(x_l))^2}{|\pi''(x_l)|}\right)$. Since $x_h > x_l$, a sufficient condition for the sign is determining if $\frac{d}{dx}\left(\frac{(\pi'(x))^2}{|\pi''(x)|}\right)$ is monotone. For the examples, note that if $\pi(x) = \ln(1 + ax)$, then

$$\pi'(x) = \frac{\alpha}{1+\alpha x} \text{ and } \pi''(x) = -\frac{\alpha^2}{(1+\alpha x)^2},$$

so that $\frac{(\pi'(x))^2}{|\pi''(x)|} = 1$, independent of x. Thus, the optimal level of investment is independent of the precision of information. Next, suppose that $\pi(x) = x^{\frac{1}{g}}$. Then,

$$\pi'(x) = \frac{1}{g}x^{\frac{1-g}{g}} \text{ and } \pi''(x) = \frac{1}{g}\left(\frac{1-g}{g}\right)x^{\frac{1-2g}{g}},$$

so that $\frac{(\pi'(x))^2}{|\pi''(x)|} = \frac{1}{g-1}x^{\frac{1}{g}}$, which is increasing in x. Therefore, the optimal total level of investment is increasing in the precision of information. Finally, let $\pi(x) = (1 - e^{-\alpha x})$. Then,

$$\pi'(x) = \alpha e^{-\alpha x} \text{ and } \pi''(x) = -\alpha^2 e^{-\alpha x},$$

so that $\frac{(\pi'(x))^2}{|\pi''(x)|} = e^{-\alpha x}$, which is decreasing in x. Therefore, the optimal total level of investment is decreasing in the precision of information.

A.2 Proof of proposition 4

Let U denote the agent's expected compensation. Then, we can write the agent's first-order conditions as

$$\frac{\partial U}{\partial x_h} = \lambda = \frac{\partial U}{\partial x_l} \qquad \textit{and} \qquad \frac{\partial U}{\partial q} - C'(q) = 0.$$

Next, we have that $\frac{dq}{dI}$, the motivational effect of the level of resources, is given by

$$\frac{dq}{dI} = \frac{\frac{\partial^2 U}{\partial q \partial x_h}\frac{dx_h}{dI} + \frac{\partial^2 U}{\partial q \partial x_l}\frac{dx_l}{dI}}{C''(q)}.$$

First, let us determine $\frac{d\lambda}{dq}$. From the agent's first-order condition for the resource allocation we get

$$\frac{dx_h}{dq} = \frac{\frac{\partial^2 U}{\partial x_h \partial q} + \frac{\partial^2 U}{\partial x_h \partial x_l}\frac{dx_l}{dq} - \frac{\partial \lambda}{\partial q}}{-\frac{\partial^2 U}{\partial x_h^2}} \qquad \textit{and} \qquad \frac{dx_l}{dq} = \frac{\frac{\partial^2 U}{\partial x_l \partial q} + \frac{\partial^2 U}{\partial x_h \partial x_l}\frac{dx_h}{dq} - \frac{\partial \lambda}{\partial q}}{-\frac{\partial^2 U}{\partial x_l^2}},$$

which we can rearrange to yield

$$\frac{dx_h}{dq} = \frac{\left(\frac{\partial^2 U}{\partial x_h \partial x_l}\frac{\partial^2 U}{\partial x_l \partial q} - \frac{\partial^2 U}{\partial x_h \partial q}\frac{\partial^2 U}{\partial x_l^2}\right) + \left(\frac{\partial^2 U}{\partial x_l^2} - \frac{\partial^2 U}{\partial x_h \partial x_l}\right)\frac{d\lambda}{dq}}{\left(\frac{\partial^2 U}{\partial x_h^2}\frac{\partial^2 U}{\partial x_l^2} - \frac{\partial^2 U}{\partial x_h \partial x_l}\frac{\partial^2 U}{\partial x_h \partial x_l}\right)}$$

$$\frac{dx_l}{dq} = \frac{\left(\frac{\partial^2 U}{\partial x_h \partial x_l}\frac{\partial^2 U}{\partial x_h \partial q} - \frac{\partial^2 U}{\partial x_l \partial q}\frac{\partial^2 U}{\partial x_h^2}\right) + \left(\frac{\partial^2 U}{\partial x_h^2} - \frac{\partial^2 U}{\partial x_h \partial x_l}\right)\frac{d\lambda}{dq}}{\left(\frac{\partial^2 U}{\partial x_l^2}\frac{\partial^2 U}{\partial x_h^2} - \frac{\partial^2 U}{\partial x_h \partial x_l}\frac{\partial^2 U}{\partial x_h \partial x_l}\right)}.$$

Finally, since it must be that $\frac{dx_h}{dq} + \frac{dx_l}{dq} = 0$, we can use the above to solve

$$\frac{d\lambda}{dq} = \frac{\frac{\partial^2 U}{\partial x_h \partial x_l}\left(\frac{\partial^2 U}{\partial x_l \partial q} + \frac{\partial^2 U}{\partial x_h \partial q}\right) - \frac{\partial^2 U}{\partial x_h \partial q}\frac{\partial^2 U}{\partial x_l^2} - \frac{\partial^2 U}{\partial x_l \partial q}\frac{\partial^2 U}{\partial x_h^2}}{\left(2\frac{\partial^2 U}{\partial x_h \partial x_l} - \frac{\partial^2 U}{\partial x_h^2} - \frac{\partial^2 U}{\partial x_l^2}\right)},$$

where $\left(2\frac{\partial^2 U}{\partial x_h \partial x_l} - \frac{\partial^2 U}{\partial x_h^2} - \frac{\partial^2 U}{\partial x_l^2}\right) > 0$ for an interior optimum to exist for x_h and x_l.[13]. Next, for the effects of resource allocation, we have

$$\frac{dx_h}{dI} = \frac{\frac{\partial^2 U}{\partial x_h \partial q}\frac{dq}{dI} + \frac{\partial^2 U}{\partial x_h \partial x_l}\frac{dx_l}{dI} - \frac{d\lambda}{dI}}{\left(-\frac{\partial^2 U}{\partial x_h^2}\right)} \qquad \frac{dx_l}{dI} = \frac{\frac{\partial^2 U}{\partial x_l \partial q}\frac{dq}{dI} + \frac{\partial^2 U}{\partial x_h \partial x_l}\frac{dx_h}{dI} - \frac{d\lambda}{dI}}{\left(-\frac{\partial^2 U}{\partial x_l^2}\right)},$$

which we can rearrange to yield the equilibrium changes as

[13] For an interior solution to exist we must have $\left(\frac{\partial^2 U}{\partial x_l^2}\frac{\partial^2 U}{\partial x_h^2} - \frac{\partial^2 U}{\partial x_h \partial x_l}\frac{\partial^2 U}{\partial x_h \partial x_l}\right) > 0$, which implies that $\left|\frac{\partial^2 U}{\partial x_h \partial x_l}\right| < \sqrt{\frac{\partial^2 U}{\partial x_l^2}\frac{\partial^2 U}{\partial x_h^2}}$, which in turn implies that $2\frac{\partial^2 U}{\partial x_h \partial x_l} - \frac{\partial^2 U}{\partial x_h^2} - \frac{\partial^2 U}{\partial x_l^2} > 0$

$$\frac{dx_h}{dI} = \frac{\left(\frac{\partial^2 U}{\partial x_h \partial x_l}\frac{\partial^2 U}{\partial x_l \partial q} - \frac{\partial^2 U}{\partial x_h \partial q}\frac{\partial^2 U}{\partial x_l^2}\right)\frac{dq}{dI} + \left(\frac{\partial^2 U}{\partial x_l^2} - \frac{\partial^2 U}{\partial x_h \partial x_l}\right)\frac{d\lambda}{dI}}{\left(\frac{\partial^2 U}{\partial x_h^2}\frac{\partial^2 U}{\partial x_l^2} - \frac{\partial^2 U}{\partial x_h \partial x_l}\frac{\partial^2 U}{\partial x_h \partial x_l}\right)}$$

$$\frac{dx_l}{dI} = \frac{\left(\frac{\partial^2 U}{\partial x_h \partial x_l}\frac{\partial^2 U}{\partial x_h \partial q} - \frac{\partial^2 U}{\partial x_l \partial q}\frac{\partial^2 U}{\partial x_h^2}\right)\frac{dq}{dI} + \left(\frac{\partial^2 U}{\partial x_h^2} - \frac{\partial^2 U}{\partial x_h \partial x_l}\right)\frac{d\lambda}{dI}}{\left(\frac{\partial^2 U}{\partial x_h^2}\frac{\partial^2 U}{\partial x_l^2} - \frac{\partial^2 U}{\partial x_h \partial x_l}\frac{\partial^2 U}{\partial x_h \partial x_l}\right)}.$$

Since all resources will be used in equilibrium, we have that $\frac{dx_h}{dI} + \frac{dx_l}{dI} = 1$, which allows us to write

$$\frac{d\lambda}{dI} = \frac{\left(\frac{\partial^2 U}{\partial x_h \partial x_l}\frac{\partial^2 U}{\partial x_l \partial q} - \frac{\partial^2 U}{\partial x_h \partial q}\frac{\partial^2 U}{\partial x_l^2} + \frac{\partial^2 U}{\partial x_h \partial x_l}\frac{\partial^2 U}{\partial x_h \partial q} - \frac{\partial^2 U}{\partial x_l \partial q}\frac{\partial^2 U}{\partial x_h^2}\right)\frac{dq}{dI} - \left(\frac{\partial^2 U}{\partial x_h^2}\frac{\partial^2 U}{\partial x_l^2} - \frac{\partial^2 U}{\partial x_h \partial x_l}\frac{\partial^2 U}{\partial x_h \partial x_l}\right)}{\left(2\frac{\partial^2 U}{\partial x_h \partial x_l} - \frac{\partial^2 U}{\partial x_h^2} - \frac{\partial^2 U}{\partial x_l^2}\right)},$$

which we can substitute back in the sensitivies, which then simplify to

$$\frac{dx_l}{dI} = \frac{\left(\frac{\partial^2 U}{\partial x_h \partial x_l} - \frac{\partial^2 U}{\partial x_h^2}\right) - \left(\frac{\partial^2 U}{\partial x_h \partial q} - \frac{\partial^2 U}{\partial x_l \partial q}\right)\frac{dq}{dI}}{2\frac{\partial^2 U}{\partial x_h \partial x_l} - \frac{\partial^2 U}{\partial x_h^2} - \frac{\partial^2 U}{\partial x_l^2}} \qquad \frac{dx_h}{dI} = \frac{\left(\frac{\partial^2 U}{\partial x_h \partial x_l} - \frac{\partial^2 U}{\partial x_l^2}\right) + \left(\frac{\partial^2 U}{\partial x_h \partial q} - \frac{\partial^2 U}{\partial x_l \partial q}\right)\frac{dq}{dI}}{2\frac{\partial^2 U}{\partial x_h \partial x_l} - \frac{\partial^2 U}{\partial x_h^2} - \frac{\partial^2 U}{\partial x_l^2}}.$$

Next, we can substitute these in $\frac{dq}{dI}$, which simplifies to

$$\frac{dq}{dI} = \frac{\frac{\partial^2 U}{\partial q \partial x_h}\left(\frac{\partial^2 U}{\partial x_h \partial x_l} - \frac{\partial^2 U}{\partial x_l^2}\right) + \frac{\partial^2 U}{\partial q \partial x_l}\left(\frac{\partial^2 U}{\partial x_h \partial x_l} - \frac{\partial^2 U}{\partial x_h^2}\right)}{\left[C''(q)\left(2\frac{\partial^2 U}{\partial x_h \partial x_l} - \frac{\partial^2 U}{\partial x_h^2} - \frac{\partial^2 U}{\partial x_l^2}\right) - \left(\frac{\partial^2 U}{\partial x_h \partial q} - \frac{\partial^2 U}{\partial x_l \partial q}\right)^2\right]}.$$

Again, for a solution to exist, we must have $C''(q)\left(2\frac{\partial^2 U}{\partial x_h \partial x_l} - \frac{\partial^2 U}{\partial x_h^2} - \frac{\partial^2 U}{\partial x_l^2}\right) - \left(\frac{\partial^2 U}{\partial x_h \partial q} - \frac{\partial^2 U}{\partial x_l \partial q}\right)^2 > 0$, and so

$$sign\left(\frac{dq}{dI}\right) = sign\left(\frac{\partial^2 U}{\partial q \partial x_h}\left(\frac{\partial^2 U}{\partial x_h \partial x_l} - \frac{\partial^2 U}{\partial x_l^2}\right) + \frac{\partial^2 U}{\partial q \partial x_l}\left(\frac{\partial^2 U}{\partial x_h \partial x_l} - \frac{\partial^2 U}{\partial x_h^2}\right)\right),$$

but recall from above that $\frac{d\lambda}{dq} = \frac{\frac{\partial^2 U}{\partial x_h \partial q}\left(\frac{\partial^2 U}{\partial x_h \partial x_l} - \frac{\partial^2 U}{\partial x_l^2}\right) + \frac{\partial^2 U}{\partial x_l \partial q}\left(\frac{\partial^2 U}{\partial x_h \partial x_l} - \frac{\partial^2 U}{\partial x_h^2}\right)}{\left(2\frac{\partial^2 U}{\partial x_h \partial x_l} - \frac{\partial^2 U}{\partial x_h^2} - \frac{\partial^2 U}{\partial x_l^2}\right)}$, so that $sign\left(\frac{dq}{dI}\right) = sign\left(\frac{d\lambda}{dq}\right)$.

A.3 Proof of proposition 5

Our first step is to establish that a contract consisting of at most two bonuses will be weakly optimal, which amounts to showing that it is weakly cheaper to induce any (q, x_h, x_l) using a contract with two bonuses than any other contract. First, approximate any non-decreasing compensation contract by a sequence of bonuses, which gives us the expected cost of such a contract as

$$q \sum B_i \left(F_L\left(\pi_i | x_h, x_l\right) - F_H\left(\pi_i | x_h, x_l\right)\right) + \sum B_i \left(1 - F_L\left(\pi_i | x_h, x_l\right)\right).$$

Now, rewrite the choice of x_h, x_l as the choice of $\Delta x = x_h - x_l$, given I. Then, for the contract to maintain the same incentives to acquire information and the resource allocation, the conditions that need to be satisfied are

$$\begin{aligned} &q : \sum B_i \left(F_L\left(\pi | \Delta x, I\right) - F_H\left(\pi | \Delta x, I\right)\right) = C'(q) \\ &x_i : \sum B_i \left(\frac{\partial \Pr(\pi > \pi_i)}{\partial \Delta x}\right) = 0, \end{aligned}$$

where $\Pr(\pi > \pi_i) = q\left(1 - F_H\left(\pi_i | \Delta x, I\right)\right) + (1-q)\left(1 - F_L\left(\pi_i | \Delta x, I\right)\right)$, the expected probability that the manager will meet a given performance threshold, conditional on the resource allocation Δx. Now, take three bonuses (B_i, B_j, B_k) and consider increasing one of them while changing the other two in a fashion that leaves both the equilibrium decisions and the equilibrium effort level unchanged. If at least one such change increases costs, that implies that three or more bonuses is suboptimal. The key element is that the first-order conditions for the manager are linear in the bonuses. From the FOC for effort, we have that

$$\left(F_L\left(\pi_i\right) - F_H\left(\pi_i\right)\right) + \frac{\partial B_j}{\partial B_i}\left(F_L\left(\pi_j\right) - F_H\left(\pi_j\right)\right) + \frac{\partial B_k}{\partial B_i}\left(F_L\left(\pi_k\right) - F_H\left(\pi_k\right)\right) = 0,$$

while from the resource allocation constraints we get that

$$\left(\frac{\partial \Pr(\pi > \pi_i)}{\partial \Delta x}\right) + \frac{\partial B_j}{\partial B_i}\left(\frac{\partial \Pr(\pi > \pi_j)}{\partial \Delta x}\right) + \frac{\partial B_k}{\partial B_i}\left(\frac{\partial \Pr(\pi > \pi_k)}{\partial \Delta x}\right) = 0,$$

which define $\frac{\partial B_j}{\partial B_i}$ and $\frac{\partial B_k}{\partial B_i}$ as

$$\begin{aligned} \frac{\partial B_j}{\partial B_i} &= \frac{\left(F_L(\pi_i) - F_H(\pi_i)\right)\left(\frac{\partial \Pr(\pi > \pi_k)}{\partial \Delta x}\right) - \left(F_L(\pi_k) - F_H(\pi_k)\right)\left(\frac{\partial \Pr(\pi > \pi_i)}{\partial \Delta x}\right)}{\left(F_L(\pi_k) - F_H(\pi_k)\right)\left(\frac{\partial \Pr(\pi > \pi_j)}{\partial \Delta x}\right) - \left(F_L(\pi_j) - F_H(\pi_j)\right)\left(\frac{\partial \Pr(\pi > \pi_k)}{\partial \Delta x}\right)} \\ \frac{\partial B_k}{\partial B_i} &= \frac{\left(F_L(\pi_i) - F_H(\pi_i)\right)\left(\frac{\partial \Pr(\pi > \pi_j)}{\partial \Delta x}\right) - \left(F_L(\pi_j) - F_H(\pi_j)\right)\left(\frac{\partial \Pr(\pi > \pi_i)}{\partial \Delta x}\right)}{\left(F_L(\pi_j) - F_H(\pi_j)\right)\left(\frac{\partial \Pr(\pi > \pi_k)}{\partial \Delta x}\right) - \left(F_L(\pi_k) - F_H(\pi_k)\right)\left(\frac{\partial \Pr(\pi > \pi_j)}{\partial \Delta x}\right)}. \end{aligned}$$

There is thus a continuum of (B_i, B_j, B_k) that are able to achieve the same behavior by the manager, with the bonuses linearly related to each other. Finally, let $\Gamma(\mathbf{B}, \boldsymbol{\pi})$ denote the expected cost of any contract. We have that the effect of changing any of the bonuses while holding the manager's behavior constant are

$$\frac{d\Gamma(\mathbf{B}, \boldsymbol{\pi})}{dB_i} = \left(1 - F_L\left(\pi_i\right)\right) + \frac{\partial B_j}{\partial B_i}\left(1 - F_j\left(\pi_j\right)\right) + \frac{\partial B_k}{\partial B_i}\left(1 - F_k\left(\pi_k\right)\right).$$

We have thus three expressions, and each of the bonuses is optimal if and only if $\frac{d\Gamma(\mathbf{B}, \boldsymbol{\pi})}{dB_i} = 0$.

But since the rates at which the bonuses need to adjust in response to each other are constant, this derivative is constant, independent of the level of B. Therefore, starting with all three bonuses greater than zero, we have that either either $\frac{d\Gamma(\mathbf{B},\boldsymbol{\pi})}{dB_i} > 0$, which implies that we can reduce B_i and lower cost, until $B_i = 0$, $\frac{d\Gamma(\mathbf{B},\boldsymbol{\pi})}{dB_i} < 0$, in which case we can lower the cost by increasing B_i, but that implies that for constant cost either B_j or B_k (or both) need to be decreasing, until one of them is zero, or $\frac{d\Gamma(\mathbf{B},\boldsymbol{\pi})}{dB_i} = 0$, in which case the bonus is irrelevant and we can set it equal to zero. Thus, the cost-minimizing contract consists of at most two bonuses.

Finally, to consider when a single bonus can be optimal, we can follow a similar logic. In particular, we will consider two- and single-bonus contracts that induce the same level of effort and cost, and consider which introduces a smaller decision distortion. First, for the effort levels to be equal, we need

$$\overline{B}\Delta F\left(\overline{\pi}\right) + \underline{B}\Delta F\left(\underline{\pi}\right) = B\Delta F\left(\pi\right),$$

where $\Delta F\left(\pi\right) = F_L\left(\pi\right) - F_H\left(\pi\right)$. Similarly, for equal cost we need that

$$\overline{B}\left(1 - F_L\left(\overline{\pi}\right)\right) + \underline{B}\left(1 - F_L\left(\underline{\pi}\right)\right) = B\left(1 - F_L\left(\pi\right)\right).$$

Together, we can use these two conditions to solve for the bonuses that lead these equalities to hold as

$$\frac{\left[(1-F_L(\pi))\Delta F(\underline{\pi}) - (1-F_L(\underline{\pi}))\Delta F(\pi)\right]}{\left[(1-F_L(\overline{\pi}))\Delta F(\underline{\pi}) - (1-F_L(\underline{\pi}))\Delta F(\overline{\pi})\right]} B = \overline{B} \geq 0$$

$$\frac{\left[(1-F_L(\overline{\pi}))\Delta F(\pi) - (1-F_L(\pi))\Delta F(\overline{\pi})\right]}{\left[(1-F_L(\overline{\pi}))\Delta F(\underline{\pi}) - (1-F_L(\underline{\pi}))\Delta F(\overline{\pi})\right]} B = \underline{B} \geq 0.$$

From the first-order conditions for the manager's resource allocation choice we get, in the case of two bonuses

$$q\left(\underline{B}f_L\left(\underline{\pi}\right)g\left(\underline{\pi}\right) + \overline{B}f_L\left(\overline{\pi}\right)g\left(\overline{\pi}\right)\right)\left(\frac{\partial \pi_H}{\partial x_h} - \frac{\partial \pi_L}{\partial x_l}\right) + (1-q)\left(\underline{B}f_L\left(\underline{\pi}\right) + \overline{B}f_L\left(\overline{\pi}\right)\right)\left(\frac{\partial \pi_L}{\partial x_h} - \frac{\partial \pi_H}{\partial x_l}\right) = 0,$$

where $g\left(\pi\right)$ is the likelihood ratio at π, which we can rearrange to give

$$\frac{q\left(\underline{B}f_L(\underline{\pi})g(\underline{\pi}) + \overline{B}f_L(\overline{\pi})g(\overline{\pi})\right)}{(1-q)\left(\underline{B}f_L(\underline{\pi}) + \overline{B}f_L(\overline{\pi})\right)} = \frac{\left(\frac{\partial \pi_H}{\partial x_l} - \frac{\partial \pi_L}{\partial x_h}\right)}{\left(\frac{\partial \pi_H}{\partial x_h} - \frac{\partial \pi_L}{\partial x_l}\right)}.$$

Similarly, for the single-bonus contract, we get

$$\frac{q(Bf_L(\pi)g(\pi))}{(1-q)Bf_L(\pi)} = \frac{\left(\frac{\partial \pi_H}{\partial x_l} - \frac{\partial \pi_L}{\partial x_h}\right)}{\left(\frac{\partial \pi_H}{\partial x_h} - \frac{\partial \pi_L}{\partial x_l}\right)}.$$

Next, as shown below, the optimal contract always induces overly aggressive investment behavior to economize on information acquisition costs, and since the distortion is increases $\frac{\left(\frac{\partial \pi_H}{\partial x_l} - \frac{\partial \pi_L}{\partial x_h}\right)}{\left(\frac{\partial \pi_H}{\partial x_h} - \frac{\partial \pi_L}{\partial x_l}\right)}$, we have that the two-bonus contract is worse if

$$\frac{q\left(\underline{B}f_L(\underline{\pi})g(\underline{\pi})+\overline{B}f_L(\overline{\pi})g(\overline{\pi})\right)}{(1-q)\left(\underline{B}f_L(\underline{\pi})+\overline{B}f_L(\overline{\pi})\right)} > \frac{q(Bf_L(\pi)g(\pi))}{(1-q)Bf_L(\pi)},$$

which then gives us

$$\frac{\left(\underline{B}f_L(\underline{\pi})g(\underline{\pi})+\overline{B}f_L(\overline{\pi})g(\overline{\pi})\right)}{\left(\underline{B}f_L(\underline{\pi})+\overline{B}f_L(\overline{\pi})\right)} > g(\pi)$$

$$\frac{f_L(\overline{\pi})(g(\overline{\pi})-g(\pi))}{f_L(\underline{\pi})(g(\underline{\pi})-g(\pi))} > \frac{\underline{B}}{\overline{B}} = \frac{[(1-F_L(\overline{\pi}))\Delta F(\pi)-(1-F_L(\pi))\Delta F(\overline{\pi})]}{[(1-F_L(\pi))\Delta F(\underline{\pi})-(1-F_L(\underline{\pi}))\Delta F(\pi)]}$$

$$= \frac{[\Delta F_L(\overline{\pi},\pi)(1-F_H(\pi))-\Delta F_H(\overline{\pi},\pi)(1-F_L(\pi))]}{[\Delta F_L(\pi,\underline{\pi})(1-F_H(\underline{\pi}))-\Delta F_H(\pi,\underline{\pi})(1-F_L(\underline{\pi}))]}.$$

Finally, to establish the equilibrium distortion, note that we can write the principal's optimization problem (given I) as

$$\max_{\underline{B},\overline{B},\underline{\pi},\overline{\pi}} qE_H(\pi|\Delta x) + (1-q)\,E_L(\pi|\Delta x)$$
$$-\underline{B}\left(q\left(1-F_H(\underline{\pi}|\Delta x)\right)+[1-q]\left(1-F_L(\underline{\pi}|\Delta x)\right)\right)-\overline{B}\left(q\left(1-F_H(\overline{\pi}|\Delta x)\right)+(1-q)\left(1-F_L(\overline{\pi}|\Delta x)\right)\right)$$

s.t. $\quad \underline{B}\Delta F(\underline{\pi}|\Delta x)+\overline{B}\Delta F(\overline{\pi}|\Delta x) = C'(q)$

$$\underline{B}\left(q\left(\frac{\partial F_H(\underline{\pi}|\Delta x)}{\partial \Delta x}\right)+[1-q]\left(\frac{\partial F_L(\underline{\pi}|\Delta x)}{\partial \Delta x}\right)\right)+\overline{B}\left(q\left(\frac{\partial F_H(\overline{\pi}|\Delta x)}{\partial \Delta x}\right)+[1-q]\left(\frac{\partial F_L(\overline{\pi}|\Delta x)}{\partial \Delta x}\right)\right)=0,$$

where the two constraints are the manager's information acquisition and investment constraints, with $\Delta x = x_h - x_l$ indicating the aggressiveness of the resource allocation. Next, note that we can adjust $(\underline{\pi},\overline{\pi}),(\underline{B},\overline{B})$ in a fashion that leaves the expected cost of the contract unchanged to induce any desired distortion. Indeed, given that MLRP holds, the expected cost of the contract is decreasing in $\overline{\pi}$. Thus, we can view the principal as directly choosing Δx, in which case we get the first-order condition

$$\left(q\frac{\partial E_H(\pi|\Delta x)}{\partial \Delta x}+(1-q)\frac{\partial E_L(\pi|\Delta x)}{\partial \Delta x}\right)+\left(E_H(\pi|\Delta x)-E_L(\pi|\Delta x)\right)\frac{\partial q}{\partial \Delta x} = \frac{dW_A}{d\Delta x},$$

where $\frac{dW_A}{d\Delta x}$ is the change in the agent's compensation, which we can write as

$$\frac{dW_A}{d\Delta x} = \frac{\partial W_A}{\partial \Delta x} + \frac{\partial W_A}{\partial q}\frac{dq}{d\Delta x},$$

but from the manager's first-order condition we know that $\frac{\partial W_A}{\partial \Delta x} = 0$ while $\frac{\partial W_A}{\partial q} = C'(q)$. Thus, the expression simplifies to

$$\left(q\frac{\partial E_H(\pi|\Delta x)}{\partial \Delta x} + (1-q)\frac{\partial E_L(\pi|\Delta x)}{\partial \Delta x}\right) + \left(E_H(\pi|\Delta x) - E_L(\pi|\Delta x) - C'(q)\right)\frac{\partial q}{\partial \Delta x} = 0$$

while from the manager's first-order condition we know that $\frac{\partial q}{\partial \Delta x} > 0$, which in turn implies that, in equilibrium

$$\left(q\frac{\partial E_H(\pi|\Delta x)}{\partial \Delta x} + (1-q)\frac{\partial E_L(\pi|\Delta x)}{\partial \Delta x}\right) < 0,$$

so that $\Delta x^M > \Delta x^{FB}$ because of the motivational benefit of over-aggressive behavior.

A.4 Proof of proposition 6

For the level of investment, we need to endogenize the investment level, which we do by assuming that the bonus threshold is now $\widetilde{\pi}(I) = \underline{\pi} + \widetilde{r}I$. Then, we can write the first-order conditions for the investment decision as

$$\begin{aligned}&\underline{B}\Big(q\left(f_H(\underline{\pi})\left(\tfrac{\partial \pi_H}{\partial x_h} - \widetilde{r}\right)\right) + (1-q)\,f_L(\underline{\pi})\left(\tfrac{\partial \pi_L}{\partial x_h} - \widetilde{r}\right)\Big)\\ &+\overline{B}\left(qf_H(\overline{\pi})\left(\tfrac{\partial \pi_H}{\partial x_h} - \widetilde{r}\right) + (1-q)\,f_L(\overline{\pi})\left(\tfrac{\partial \pi_L}{\partial x_h} - \widetilde{r}\right)\right) = 0,\end{aligned}$$

which we can write as

$$q\frac{\left(\underline{B}f_H(\underline{\pi}) + \overline{B}f_H(\overline{\pi})\right)}{\left(\underline{B}f_L(\underline{\pi}) + \overline{B}f_L(\overline{\pi})\right)}\frac{\partial \pi_H}{\partial x_h} + (1-q)\frac{\partial \pi_L}{\partial x_h} = \left[q\frac{\left(\underline{B}f_H(\underline{\pi}) + \overline{B}f_H(\overline{\pi})\right)}{\left(\underline{B}f_L(\underline{\pi}) + \overline{B}f_L(\overline{\pi})\right)} + (1-q)\right]\widetilde{r}$$

and symmetrically for x_l. Now, define $y = \frac{\left(\underline{B}f_H(\underline{\pi}) + \overline{B}f_H(\overline{\pi})\right)}{\left(\underline{B}f_L(\underline{\pi}) + \overline{B}f_L(\overline{\pi})\right)} \geq 1$ as the weighted average likelihood ratio, which determines the equilibrium aggressiveness of the allocation, and note that any changes with respect to y are going to be equivalent to changes in $\underline{\pi}$ and $\overline{\pi}$ given our assumption of increasing likelihood ratio. Then, we have that

$$\begin{aligned}&qy\frac{\partial \pi_H}{\partial x_h} + (1-q)\frac{\partial \pi_L}{\partial x_h} = [qy + (1-q)]\,\widetilde{r}\\ &qy\frac{\partial \pi_L}{\partial x_l} + (1-q)\frac{\partial \pi_H}{\partial x_l} = [qy + (1-q)]\,\widetilde{r}.\end{aligned}$$

We can immediately observe that if $y = 1$, then $\widetilde{r} = r$ will induce first-best investment decisions. However, we know that for the equilibrium contract, $y > 1$ because it will tolerate some over-aggressiveness and thus if we can show that $\frac{\partial(x_h + x_l)}{\partial y} > 0$, we know that the manager will, in equilibrium, want to invest more than the first-best level and that distortion will become increasingly severe as we increase the performance threshold (i.e. increase the likelihood ratio at the bonus threshold).

Since $\frac{\partial \pi_H}{\partial x_h} > \frac{\partial \pi_L}{\partial x_h}$ and $\frac{\partial \pi_L}{\partial x_l} < \frac{\partial \pi_H}{\partial x_l}$, we can immediately see that increasing y increases the value of investing in x_h by $q\left(\frac{\partial \pi_H}{\partial x_h} - \widetilde{r}\right)$ while it decreases the marginal value of investing in x_l by $q\left(\widetilde{r} - \frac{\partial \pi_L}{\partial x_l}\right)$. Note also that the distortion will disappear if $q \to 1$ because then all uncertainty will disappear. From here, we then have that

$$\frac{\partial x_h}{\partial y} = -\frac{q\left(\frac{\partial \pi_H}{\partial x_h} - \widetilde{r}\right)}{dD_H} \qquad and \qquad \frac{\partial x_l}{\partial y} = -\frac{q\left(\frac{\partial \pi_L}{\partial x_l} - \widetilde{r}\right)}{dD_L},$$

where dD_H, dD_L are the second derivatives of the manager's objective function with respect to x_h and x_l, respectively. But we also know that

$$\frac{\partial x_h}{\partial q} = -\frac{\left(\frac{\partial \pi_H}{\partial x_h} - \frac{\partial \pi_L}{\partial x_h}\right) - (y-1)\widetilde{r}}{dD_H} \qquad and \qquad \frac{\partial x_l}{\partial q} = -\frac{\left(\frac{\partial \pi_L}{\partial x_l} - \frac{\partial \pi_H}{\partial x_l}\right) - (y-1)\widetilde{r}}{dD_L},$$

so that $\frac{\partial x_h}{\partial y} + \frac{\partial x_l}{\partial y} = \frac{q\left(\frac{\partial \pi_H}{\partial x_h} - \widetilde{r}\right)}{\left(y\frac{\partial \pi_H}{\partial x_h} - \frac{\partial \pi_L}{\partial x_h}\right) - (y-1)\widetilde{r}} \frac{\partial x_h}{\partial q} + \frac{q\left(\frac{\partial \pi_L}{\partial x_l} - \widetilde{r}\right)}{\left(y\frac{\partial \pi_L}{\partial x_l} - \frac{\partial \pi_H}{\partial x_l}\right) - (y-1)\widetilde{r}} \frac{\partial x_l}{\partial q}$.

The rest is then just manipulation of the first-order conditions to simplify the expressions. First, note that

$$qy\frac{\partial \pi_H}{\partial x_h} + (1-q)\frac{\partial \pi_L}{\partial x_h} = [qy + (1-q)]\,\widetilde{r}$$

can be rearranged as

$$\left(y\frac{\partial \pi_H}{\partial x_h} - \frac{\partial \pi_L}{\partial x_h}\right) - [y-1]\,\widetilde{r} = \frac{1}{q}\left(\widetilde{r} - \frac{\partial \pi_L}{\partial x_h}\right)$$

and similarly for $qy\frac{\partial \pi_L}{\partial x_l} + (1-q)\frac{\partial \pi_H}{\partial x_l} = [qy + (1-q)]\,\widetilde{r}$, giving

$$\left(y\frac{\partial \pi_L}{\partial x_l} - \frac{\partial \pi_H}{\partial x_l}\right) - [y-1]\,\widetilde{r} = \frac{1}{q}\left(\widetilde{r} - \frac{\partial \pi_H}{\partial x_l}\right),$$

which allows us to simplify the above to

$$q^2\left[\frac{\left(\frac{\partial \pi_H}{\partial x_h} - \widetilde{r}\right)}{\left(\widetilde{r} - \frac{\partial \pi_L}{\partial x_h}\right)}\frac{\partial x_h}{\partial q} + \frac{\left(\frac{\partial \pi_L}{\partial x_l} - \widetilde{r}\right)}{\left(\widetilde{r} - \frac{\partial \pi_H}{\partial x_l}\right)}\frac{\partial x_l}{\partial q}\right].$$

Similarly, we can rearrange the first-order conditions to

$$\frac{\left[\frac{\partial \pi_H}{\partial x_h} - \widetilde{r}\right]}{\left[\widetilde{r} - \frac{\partial \pi_L}{\partial x_h}\right]} = \frac{(1-q)}{qy} \qquad and \qquad \frac{\left(\frac{\partial \pi_L}{\partial x_l} - \widetilde{r}\right)}{\left(\widetilde{r} - \frac{\partial \pi_H}{\partial x_l}\right)} = \frac{[(1-q)]}{qy},$$

so that in equilibrium,

$$\frac{\partial x_h}{\partial y} + \frac{\partial x_l}{\partial y} = \frac{q(1-q)}{y}\left[\frac{\partial x_h}{\partial q} + \frac{\partial x_l}{\partial q}\right],$$

and so the impact on total investment is proportional to the impact of information on the total level of investment desired.

B Mechanism design

This appendix derives the solution to the mechanism design problem where the firm is able to commit to the investment levels x_h and x_l, but the task of information acquisition must be undertaken by the manager who is then offered a contract $w(\pi)$ to motivate that information acquisition. To simplify the notation, let $f_H(\pi|x_h, x_l) = g(\pi - \pi_H(x_h) - \pi_L(x_l))$ denote the distribution of profits when the capital allocation is right and $f_L(\pi|x_h, x_l)$ when the capital allocation is wrong. The design problem is then

$$\max_{x_h, x_l, w(\pi)} \int_0^\infty (\pi - w(\pi))\,(q f_H(\pi|x_h, x_l) + (1-q)\, f_L(\pi|x_h, x_l))\, d\pi - r(x_h + x_l)$$

$$\begin{aligned} \text{s.t.} \qquad & q \in \arg\max \int_0^\infty w(\pi)\,(q f_H(\pi|x_h, x_l) + (1-q)\, f_L(\pi|x_h, x_l))\, d\pi - C(q) \\ & w(\pi), w'(\pi) \geq 0. \end{aligned}$$

The solution to this design problem is given by the following proposition.

Proposition 7 ***Optimal mechanism:***

(i) The optimal wage contract consists of a single bonus B paid for performance exceeding $\overline{\pi}$, where $\overline{\pi}$ solves $\frac{1-F_H(\overline{\pi})}{1-F_L(\overline{\pi})} = \frac{f_H(\overline{\pi})}{f_L(\overline{\pi})} > 1$.

(ii) The optimal investment levels solve

$$q\frac{\partial(\pi_H(x_h)+\pi_L(x_l))}{\partial x_i} + (1-q)\frac{\partial(\pi_H(x_l)+\pi_L(x_h))}{\partial x_i} = r + B\frac{\left((1-F_L(\overline{\pi}))\frac{\partial F_H(\overline{\pi})}{\partial x_i} - (1-F_H(\overline{\pi}))\frac{\partial F_L(\overline{\pi})}{\partial x_i}\right)}{(F_L(\overline{\pi}) - F_H(\overline{\pi}))}.$$

(iii) $x_h^* > x_h^{FB}$ and $x_l^* < x_l^{FB}$, with $(x_h^* + x_l^*) \gtreqless (x_h^{FB} + x_l^{FB})$.

(iv) If the monotone likelihood ratio holds, then $\overline{\pi}, B \to \infty$ and the bonus alone achieves

first-best information acquisition. A sufficient condition for distortions in the capital allocation to disappear is that $\frac{f_H(\pi)}{f_L(\pi)} \to \infty$ *as* $\pi \to \infty$.

Let us begin with the optimal $w(\pi)$. We can approximate any increasing function with a sequence of bonuses paid for exceeding performance thresholds π_i, so that

$$E(w(\pi)) = \sum qB_i [(F_L(\pi_i) - F_H(\pi_i))] + \sum B_i (1 - F_L(\pi_i)),$$

where $F_H(\pi_i)$ and $F_L(\pi_i)$ are the CDFs of the probability distribution functions $f_H(\pi|x_h, x_l)$ and $f_L(\pi|x_h, x_l)$. Since the manager has no control over (x_h, x_l), we suppress them from the notation. The manager wants to maximize his expected compensation, which gives the first-order condition

$$\sum B_i [F_L(\pi_i) - F_H(\pi_i)] = C'(q),$$

which has a unique optimum as long as $C''(q) > 0$. And since the manager's problem is concave, the first-order approach used below for the rest of the analysis is non-problematic. Also, given the requirement that $B_i \geq 0$ and that $C''(q) > 0$, the IC constraint will be binding while the IR constraint ($EU \geq 0$) is slack. Of course, for sufficiently high outside options the participation constraint would be binding, but that is not allowed here. The shape of the optimal compensation contract then minimizes $E(w(\pi))$ subject to holding q constant. Thus, the minimization problem is

$$\min_{\{B_i\},\{\pi_i\}} \sum B_i (1 - F_L(\pi_i))$$

$$\text{s.t.} \sum B_i [F_L(\pi_i) - F_H(\pi_i)] = K.$$

Now, consider changing π_i and B_i so that the effort incentives are unchanged. The change in the cost is $\frac{\partial B_i}{\partial \pi_i}(1 - F_L(\pi_i)) - B_i f_L(\pi_i)$, so the location of the bonus is optimal if

$$\frac{\partial B_i}{\partial \pi_i}(1 - F_L(\pi_i)) = B_i f_L(\pi_i),$$

while from the effort IC, we have that $\frac{\partial B_i}{\partial \pi_i} = -B_i \frac{[f_L(\pi_i) - f_H(\pi_i)]}{[F_L(\pi_i) - F_H(\pi_i)]}$, which implies that the optimal location of the bonus is

$$\frac{f_H(\pi_i)}{f_L(\pi_i)} = \frac{(1 - F_H(\pi_i))}{(1 - F_L(\pi_i))}.$$

And since this choice doesn't depend on the rest of the compensation structure, there will thus be only one bonus paid after performance exceeding this level. The basic logic is then simple. Following the standard moral hazard intuition, the manager should be paid in the outcome states that are most informative about his effort. In the present case, we obtain the solution familiar from hypothesis testing, whereby the manager receives his bonus only after a threshold where the hazard rates are equal - it is after this point that the probability of the particular profit realization coming from the right resource allocation starts to exceed that of the wrong resource allocation, conditional on being at least $\overline{\pi}$.

For later, note that the first-order condition for the optimal choice of B solves

$$\begin{array}{l}\frac{\partial}{\partial B}:\left[E_H\left(\pi|x_h,x_l\right)-E_L\left(\pi|x_h,x_l\right)\right]\frac{\partial q}{\partial B}\\ -B\left[\left(F_L\left(\overline{\pi}\right)-F_H\left(\overline{\pi}\right)\right)\right]\frac{\partial q}{\partial B}-\left[q\left(F_L\left(\overline{\pi}\right)-F_H\left(\overline{\pi}\right)\right)+\left(1-F_L\left(\overline{\pi}\right)\right)\right]=0,\end{array}$$

where $E_H\left(\pi|x_h,x_l\right)=\left(\pi_H\left(x_h\right)+\pi_L\left(x_l\right)\right)$ and $E_L\left(\pi|x_h,x_l\right)=\left(\pi_H\left(x_l\right)+\pi_L\left(x_h\right)\right)$ are shorthand for the expected returns under correct and wrong capital allocations, while from the manager's FOC we know that $\frac{\partial q}{\partial B}=\frac{\left(F_L(\overline{\pi})-F_H(\overline{\pi})\right)}{C''(q)}$. Next, consider the optimal choice of x_h and x_l. Taking the first-order condition with respect to x_i, we have

$$q\frac{\partial E_H(\pi|x_i)}{\partial x_i}+\left(1-q\right)\frac{\partial E_L(\pi|x_i)}{\partial x_i}-r-\frac{dE(w(\pi))}{dx_i}+\left[E_H\left(\pi|x_h,x_l\right)-E_L\left(\pi|x_h,x_l\right)\right]\frac{\partial q}{\partial x_i}=0,$$

which is equivalent to the case under the first-best solution, plus the impact of the investment level on expected compensation, $E\left(w\left(\pi\right)\right)$, and the quality of information, q. Further, the change in the expected compensation is equal to

$$\frac{dE(w(\pi))}{dx_i}=B\left[q\left(\frac{\partial F_L(\overline{\pi})}{\partial x_i}-\frac{\partial F_H(\overline{\pi})}{\partial x_i}\right)-\frac{\partial F_L(\overline{\pi})}{\partial x_i}\right]+B\left(F_L\left(\overline{\pi}\right)-F_H\left(\overline{\pi}\right)\right)\frac{\partial q}{\partial x_i},$$

so we can write the first-order condition as

$$\begin{array}{l}q\frac{\partial E_H(\pi|x_i)}{\partial x_i}+\left(1-q\right)\frac{\partial E_L(\pi|x_i)}{\partial x_i}=r+B\left[q\left(\frac{\partial F_L(\overline{\pi})}{\partial x_i}-\frac{\partial F_H(\overline{\pi})}{\partial x_i}\right)-\frac{\partial F_L(\overline{\pi})}{\partial x_i}\right]\\ +\left[B\left(F_L\left(\overline{\pi}\right)-F_H\left(\overline{\pi}\right)\right)-\left[E_H\left(\pi|x_h,x_l\right)-E_L\left(\pi|x_h,x_l\right)\right]\right]\frac{\partial q}{\partial x_i}.\end{array}$$

Next, from the manager's FOC we have that $B\left[F_L\left(\overline{\pi}\right)-F_H\left(\overline{\pi}\right)\right]=C'(q)$, which also implies that

$$\frac{\partial q}{\partial x_i}=\frac{B\left(\frac{\partial F_L(\overline{\pi})}{\partial x_i}-\frac{\partial F_H(\overline{\pi})}{\partial x_i}\right)}{C''(q)}\text{ and }\frac{\partial q}{\partial B}=\frac{\left(F_L(\overline{\pi})-F_H(\overline{\pi})\right)}{C''(q)},$$

which allows us to write $\frac{\partial q}{\partial x_i}=\frac{B\left(\frac{\partial F_L(\overline{\pi})}{\partial x_i}-\frac{\partial F_H(\overline{\pi})}{\partial x_i}\right)}{\left(F_L(\overline{\pi})-F_H(\overline{\pi})\right)}\frac{\partial q}{\partial B}$. Finally, we can rearrange the first-order condition for the optimal choice of B as

$$\frac{\partial q}{\partial B}=\frac{[q(F_L(\overline{\pi})-F_H(\overline{\pi}))+(1-F_L(\overline{\pi}))]}{[[E_H(\pi|x_h,x_l)-E_L(\pi|x_h,x_l)]-B[(F_L(\overline{\pi})-F_H(\overline{\pi}))]]},$$

which allows us to write the first-order condition for the optimal investment level as

$$q\frac{\partial E_H(\pi|x_i)}{\partial x_i}+(1-q)\frac{\partial E_L(\pi|x_i)}{\partial x_i}=$$
$$r+B\left[q\left(\frac{\partial F_L(\overline{\pi})}{\partial x_i}-\frac{\partial F_H(\overline{\pi})}{\partial x_i}\right)-\frac{\partial F_L(\overline{\pi})}{\partial x_i}\right]-\frac{B\left(\frac{\partial F_L(\overline{\pi})}{\partial x_i}-\frac{\partial F_H(\overline{\pi})}{\partial x_i}\right)}{(F_L(\overline{\pi})-F_H(\overline{\pi}))}\left[q\left(F_L\left(\overline{\pi}\right)-F_H\left(\overline{\pi}\right)\right)+\left(1-F_L\left(\overline{\pi}\right)\right)\right],$$

which finally simplifies to

$$q\frac{\partial E_H(\pi|x_i)}{\partial x_i}+(1-q)\frac{\partial E_L(\pi|x_i)}{\partial x_i}=r+B\frac{\left[(1-F_L(\overline{\pi}))\frac{\partial F_H(\overline{\pi})}{\partial x_i}-(1-F_H(\overline{\pi}))\frac{\partial F_L(\overline{\pi})}{\partial x_i}\right]}{(F_L(\overline{\pi})-F_H(\overline{\pi}))}.$$

Finally, from the additive nature of uncertainty, we have that

$$\begin{array}{ll}\frac{\partial F_H(\pi)}{\partial x_H}=-f_H\left(\pi|x_h,x_l\right)\frac{\partial \pi_H}{\partial x_h} & \frac{\partial F_L(\pi)}{\partial x_H}=-f_L\left(\pi|x_h,x_l\right)\frac{\partial \pi_L}{\partial x_h}\\ \frac{\partial F_H(\pi)}{\partial x_L}=-f_H\left(\pi|x_h,x_l\right)\frac{\partial \pi_L}{\partial x_l} & \frac{\partial F_L(\pi)}{\partial x_L}=-f_L\left(\pi|x_h,x_l\right)\frac{\partial \pi_H}{\partial x_l},\end{array}$$

while the optimal compensation contract has $f_H\left(\overline{\pi}\right)\left(1-F_L\left(\overline{\pi}\right)\right)=\left(1-F_H\left(\overline{\pi}\right)\right)f_L\left(\overline{\pi}\right),$ which allows us to write further that the first-order conditions for x_h and x_l equal

$$\begin{array}{l}q\frac{\partial \pi_H}{\partial x_h}+(1-q)\frac{\partial \pi_L}{\partial x_h}=r+B\frac{f_H(\overline{\pi})(1-F_L(\overline{\pi}))}{(F_L(\overline{\pi})-F_H(\overline{\pi}))}\left[\frac{\partial \pi_L}{\partial x_h}-\frac{\partial \pi_H}{\partial x_h}\right]\\ q\frac{\partial \pi_L}{\partial x_l}+(1-q)\frac{\partial \pi_H}{\partial x_l}=r+B\frac{f_H(\overline{\pi})(1-F_L(\overline{\pi}))}{(F_L(\overline{\pi})-F_H(\overline{\pi}))}\left[\frac{\partial \pi_H}{\partial x_l}-\frac{\partial \pi_L}{\partial x_l}\right].\end{array}$$

From the original assumptions, we have that $\frac{\partial \pi_H}{\partial x}>\frac{\partial \pi_L}{\partial x}$, so the marginal cost of capital is below r for x_h while above r for x_l, which implies that $x_h^*\geq x_h^{FB}$ while $x_l^*\leq x_l^{FB}$. Finally, to consider the distortion in $x_h^*+x_l^*$, note that the magnitude of both disortions is driven by B, with the solution converging to first-best when $B\rightarrow 0$. Thus, we can establish the sign of the distortion by establishing the sign of $\frac{\partial\left(x_h^*+x_l^*\right)}{\partial B}$. Let

$$\begin{array}{l}D_H:q\frac{\partial \pi_H}{\partial x_h}+(1-q)\frac{\partial \pi_L}{\partial x_h}-r-B\frac{f_H(\overline{\pi})(1-F_L(\overline{\pi}))}{(F_L(\overline{\pi})-F_H(\overline{\pi}))}\left[\frac{\partial \pi_L}{\partial x_h}-\frac{\partial \pi_H}{\partial x_h}\right]=0\\ D_L:q\frac{\partial \pi_L}{\partial x_l}+(1-q)\frac{\partial \pi_H}{\partial x_l}-r-B\frac{f_H(\overline{\pi})(1-F_L(\overline{\pi}))}{(F_L(\overline{\pi})-F_H(\overline{\pi}))}\left[\frac{\partial \pi_H}{\partial x_l}-\frac{\partial \pi_L}{\partial x_l}\right]=0\end{array}$$

denote the two implicit functions. Then, from the implicit function theorem we know that

$$\frac{\partial x_h}{\partial B}=\frac{\frac{f_H(\overline{\pi})(1-F_L(\overline{\pi}))}{(F_L(\overline{\pi})-F_H(\overline{\pi}))}\left[\frac{\partial \pi_L}{\partial x_h}-\frac{\partial \pi_H}{\partial x_h}\right]}{\frac{\partial D_H}{\partial x_h}}\qquad and\qquad \frac{\partial x_l}{\partial B}=\frac{\frac{f_H(\overline{\pi})(1-F_L(\overline{\pi}))}{(F_L(\overline{\pi})-F_H(\overline{\pi}))}\left[\frac{\partial \pi_H}{\partial x_l}-\frac{\partial \pi_L}{\partial x_l}\right]}{\frac{\partial D_L}{\partial x_l}},$$

while $\frac{\partial x_h}{\partial q}=-\frac{\left(\frac{\partial \pi_H}{\partial x_h}-\frac{\partial \pi_L}{\partial x_h}\right)}{\frac{\partial D_H}{\partial x_h}}$ and $\frac{\partial x_l}{\partial q}=-\frac{\left(\frac{\partial \pi_H}{\partial x_l}-\frac{\partial \pi_L}{\partial x_l}\right)}{\frac{\partial D_L}{\partial x_l}}$, so that

$$\frac{\partial x_h}{\partial B}+\frac{\partial x_l}{\partial B}=-\frac{\frac{f_H(\overline{\pi})(1-F_L(\overline{\pi}))}{(F_L(\overline{\pi})-F_H(\overline{\pi}))}\left[\frac{\partial \pi_L}{\partial x_h}-\frac{\partial \pi_H}{\partial x_h}\right]}{\left(\frac{\partial \pi_H}{\partial x_h}-\frac{\partial \pi_L}{\partial x_h}\right)}\frac{\partial x_h}{\partial q}-\frac{\frac{f_H(\overline{\pi})(1-F_L(\overline{\pi}))}{(F_L(\overline{\pi})-F_H(\overline{\pi}))}\left[\frac{\partial \pi_H}{\partial x_l}-\frac{\partial \pi_L}{\partial x_l}\right]}{\left(\frac{\partial \pi_H}{\partial x_l}-\frac{\partial \pi_L}{\partial x_l}\right)}\frac{\partial x_l}{\partial q}.$$

Thus, $\frac{\partial x_h}{\partial B}+\frac{\partial x_l}{\partial B}<0$ iff

$$\frac{f_H(\overline{\pi})(1-F_L(\overline{\pi}))}{(F_L(\overline{\pi})-F_H(\overline{\pi}))}\left(\frac{\partial x_h^*}{\partial q}+\frac{\partial x_l^*}{\partial q}\right)<0.$$

Therefore, the basic results of the model are the same whether we consider the mechanism design solution or the main setting where the resource allocation decision is delegated to the manager. The reason is that the resource allocation decision plays the same role in both settings: by making the equilibrium resource allocation more aggressive than the first-best solution, the principal can increase the value of information to the manager. In the mechanism design case, this aggressiveness can be dictated directly by the choice of (x_l^*, x_h^*). In the main case, this effect needs to be induced through the compensation structure of the manager. The same logic applies to the level of resources: if $\frac{\partial\left(x_h^*+x_l^*\right)}{\partial q}>0$, then it is optimal to give the manager access to an excessive amount of resources while if $\frac{\partial\left(x_h^*+x_l^*\right)}{\partial q}<0$, it is optimal to give the manager less than the first-best level of resources, again to increase the marginal value of information to the manager.

Now, there are two caveats to the derivation. First, because $\frac{f_i(\pi|x_h,x_l)}{1-F_i(\pi|x_h,x_l)}$ depends on (x_h, x_l), it is possible that $(\overline{\pi}, x_h, x_l)$ has multiple solutions, while the comparative statics apply around a given solution. The basic logic of the analysis is, however, unchanged, as the distortion itself applies to all potential equilibria. Second, recall that if MLRP holds, then $\frac{f_H(\pi)}{(1-F_H(\pi))}<\frac{f_L(\pi)}{(1-F_L(\pi))}$ $\forall\pi$. In this case, the optimal contract becomes $(B,\overline{\pi})\to\infty$ and the solution will achieve first-best effort incentives. To derive the solution and the associated distortions in this case, we have that the optimal size of the bonus is given by

$$\begin{aligned}&\frac{\partial}{\partial B}:\left[E_H\left(\pi|x_h,x_l\right)-E_L\left(\pi|x_h,x_l\right)\right]\frac{\partial q}{\partial B}\\&-B\left[\left(F_L\left(\pi\right)-F_H\left(\pi\right)\right)\right]\frac{\partial q}{\partial B}-\left[q\left(F_L\left(\pi\right)-F_H\left(\pi\right)\right)+\left(1-F_L\left(\pi\right)\right)\right]=0.\end{aligned}$$

Now, if $\pi\to\infty$, the last component converges to zero while from the manager's FOC we get $B=\frac{C'(q)}{[F_L(\pi)-F_H(\pi)]}$, so that the principal's first-order condition becomes

$$\left[\left[E_H\left(\pi|x_h,x_l\right)-E_L\left(\pi|x_h,x_l\right)\right]-C'(q)\right]\frac{\partial q}{\partial B}=0.$$

Next, recall that the FOC for the choice of x_i was given by

$$q\frac{\partial E_H(\pi|x_i)}{\partial x_i}+(1-q)\frac{\partial E_L(\pi|x_i)}{\partial x_i}=r+\frac{\partial E(w(\pi))}{\partial x_i}+\left(\frac{\partial E(w(\pi))}{\partial q}-\left[E_H\left(\pi|x_h,x_l\right)-E_L\left(\pi|x_h,x_l\right)\right]\right)\frac{\partial q}{\partial x_i},$$

Next, note that the manager's compensation is given by

$$B\left(q\left(F_L\left(\pi\right)-F_H\left(\pi\right)\right)+\left(1-F_L\left(\pi\right)\right)\right).$$

Thus, $\frac{\partial E(w(\pi))}{\partial q}=B\left(F_L\left(\pi\right)-F_H\left(\pi\right)\right)$, while from the manager's FOC we have that $B=\frac{C'(q)}{(F_L(\pi)-F_H(\pi))}$ and from the principal's optimal choice of B we have that $\left[E_H\left(\pi|x_h,x_l\right)-E_L\left(\pi|x_h,x_l\right)\right]=C'(q)$, we get that

$$\frac{\partial E(W)}{\partial q}-\left[E_H\left(\pi|x_h,x_l\right)-E_L\left(\pi|x_h,x_l\right)\right]=0,$$

so that the optimal resource allocation solves

$$q\frac{\partial E_H(\pi|x_i)}{\partial x_i}+\left(1-q\right)\frac{\partial E_L(\pi|x_i)}{\partial x_i}=r+\frac{\partial E(W)}{\partial x_i}.$$

Since giving access to any resources improves the likelihood of meeting a given performance target, $\frac{\partial E(w(\pi))}{\partial x_i}\geq 0$ and thus the manager will be provided resources that are below the first-best level to economize on the wage bill. To see when this distortion disappears, we can write the manager's expected compensation, by using $B=\frac{C'(q)}{(F_L(\pi)-F_H(\pi))}$, as

$$\frac{C'(q)}{(F_L(\pi)-F_H(\pi))}\left(q\left(F_L\left(\pi\right)-F_H\left(\pi\right)\right)+\left(1-F_L\left(\pi\right)\right)\right)=C'(q)q+\frac{C'(q)(1-F_L(\pi))}{(F_L(\pi)-F_H(\pi))},$$

where $C'(q)\frac{(1-F_L(\pi))}{(F_L(\pi)-F_H(\pi))}$ is the rents earned by the manager and thus the potential source of the distortion. To see how $\frac{f_H(\pi)}{f_L(\pi)}\to\infty$ as $\pi\to\infty$ is a sufficient condition for this distortion to disappear, note that we can write this as

$$\frac{(1-F_L(\pi))}{(F_L(\pi)-F_H(\pi))}=\frac{(1-F_L(\pi))}{((1-F_H(\pi))-(1-F_L(\pi)))}=\frac{1}{\left(\frac{(1-F_H(\pi))}{(1-F_L(\pi))}-1\right)},$$

while from the MLRP property we know that $\frac{f_H(\pi)}{f_L(\pi)}<\frac{(1-F_H(\pi))}{(1-F_L(\pi))}$. Thus, if $\frac{f_H(\pi)}{f_L(\pi)}\to\infty$, $\frac{(1-F_H(\pi))}{(1-F_L(\pi))}\to\infty$ and as a result,

$$\frac{C'(q)(1-F_L(\pi))}{(F_L(\pi)-F_H(\pi))}\to 0$$

and managerial rents disappear. And once the rents disappear, the role for distortions in the capital allocation is eliminated and the first-best solution is obtained. Intuitively, since the extreme events become perfectly informative of effort, the manager obtains no rents and thus there is no scope for the use of resources to reduce those rents.

Such extreme contracts, however, are rarely observed in practice. A more realistic assumption would be to assume that the limited liability constraint binds in both directions, which eliminates the principal's ability to use such extreme contracts to achieve efficiency and leaves the manager with positive rents even under the mechanism design solution. And once those positive rents remain, the basic distortions of over-aggressiveness and either over- or undersupply of resources to limit those rents are restored as equilibrium phenomena. Since the informativeness of the signal is increasing π, the constrained solution will have the manager receiving all the returns after a given threshold $\underline{\pi}$, giving $E(w) = \int_{\underline{\pi}}^{\infty} \pi \left(q f_H(\pi) + (1-q) f_L(\pi)\right) d\pi$. But such a contract still faces the challenge that the principal's payoff drops at $\underline{\pi}$, and thus risks sabotage by the principal. So if we introduce the additional constraint of a non-negative slope for the principal's compensation, then the logic of mostly informative compensation structure reduces to a debt contract, where the expected payment for the manager is $\int_{\underline{\pi}}^{\infty} (\pi - \underline{\pi}) \left(q f_H(\pi) + (1-q) f_L(\pi)\right) d\pi$, while the principal receives the rest.