OPTIMALITY OF NON-COMPETITIVE ALLOCATION RULES

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Abstract. We study an allocation problem with asymmetric information, no monetary transfers, and ex-post verifiability. We show that optimal allocation has a number of anti-competitive features: participation might be restricted to a select group of agents; allocation is stochastic, occasionally favoring low-value agents; agents might be required to undertake costly wasteful activities prior to participation. We also show that, in contrast with the classic insight of Bulow and Klemperer (1996) for environments with monetary payments, expanding the market can be counterproductive and the principal should focus on learning details of the environment and designing an optimal mechanism for a small number of agents.

Keywords: matching with incomplete information, sequential search with incomplete information, mechanism design without transfers, negative effects of competition

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1. Introduction

Non-competitive allocation mechanisms are ubiquitous in practice. Municipal governments assign affordable housing, tax credits, construction permits, and other resources using queues and lotteries. New job positions, equipment, and office space in organizations are distributed based on reported need and waiting times. Recipients of foreign aid are determined based on prior allocation or complex bureaucratic procedures. These allocation processes are frequently accompanied by ex-post inefficiencies, barriers to participation, and diverse costly wasteful activities imposed on parties seeking these resources.

Such mechanisms can come across as suboptimal and indicative of incompetence or corruption problems within a government or an organization and a call for alternative, more competitive, procedures can be in order. Indeed, competition and expanding participation are among the underlying principles of *New Public Management* doctrine that has been pervasive in public administration since 1980s (see, e.g., Hood (1991), Table 1). The doctrine is credited with the broad push towards market-oriented mechanisms of service provision and resource allocation.[1]

The logic behind optimality of competitive allocation mechanisms is compelling. If the government can sell a resource and is concerned with allocating it to the party with the highest private valuation, it should attract as many bidders as possible and allocate the resource through a competitive mechanism. A larger pool of bidders will increase the maximal potential surplus by improving the first-order statistics of the valuation of the resource, while competition among the bidders will ensure efficient allocation, since higher valuations generally translate into higher bids in competitive mechanisms.

Nevertheless, in many economic environments, including those mentioned above, selling the resource is infeasible and the notion of efficiency is different from that of allocating the resource to the party with the maximal private value. Consider, for instance, a municipal government that would like to allocate a subsidy to a firm with the highest *social* rather than the private value of resource.[2] Each firm privately knows the social value it will generate but does not internalize it. Furthermore, payments to the government are infeasible or non-sensible because the firms have budget constraints, value the subsidy at its face value or the private valuation of

[1]For a discussion of New Public Management doctrine, its effects, and critique, see Batley and Larbi (2004), Dent, Chandler and Barry (2004), Dibben, Wood and Roper (2004), and OECD (2005).
[2]For optimal auction design with externalities, see, e.g., Jehiel, Moldovanu and Stacchetti (1996, 1999), Aseff and Chade (2008), Figueroa and Skreta (2009), and Brocas (2012).

the subsidy is independent from its social value, and are not allowed to offer the government cutbacks for agency reasons.[3]

In this environment, the government can choose the recipient of the subsidy based on the reports of the interested firms about their social value, and impose a financial or legal penalty that destroys part of the firm's surplus if the realized social value is inconsistent with the firm's promise. The penalty, however, will be bounded because the firms are legally protected by limited liability (Sappington 1983, Brander and Lewis 1986), can declare bankruptcy and divert some of the assets (Diamond 1984, 2004), courts might be unwilling to impose harsh penalties (Andreoni 1991), enforcement of contracts is imperfect (Gibbons 1988, Krasa and Villamil 2000, Laffont 2003), or monitoring of realized social value is imprecise.

We show that optimal allocation rules in these environments have a number of anti-competitive features: participation might be restricted to a select group of firms, allocation is stochastic, occasionally favoring firms with social values lower than of their competition, and firms might be required to undertake costly wasteful activities, such as, e.g., lobbying and bureaucratic paperwork, prior to participation. Thus, these seemingly inefficient features of allocation rules can be understood as an optimal response to the informational and enforcement frictions faced by the government.

In our model, there is a principal (government) who has to choose one agent (firm) from a pool of ex-ante identical agents. The agents' value for the principal is their private information and all agents would like to be selected. The utility is not transferrable, but the principal can learn the value of the selected agent ex-post and there are penalties that can be imposed on this agent. Municipal government deciding on allocation of a subsidy is one application of this model. Other applications include organizational decisions, employee search, choice of a public project, political appointments, business rankings, and allocation of foreign financial aid, among others.

We start our analysis by studying two simple and natural allocation rules: auctions, in which the government chooses the firm with the highest reported social value, and cutoff rules, in which the government randomly chooses among firms that report a value above a cutoff (Section 4). Under both rules, the government attempts to deter lies by imposing maximal possible penalty for the false reports. However, since the penalty is limited, the low value firms will lie and exaggerate their values if competition is fierce and their chances of winning the resource by truthful reporting

[3]For recent papers on mechanism design without transfers, see Alonso and Matouschek (2008), Mylovanov (2008), Börgers and Postl (2009), Kováč and Mylovanov (2009), Miralles (2012), Schmitz and Tröger (2012), Azrieli and Kim (2011), and Gershkov, Moldovanu and Shi (2013).

are slim. We show that if the firms' values are continuously distributed, then the unique equilibrium outcome in the auction is equivalent to selecting a firm at random. In cutoff rules, informative reporting is consistent with an equilibrium if the number of participating firms is not too big, providing a rationale for restricting participation.

In Sections 5-7, we study optimal allocation rules. The main theme emerging from the analysis is that competitive forces have to be checked even if the government can optimally adjust the allocation rule based on the number of participating firms. An optimal allocation rule is a *shortlisting procedure*: each firm is shortlisted with probability that is increasing in its report, and the government chooses a firm at random from the shortlist. Naturally, after the government observes the true value of the selected firm, it imposes the maximum feasible penalty whenever the firm's report is inconsistent with the realized value. These rules are ex-post inefficient since the firms with the highest value might not be selected or even shortlisted.

The central feature of the optimal rule is that the government maximizes the probability of selecting high-value firms subject to the constraint that the low-value firms are chosen frequently enough so that they do not want to misreport their information. That is, the government commits to choose low-social-value firms with some probability, even when better firms are available. This incentive constraint implies that the firm's probability to win the resource increases in its social value slower than would be optimal in the world with complete information. Surprisingly, this induces a neutrality result: the maximal attainable payoff for the government is independent of the size of the market as long as there are at least $\bar{n}$ firms, where $\bar{n}$ depends on the penalty size that can be imposed on the firms. The value of $\bar{n}$ could be quite small. For instance, if the penalty does not exceed half of the firm's surplus from being selected, then $\bar{n} = 2$.

The observation that expanding the market beyond a certain point confers no additional benefit for the principal stands in contrast with the standard environments with private values in which the surplus can be fully internalized through payments (Bulow and Klemperer 1996). In those environments, market expansion simultaneously achieves two objectives: it increases the maximal expected surplus and strengthens competition among agents forcing them to redistribute a higher share of the surplus to the principal. In our environment, the second force is not present, which leads to the different conclusion.

An important implication of Bulow and Klemperer (1996) is that design of optimal allocation rules is a second-order issue and the principal should focus on increasing agents' participation. In our environment, however, the principal's priorities are reversed and the focus should be optimal design of an allocation rule for a small number of agents.

Taken from a different perspective, our results suggest that in environments in which transfers are banned and the government's purposed objective is social welfare, the government should not be aggressively advocating use of competitive mechanisms and expanding competition among participants; and if the government does so, it is indicative of private transfers such as bribes, cutbacks, and favors between the government and the firms. This observation bears similarity to the conclusion in Esteban and Ray (2006) that both poorer economies and unequal economies can display greater misallocation of public resource even if the government is honest.[4]

The effect that the principal cannot benefit from a larger market is general and can be obtained in many other variations of the model. The result is due to two forces: externalities that cannot be priced and the ability to provide incentives based on the realized value of the principal's payoff. The value of competition is limited because of externalities, but it is optimal to sample more than one agent because the government can provide incentives ex-post.

There is a connection between the value of competition and the value of recall in a sequential search interpretation of our model.[5] Instead of using the shortlisting procedure outlined above, the principal can sample the agents sequentially, selecting each sampled agent with probability one if she reports high value and, otherwise, selecting her with minimal probability sufficient to ensure truthtelling. This rule attains the optimal payoff but requires an infinite pool of agents. The optimal shortlisting rule that needs only a handful of agents can be viewed as sequential search procedure with recall.

The optimal shortlisting rule requires commitment on the part of the principal to implement allocations that are ex-post inefficient. In Section 9, we observe that in some environments the principal needs much less commitment power to implement

[4]In Esteban and Ray (2006), the social value of allocating resource is private information of the agents who also have privately known differential wealth; the agents engage in costly lobbying for access to the resource.

[5]For recent work on search and matching with incomplete information, see, e.g., Menzio (2007), Hoppe, Moldovanu and Sela (2009), Chakraborty, Citanna and Ostrovsky (2010), Guerrieri, Shimer and Wright (2010), Galenianos, Pacula and Persico (2012), and Liu, Mailath, Postlewaite and Samuelson (2012), Cremer, Spiegel and Zheng (2006, 2007), and Moldovanu and Shi (forthcoming).

optimal rules, by approaching the agents sequentially and committing to when to stop as a function of received reports.

Finally, we demonstrate that in our model the principal might benefit from making participation costly for the agents if the principal' and the agents' payoffs are aligned in a sense (defined in Section 11). In this case, positive participation costs will turn away the agents with low values because their expected payoff from participation is low. We show that there exist allocation rules such that the principal's payoff converges to that of the environment with complete information as the number of agents converges to infinity. It is interesting that participation costs may restore the value of competition — the principal attains the maximal payoff only in the limit as the agents' pool becomes infinitely large.

If we interpret the participation costs as lobbying activity by the firms for the right to be in the pool, our results provide a warning against attempts to eradicate lobbying. In our model, the positive effect of lobbying consists in discouraging "social" speculation, whereby firms with negligible social value attempt to obtain the resource and distort the final allocation.

The rest of the paper is organized as follows. We discuss the related literature in Section 2, including the relationship between this paper and the recent paper by Ben-Porath, Dekel and Lipman (2013). The model is presented in Section 3. Section 4 studies two simple rules: auctions and simple shortlisting procedures, and shows that these rules fail to be incentive compatible if the number of participating agents is sufficiently large. We derive an upper bound on the principal's payoff in our environment in Section 5. In Section 6, we show that the upper bound can be achieved by a rule that sequentially samples agents without recall and by a shortlisting rule. Section 7 presents the result that the principal cannot benefit from expanding the pool of agents beyond a certain point. In Section 8, we consider a sequential search variation of our model. We relax the commitment power of the principal in Section 9. Section 10 considers an environment in which the principal can replace the agent after learning his value. In Section 11, we extend the model to allow for participation costs. We conclude with discussion our results and directions for future research in Section 12. Some of the proofs are in the Appendix.

2. Literature

Ben-Porath, Dekel and Lipman (2013) (henceforth, BDL) study a related model, with the key difference in modeling verification of agents' types by the principal. In

BDL the principal can pay a cost and acquire information about agents' types before making an allocation decision, that is, types can be verified *ex ante*.[6] Our paper takes a different approach: after having selected an agent, the principal learns the type and can impose a penalty on that agent, that is, types are verified *ex post*.[7]

As a consequence, the structures of feasible mechanisms and underlying incentives are distinct, giving rise to drastically different optimal mechanisms. BDL feature an intriguing and surprisingly simple optimal rule called *favored-agent mechanism*. The principal picks a favored agent, i, and a threshold. If the highest report among agents other than i is above the threshold, then that report is checked and, if confirmed, the agent with that report is selected; in any other event the favored agent is selected. In our paper the optimal mechanism is a *shortlisting procedure*, where each agent is shortlisted with a probability that depends on its reported value, and then the choice is made from the short list at random, equally likely. One major difference between these two approaches is that BDL's optimal mechanism is deterministic and asymmetric, while ours is stochastic and symmetric; in fact, in our model deterministic allocations are generally not optimal. Another major difference concerns the value of competition, the focal issue of this paper: in BDL the principal's optimal payoff is increasing in the number of (symmetric) agents, so additional competition is always valuable, while in our model this is not the case. We discuss reasons for different effect of competition in our and BDL model in Conclusions.

There is a recent literature on mechanism design with partial transfers; in this literature the agents' information is non-verifiable.[8] In Chakravarty and Kaplan (2013) and Condorelli (2012b), a benevolent principal would like to allocate an object to the agent with the highest valuation, and the agents signal their private types by exerting socially wasteful effort. The central issue is the trade-off between efficient

[6]A growing literature studies environments in which evidence that can be presented *before* an allocation decision is made, e.g., Townsend (1979), Grossman and Hart (1980), Grossman (1981), Milgrom (1981), Green and Laffont (1986), Postlewaite and Wettstein (1989), Lipman and Seppi (1995), Seidmann and Winter (1997), Glazer and Rubinstein (2004, 2006, 2012, 2013), Forges and Koessler (2005), Bull and Watson (2007), Severinov and Deneckere (2006), Deneckere and Severinov (2008), Kartik, Ottaviani and Squintani (2007), Kartik (2009), Sher (2011), Sher and Vohra (2011), Ben-Porath and Lipman (2012), Dziuda (2012), and Kartik and Tercieux (2012).

[7]In our model, the utility is not transferable. Optimal mechanism design with transfers that can depend on ex-post information has been studied in, e.g., Mezzetti (2004), DeMarzo, Kremer and Skrzypacz (2005), Eraslan and Yimaz (2007), Dang, Gorton and Holmström (2013), Deb and Mishra (2013), and Ekmekci, Kos and Vohra (2013). This literature is surveyed in Skrzypacz (2013).

[8]An exception is Bar and Gordon (forthcoming), discussed below, who consider an extension with ex-post verifiable types.

allocation and wasted resources: if truthful communication is expensive, it might be optimal to select a winner randomly or based only on the publicly available information. Condorelli (2012b) studies a very general model with heterogeneuos objects and agents and characterizes optimal allocation rules where a socially wasteful cost is a part of mechanism design. Chakravarty and Kaplan (2013) restrict attention to homogeneous objects and agents, and consider environments in which socially wasteful cost has two components: an exogenously given type and a component controlled by the principal. In particular, they demonstrate conditions under which, surprisingly, the uniform lottery is optimal.[9]

Bar and Gordon (forthcoming) consider a problem of project selection. For each project, the principal's and the project manager's values of the potential match are privately known to the manager. Transfers are permitted in one direction: the principal can subsidize but cannot tax projects. Even though the problem of efficient matching is isomorphic to Myerson (1981), the problem of the principal's revenue-maximizing project selection is different and yields an unexpected result. The optimal mechanism features randomization over projects whose reported value for the principal exceeds some threshold; the highest type is not guaranteed to be chosen.

In Manelli and Vincent (1995), a principal would like to procure a good from suppliers whose quality is uncertain. In their environment, a trading mechanism that selects the bidder with the lowest price might result in only low-quality goods being offered for sale, so competitive mechanisms might price out high quality suppliers. Che, Gale and Kim (2013) consider a problem of efficient allocation of resource to budget constrained agents and show that a random allocation with resale can outperform competitive market allocation. In an allocation problem in which the private and the social values of the agents' are private information, Condorelli (2012a) characterizes conditions under which optimal mechanism is stochastic and does not employ payments.

The literature has identified multiple reasons for restricting participation in allocation mechanisms. In auctions with entry costs, large number of bidders might be inefficient, as low-value agents have low probability to win and thus lack incentives to participate (Levin and Smith 1994, Gilbert and Klemperer 2000, Ye 2007). In our model, on the other hand, participation costs can restore the value of competition

[9]See also McAfee and McMillan (1992), Hartline and Roughgarden (2008), Yoon (2011) for environments without transfers and money burning. In addition, money burning is studied in Ambrus and Egorov (2012) in the context of a delegation model.

(Proposition 11). Compte and Jehiel (2002) study auctions in affiliated value environments and show that the uncertainty about the common value component might imply that more bidders need not lead to higher welfare.

If the value of surplus is endogenous and is determined by the actions of the agents prior to the allocation decision, excessive thickness of the market might weaken their incentives to undertake costly actions that increase the total surplus. For example, in research and development contests, it might be optimal to limit the number of participants to improve their incentives to invest in developing new technology (Taylor 1995, Fullerton and McAfee 1999, Che and Gale 2003). In financial settings, it may also be desirable to limit the number of banks to keep their incentives to screen loan applicants (Cao and Shi 2001).

3. Model

There is a principal who has to select one of many agents. The principal's payoff from a match with agent i is $x_i \in X \equiv [a, b]$, where x_i is private to agent i. The values of x_i's are i.i.d. random draws, with continuously differentiable c.d.f. F on X, whose density f is positive almost everywhere on X. Each agent i can make a statement $y_i \in X$ about his x_i.

If an agent is not selected, his payoff is 0. Otherwise, he obtains a payoff of $v(x_i) > 0$. Thus, a match with agent i generates the surplus $S(x_i) = x_i + v(x_i)$, and the distribution of the surplus between the parties is exogenously fixed.

In addition, we assume that if the agent is selected, the principal can verify x_i and impose a penalty equal to the fraction $c(x_i) \geq 0$ of the agent's share of the surplus, where c is measurable.[10] Note that c can be non-monotonic and greater than one. To save on notation, w.l.o.g. we normalize $v_i(x_i)$ to a unit of utility for agent i, $v_i(x_i) \equiv 1$.

Our primary interpretation of c is the *maximal* penalty that can be imposed on the agent, conditional on his report y_i and type x_i; that is, we assume that x_i of the selected agent can be verified ex-post. The main message of the paper is robust to extensions that allow for stochastic verification of x_i, assume that the value of x_i is verified with some noise, or make verification costly for the principal or the agent. The value $c(x_i)$ can also capture the share of the lost surplus from the match if the principal can burn the surplus or break up the match with some probability or delay after learning the payoff (see Section 10). An alternative interpretation consistent

[10]We could allow c to depend on the entire type profile, $c(x_1, x_2, \ldots, x_n)$, where n is the number of agents, without affecting any of the results. In that case, $c(x_i)$ should be thought of as the expected value of $c(x_1, x_2, \ldots, x_n)$ conditional on x_i.

with the results is that c represents the intrinsic disutility borne by the agent for being dishonest in case he is selected.

The principal has full commitment power and can choose any stochastic allocation rule that determines a probability of selecting each agent conditional on the report profile and the penalty conditional on the report profile and the type of the selected agent after it is verified ex-post.[11] The allocation rule is common knowledge among the agents. The solution concept is perfect Bayesian equilibrium.

3.1. **Applications.** *Resource allocation.* The principal is a government that has a subsidy or a construction permit to allocate to one of several firms. There are no payments between the firms and the government, for example, because of legal or political constraints. The firms have private information about the social impact of their use of the resource. With some probability, the government can verify this information ex-post, in which case it can impose a penalty on the firm if it lied in its application.

Organizational decisions. A related application (Ben-Porath, Dekel and Lipman 2013) is a dean who has a job slot to allocate to one of the departments. Each department has private information that determines the value to the dean of giving the post to the department. There are no monetary transfers, but the dean will eventually learn the value of the hired faculty and can penalize the department in the long run.

Contest. The principal is a ranking agency that selects the best business in an industry. Her objective is to select the best business based on customer opinions and referee reports. The businesses can manipulate the ranking algorithm by establishing cozy connections with referees and encouraging multiple votes from their patrons. The winner gets the spotlight; the relevant disutility cost for the winner is the probability of being caught manipulating the ranking and excluded in the future.

Public project. A regulator would like to reorganize a failing bank. Its objective is to maximize the social value of the restructured bank. There are multiple stakeholders who can make proposals about how to reorganize the bank. The quality of their proposals is their private information. Once a proposal is adopted, its quality is revealed; there is a legal penalty for deliberate distortion of information if there are negative consequences for public and/or other stakeholders.

Political appointments. The principal is a politician that would like to appoint a loyal and competent bureaucrat to an agency position. The payoff to the bureaucrat

[11]We consider the environment with limited commitment in Section 9.

is the experience, visibility, and connections he or she acquires in the position. The competence and loyalty of the bureaucrat are eventually revealed and the principal can choose to fire the bureaucrat. In particular, the agent can be fired and the penalty can be the unrealized share of the match surplus. Furthermore, the characterization of the optimal rule is not affected if we allow the principal to restart the search after firing the agent to recover the unrealized share of the surplus. We discuss this more formally in Section 10. A related example is that of a prime minister of a country who would like to choose a judge to lead a judicial inquiry into a political scandal. Prime minister is interested in selecting a judge whose preferences and views are aligned with those of the government. If the final report is unpleasant to the government, it can penalize the judge by excluding her from lucrative appointments in the future.

Foreign financial assistance. A foreign country is undergoing a political transformation as a result of, e.g., a civil war or a revolution. The government of the United States has the ability to influence who will come to power and would like to support an opposition or rebel faction whose interests are most aligned with those of the US objectives; the US will channel financial and political support to this faction. The faction's true allegiance and goals are, however, uncertain, and will come to light after, and if, it comes to power, at which point the US can choose whether to discontinue financial aid. For example, the emergency appropriation bill H.R.1591, Section 1054, passed by the US Congress in 2007 imposes a number of benchmarks that have to be met by the Iraqi government and threatens with early withdrawal of the US troops and cessation of further financial aid if the benchmarks are not met.[12] Similar aid cutoff provisions are present in many other bills, including recent bills concerning Arab Spring countries.

4. Simple Rules: Auctions and Shortlisting

In this section, we explore two simple rules: auctions and simple shortlisting procedures. In auctions, the agents report their information and the principal chooses an agent with the highest report. Under shortlisting, the principal shortlists any agent whose report exceeds a certain cutoff and choose at random an agent from the shortlist. We show that both rules are outcome equivalent to selecting an agent at random

[12]The conditions include "whether the Government of Iraq has given United States Armed Forces and Iraqi Security Forces the authority to pursue all extremists, including Sunni insurgents and Shiite militias, ..., reform of current laws governing the deBaathification process, ..., whether the Government of Iraq is ensuring the rights of minority political parties in the Iraqi Parliament are protected."

if the number of participating agents is sufficiently high. Hence, if the principal is restricted to use these rules, it might be optimal to limit the number of participating agents. In applications, exclusion can take a variety of forms, including imposing irrelevant qualifying requirements, informing about the possibility to participate only a select group of agents, and imposing early participation application deadlines.

4.1. **Auctions.** If the value of x_i were commonly known, the optimal rule for the principal would be to choose the agent with the highest x_i. When the agents' values are not observable, the rules must rely on the agents' reports. So, the principal can attempt to take advantage of the competition among the agents by running an auction in which the agents report (bid) their x_i and the principal chooses an agent with the highest report; the ties are split randomly; and the winner is penalized if and only if his report is incorrect. If agents report their x_i truthfully in the auction, then it is, clearly, an optimal allocation rule for the principal.

Let us consider a two-type example: each agent is either high type (x_H) or low type (x_L), where $x_H > x_L > 0$. The prior belief puts probability $\alpha \in (0,1)$ on low type. The relevant incentive constraint in the auction is that low type finds it optimal to report his true type

$$1 \cdot \Pr(\text{match}|x_L) \geq (1 - c(x_L)) \Pr(\text{match}|x_H)$$

If all agents report their types truthfully,

$$\begin{aligned}
\Pr(\text{match}|x_L) &= \frac{\alpha^n}{n}, \\
\Pr(\text{match}|x_H) &= \sum_{k=1}^{n} \frac{1}{k} \frac{(n-1)!}{(k-1)!(n-k)!} (1-\alpha)^{k-1} \alpha^{n-k} \\
&= \frac{1}{n(1-\alpha)} \sum_{k=1}^{n} \frac{n!}{k!(n-k)!} (1-\alpha)^k \alpha^{n-k} \\
&= \frac{1-\alpha^n}{n(1-\alpha)}.
\end{aligned}$$

We obtain

Remark 1. In a two-type environment, the auction is incentive-compatible if and only if

$$c(x_L) \ \geq \ 1 - \frac{\alpha^n}{1-\alpha^n}(1-\alpha).$$

For any $c(x_L)$, the auction is not incentive compatible if n is sufficiently large, in which case the unique equilibrium outcome is for all agents to report x_H regardless of their types. As a result, the principal might want to restrict participation in auction in order to make reports informative.

Corollary 1. *If $c(x_L) < 1$ and the principal is restricted to use auction in a two-type environment, it can be optimal to exclude agents from participating.*

The above observations can be easily generalized to discrete type environments with more than two types. However, as we show in the following proposition, auction is not incentive-compatible if types are continuous.

Proposition 1. *Let types be distributed continuously and let $\sup c(x) < 1$. Then in the unique symmetric equilibrium of the n-agent auction every agent $i \in \{1, \ldots, n\}$ reports the highest value b irrespective of his type.*

The existence argument is straightforward. If everyone is bidding the highest value, b, for all $j \neq i$, then agent i also finds it optimal to bid the highest value since this bid ensures $\frac{1}{n}(1 - c(x_i)) > 0$ and every other bid has no chance of winning. The uniqueness is shown by, first, observing that for low values of x bidding truthfully is dominated by paying penalty $c(x)$ and outbidding, possibly weakly, everyone else and, then, applying this argument inductively for other values of x. The complete proof is in the Appendix.

4.2. **Shortlisting rules.** In order to provide incentives for the agents to report their types truthfully, we can modify the auction in continuous type environments to mimic the incentive structure of the two-type example. Consider the following *simple shortlisting rule*: All agents who report types above some cutoff $\bar{x}$ are shortlisted and the principal chooses an agent at random from the shortlisted agents. If no one is shortlisted, the principal randomly picks an agent. As in auction, the chosen agent is penalized if and only if his report is a lie.

The following result follows from the argument identical to the one for Remark 1.

Proposition 2. *There exists a truthtelling equilibrium in the simple shortlisting rule with cutoff $\bar{x}$ if and only if*

$$c(x) \geq 1 - \frac{F^n(\bar{x})}{1 - F^n(\bar{x})}(1 - F(\bar{x})) \quad \text{for all } x \leq \bar{x}. \tag{1}$$

Note that if $\sup c(x) < 1$, then for any given $\bar{x}$ the rule ceases to have a truthtelling equilibrium for a sufficiently large n. Hence, we have a result similar to the one for auction in two-type environments.

Corollary 2. *If the principal is restricted to use simple shortlisting rules, it can be optimal to exclude agents from participating.*

We have established that it might be optimal to restrict competition among agents if the principal is limited to use a certain class of rules. It is not clear, however, if it continues to be optimal to exclude agents if there is no restriction on the rules that can be used for the principal. Intuitively, as the number of agents increases, the principal might be able to adjust the rules to maintain incentive compatibility and benefit from superior distribution of the highest type in the agents' pool.

5. Upper bound

In this section, we allow the principal to consider any rule and derive an upper bound on the principal's payoff. An allocation rule (p, ξ) associates with every profile of statements $y = (y_1, \ldots, y_n)$ a probability distribution $p(y)$ over $\{1, 2, \ldots, n\}$, where for each agent i, $p_i(y)$ stands for the probability of choosing i, and a family of functions $\xi_i(x_i, y) \in [0, 1]$, $i = 1, \ldots, n$, which determine the probability that agent i is penalized if he is selected given his type and the report profile.

Let x^* be the unique solution of

$$\int_a^{x^*} (x^* - x) \max\{1 - c(x), 0\} f(x) \mathrm{d}x = \int_{x^*}^b (x - x^*) f(x) \mathrm{d}x. \tag{2}$$

Proposition 3. *In any allocation rule, the principal's payoff is at most x^*.*

To understand (2), note that by the revelation principle, we can focus on incentive-compatible rules in which truthtelling is an equilibrium. Since x_i of the selected agent is verifiable, it is optimal to penalize the selected agent whenever he lies, $y_i \neq x_i$, and not penalize him otherwise.

If the penalty exceeds the surplus, $c(x) \geq 1$ for all x, then this is sufficient to deter lying. Then, the principal payoff is uncapped and $x^* = b$. Otherwise, the penalty might be insufficient to ensure truthful reporting. Since there are no transfers, the incentives have to be provided through allocative distortion: each agent must be chosen with a high enough probability so that the benefit of making a false report is less than the penalty.

Now, let $g_i(\hat{x})$ be the expected probability that agent i is selected if he reports $\hat{x}$. Hence, $g_i(x_i)$ is the agent's payoff from truthful report. The most profitable deviation for the agent is to report a type that maximizes the probability of selection, in which case he obtains $\sup g_i \cdot (1 - c(x_i))$ since he will be penalized after his lie is found out. Thus, incentive compatibility requires that $g_i(x) \geq \sup g_i \cdot \max\{1 - c(x), 0\}$ (c.f. the left-hand side of (2)).

The principal's payoff is maximized by a cutoff rule that maximizes probability of selecting agents with types above x^* subject to the constraint that the types below x^* are selected with high enough probability to provide incentives for truthful reporting. The left-hand side of (2) is the marginal incentive cost as a function of x^* due to the rents that have to be given to the agents below x^*, while the right-hand side of (2) is the marginal value of selecting agents with types above x^*.

Proof of Proposition 3. By the revelation principle, it is sufficient to consider allocation rules in which it is a perfect Bayesian equilibrium for all agents to make truthful statements. Furthermore, without loss of generality, we focus on the rules in which reports are punished if and only if they are dishonest. So, henceforth, we set $\xi_i(x_i, y) = 0$ if $y_i = x_i$ and 1 otherwise and drop ξ in the description of the allocation rules.

Fix allocation rule p. Denote by $\bar{F}$ the joint c.d.f. of all n agents and by $\bar{F}_{-i}$ the joint c.d.f. of all agents except i. Denote by $g_i(y_i)$ the expected probability that agent i is chosen in the truthful equilibrium after reporting y_i,

$$g_i(y_i) = \int_{x_{-i} \in X^{n-1}} p_i(y_i, x_{-i}) \mathrm{d}\bar{F}_{-i}(x_{-i}). \tag{3}$$

The payoff of agent i whose type is x_i and who reports y_i is equal to

$$V_i(x_i, y_i) = g_i(y_i)(1 - \mathbf{1}_{y_i \neq x_i} c(x_i))$$

where $\mathbf{1}$ is the indicator function that equals one if the selected agent is punished, $y_i \neq x_i$, and zero otherwise. Hence, each i's incentive constraint is $V_i(x_i, x_i) \geq V_i(x_i, y_i)$ for all $x_i, y_i \in X$, or equivalently,

$$g_i(x) \geq (1 - c(x))\bar{g}_i \quad \text{for all } x \in X, \tag{4}$$

where $\bar{g}_i = \sup g_i(x)$. The principal's problem is to

$$\max_p \sum_{i=1}^n \int_{x_i \in X} x_i g_i(x_i) f(x_i) \mathrm{d}x_i$$
$$\text{s.t. (3) and (4) for all } i = 1, \dots, n.$$

By symmetry, we can use the same g_i for all agents, $g_i = g$. The relevant incentive constraint for agent i is, therefore,

$$g(x) \geq (1 - c(x))\bar{g} \quad \text{for all } x \in X. \tag{4'}$$

The expected payoff for the principal is equal to

$$W(g) = n \int_{x \in X} x g(x) f(x) \mathrm{d}x.$$

Let g^* be a maximizer of the relaxed principal's problem in which we maximize $W(g)$ over g subject to (4′) and the interim feasibility constraint[13]

$$n \int_{x \in X} g(x) f(x) \mathrm{d}x = 1. \tag{5}$$

We optimize $W(g)$ by assigning the highest feasible weight (a constant $\bar{g}$) to high values and the lowest feasible, incentive compatible weight (equal to $\bar{g} \max\{1 - c(x), 0\}$) to low values. So, there exists threshold $x^* \in X$ such that almost everywhere on X

$$g^*(x) = \begin{cases} \max\{1 - c(x), 0\}\bar{g}^*, & \text{if } x < x^*, \\ \bar{g}^*, & \text{if } x \geq x^*. \end{cases}$$

Otherwise, we can redistribute the probability mass from low values of x for which there is slack in (4′) to higher values of x for which $g^*(x) < \bar{g}^*$ and increase the principal's payoff value.

Denote by h_x^* the transformed density function which coincides with the original density f for values above x^* and is minimized s.t. the incentive constraint for values below x^*:

$$h_{x^*}(x) = \begin{cases} \max\{1 - c(x), 0\}, & x < x^*, \\ 1, & x \geq x^*. \end{cases}$$

Let

$$H(x^*) = \int_X h_{x^*}(x) f(x) dx \quad \text{and} \quad V(x^*) = \frac{1}{H(x^*)} \int_X x h_{x^*}(x) f(x) dx. \tag{6}$$

[13] We relax the constraint that there should exist p consistent with g.

Note that $\frac{h_{x^*}(x)f(x)}{H(x^*)}$ is a probability density, and $V(x^*)$ is the expectation w.r.t. that density.

Since $g^*(x) = \bar{g}^* h_{x^*}(x)$, we have $W(g^*) = n\bar{g}^* H(x^*)V(x^*)$, and feasibility constraint (5) reduces to

$$n\bar{g}^* H(x^*) = 1. \tag{7}$$

Substituting (7) into the payoff, we get

$$W(g^*) = \max_{x^* \in X} V(x^*).$$

The first-order condition reduces to (2). By the standard argument, the solution is unique.[14] A straightforward calculation establishes that $W(g^*) = x^*$.[15] The value of x^* is the upper bound on the principal's payoff, because we have not verified that there exists p consistent with g^*. ■

Comparative statics for x^*. We now provide several intuitive comparative statics results for x^*. The value of x^* increases if the principal has access to stricter penalties and if higher types are more likely.

Corollary 3. *Let $\tilde{c}(x) > c(x)$ for almost all $x \in [a, x^*]$. Then, $x^*(\tilde{c}, F) > x^*(c, F)$.*

The proof is immediate by (2).

Corollary 4. *Let $c(x)$ be nondecreasing and a.e. differentiable. Suppose that F first-order stochastically dominates $\tilde{F}$. Then $x^*(c, F) > x^*(c, \tilde{F})$.*

Proof. Rewrite (2) as

$$\int_a^{x^*} \left(x^* - \max\{1 - c(x), 0\}(x^* - x)\right) f(x)\mathrm{d}x + \int_{x^*}^b x f(x)\mathrm{d}x = x^*.$$

Integrating by parts and rearranging the terms, we obtain

$$\int_a^{x^*} \left(\max\{1 - c(x), 0\} - \frac{\mathrm{d}}{\mathrm{d}x} \max\{1 - c(x), 0\}(x^* - x)\right) F(x)\mathrm{d}x + \int_{x^*}^b F(x)\mathrm{d}x = b - x^*.$$

Note that $\frac{\mathrm{d}}{\mathrm{d}x} \max\{1 - c(x), 0\}$ is a.e. well defined and nonpositive. Hence, replacing F by $\tilde{F}$ makes the left-hand side smaller, and consequently, x^* greater. ■

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[14]Since f is a.e. positive, the left-hand side of (2) is strictly increasing in x^*, moreover, for $x^* = a$ we have $-(\int_a^b x f(x)\mathrm{d}x - a) < 0$ and for $x^* = b$ we have $b - \int_a^b x \max\{1 - c(x), 0\} f(x)\mathrm{d}x > 0$.

[15]$W(g^*) = x^* + \frac{1}{H(x^*)}\left(-\int_a^{x^*}(x^* - x)\max\{1 - c(x), 0\} f(x)\mathrm{d}x + \int_{x^*}^b (x - x^*) f(x)\mathrm{d}x\right) = x^*$ since the second term is zero by (2).

Example. Let F be uniform on $[a,b]$ and let c be constant. Then x^* solves

$$\int_a^b (x^*-x)\mathrm{d}x = c\int_a^{x^*}(x^*-x)\mathrm{d}x,$$

which reduces to $x^*(b-a)-\left(\frac{b^2}{2}-\frac{a^2}{2}\right) = c\left(x^*(x^*-a)-\left(\frac{(x^*)^2}{2}-\frac{a^2}{2}\right)\right)$. If $[a,b]=[0,1]$, the expression is equal to $c(x^*)^2-2x^*+1=0$. The relevant root is $x^*=\frac{1-\sqrt{1-c}}{c}$.

6. Implementation

How many agents are required to attain the payoff of x^*? Let $n=\infty$ and consider the allocation rule p^*_∞ that samples agents sequentially, until some agent is selected. In every period a new agent is drawn and selected with probability

$$h_{x^*}(y) = \begin{cases} \max\{1-c(y),0\}, & y<x^*, \\ 1, & y\geq x^*, \end{cases}$$

given his report y.[16] This rule achieves x^*. To see this, note that if the first agent is selected, then the principal's expected payoff is equal to x^*:[17]

$$V_1 \;=\; \int_a^{x^*} x\max\{1-c(x),0\}f(x)\mathrm{d}x + \int_{x^*}^b xf(x)\mathrm{d}x = x^*.$$

This continues to hold for any agent: conditional on nth agent being selected, the principal's expected payoff is x^*. Since there are infinitely many agents, the principal will select an agent eventually, implying that the rule implements the payoff of x^*.

Proposition 4. *The rule p^*_∞ attains the payoff of x^*.*

Proof. Let $H(x^*)$ and $V(x^*)$ be as in (6). The principal's payoff $W(p^*_\infty)$ is stationary:

$$W(p^*_\infty) \;=\; \int_X \Big(h_{x^*}(x)x+(1-h_{x^*}(x))W(p^*_\infty)\Big)f(x)\mathrm{d}x,$$

hence $W(p^*_\infty)=\frac{1}{H(x^*)}\int_X xh_{x^*}(x)f(x)\mathrm{d}x = V(x^*)=x^*$. ■

Could the same payoff be achieved with a smaller number of agents? For a given penalty function c and a given c.d.f. F, denote by $\bar{n}$ the least integer that satisfies

$$c(x)\leq \frac{1-F^{\bar{n}-1}(x^*)}{1-F^{\bar{n}}(x^*)} \text{ for all } x\leq x^*. \tag{8}$$

[16]Note that this rule is asymmetric; the proof of the upper bound x^* applies to all rules, including asymmetric ones.

[17]C.f. footnote 15.

This is a condition on primitives: F and c determine x^* and, consequently, $\bar{n}$. Observe that $\bar{n}$ exists if and only if $\sup\{c(x)|x \leq x^*\} < 1$.

Consider the following *shortlisting procedure.* Let each agent $i = 1, \ldots, n$ be short-listed with some probability $q(y_i)$ given report y_i. The rule chooses an agent from the shortlist with equal probability. If the short list is empty, then the choice is made at random, uniformly among all n agents.

Proposition 5. *If $n \geq \bar{n}$, then there exists a shortlisting procedure that attains the payoff of x^*.*

The proof is relegated to the Appendix.

To obtain the intuition why the principal can achieve the payoff of x^* with fewer than $n = \infty$ agents, recall that in the sequential sampling rule described in Proposition 4, the principal achieves the payoff x^* conditional on each selected agent. This rule selects kth agent with probability $(1 - H(x^*))^{k-1}H(x^*)$ and, thus, the expected contribution of this agent to the principal's payoff is $x^*(1 - H(x^*))^{k-1}H(x^*)$.

The proof of Proposition 3 has established that each agent can contribute up to $x^*H(x^*)$ to the payoff of the principal. The proof of Proposition 5 constructs a symmetric rule in which each agent contributes the same amount to the principal's payoff and their total contribution is equal to x^*.

The shortlisting procedure is an indirect implementation of this rule. It is obtained by setting the probability of shortlisting to be equal to

$$q(x) \;=\; (Ah_{x^*}(x) - B)/(A - B), \tag{9}$$

where A and B are the expected probabilities to be chosen conditional on being shortlisted and conditional on not being short-listed that depend on q and the number of agents n. Hence, the solution of q is a fixed point. The values of A and B are decreasing in n. The proof then shows that there exists a finite $\bar{n}$ such that there exists finite n such that (9) has a solution.

Note that (9) implies that an agent's payoff conditional on his truthful report x is equal to the probability to be chosen,

$$q(x)A + (1 - q(x))B = Ah_{x^*}(x),$$

whereas his payoff from reporting $y \geq x^*$, $y \neq x$, is equal to $A(1 - c(x)) \leq Ah_{x^*}(x)$. Hence, the rule is incentive-compatible. Furthermore, it delivers the principal the payoff of x^* as

$$W = \frac{\int_X x\,(q(x)A + (1-q(x))B)\,f(x)\mathrm{d}x}{\int_X (q(x)A + (1-q(x))B)\,f(x)\mathrm{d}x} = \frac{1}{H(x^*)}\int_X x h_{x^*}(x) f(x) = V(x^*) = x^*.$$

The optimal shortlisting rule requires commitment by the principal to sometimes forgo the agent with the highest report. We discuss the role of commitment power in more detail Section 9 and argue that the principal requires much less commitment power. In particular, we show for two type environments that an optimal rule can be implemented by a principal who can commit to stop sampling agents with some probability.

Condition (8) is not particularly elegant. Here is a much simpler condition that is independent of F and x^*. Let $\tilde{n}$ the smallest positive integer such that[18]

$$c(x) \leq 1 - \frac{1}{\tilde{n}} \quad \text{for all } x \in X \tag{10}$$

By construction $\tilde{n} \geq \bar{n}$[19] and hence

Corollary 5. *If $n \geq \tilde{n}$, then there exists a shortlisting procedure that attains the payoff of x^*.*

7. Value of competition

Thus, the value of competition is limited and expanding the market beyond $\bar{n}$ agents confers no benefit to the principal. Propositions 3 and 5 imply our central result:

Theorem 1. *The optimal rule with $\bar{n}$ agents is superior to any rule with $n > \bar{n}$ agents.*

If expanding the market is costly, this result strengthens: the optimal rule with $\bar{n}$ agents is *strictly* superior to any rule with $n > \bar{n}$ agents.

Thus, in contrast with the classic insight in Bulow and Klemperer (1996), competition has limited value, and the principal should focus on learning details of the environment and designing an optimal mechanism for a small number of agents.

[18]Note that $\tilde{n}$ also exists if and only if $\sup_{x\in X} c(x) < 1$.

[19]Indeed, for any $z \in (0,1)$

$$\frac{1-z^{n-1}}{1-z^n} = \frac{1+z+\ldots+z^{n-2}}{1+z+\ldots+z^{n-2}+z^{n-1}} = 1 - \frac{1}{z^{-(n-1)}+z^{-(n-2)}+\ldots+z^{-1}+1} \geq 1 - \frac{1}{n}.$$

Moreover, under fairly broad conditions, $\bar{n}$ is very small. For example, suppose that the penalty is bounded by half of the utility of matching:

$$c(x) \leq \tfrac{1}{2} \text{ for all } x \in X.$$

Then it is optimal for the principal to look at most *two* agents. On the other hand, $\bar{n} \to \infty$ as $c(x)$ converges to one for all x.

8. Sequential search

Time is discrete, $t = 1, 2, \dots, \infty$, and the principal can sample agents at cost $\gamma \geq 0$ per agent. The rest of the model is identical. If $\gamma = 0$ and there is no recall of agents, the optimal rule for the principal is to choose the agent with probability one if he reports $x \geq x^*$ and with probability $\max\{1-c(x), 0\}$ otherwise. This rule implements the payoff of x^*. For positive sampling costs, the optimal rule without recall has the same cutoff structure but a lower value of cutoff, which is the unique solution of

$$\int_a^{x^*} (x^* - x) \max\{1 - c(x), 0\} f(x) \mathrm{d}x + \gamma = \int_{x^*}^{b} (x - x^*) f(x) \mathrm{d}x. \tag{11}$$

Proposition 6. *Assume the principal samples agents sequentially at cost* $\gamma \leq \mathbb{E}[x]$, *and recall is not allowed. Then the principal's optimal rule is to choose the sampled agent with certainty if his report is above* x^* *and to choose him with probability* $\max\{1 - c(x), 0\}$ *if his report is below* x^*, *and sample a new agent with the complementary probability.*

Note that in the standard sequential model with observable types, the optimal cutoff is given by the solution of $\gamma = \int_{x^*}^{b} (x - x^*) f(x) \mathrm{d}x$ and the optimal rule chooses the first agent with type exceeding x^*. Hence, $\int_a^{x^*} (x^* - x) \max\{1 - c(x), 0\} f(x) \mathrm{d}x$ can be interpreted as the search cost incurred by the principal due to privacy of the agents' types.

Since sampling agents is costly, recall is valuable. Furthermore, the principal will sample fewer agents that would be optimal if there were no sampling costs, implying that the optimal rule samples at most $\bar{n}$ agents. Hence, the result that competition has limited value can be viewed from the perspective of the value of recall in the search interpretation of our model.[20] Thus:

Proposition 7. *Assume the principal has to sample agents sequentially at cost* γ *and recall is allowed. Then, it is optimal for the principal to sample at most* $\bar{n}$ *agents.*

[20] For a survey of search literature, see Rogerson, Shimer and Wright (2005).

9. Limited commitment

The optimal shortlisting rule requires commitment on the part of the principal to implement allocation that is ex-post inefficient. To implement optimal rules, however, the principal requires much less commitment power.

Consider the class of *stopping rules* that sample agents sequentially and prescribe when to stop as a function of received reports. Once the search is stopped, the principal makes an ex-post optimal choice among the sampled agents. That is, the principal can commit to stop sampling with some probability, but she cannot commit to choose an ex-post inefficient agent.

In two-type environments, optimal rules can be implemented as stopping rules: after sampling an agent, the principal commits to stop search with certain probability. We state this observation without proof (see the working paper version of this paper).

Proposition 8. *If the environment has two types and the principal can commit to stopping rules, then the principal can attain the optimal payoff.*

Another way to relax commitment is to consider indirect rules in which the agents do not report their types, but report whether the types belong to a subset of X. Clearly, the simple shortlisting rules permit an equilibrium in which the agents report if their types is above or below a cutoff and the chosen agent is punished if he lies.

Proposition 9. *The principal can implement simple shortlisting rules even if she cannot commit to choose an inefficient agent conditional on available information at the moment of selection.*

The same idea will allow to implement a somewhat more complex rules, in which X is partitioned, and the agents report elements of the partition. The principal then selects at random any agent that reported the highest element of the partition. If the number of agents is not too high and the partition is sufficiently coarse, this rules are incentive compatible and do not require the principal to commit to select ex-post inefficient agents.

We leave the question of whether these constructions can be adjusted to implement optimal rules in general environments and, otherwise, how much is the principal's loss due to lack of commitment power for future research.

10. Replacement

In this section, we would like to substantiate the claim we made after describing the model in Section 3 that making the punishment costly for the principal does not affect the results. Let us look at the model in which the principal and the agent interact for $T > 1$ periods, the principal learns the agent's type at the end of the first period, and can punish the agent by firing him. Firing the agent, however, is costly for the principal, as the principal need to replace the agent in the second period. The agent's utility per period is 0 if unemployed and $\frac{1}{T}$ otherwise. The principal's payoff from employing agent i is $\frac{x_i}{T}$ per period. If the principal does not employ the agent, she obtains the outside option of $\frac{\bar{x}}{T}$ per period. We can think of $\bar{x}$ as the expected quality of another agent chosen at random from the pool of remaining agents.

The arguments for the upper bound of the principal's payoff extend immediately to this model. Since the cost of lying in period 1 is the agent's foregone payoff in all subsequent periods, $c(x) = (T-1)/T$, the upper bound of the principal's payoff is the max of $\bar{x}$ and x^*, where x^* is the unique solution of

$$\frac{1}{T}\int_a^{x^*}(x^* - x)f(x)\mathrm{d}x = \int_{x^*}^b (x - x^*)f(x)\mathrm{d}x. \tag{12}$$

If $x^* \geq \bar{x}$, the shortlisting rule described in the proof of Proposition 5 attains this payoff. Furthermore,

Proposition 10. *For any T, the principal can attain the payoff of x^* with T agents.*

Proof. By (10) and Corollary 5, the upper bound of the minimal number of agents required to attain x^* is less than or equal to T. ∎

11. Costly Participation

In this section, we demonstrate that in our model the principal might benefit from making participation costly for the agents. For example, a government can impose costly bureaucratic requirements on the companies that would like to apply for a subsidy or a construction permit and the selection procedure can be made lengthy, require physical presence of top management of the companies, and the companies can be burdened with unnecessary paperwork. These requirements are socially wasteful, but in some environments can discourage participation of companies with low value for the principal and increase the efficiency of the final outcome.

Recall that $v(x_i)$ is the payoff of an agent i from the match, which has been normalized to one. In the environments with participation costs, this normalization is no longer valid and, hencenforth, we allow for $v(x_i)$ different from 1. Let $\gamma \geq 0$ be the utility loss the the agent has to incur in order to participate in an allocation rule.

Participation costs can be beneficial for the principal if the principal and the agents payoffs are aligned and $v(x)$ is increasing. In this case, positive participation costs will turn away the agents with low types because their expected payoff from participation is low. The downside is that with some probability all agents may happen to be low types and no one participates, so the principal ends up with no match. In an optimal rule with participation costs, the principal will balance these forces pretty much as a monopolist trades off the probability of sale with price. The proposition below exhibits an environment and an allocation rule such that the principal's payoff converges to that of the environment with complete information as the number of agents converges to infinity. The rule has is qualitatively different from the optimal rule identified in the previous sections: the agents are not asked to report their types, the chosen agent is always penalized, and the penalty is less than the maximum and varies with the agent's type. It is interesting that participation costs may restore the value of competition — the principal attains the maximal payoff only in the limit as the agents' pool becomes infinitely large. These results, however, require a joint assumption on the preferences of the agent and penalty function c.

Proposition 11. *Suppose that there exists a strictly increasing function $t(x)$ such that $v(x)(1-c(x)) \leq t(x) \leq v(x)$ for all $x \in X$. Then, for every γ, $0 < \gamma < \sup t(x)$, the principal's optimal payoff approaches the maximal value of the agent's type b as $n \to \infty$.*

The proof of this proposition is below. The conditions of the proposition on $v(x)$ and $c(x)$ can not be discarded with easily. In particular, if the principal's and agents' payoffs from a match are misaligned and $v(x)$ is decreasing, then imposing a participation cost need not be beneficial for the principal. For example, if $v(x)$ is decreasing fast enough, positive participation costs will only have a negative effect by excluding high type agents. Thus, in the environments in which the principal is eager to select the agents who are not eager to be selected and the opposite is true for the agents, the principal will continue to benefit from limiting competitive pressure on the agents but will not use participation costs to exclude them.

We conjecture that there is a relationship between the conditions under which the principal can benefit from imposing participation costs and conditions under which other forms of transfers can be valuable. For example, Condorelli (2012a) shows that monetary transfers are not valuable as an incentive tool[21] if the principal's objective

[21]Ex-post verification is not possible.

function is negatively correlated with the agents' willingness to pay for the object, which, in spirit, is the opposite of the condition in Proposition 11.

Proof of Proposition 11. Consider the rule that makes a uniform choice from participating agents and imposes penalty of $1 - t(x)/v(x)$ on the selected agent. In this rule, conditional on being selected, the agent obtains the payoff of $t(x)$. Then, an agent with type x participates if and only if

$$Q_n t(x) \geq \gamma,$$

where Q_n is the probability of being chosen. Since $t(x)$ is increasing, for any Q_n there exists $\tilde{x} \in [a, b]$ such that $Q_n t(\tilde{x}) < \gamma$ for any type below $\tilde{x}$ and no type below $\tilde{x}$ participates. Then,

$$Q_n = \sum_{k=1}^{n} \frac{1}{k} \binom{n-1}{k-1} (1-F(\tilde{x}))^{k-1} F^{n-k}(\tilde{x}) = \frac{1 - F^n(\tilde{x})}{n(1 - F(\tilde{x}))} = \frac{1 + F(\tilde{x}) + \ldots + F^{n-1}(\tilde{x})}{n}.$$

Thus, $\tilde{x}$ is the unique solution of

$$\frac{1 + F(\tilde{x}) + \ldots + F^{n-1}(\tilde{x})}{n} t(\tilde{x}) = \gamma$$

if $n > t(a)/\gamma$ and $\tilde{x} = a$ otherwise. Observe that, for a given $\tilde{x}$, the left-hand side is decreasing and converges to zero as $n \to \infty$. Consequently, $\tilde{x}$ is increasing in n and approaches b as $n \to \infty$. The payoff of the principal is equal to

$$(1 - F^n(\tilde{x}))\mathbb{E}[x|x > \tilde{x}],$$

and approaches b as $\tilde{x} \to b$. ■

12. Conclusions

In this paper, we study an allocation problem in which a principal has to choose one among several agents. We have shown that if the agents have private information about their value for the principal and transferability of utility is limited, the optimal rules for the principal exhibit a number of interesting features. The principal might want to limit competitive pressure on the agents by excluding some of them from participation in the allocation mechanism, can benefit from using lotteries to choose agents, occasionally favoring an agent with lower value than his competitors, as well as from requiring the participating agents to undertake socially wasteful and individually costly activities. Limited punishment requires leaving information rents to low value agents and flattens the interim expected probability of selecting an agent relative to the complete information benchmark in which the agents do not have private information.

We have characterized optimal rules and demonstrated that the principal can obtain the optimal payoff with finitely many agents if all agents can simultaneously report their information. On the other hand, if the agents must be sampled sequentially, the principal's maximal payoff is attained within finite pool of agents if and only if the agents can be recalled.

Our model is an idealization of environments with non-pecuniary externalities: if selected, agent i imposes externality x_i on the principal. In order to clarify the economics of the problem, we have shut down other considerations. A natural extension is an auction environment with externalities in which the agents have private information about their valuation of the good as well as about the externality they impose on the principal and the other agents if they win the auction. (A symmetric problem is that of a principal (a firm) who solicits financing from the agents and can impose an externality on the agents' payoffs, c.f. Diamond (2004).)

In Section 9, we have demonstrated that the principal requires very little commitment to implement optimal rules in environments with two types. An open question is whether this results extends to more general environments.

In our model, the principal can verify the information of an agent after he is selected. The polar assumption is made in Ben-Porath, Dekel and Lipman (2013) (BDL), where the principal can verify, at a cost, any agent's information before selection. In BDL, the principal always benefits from increasing the agents' pool. Thus, it would be valuable to better understand the role of different assumptions about incentive tools available to the principal in these environments. In BDL, the monitoring technology is perfect and, hence, the agent's surplus acquired by a lie is fully burned if the agent is monitored. This corresponds to the case of $c \equiv 1$ in our model. It is possible that incentive issues similar to those in our model will reappear in BDL if monitoring becomes imperfect.

In our model, the principal is patient and attracting agents is either free or imposes a fixed cost of $\gamma \geq 0$. A natural alternative is to allow for sequentially arriving agents and an impatient principal. In this environment, the principal will face the tradeoff between the quality of the selected agent and the time of match. One can also consider richer extensions with impatient principal, along the lines of Section 10.

Appendix

Proof of Proposition 1. The existence is straightforward. To prove uniqueness, consider the partition of $X = [a, b]$ to intervals with endpoints $(x^0, x^1, \dots, x^S)$ with $a = x^0 < x^1 < \dots < x^S = b$ that satisfy for every $s = 1, \dots, S-1$

$$n(F(x^s) - F(x^{s-1}))^{n-1} < 1 - \bar{c}$$

where $\bar{c} = \sup c(x)$. By assumption, $\bar{c} < 1$, and density f is assumed to be a.e. positive, so this partition is well defined and has a finite size.

Observe that in n-agent auction, since all lies are penalized, each agent i either states the truth or the value that maximizes the probability of winning.

We proceed by induction in $s = 0, 1, \dots, S-1$. Suppose that for types in $[a, x^s)$ stating the true value is not optimal (for $s = 0$ this is trivial as it applies to the empty inteval). Then, if a type $x_i \geq x^s$ tells the truth, her payoff is almost surely at most $(F(x_i) - F(x^s))^{n-1}$. On the other hand, if i lies, she can ensure the payoff of at least $\frac{1}{n}(1 - c(x_i))$ by reporting b whenever $x_i < b$. Hence, i strictly prefers to lie if $(F(x_i) - F(x^s))^{n-1} < \frac{1}{n}(1 - c(x_i))$, or, equivalently,

$$n(F(x_i) - F(x^s))^{n-1} < 1 - c(x_i).$$

Since $c(x_i) \leq \bar{c}$, the above inequality holds for all $x_i \in [x^s, x^{s+1})$. Consequently, types in $[a, x^{s+1})$ do not state the truth. Since everyone lies and pays the penalty that is independent of the report, the unique equilibrium outcome is for everyone to submit the highest possible report b. ■

Proof of Proposition 5. Define the probability of shortlisting an agent who reports x as

$$q(x) = \frac{Ah_{x^*}(x) - B}{A - B},$$

where A and B are the expected probabilities to be chosen conditional on being short-listed and conditional on not being short-listed, respectively. Let $Q = \int_X q(x)f(x)\mathrm{d}x$ be the ex-ante probability to be short-listed. Then,

$$A = \sum_{k=1}^{n} \frac{1}{k}\binom{n-1}{k-1}Q^{k-1}(1-Q)^{n-k} \quad \text{and} \quad B = \frac{1}{n}(1-Q)^{n-1}.$$

The value of Q is implicitly defined by

$$Q = \frac{AH(x^*) - B}{A - B}.$$

Lemma 1. *There is a unique solution of equation* $Q = \frac{AH(x^*)-B}{A-B}$ *w.r.t.* Q.

Proof. Note that $A > B$ for all $Q \in [0,1]$, since $n \geq 2$. Thus, if $H(x^*) = 1$, then the unique solution is $Q = 1$.

Assume $H(x^*) < 1$. Then, for $Q = 0$ we have $A = 1$ and $B = 1/n$, so LHS<RHS, since $H(x^*) > 1/n$ by (10) and $x^* < b$. Also, for $Q = 1$ we have $A = 1/n$ and $B = 0$, so LHS>RHS. Thus, neither $Q = 0$ or $Q = 1$ are solutions.

Now, we establish uniqueness of the solution. Assume $0 < Q < 1$. Simplify A:

$$A = \sum_{k=1}^{n} \frac{1}{k} \frac{(n-1)!}{(k-1)!(n-k)!} Q^{k-1}(1-Q)^{n-k} = \frac{1}{nQ} \sum_{k=1}^{n} \frac{n!}{k!(n-k)!} Q^{k}(1-Q)^{n-k}$$

$$= \frac{1}{nQ}\left(1-(1-Q)^n\right) = \frac{(1-Q)^n}{nQ}\left(\frac{1}{(1-Q)^n} - 1\right) = B\frac{(1-Q)}{Q}\left(\frac{1}{(1-Q)^n} - 1\right).$$

Rewriting $Q = \frac{AH(x^*)-B}{A-B}$, we have

$$\left(\frac{A}{B} - 1\right) Q = \frac{A}{B} H(x^*) - 1,$$

or

$$\frac{A}{B}(H(x^*) - Q) = 1 - Q.$$

Substituting the expression for A, we obtain

$$\frac{(1-Q)}{Q}\left(\frac{1}{(1-Q)^n} - 1\right)(H(x^*) - Q) = 1 - Q$$

or

$$\left(\frac{1}{(1-Q)^n} - 1\right)(H(x^*) - Q) = Q$$

or

$$\frac{H(x^*) - Q}{(1-Q)^n} = H(x^*)$$

or

$$H(x^*) - Q = (1-Q)^n H(x^*).$$

or

$$(1-Q)^n H(x^*) + Q - H(x^*) = 0.$$

Since $n \geq 2$, the LHS is strictly convex in Q, so there are at most two solutions on $\mathbb{R}$. One of them is $Q = 0$. The derivative of the LHS evaluated at $Q = 0$ is equal to $-nH(x^*) + 1$, which is less than 0 by (10) and $x^* < b$. Hence the other remaining solution is strictly positive. Moreover, it must be less than 1, since for $Q = 1$ the LHS>0 as we have assumed $H(x^*) < 1$. ■

The rule is feasible. It is straightforward that $q(x) \leq 1$ for all x and that $q(x) \geq 0$ for $x \geq x^*$. To verify $q(x) \geq 0$ for $x < x^*$, observe that $nB \leq A$, and hence, by (10)

$$Ah_{x^*}(x) - B \geq \frac{A}{n} - B \geq 0.$$

The rule is incentive compatible. An agent's payoff conditional on his truthful report x is equal to the probability to be chosen,

$$q(x)A + (1 - q(x))B = Ah_{x^*}(x),$$

whereas his payoff from reporting $y \geq x^*$, $y \neq x$, is equal to $A(1 - c(x)) \leq Ah_{x^*}(x)$.

The principal's expected payoff is equal to

$$W = \frac{\int_X x\,(q(x)A + (1 - q(x))B)\,f(x)\mathrm{d}x}{\int_X (q(x)A + (1 - q(x))B)\,f(x)\mathrm{d}x} = \frac{1}{H(x^*)}\int_X xh_{x^*}(x)f(x) = V(x^*) = x^*.$$

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