Declining Discount Rates

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In this paper we ask whether the US government should replace its current discounting practices with a declining discount rate schedule, as the UK and France have done, or should continue to discount the future at a constant exponential rate. To address the question, we briefly present the theoretical basis for a declining discount rate (DDR) schedule, but focus on how, in practice, a DDR could be estimated for use by policy analysts. We discuss the empirical approaches in the literature and review how the UK and France estimated their DDR schedules. We conclude with advice on how the US might proceed to consider modifying its current discounting practices.

I. The Expected Net Present Value Approach to DDRs

The declining discount rate was developed by Weitzman (1998, 2001) who proved that computing the expected net present value of a project (ENPV) with an uncertain but constant discount rate is equivalent to computing the NPV with a certain but decreasing "certainty-equivalent" discount rate. Suppose that net benefits at time t, $Z(t)$, are discounted to the present at a constant exponential rate r, so that the present value of net benefits at time t equals $Z(t)exp(-rt)$.[2] If the discount rate r is fixed over time but uncertain, then the expected value of net benefits is given by $p(t)Z(t) = E(exp(-rt))Z(t)$ where $p(t)$ is the expected discount factor. The certainty-equivalent discount rate R_t used to discount $Z(t)$ to the present is defined by

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[2] We assume that $Z(t)$ represents certain benefits. If benefits are uncertain we assume that they are uncorrelated with r and that $Z(t)$ represents certainty-equivalent benefits.

(1) $exp(-R_t t) = E(exp(-rt))$

implying $R_t = -1/t\ ln[p(t)]$. To illustrate, if r =1% and 7% each with probability 0.5, the certainty equivalent discount rate ranges from 3.96% in year 1 to 1.69% in year 100, declining to 1.17% in year 400. Jensen's inequality and the convexity of the discount factor guarantee that the certainty-equivalent discount rate is always less than $E(r)$ and that it declines over time.[3]

The *instantaneous* certainty-equivalent discount rate, or forward rate, is given by the rate of change in the expected discount factor $-(dp_t/dt)/p_t \equiv F_t$. This is the rate at which benefits in period t would be discounted back to period t-1. Figures 1 and 2 show the forward rates used by the UK and France, and the corresponding certainty-equivalent rates (labeled "Effective Term Structure").

The declining certainty-equivalent discount rate in the above example follows directly from Jensen's inequality and a constant but uncertain discount rate. In the more general case in which the discount rate varies over time

(2) $p(t) = E[exp(-\sum_{\tau=1 \ldots t} r_\tau)]$.

In this case, the shape of the R_t path depends on the distribution of the per-period discount rates $\{r_\tau\}$. If $\{r_\tau\}$ are independently and identically distributed, the certainty-equivalent discount rate is constant. There must be persistence in uncertainty about the discount rate for the certainty-equivalent rate to decline. If, for example, shocks to the discount rate are correlated over time, as in equation (3),

(3) $r_t = \pi + e_t$ and $e_t = ae_{t\text{-}1} + u_t$, $|a| \leq 1$

[3] Formally, $E(exp(-rt)) > exp(-E(r)t)$.

the certainty-equivalent discount rate will decline over time (Newell and Pizer 2003).

There is a growing empirical literature that uses the yield on government bonds to estimate econometric models of interest rate behavior which are, in turn, used to forecast r_t. Models estimated using two centuries of data for the US suggest persistence in shocks to the interest rate, suggesting a declining DDR. In their relatively simple model, Newell and Pizer (2003) conclude that a random walk model (a=1) fits the US data better than a mean-reverting model ($0<a<1$). Groom et al. (2007) estimate more flexible reduced-form models for the United States using the same data as Newell and Pizer (2003). They suggest that a state space model performs better than either a random walk or mean-reverting model. Freeman et al. (2013) offer several improvements to the data series and specification used by Newell and Pizer and Groom et al. (2007). Figure 3 uses the results from the preferred specification in each paper to simulate the path of forward rates for the US for 400 years.

The econometric ENPV literature implicitly assumes that the stochastic process generating future interest rates can be estimated from historic data. An alternative approach within the ENPV framework is to elicit forecasts of future interest rates from experts. Freeman and Groom (forthcoming) argue that these should be combined to reduce forecasting error, as is typical in the literature on combining forecasts (e.g., Bates and Granger 1969).

II. A Consumption-Based Approach to DDRs

The ENPV literature has been criticized for its lack of connection to the theory of benefit-cost analysis, which traditionally follows a representative-agent model. If the social planner has an additively separable utility function over consumption each period c_t, with $u_t = u(c_t)$, and discounts future utility at rate δ he will be indifferent between receiving ε dollars today and \$1 at time t if the marginal utility of the two are equal,

$$(4) \qquad u'(c_0)\varepsilon = e^{-\delta t}u'(c_t).$$

Equation (4) can be solved to yield the consumption rate of discount. In the case of an iso-elastic utility function, $u(c_t) = c_t^{(1-\eta)}/(1-\eta)$, solving equation (4) for ε yields $\varepsilon = exp(-\rho_t t)$, where the consumption discount rate ρ_t is given by the Ramsey formula,

$$(5) \qquad \rho_t = \delta + \eta \cdot g_t .$$

In (5) η is (minus) the elasticity of marginal utility with respect to consumption, and g_t is the annualized growth rate of consumption between time 0 and time t.[4]

Allowing for uncertainty in the rate of growth in consumption leads to the extended Ramsey formula. If the growth rate of consumption is independently and identically normally distributed with mean μ and variance σ^2, this uncertainty adds a third term to the Ramsey formula (Mankiw 1981):

$$(6) \qquad \rho_t = \delta + \eta\mu - 0.5\, \eta^2\sigma^2.$$

The last term in (6) is a precautionary effect: uncertainty about the rate of growth in consumption reduces the discount rate, causing the social planner to invest more for the future.[5] However, in (6) the consumption rate of discount is constant.

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[4] Formally, $g_t = t^{-1} \ln(c_t/c_0)$.

[5] A necessary condition for this to hold is that the planner be prudent (i.e., that the third derivative of $u(c)$ be positive), which is satisfied by the iso-elastic utility function.

The consumption rate of discount may decline over time if shocks to consumption growth are positively correlated over time rather than being independent, or if mean or variance of the shocks are themselves uncertain. Gollier (2012, Chapter 8) proves that if shocks to consumption growth are positively correlated and $u(c)$ is iso-elastic, ρ_t will decline. The intuition behind this is that positive correlation among shocks to consumption growth make future consumption riskier, increasing the strength of the precautionary effect in equation (6) for distant time horizons. To illustrate, a possible form that shocks to consumption could take is for $ln(c_t/c_{t-1}) \equiv x_t$, the percentage growth in consumption at t, to follow an AR(1) process $x_t = \varphi x_{t-1} + (1- \varphi)\mu + u_t$ where u_t is independently and identically normally distributed with constant variance. This will generate a declining discount rate, provided $0 < \varphi < 1$.

It has also been argued that the stochastic consumption-growth process cannot be adequately characterized by econometric models estimated using historic data with non-stochastic parameter values. Instead, either μ or σ should be treated as uncertain (Gollier 2012; Weitzman 2007). To illustrate, Gollier (2008) proves that, when log consumption follows a random walk and the mean rate of growth depends on θ [$\mu = \mu(\theta)$], a parameter that could capture technological uncertainty, the certainty-equivalent discount rate, R_t, is given by

(7) $$R_t = \delta + \eta M_t$$

where M_t is defined by $exp(-\eta t\ M_t) = E_\theta\ exp\ [-\eta t(\mu(\theta) - 0.5\eta\sigma^2)]$. As a result of Jensen's inequality, M_t (and R_t) will decline over time.

Empirically implementing a DDR using the extended Ramsey formula requires estimating δ and η and the process generating the stochastic rate of consumption growth. The parameter δ represents the pure rate of time preference plus the likelihood of a catastrophe great enough to obliterate future consumption. There is disagreement among economists as to the appropriate value of δ. Heal (2012) proposes the median value as a fair resolution of any disagreement, since it would arise under certain appealing social choice rules, such as majority voting. Other literature (Gollier and Zeckhauser 2005; Heal and Millner 2013) examines how a social planner would efficiently aggregate heterogeneous time preferences.[6]

The parameter η represents the elasticity of marginal utility and embodies several concepts including the inter-temporal elasticity of substitution between consumption today and consumption in the future, the coefficient of relative risk aversion, and inter- or intra-generational inequality aversion.[7] If one focuses on inequality aversion, one approach to estimating η is to infer the value implied by decisions that society makes to redistribute income through progressive income taxes. In the United Kingdom, the value of η based on income tax schedules has fluctuated considerably since the Second World War, with a mean of 1.6 (Groom and Maddison 2013). Again, there is wide disagreement on the value of this parameter and the appropriate source of information.

To explicitly implement a Ramsey-based DDR also requires characterizing the uncertain rate of growth in future consumption. This could be based on historic consumption data, using an approach similar to the analysis of interest rates on government bonds in the empirical ENPV literature. For example, Gollier (2008) uses the above formulation where consumption growth is AR(1) and reports an estimate of $\varphi = 0.3$, based on the US literature, which implies a very gradual decline in the discount rate. For the UK, Groom and Maddison (2013) estimate that $\varphi =$

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[6] Jouini, Marin and Napp (2010) characterize equilibrium discount rates in an economy in which agents differ in their rate of time preference and in their assumptions about future growth in consumption.

[7] There is a large literature that estimates η and obtains different values depending on the interpretation given to η. Groom and Maddison (2013) summarize the literature and provide updated estimates of η for the UK.

0.9 using growth data with the cyclical component removed (Hodrick-Prescott), raising further data issues. Other models of consumption growth in the US (e.g., the regime-switching model of Cecchetti et al. (2000)) also suggest a very slowly declining DDR if models based on historic data are used to forecast future consumption growth. In contrast, if the model in (7) is used, a simple distribution over μ can generate a DDR that declines more rapidly. Suppose, for example, that the mean rate of growth in consumption is assumed to equal 1 percent or 3 percent with equal probability and that $\delta = 0$, $\eta = 2$ and $\sigma = 3.6\%$. This yields a certainty-equivalent discount rate that declines from 3.8% today to 2% after 300 years.

III. DDRs in the UK, France and the US Practice

The governments of the UK and France have both adopted DDRs for project evaluation. The UK schedule, implemented in 2003 (HM Treasury 2003), uses the Ramsey formula with $\delta = 1.5$, $\eta = 1.0$ and $g_0 = 2\%$ to set the initial discount rate of 3.5%. The forward rate is a step function patterned after Newell and Pizer's (2003) random walk model. The resulting certainty-equivalent discount rate falls to 2% after 300 years. The French schedule (Lebègue 2005) sets the forward rate equal to 4% for maturities of up to 30 years, falling to 2% thereafter. This results in the certainty equivalent rate shown in Figure 1, which begins at 4% and falls to 2.2% after 300 years. The latter loosely approximates an extended Ramsey model with uncertainty about the mean rate of growth in per capita consumption. Lebègue (2005) states that it is broadly consistent with $\delta = 1$, $\eta = 2$ and $\mu = 0.5$ with probability 1/3 and = 2.0 with probability 2/3.

The French and UK schedules both appeal to the Ramsey model as their theoretical foundation. The French DDR is, however, only loosely tied to the Ramsey formula; i.e., δ and η are not explicitly estimated, nor is the data generating process for per capita consumption. The recommendation represents a compromise among a range of acceptable values. The British DDR uses estimates of η based on a variety of empirical methods including the analysis of the UK personal income tax structure. δ is estimated as the sum of the "likelihood that there will be some event so devastating that all returns from policies, programmes or projects are eliminated" and the pure rate of time preference, both from UK sources. The rate of decline in the forward discount rate is, however, based on Newell and Pizer's analysis of government bond rates in the US. The UK approach thus represents a hybrid of Ramsey- and ENPV-based approaches (HM Treasury 2003, Annex 6).

In the United States, OMB (2003) recommends that benefit-cost analyses be performed using a discount rate of 7%, representing the pre-tax real return on private investments and a discount rate of 3%, representing the "social rate of time preference." The latter is measured by the real rate of return on Treasury bonds.[8] The justification for using 7% is that, although a consumption-based approach is theoretically preferred, converting costs and benefits to consumption equivalents is, in practice, difficult.

Should the OMB consider revising its current discounting practice? We make two observations: The first is that, even if the current market-based approach is used to determine a constant exponential discount rate, the recommended rate should be revisited at regular intervals. OMB already does this for discount rates used for lease-purchase agreements and cost-effectiveness analyses, but not for rates used for regulatory analysis or benefit-cost analysis of public investment (OMB 2003). This process needs to be regular, but not too frequent, and there

[8] A lower constant discount rate may be used as a sensitivity analysis when evaluating projects involving intergenerational benefits or costs. In applying this, it is important that all benefits and costs with the same systemic risk be discounted at the same rate. See Arrow et al. (2013) for a discussion.

needs to be an emphasis on gradual adjustments, so recent benefit-cost analyses are not suddenly made irrelevant.

The second observation is that the US should consider the use of a DDR. The use of declining discount rates for risk-free projects is now well established in both the academic literature and international policy circles.[9] As described above, there are a plethora of techniques that are available to the OMB to estimate a specific DDR schedule for the US. Unfortunately, the resulting term structure can be highly sensitive to the specific choice. Some schedules decline so slowly that results barely differ from a flat term structure, while others have long-term rates that are significantly below their short-term equivalent counterparts (see, for example, Freeman and Groom, forthcoming).

The UK and French authorities have arrived at their DDRs through interactions between experts and government officials. This would, we believe, be a fruitful approach in the US. In particular, it would be useful to explore the implications of various approaches to estimating a DDR using US data—from both ENPV and consumption-based perspectives. Open and informed discussion can then take place about the appropriateness of assumptions for data selection and protocols for model selection.

[9] In addition to the UK and France, Norway and Denmark have adopted DDRs and The Netherlands and Sweden are considering adopting them.

Figures

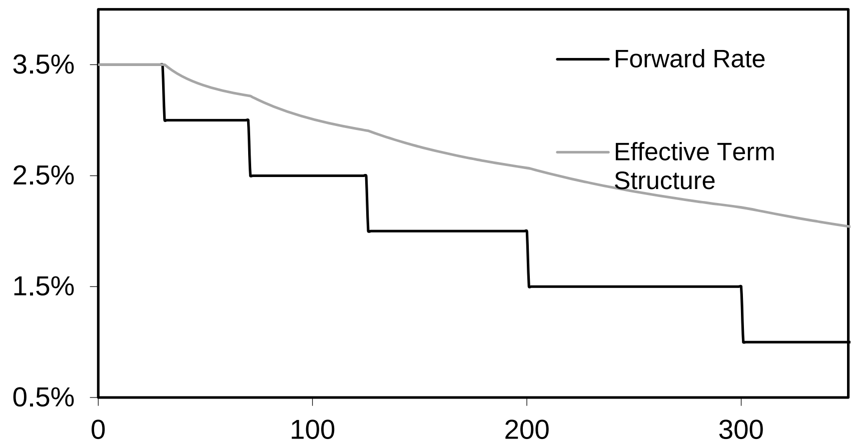

Figure 1. The UK Government Social Discount Rate Term Structure. HMT (2003)

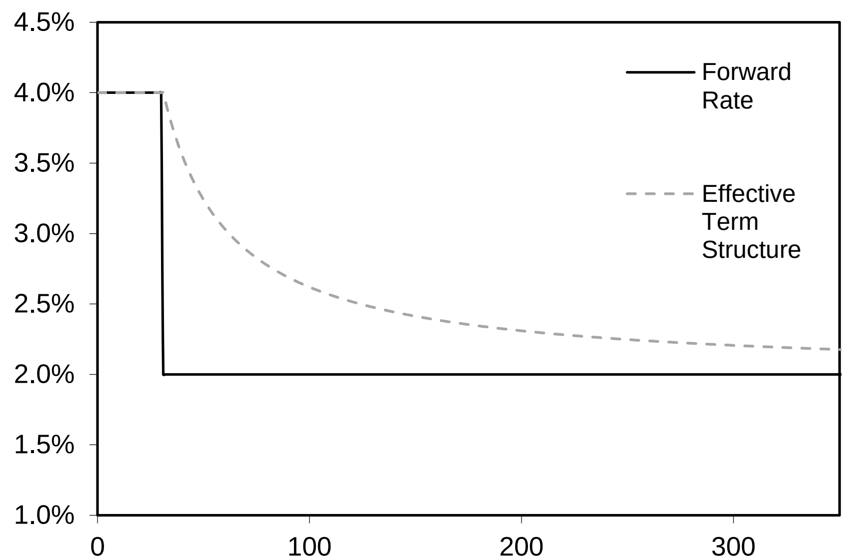

Figure 2. The French Government Social Discount Rate Term Structure. Lebègue (2005)

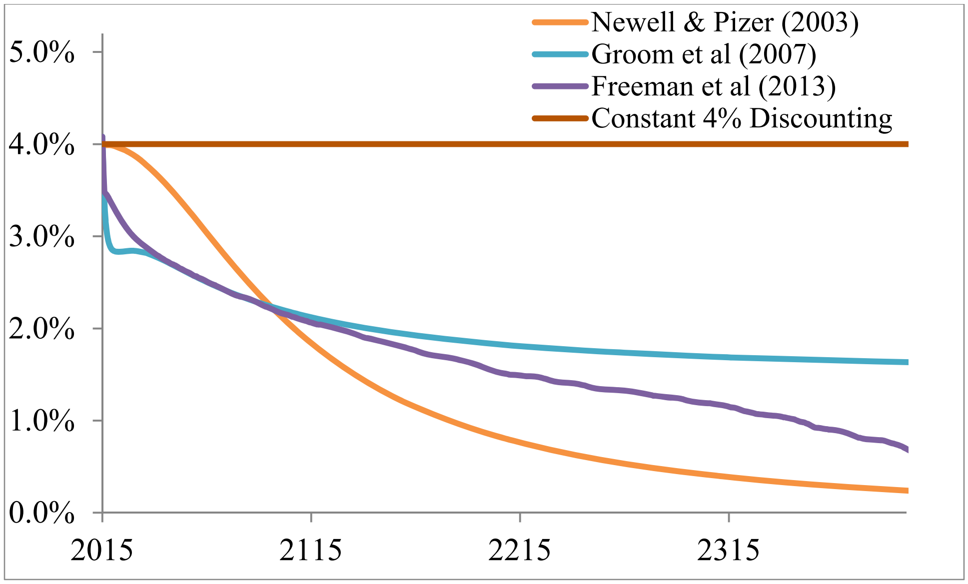

Figure 3. Estimates of Forward Rates for the United States

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