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Productivity Insurance: The Role of Unemployment Benefits in a Multi-Sector Model*

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Abstract

We construct a multi-sector search and matching model where the unemployed receive idiosyncratic productivity shocks that make working in certain sectors more productive than in the others. Agents must decide which sector to search in and face moving costs when leaving their current sector for another. In this environment, unemployment is associated with an additional risk: low future wages if mobility costs preclude search in the appropriate sector. This introduces a new role for unemployment benefits – productivity insurance while unemployed. Analytically, we characterize two competing effects of benefits on productivity, a moral hazard effect and a consumption effect. In a stylized quantitative analysis, we show that the consumption effect dominates, so that unemployment benefits increase per-worker productivity. We also analyze the welfare-maximizing benefit level and find that it decreases as moving costs increase.

Keywords: unemployment insurance, search, mobility, productivity

JEL classification: J62, J63, J64, J65

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1 Introduction

The existing literature on the provision of unemployment insurance (hereafter UI) has focused on the role of UI in smoothing consumption between employment states. At the center of such analyses is a fundamental trade-off between insurance and incentives (e.g., [Hopenhayn and Nicolini, 1997](#)). More insurance implies reduced output as the duration and incidence of unemployment increases. The relationship between UI and output, however, can change if one recognizes that UI also encourages unemployed workers to seek higher productivity jobs. Allowing the composition of jobs to respond endogenously, [Acemoglu and Shimer \(1999, 2000\)](#) show that UI benefits encourage firms to create higher productivity jobs, which in turn might lead to an increase in aggregate output.

In this paper, we introduce an additional role for unemployment benefits: insurance against idiosyncratic sector-specific productivity shocks while unemployed. Similarly to [Acemoglu and Shimer \(1999, 2000\)](#), we allow the composition of jobs to be endogenously determined. In particular, we consider an environment in which, upon becoming unemployed, individuals are subject to idiosyncratic shocks that render their current skills less suitable for their most recent sector of employment. If a move from one sector to another is costless, such a shock poses no additional risk to the unemployed; they simply move to the most productive sector for their particular skills. If, however, the move requires paying moving costs, then unemployment poses a risk to future wage prospects if unemployed workers search in the relatively less productive sector. In such an environment, unemployment benefits may help insure individuals against this risk by effectively reducing the costs of moving.

Specifically, we analyze a directed search model with matching frictions, multiple sectors, and risk-averse agents. An unemployed agent receives an idiosyncratic productivity shock that makes the agent more productive in one sector relative to the other sectors. The unemployed agent must decide which sector to search in, and she faces moving costs when leaving her current sector for another. We consider two alternative ways of modeling moving costs: a utility cost of moving and a pecuniary one (i.e., the cost in terms of consumption

goods). Mobility between sectors is directed. Workers know their productivity in another sector before leaving their current sector (as in [Roy \(1951\)](#) and [Heckman and Taber \(2008\)](#)). In each sector, firms post wages and agents direct their search to a specific job as in [Moen \(1997\)](#) and [Rogerson, Shimer, and Wright \(2005\)](#). Therefore, the model represents a blend of a dynamic Roy model and a competitive search model.

In contrast to standard sectoral reallocation theory (e.g., [Lucas and Prescott, 1974](#)), the model allows for explicit distinction between inter-sectoral mobility and within-sector trading frictions. Such distinction is essential for examining the link between UI and sectoral mobility.¹

We first analyze how unemployment benefits affect equilibrium outcomes, focusing on the productivity effects. We show that the mobility decision is characterized by a reservation rule for productivity shocks. For idiosyncratic shocks above the reservation value, the agent moves sectors. For idiosyncratic shocks below the reservation value, the agent remains in the current sector. This feature implies that the effect of benefits on productivity depends on how changes in benefits affect the reservation value. Analytically, we show that benefits have two main effects on the reservation value, which we refer to as a “moral hazard effect” and a “consumption effect.”

The moral hazard effect occurs because the benefit acts as a subsidy to search. Increasing benefits increases the value of unemployment, reducing the gain from moving to higher productivity sectors. Agents require larger idiosyncratic shocks to be willing to move sectors. The moral hazard effect increases the reservation productivity, decreasing per-worker productivity. This effect resembles the moral hazard effects in a McCall search model ([McCall, 1970](#)): a higher value of unemployment implies workers become more selective, resulting in longer unemployment durations. The difference between the effect in [McCall \(1970\)](#) and in our model is that in our multi-sector model, the increased selectiveness of when to switch

¹[Lkhagvasuren \(2012a\)](#) shows that the interaction of between-sector mobility and within-sector trading frictions might be key to accounting for the negative correlation between unemployment and sectoral mobility reported by [Moscarini and Thomsson \(2007\)](#) and [Kambourov and Manovskii \(2009\)](#).

sectors implies a decrease in the productivity of matches that do occur. In the McCall search model, in contrast, workers being more selective implies higher wages/productivity once employed.

An increase in unemployment benefits may also decrease the reservation productivity, which we refer to as the consumption effect. When benefits increase, the marginal flow utility of consumption from moving increases faster than the marginal flow utility of remaining in the current sector. That is, the lifetime utility of moving is closer to the lifetime utility of remaining in the current sector, for all values of the idiosyncratic productivity shock. This necessarily implies that the reservation productivity decreases, which increases per-worker productivity.

We analytically characterize the moral hazard and consumption effects. The combined effect of benefits on productivity remains ambiguous. To quantitatively illustrate our theory and determine which effect dominates, we calibrate the model to the U.S. economy. We find that the consumption effect dominates, so that increasing benefits increases per-worker productivity. The dominant consumption effect implies that workers move more frequently in response to idiosyncratic productivity shocks. Unemployment increase, while vacancies decrease.

Quantitatively, we find that with additively separable moving costs, the overall effect on per-worker productivity is relatively small. A 25% increase in the benefit level increases per-worker productivity by 0.1%. With a pecuniary moving cost (which directly reduces the consumption of the unemployed upon moving), we find that the effect on productivity may be quite large. In the calibrated version, a 12.5% increase in benefits implies a nearly 2% increase in per-worker productivity. When the moving costs are initially prohibitive (so that mobility is zero), a 12.5% increase in benefits implies a 9% increase in per-worker productivity.

We also analyze the welfare-maximizing unemployment benefit in our multi-sector setting and find that the optimal level of benefits depends on the size of the moving cost. As moving

costs increase, the optimal benefit decreases. Determining the optimal benefit requires managing the aforementioned trade-off between the moral hazard and consumption effects. As the moving cost increases, the moral hazard effect becomes stronger, which puts downward pressure on the optimal benefit level. We find that in response to a 1 percent increase in the moving cost, the benefit decreases by 6 percent. Thus, while in the case of additively separable moving costs the productivity effects are relatively small, the overall role of UI as productivity insurance may be quite important.

The productivity results in our model relate to the efficient UI literature, most notably, [Acemoglu and Shimer \(1999, 2000\)](#). In their model, there exists an endogenous trade-off between productivity and the arrival rate of job offers: more productive jobs arrive less frequently than lower productivity jobs. Since the unemployment risk consists of the duration of unemployment (i.e., low income during a longer period), benefits help workers endure the longer durations necessary to search for higher productivity jobs. [Alvarez-Parra and Sanchez \(2009\)](#) show similar effects in a model of optimal unemployment insurance (without firms) where the unemployed may work in an informal sector.

In contrast, in our model, higher wage jobs are offered more frequently, given the agent searches in the appropriate sector. More importantly, we introduce another role for UI beyond smoothing income shocks. In particular, UI serves as insurance against the risk of lower future wages if the search is in a “wrong” sector. Per-worker productivity increases because benefits help workers incur the costs of moving. That is, the benefits provide insurance against the productivity shocks associated with unemployment.

The remainder of the paper is as follows. Section 2 describes the environment and agent decisions. Section 3 characterizes the equilibrium of the model economy. Section 4 characterizes the productivity effects of benefits, and Section 5 presents our quantitative analysis. We discuss the pecuniary moving cost in Section 6, and Section 7 concludes. All proofs are contained in [Appendix A](#).

2 Model

2.1 Environment

The economy is composed of two sectors, denoted by 0 and 1, populated by a measure one of risk-averse workers and a continuum of risk-neutral firms. Individuals are either employed or unemployed. Employed workers are matched with a firm. In each period an unemployed worker chooses to search in the current sector or move to the other sector to search. While employed, workers do not engage in on-the-job search. Therefore, every mover is unemployed, while not all unemployed workers are movers.²

Each period, firms search for workers by creating vacancies. The flow cost of a vacancy is k . Free entry drives the expected present value of an open vacancy to zero. Vacant jobs and unemployed workers are matched according to a matching technology. Without loss of generality, we assume that each firm employs at most one worker. All matches are dissolved exogenously with probability λ .

Let b denote per-period income of a worker while unemployed. Flow utility of an unemployed worker searching for a job within his own sector is $\log(b)$, while that of an unemployed worker moving across sectors is $\log(b) - c$, where $c > 0$ is the utility cost of moving. The flow utility of a worker is $\log(w)$, where w denotes the wage. Workers and firms discount the future at the same rate, β .

2.2 Production Technology

Let $y_i(x)$ be the production function describing per-period output produced by a firm that employs a worker with productivity x and operates in sector $i \in \{0, 1\}$. We assume that

$$y_0(x) = 1 - x \tag{1}$$

²This is not inconsistent with the fact that unemployment among recent movers is much higher than that among local workers, even after controlling for age and education of the labor force. See [Lkhagvasuren \(2012a\)](#) for details.

and

$$y_1(x) = 1 + x. \tag{2}$$

Equations (1) and (2) imply that individual productivity is perfectly negatively correlated across sectors: the best workers of sector 0 are the worst workers of sector 1.³ We further assume that x remains unobservable to the UI agency.

We present the model in terms of two sectors, $i \in \{0, 1\}$. The model, however, can be generalized to an economy with N sectors by interpreting $y_i(x)$ as the agent's productivity shock in the current sector, and $y_{1-i}(x)$ as the highest of the $N - 1$ productivity shocks from the remaining $N - 1$ sectors.

By construction, idiosyncratic productivity does not change within a given match. If an employed worker becomes unemployed, she draws her new productivity from the uniform distribution on the interval $[-\omega, \omega]$.⁴ For relatively high values of the productivity shock x , the unemployed worker prefers to search in the current sector; for relatively low values of x , the worker prefers to move and search in another sector. We also assume that $0 < b < 1 - \omega$. If upon unemployment the worker decides to search in another sector, she incurs the moving cost c .

2.3 Wages

We assume that wages are determined through competitive search, as in [Moen \(1997\)](#) and [Rogerson et al. \(2005\)](#). A firm decides whether or not to post a vacancy. A vacancy is fully characterized by the productivity level, x , the wage, w , and the sector, i . If a firm decides

³See [Moscarini and Vella \(2008\)](#) and [Lkhagvasuren \(2012b\)](#) for related dynamic extensions of [Roy's \(1951\)](#) framework. One can consider labor income shocks that are not perfectly negatively correlated across sectors. For example, suppose that e_0 and e_1 are productivity of a worker in sectors 0 and 1, respectively. Further suppose that these two shocks are not perfectly negatively correlated; i.e., $\text{Corr}(e_0, e_1) > -1$. Then, consider the following decomposition: $e_0 = z + x$ and $e_1 = z - x$, where z and x are uncorrelated shocks. Since z is common across locations and not affected by mobility, it is not essential for the productivity distribution.

⁴To increase the tractability of the model, we adopt this specification of the persistent idiosyncratic shock from [Andolfatto and Gomme \(1996\)](#) and [Merz \(1999\)](#). Note that when a worker draws an idiosyncratic shock upon separation, this can also be reinterpreted as an all-in-one shock capturing the wage risk over the expected employment duration.

to post a vacancy, it chooses these three variables in order to maximize its expected profits. An unemployed worker directs her search towards the most attractive job. Let $\mathbb{W}_i(x)$ denote the set of wages posted at the productivity level x in sector i .

2.4 Matching Technology

Let n_τ denote the number of unemployed workers searching for a job of type $\tau = (w, x, i)$, and v_τ denote the number of vacant type τ jobs. The total number of type τ matches is given by

$$M_\tau = \mu n_\tau^\eta v_\tau^{1-\eta}$$

where $0 < \eta < 1$ and $\mu > 0$. Let $q_\tau = n_\tau/v_\tau$. We refer to q_τ as the queue length for a vacancy of type τ . A type τ vacant job is filled with probability $\alpha(q_\tau) = \mu q_\tau^\eta$, and any of the n_τ workers finds a job with the probability $f(q_\tau) = \mu/q_\tau^{1-\eta}$.

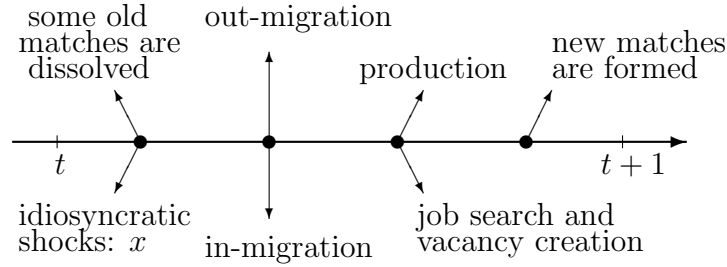
2.5 Timing of the Events

Figure 1 displays the timing of events. Each period consists of four stages. At the beginning of each period, fraction λ of the existing matches is dissolved. The workers separated from these matches become unemployed, observe their idiosyncratic shock, x , and decide which sector to search in. In the second stage, moving takes place, i.e., the unemployed workers who have decided to search in other than their current sector move. In the third stage, production takes place in the surviving matches, firms post vacancies, and unemployed workers search for jobs. In the last stage, new matches are formed.

2.6 Discussion

We have added two elements to the textbook model described in Rogerson et al. (2005): sector-specific productivity and moving costs. Specifically, if there is no sector-specific productivity dispersion (i.e., $\omega = 0$), the economy converges to the standard one-sector model

Figure 1: Timing of the Events



described in Rogerson et al. (2005). Moreover, if moving across sectors is costless (i.e., $c = 0$) or prohibitive ($c = \infty$), the economy is equivalent to a one-sector model with exogenous idiosyncratic productivity (e.g., Bils, Chang, and Kim, 2011). Therefore, endogenous idiosyncratic productivity in the presence of costly mobility is the key equilibrium channel considered in this paper.

In this economy, unemployment imposes two risks. First, a worker loses her employment earnings; i.e. income drops from w to b . Second, an unemployed worker also risks an idiosyncratic productivity shock that renders her skills unsuitable for the sector the worker is currently in. Since there exist moving costs ($c > 0$), the unemployed worker may prefer to continue searching in the relatively less productive sector. Below we show that the productivity risk affects the future lifetime earnings of an unemployed worker in two ways: a lower future wage *and* a lower job-finding probability.

2.7 Value Functions

2.7.1 Workers

Consider a worker who is unemployed at the beginning of the current period, in sector i with productivity x . Let $S_i(x)$ denote the lifetime utility value to the worker of searching for a job in the current sector, i . Let $M_i(x)$ denote the lifetime value to the worker of moving

from sector i to sector $1 - i$. The value function of the unemployed worker is

$$U_i(x) = \max \{S_i(x), M_i(x)\}. \quad (3)$$

Given the moving cost, c , and the timing of mobility, the value of moving from sector i to sector $1 - i$ is given by

$$M_i(x) = \log(b) - c + \beta S_{1-i}(x). \quad (4)$$

Let $W_i(w, x)$ denote the value of being employed, in sector i with productivity x , by a firm who pays wage w :

$$W_i(w, x) = \log(w) + \beta(1 - \lambda)W_i(w, x) + \beta\lambda \int_{-\omega}^{\omega} U_i(x')dG(x'), \quad (5)$$

where G denotes the uniform distribution function on the interval $[-\omega, \omega]$. Then, the expected lifetime utility value of searching for a job in sector i is given by

$$S_i(x) = \max_{w \in \mathbb{W}(x, i)} \{ \log(b) + \beta f(q_{w, x, i})W_i(w, x) + \beta(1 - f(q_{w, x, i}))S_i(x) \}. \quad (6)$$

As in Rogerson et al. (2005), a worker takes q_τ as given.

2.7.2 Firms

Now consider a matched firm operating at productivity level x in sector i . Given the wage w , the value of the match to the firm is given by

$$J_i(w, x) = y_i(x) - w + \beta(1 - \lambda)J_i(w, x). \quad (7)$$

Let $V_i(x)$ denote the value of posting a vacancy at productivity level x in sector i . $V_i(x)$ is defined by

$$V_i(x) = \max_w \{ -k + \beta\alpha(q_{w, x, i})J_i(w, x) \} \quad (8)$$

Due to free entry and profit maximization, all rents from vacancy creation are exhausted in the economy. Thus, for any pair (x, i) :

$$V_i(x) = 0. \quad (9)$$

2.8 Unemployment and Mobility

Let $\psi_i^u(x)$ denote the number of unemployed workers searching for jobs in sector i at productivity level x . Similarly, let $\psi_i^e(x)$ denote the number of workers employed at productivity level x in sector i . Since the total population is normalized to one,

$$\sum_i \int_{-\omega}^{\omega} (\psi_i^u(x) + \psi_i^e(x)) dx = 1. \quad (10)$$

The economy-wide unemployment rate is given by

$$u = \sum_i \int_{-\omega}^{\omega} \psi_i^u(x) dx. \quad (11)$$

Let Ω_i denote the decision rule governing whether an unemployed worker in sector i stays in her current sector:

$$\Omega_i(x) = \begin{cases} 1 & \text{if } S_i(x) \geq M_i(x), \\ 0 & \text{otherwise.} \end{cases} \quad (12)$$

Then, the measure of workers moving from sector i to $1-i$ is given by $\psi_i^m = (1 - \Omega_i(x))\psi_i^u(x)$.

Therefore, the economy-wide mobility rate is given by

$$m = \sum_i \int_{-\omega}^{\omega} \psi_i^m(x) dG(x). \quad (13)$$

2.9 Definition of the Equilibrium

The equilibrium consists of a set of value functions $\{U_i, S_i, W_i, J_i, V_i\}$, a decision rule Ω_i , sets of posted wages \mathbb{W}_i for any $i \in \{0, 1\}$, and measures $\{n, v\}$ such that

1. Given (S_0, S_1) , the decision rule $\Omega_i(x)$ and the value function $U_i(x)$ solve (3);
2. Given U_i , the value function $W_i(w, x)$ solves (5);
3. Given q_i, U_i and W_i , the value function $S_i(x)$ solves (6) for each $w \in \mathbb{W}_i(x)$;
4. The value function $J_i(w, x)$ solves (7);
5. Given J_i, n and v , the value function $V_i(x)$ solves (8) for each $w \in \mathbb{W}(x)$; and
6. Free entry:

$$\begin{cases} v(w, x, i) > 0 \text{ and } V_i(x) = 0 \text{ if } w \in \mathbb{W}_i(x), \\ v(w, x, i) = 0 \text{ and } V_i(x) = 0 \text{ if } w \notin \mathbb{W}_i(x) \text{ or } \mathbb{W}_i(x) \text{ is an empty set.} \end{cases}$$

3 Equilibrium Characterization

We solve the model in two steps. First, we find the local labor market equilibrium, treating $\bar{U}_i = \int U_i(x) dG(x)$ as a parameter. After obtaining workers' and firms' decisions within a local market, we determine \bar{U}_i using equation (3).

3.1 Queue Length and Wages

Taking \bar{U}_i as given, equation (5) can be re-written as

$$W_i(w, x) = \frac{\log(w) + \beta\lambda\bar{U}_i}{A}, \quad (14)$$

where $A = 1 - \beta(1 - \lambda)$. Inserting the latter into (6), we have

$$\log(w) = \frac{A(1 - \beta)S_i(x) - A\log(b)}{\beta f(q_{w,x,i})} + AS_i(x) - \beta\lambda\bar{U}_i \quad (15)$$

Using equations (7) and (8), a firm's problem can be written as:

$$\max_{q_{w,x,i}} \{ \alpha(q_{w,x,i}) (y_i(x) - w) \}. \quad (16)$$

A firm posting a vacancy at the productivity level x_i takes $S_i(x)$ and \bar{U}_i as given. Therefore, combining equations (15) and (16), a firm's problem becomes

$$\max_{q_{w,x,i}} \left\{ \alpha(q_{w,x,i}) \left(y_i(x) - \exp \left(\frac{A(1-\beta)S_i(x) - A \log(b)}{\beta f(q_{w,x,i})} + AS_i(x) - \beta \lambda \bar{U}_i \right) \right) \right\}. \quad (17)$$

Taking the FOC in (17) and combining the result with the free entry condition, it can be shown that

$$q_{w,x,i} = \frac{\eta k}{1-\eta} \frac{\exp \left(-\frac{A(1-\beta)S_i(x) - A \log(b)}{\beta f(q_{w,x,i})} - AS_i(x) + \beta \lambda \bar{U}_i \right)}{(1-\beta)S_i(x) - \log(b)}. \quad (18)$$

Proposition 1 (Queue length). *All firms creating a vacancy at the productivity level x in sector i choose the same queue length $q_i(x)$.*

Corollary 1 (Wage). *The free entry condition, $V_i(x) = 0$, and Proposition 1 imply that the wage is also unique for each pair (x, i) and is given by*

$$w_i(x) = y_i(x) - \frac{kA}{\beta \alpha(q_i(x))}. \quad (19)$$

Therefore, each job is fully characterized by the productivity level, x , and the sector, i . To summarize, given \bar{U}_0 and \bar{U}_1 , the local labor market equilibrium is given by (15), (18) and (19). In Appendix, we show that the wage, $w_i(x)$, and the value of searching for a job in the current sector, $S_i(x)$, increase with productivity, $y_i(x)$, while the queue length, $q_i(x)$, and the value of moving, $M_i(x)$, decrease with productivity for each i .

These results also imply the following two corollaries:

Corollary 2 (Queue length). *The queue length $q_{x,i}$ decreases with productivity $y_i(x)$ for*

each i .

Corollary 3 (Wage). *The wage $w_{x,i}$ increases with productivity $y_i(x)$ for each i .*

These two corollaries highlight the productivity insurance aspect of UI. Specifically, a shock x that implies higher productivity in sector i , $y_i(x)$, also implies a higher wage and a higher job finding probability. Notice, this represents a different trade-off from the models of Acemoglu and Shimer (1999, 2000). There, the trade-off is between higher wages and higher job-finding probabilities. In that sense, unemployment benefits allow the worker to endure longer durations to achieve a more productive match. In contrast, in our model, higher wages are associated with higher job-finding rates, provided the worker searches in the appropriate sector.

Given the results above, we also characterize the effects of productivity on the value of staying in the current sector or moving, respectively. These results are important for understanding the impact of unemployment benefits on mobility (and thus on productivity).

Corollary 4 (Value of staying). *The value of searching in the current sector, $S_i(x)$, increases with productivity $y_i(x)$ for each i .*

Corollary 5 (Value of moving). *The value of moving from sector i to sector $1 - i$, $M_i(x)$, decreases with productivity $y_i(x)$ for each i .*

3.2 Mobility Decision

Next we characterize a worker's mobility decision and determine the equilibrium values of \bar{U}_0 and \bar{U}_1 . If the moving cost is too high, mobility is zero. Let \bar{c} denote the lowest moving cost that prohibits mobility. For moving costs below \bar{c} , each period a certain fraction (but not all) of unemployed workers move between the two sectors. Thus, for each i , there exists a minimum productivity level \hat{x}_i such that

$$S_i(\hat{x}_i) = M_i(\hat{x}_i). \tag{20}$$

Productivity level \hat{x}_i represents a *reservation value* for the mobility decision. Depending on the current sector, for values of x above (sector $i = 0$) or below (sector $i = 1$) \hat{x}_i , the agent prefers to search in the other sector. In a frictionless environment (i.e. without moving costs), the workers switch sectors in response to any positive (sector $i = 0$) x ; however, with positive moving costs, workers do not always move to the most productive sector.

Figure 2 shows the determination of \hat{x}_i , for $i = 0, 1$. Note that $0 \leq \hat{x}_0 < \omega$ and $-\omega < \hat{x}_1 \leq 0$. Given the symmetry of the productivity shock in equations (1) and (2), the curve $S_1(x)$ is a reflection of the curve $S_0(x)$ with respect to a vertical line $x = 0$:

$$S_1(x) = S_0(-x). \quad (21)$$

Moreover, $M_1(x)$ is also a reflection of $M_0(x)$ with respect to the same line. Consequently, as shown in Figure 2, the decision rule for moving across sectors is symmetric with respect to 0: $\hat{x}_0 = -\hat{x}_1$. The minimum per-period match output is also the same between the sectors, i.e., $y_{\min} = 1 - |\hat{x}_1| = 1 - \hat{x}_0$. In the event of a transition from employment to unemployment, the probability of moving to another sector upon job separation is $p = (\omega - |\hat{x}_1|)/(2\omega) = (\omega - \hat{x}_0)/(2\omega)$.⁵

Given \hat{x}_0 and \hat{x}_1 , the continuation values \bar{U}_0 and \bar{U}_1 are given by

$$\bar{U}_0 = \int_{-\omega}^{\hat{x}_0} S_0(x) dG(x) + \int_{\hat{x}_0}^{\omega} M_0(x) dG(x) \quad (22)$$

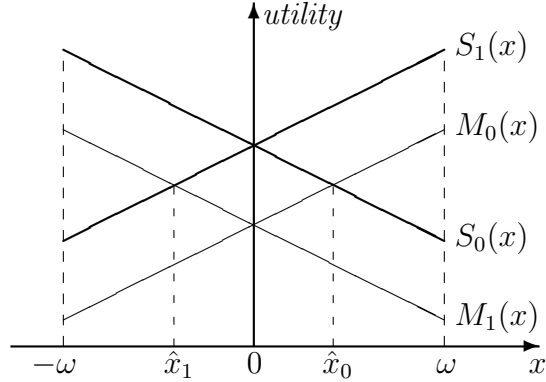
and

$$\bar{U}_1 = \int_{-\omega}^{\hat{x}_1} M_1(x) dG(x) + \int_{\hat{x}_1}^{\omega} S_1(x) dG(x). \quad (23)$$

Equilibrium is fully characterized by equations (15), (18) to (20), (22) and (23). Unemployment benefits affect equilibrium outcomes primarily through two channels. First, as in a standard one-sector search and matching model, the benefit level b affects the queue length $q_i(x)$ for each (x, i) , and thus also affects the job-finding rate. Second, the level of b affects

⁵Recall that $G(x)$ is a uniform distribution.

Figure 2: Mobility Decision



Notes: $S_i(x)$ denotes the lifetime utility value to the worker of searching for a job on her current island i when her productivity level is $x \in [-\omega, \omega]$. $M_i(x)$ denotes the lifetime value to the worker of moving from sector i to sector $1-i$ when her productivity level is $x \in [-\omega, \omega]$. The value of searching for a job in the current sector $S_i(x)$ increases with productivity $y_i(x)$, while $M_i(x)$ decreases with productivity for each i (see Corollaries 4 and 5). Therefore, an unemployed worker of sector 1 moves to sector 0 if her productivity shock is below \hat{x}_1 . Analogously, an unemployed worker of sector 0 moves to sector 1 if her productivity shock is above \hat{x}_0 .

\hat{x}_i for each i , which determines the mobility decision. Below we characterize the role of each factor to determine the impact of benefits on productivity.

4 Impact of Benefits on Productivity

Mobility in response to idiosyncratic productivity shocks represents the key addition of our model, relative to the standard one-sector model. Thus, to determine whether or not unemployment benefits can insure unemployed workers against these shocks, we need to understand how the mobility decision (i.e. \hat{x}_i) responds to benefits.

Benefits affect mobility through two channels. To highlight these effects, we combine equations (4) and (20) to get, for each i ,

$$S_i(\hat{x}_i; b) = \log(b) - c + \beta S_{1-i}(\hat{x}_i; b). \quad (24)$$

Then, using the symmetry property in equation (21), for each i ,

$$S_i(\hat{x}_i; b) = \log(b) - c + \beta S_i(-\hat{x}_i; b). \quad (25)$$

Using equation (25), we characterize how an increase in b affects the mobility decision \hat{x}_i .

Without loss of generality, we consider the mobility decision of workers in sector 1, as the case of a sector 0 worker is symmetric. Let $S'_{1,x}(x; b)$ denote the derivative of $S_1(x; b)$ with respect to x . Then, taking the first order Taylor approximation, we have,

$$S_1(x; b) \simeq S_1(0; b) + \Gamma(b)x, \quad (26)$$

where $\Gamma(b)$ denotes the slope of $S_1(x, b)$ with respect to x at $x = 0$:

$$\Gamma(b) = S'_{1,x}(0; b). \quad (27)$$

It can be seen that $\Gamma(b) > 0$ (see Corollary 4 in Appendix).

Combining equations (25) and (26), we have:

$$|\hat{x}_1| \simeq \frac{\Pi(b)}{(1 + \beta)\Gamma(b)}, \quad (28)$$

where

$$\Pi(b) = (1 - \beta)S_1(0, b) - (\log(b) - c). \quad (29)$$

What does $\Pi(b)$ measure? Recall that $\log(b) - c$ is the flow utility of a mover. Given the discount factor β , $(1 - \beta)S_1(0, b)$ is the average flow utility associated with searching at the average productivity level 0, $S_1(0, b)$. Therefore, $\Pi(b)$ measures the value of staying in the current sector relative to the value of moving across sectors. Notice that $\Pi(b)$ is also positive, since the value of searching for a job, $S_1(0, b)$, is higher than the value of not searching, $\log(b)/(1 - \beta)$.

Using equation (28), taking the log of each side, we have

$$\log |\hat{x}_1| \simeq -\log(1 + \beta) - \log(\Gamma(b)) + \log(\Pi(b)). \quad (30)$$

In equation (30), clearly the benefit b affects \hat{x}_1 .⁶ Whether an increase in benefits works to increase mobility, and thus help insure the unemployed against the risk of productivity shocks, depends on the relative size of the two effects. We characterize these effects below.

4.1 Moral Hazard Effect

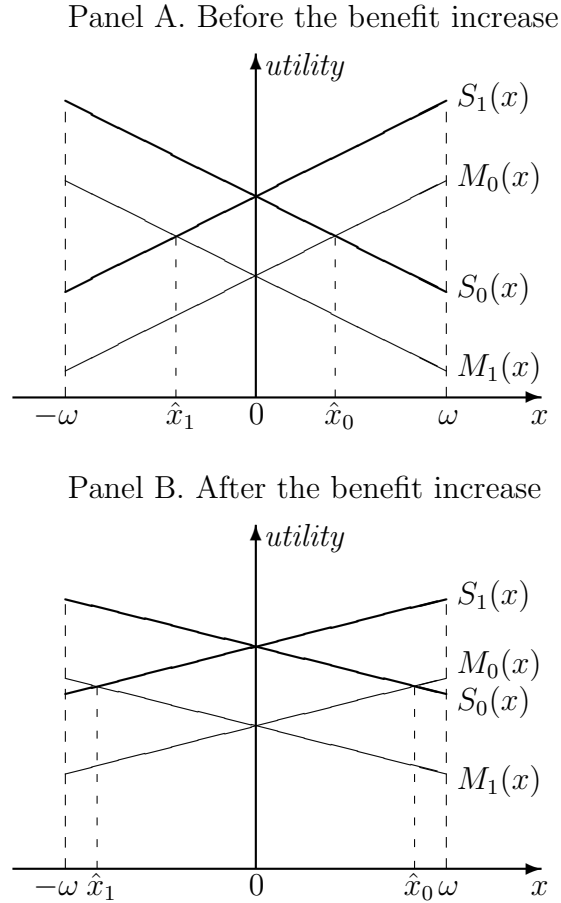
The first channel concerns within-sector trading frictions. Specifically, unemployment benefits affect $\Gamma(b)$, the slope of the value function $S_1(x; b)$ with respect to x : higher benefits lower the utility differences between high and low productivity jobs, making $S_1(x; b)$ flatter with respect to x . In other words, $|\partial S_i(x; b)/\partial x|$ and $|\partial M_i(x; b)/\partial x|$ decrease for each (x, i) . Figure 3 displays this effect.

This effect occurs because the attractiveness of high wage jobs diminishes relative to low wage jobs. Consequently, moving across sectors become less rewarding, and individuals become more selective. Thus, as benefits increase, the slope $\Gamma(b)$ goes down, putting upward pressure on $|\hat{x}_1|$ (see equation (30)). Indeed, Figure 3 shows that a decrease in the slope lowers \hat{x}_1 (raises \hat{x}_0).

When \hat{x}_1 decreases, mobility also decreases. This implies that unemployed workers become less responsive to the idiosyncratic productivity shocks. That is, more unemployed remain in a relatively unproductive sector for their particular skills; as a result, average productivity decreases. We refer to this as the *moral hazard effect*.

⁶Using the symmetry property, one can show that $\hat{x}_0 = |\hat{x}_1|$.

Figure 3: Moral Hazard Effect



Notes: A higher benefit level lowers the value of searching for high productivity jobs relative to that for low productivity jobs (i.e., it makes the curves $S_i(x)$ and $M_i(x)$ flatter for each x and i), putting downward pressure on productivity. We refer to this as the *moral hazard effect*.

4.2 Consumption Effect

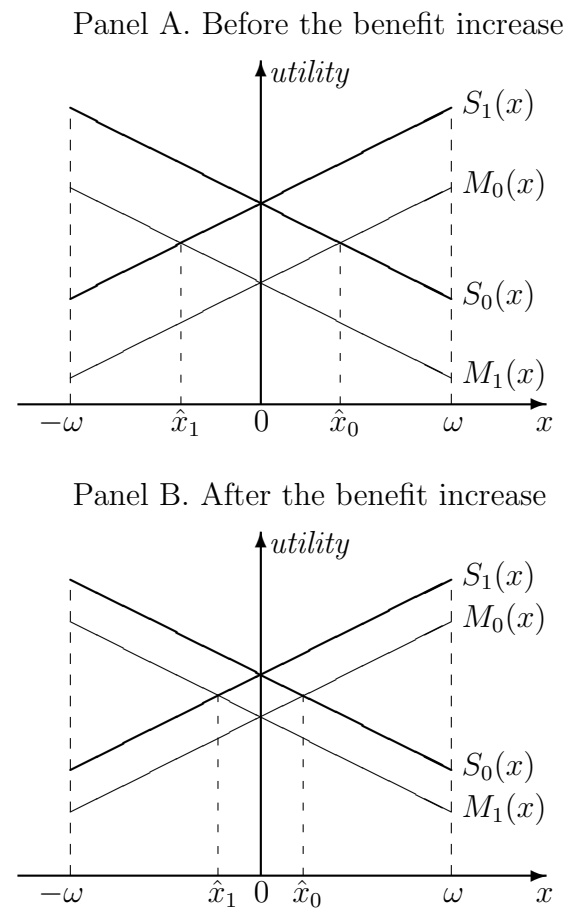
We now describe the potential role unemployment benefits can play in insuring workers against the idiosyncratic shocks. This second effect works through the marginal utility of consumption, and we refer to it as the *consumption effect*.

Specifically, benefits affect $\Pi(b)$, the value of staying in the current sector relative to the value of moving across sectors. As benefits increase, the flow utility of moving across sectors, $\log(b)$, grows faster than the constant flow utility associated with staying in the

current sector, $(1 - \beta)S_1(0, b)$. That is, differentiating $\Pi(b)$ with respect to b gives $\frac{-\beta}{b}$. Thus, the relative value of staying in the current sector, $\Pi(b)$, goes down. This reduces $|\hat{x}_1|$ (see Figure 4), increasing mobility, and thus average productivity.

Analytically the total effect on productivity remains ambiguous, depending on whether the moral hazard or consumption effect dominates. For the remainder of the paper, we evaluate these effects quantitatively.

Figure 4: Consumption Effect



Notes: Higher benefits reduce the gap between $S_i(x)$ and $M_i(x)$ for each x and i , putting upward pressure on productivity.

5 Quantitative Evaluation

Below we first describe our baseline parametrization, and then we present our quantitative results.

5.1 Calibration

The time period is one month. We set the discount factor $\beta = 1/1.05^{1/12}$, consistent with an annual interest rate of 5%, and the separation rate to $\lambda = 0.033$, consistent with Shimer (2005). The elasticity of the matching function, η , is set to 0.5, which is in the range of estimates in Petrongolo and Pissarides (2001). The flow utility of unemployed workers staying in their current sector is $b = 0.4$ (Shimer, 2005; Mortensen and Nagypál, 2007). The volatility of the idiosyncratic shocks is set to $\omega = 0.2$. This value gives us approximately 10 percent wage variation.

Empirically, sectors can be thought of in terms of geographical locations, industries, occupations, or a combination of these. The cost of moving across sectors is chosen to target an annual mobility rate of 10 percent.⁷ The moving cost is $c = 4.66$, while the average wage defined as $\bar{w} = \frac{1}{1-u} \sum_i \int_{-\omega}^{\omega} w_i(x) \psi_i^e(x) dx$ is slightly greater than one. This implies a moving cost equal to approximately four months of labor income.

Given the rest of the parameter values, the coefficient of the matching function μ is set to 0.4875 by targeting an economy-wide unemployment rate of 6 percent (Shimer, 2005). The vacancy cost is set to 1.27, targeting overall labor market tightness of 0.6, which is between the values obtained by Hall (2005) and Hagedorn and Manovskii (2008). Table 1 summarizes the key parameters. The column labeled *benchmark* in Table 2 displays the predictions of the baseline model.

⁷In terms of the empirical mobility rates, Murphy and Topel (1987) report industry annual mobility rates at 6-10 percent, Jovanovic and Moffitt (1990) report industry bi-annual mobility rates at 15 percent, Kambourov and Manovskii (2009) report annual occupational mobility rate at 16 percent in the early 1970s and at 21 percent in the mid-1990s. For geographical labor markets, Ihrke, Faber, and Koerber (2011) estimate that the annual inter-county mobility rate is approximately 6 percent.

Table 1: Parameters of the Benchmark Model

<i>Parameters</i>	<i>Values</i>	<i>Description</i>
β	0.9959	the time-discount factor
λ	0.033	the job separation rate
η	0.50	the elasticity of the matching technology
b	0.4	flow utility of unemployment
μ	0.4875	the efficiency of the matching technology
k	1.2661	the vacancy cost
ω	0.2	volatility of the sector-specific shock
c	4.6574	moving cost

Notes: This table summarizes the key parameters of the model.

Table 2: Prediction of the Model

<i>Variables</i>	<i>Benchmark</i>	<i>Higher benefit levels</i>		<i>Description</i>
	$b = 0.40$	$b = 0.45$	$b = 0.50$	
u	0.0586	0.0642	0.0703	unemployment
m	0.0990	0.0996	0.1000	annual mobility
v/u	0.6198	0.5398	0.4679	the vacancy-unemployment ratio
y_{\min}	0.9052	0.9065	0.9076	minimum observed productivity
\bar{y}	1.0770	1.0776	1.0781	per-worker output
\bar{w}	1.0004	1.0061	1.0115	the average wage
γ	n/a	-0.0062	-0.0122	the moral hazard effect
π	n/a	-0.0197	-0.0375	the consumption effect

5.2 Policy Experiments

We first explore if unemployment benefits can insure workers against the idiosyncratic productivity shocks that they receive upon becoming unemployed. The answer to this question depends on which effect dominates: moral hazard or consumption. If the consumption effect dominates, then unemployment benefits work to increase average productivity by encouraging workers to search in the relatively more productive sector. To analyze these effects, we first simulate the benchmark model for different levels of b . Table 2 summarizes the results.

5.2.1 Productivity

Table 2 indicates that benefits lower $|\hat{x}_j|$. As a result, higher benefits raise both the minimum productivity $y_{\min} = 1 - |\hat{x}_1|$ and average productivity $\bar{y} = \frac{1}{1-u} \sum_i \int_{-\omega}^{\omega} y_i(x) \psi_i^e(x) dx$. Therefore, the results imply that the consumption effect $\Pi(b)$ dominates the moral hazard effect $\Gamma(b)$ (see Figure 5). To quantify the relative magnitudes of these two effects, we calculate the responses of $\Gamma(b)$ and $\Pi(b)$ to b . For this purpose, we consider the following two values:

$$\gamma = \log(\Gamma(b)) - \log(\Gamma(b^{BM})) \quad (31)$$

and

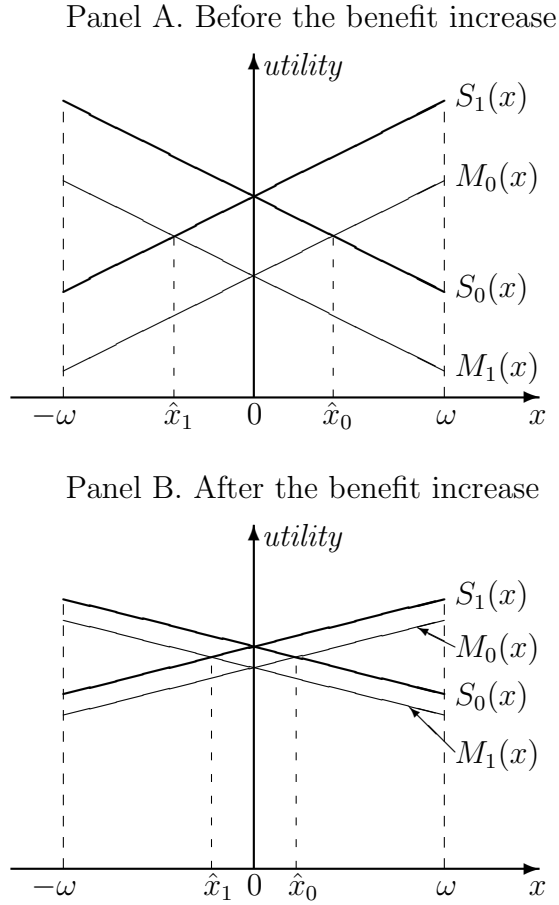
$$\pi = \log(\Pi(b)) - \log(\Pi(b^{BM})), \quad (32)$$

where b^{BM} denotes the benchmark value of unemployment benefits. These two variables are summarized in the last two rows of Table 2. The Table implies that the consumption effect (π) is three times larger than the moral hazard effect (γ).

5.2.2 Unemployment

As can be seen in Table 2, as benefits increase, unemployment increases, the ratio of vacancies to unemployment decreases and the average productivity increases. While qualitatively similar to the results in Acemoglu and Shimer (1999, 2000), the mechanism driving the

Figure 5: Moral Hazard and Consumption Effects



Notes: Case where the consumption effect dominates the moral hazard effect.

results in this paper is quite different.

As $|\hat{x}_j|$ decreases with benefits, unemployed workers become more selective and, on average, search for a job at a higher productivity level. This puts upward pressure on the probability of finding a job. As $|\hat{x}_j|$ decreases, the probability of moving, given a transition from employment to unemployment, increases. Since unemployed workers move to a sector where their productivity is higher, job offers are more likely to occur. This represents an important distinction between the mechanism in our paper relative to [Acemoglu and Shimer \(1999, 2000\)](#). In [Acemoglu and Shimer \(1999, 2000\)](#), more productive jobs take longer to find. Thus, the fundamental role of unemployment benefits in their environment is to help

smooth consumption and thus encourage workers to endure longer durations of unemployment in favor of higher productivity jobs. In contrast, in our model, the unemployed workers who move to their higher productivity sector have a higher probability of finding job. The risk to the unemployed in searching in this sector is that it requires mobility across sectors. Such mobility is costly. Unemployment benefits help insure agents against this risk, thus increasing average productivity.

As in standard one-sector search and matching models (Pissarides, 2000), as benefits increase the vacancy-unemployment ratio decreases at each productivity level, exerting downward pressure on the job finding rate. Table 2 shows that as the benefits rise, unemployment rises, indicating that the effect through the vacancy-unemployment ratio is much stronger than the other two effects. This is not surprising since the fraction of unemployed workers searching for a job in their own sector is much larger than those moving across sectors and, thus, unemployment is mainly determined by within-market search frictions.

5.2.3 Mobility

What is the impact of an increase in benefits on mobility? Using the accounting equations of labor market flows and stocks in Section 2.8, the mobility rate is given by

$$m = \lambda(1 - u)p, \tag{33}$$

where $1 - u$ is employment and $p = (\omega - |\hat{x}_1|)/(2\omega)$ is the probability of moving across sectors (given a transition from employment to unemployment).

The results in Table 2 show that higher benefits lower employment, $1 - u$. At the same time, workers become more selective (i.e., $|\hat{x}_j|$ decreases), which increases p , the probability of moving upon separation. Therefore, the impact of benefits on overall mobility is analytically uncertain. Table 2 shows that benefits raise overall mobility, indicating that the probability of moving across sectors, given a transition from employment to unemployment, responds to benefits more than employment, in percentage terms.

Table 3: Optimal Benefit Level by Moving Cost

Moving cost, c	Optimal Benefit Level	Tax, τ ,	Welfare Gain, %
0	0.6909	0.0284	0.0058
1	0.6788	0.0268	0.0057
3	0.6655	0.0253	0.0055
7	0.6509	0.0237	0.0049
12	0.6148	0.0189	0.0040

Notes: The welfare gain refers to the percentage increase in the average flow utility measured by $\exp((1 - \beta)(\mathcal{U}(b) - \mathcal{U}(b^{BM}))) - 1$ where $\mathcal{U}(b)$ is given by equation (34).

5.3 Moving Costs and Optimal Benefit Level

In this section, we analyze the relationship between moving costs and the welfare-maximizing benefit level. Determining the optimal benefit level is an exercise in balancing the traditional insurance vs. incentives (i.e. moral hazard effect), and the additional insurance motive provided by the consumption effect. We thus determine the optimal benefit level and how it changes with moving costs. In the welfare comparisons, we consider benefits above the benefit level in the benchmark economy, b^{BM} , that are financed by a lump sum tax τ . Then, the aggregate welfare is given by:

$$\mathcal{U} = u \log(b - \tau) - mc + \sum_i \int_{-\omega}^{\omega} \log(w_i(x) - \tau) \psi_i^e(x) dx. \quad (34)$$

The optimal benefit level is determined by finding the combination of τ and b that maximizes \mathcal{U} , subject to the budget constraint $\tau = u(b - b^{BM})$.

Table 3 describes how the optimal benefit level varies with moving costs. Our main finding from this experiment is that the optimal benefit level is decreasing in moving costs. Moreover, the total welfare gains from adopting the optimal benefit level are also decreasing in moving costs. There are many factors behind this result. The basic idea, however, can be explained by comparing the two extreme cases: 1) zero moving cost and 2) prohibitive moving cost (i.e. no mobility). In both economies, unemployment benefits do not affect mobility.

In the case of zero moving costs, agents continue to move to the most productive sector, while in the case of prohibitive moving costs, no one moves regardless of their productivity shock. With zero moving costs, the insurance vs. incentives trade-off becomes relatively tilted in favor of insurance. Since agents are moving to more productive sectors, often with more vacancies available per searcher, the optimal benefit can tolerate more insurance before the moral hazard problem begins to reduce welfare. In contrast, in the economy with high moving costs and no mobility, the moral hazard problem is more troublesome, as agents simply take more time to find a job in a relatively unproductive sector.

6 Alternative Moving Cost

In the analysis above, we model moving costs as an additively separable flow utility. We now consider an alternative specification. Specifically, we consider the possibility that switching industries or moving across locations may result in a loss of consumption, i.e., flow utility of a mover is given by $\log(b - c)$ rather than $\log(b) - c$.

An example of such costs is the costs associated with selling or buying a house when moving across locations. Such costs can be especially sizeable if the house price falls below the amount owed on the mortgage. A worker whose house price falls below the amount he owes on the mortgage may choose to remain in his current sector instead of moving to the sector where he is more productive. When a sector refers to an industry or an occupation, then pecuniary costs may include costs associated with schooling or other training programs.

6.1 Within-Sector Frictions

The characterization of the local labor market equilibrium follows that in Section 3. The reason is that conditional on \bar{U}_0 and \bar{U}_1 , the wage and the queue length are characterized as in 3.1. Therefore, the uniqueness results in Proposition 1 and Corollary 1 also hold in this case.

6.2 Mobility

The mobility decision is characterized analogously to that in the previous cases. However, now the value of moving from sector i to sector $1 - i$ is given by

$$M_i(x) = \log(b - c) + \beta S_{1-i}(x). \quad (35)$$

As in the previous case, the impact of benefits on productivity is given by equation (30). The only difference is that now we have

$$\Pi(b) = (1 - \beta)S_1(0, b) - \log(b - c) \quad (36)$$

instead of equation (29). Comparing equations (29) and (36), the consumption effect $\Pi(b)$ becomes even stronger, as an increase in benefits will have a much higher marginal effect on movers than on stayers because the impact of a marginal increase in benefits on the flow utility of a mover is proportional to $\frac{1}{b-c}$ instead of $\frac{1}{b}$.

6.3 Numerical Results

We consider the case where all the parameters except for the moving cost are fixed at their benchmark values. The moving cost is adjusted to target the annual mobility rate of 10%. The moving cost is then $c = 0.396$. The model predictions are summarized in Table 4. It shows that the benefits have a much larger effect on unemployment than in the previous case. Moreover, the response of productivity to benefits is now much larger. The consumption effect is now 15 times higher than the moral hazard effect.

Clearly, in equation (35), if the benefit level b is very close to the moving cost c , there is no mobility. Under such circumstances, a small increase in the benefit level b can sharply raise productivity. To illustrate this point numerically, we set the moving cost to $c = b - 10^{-6}$ and analyze the impact of higher benefits on mobility and productivity. The results are in

Table 4: Economy with Alternative Moving Cost

<i>Variables</i>	<i>Benefit levels</i>			<i>Description</i>
	0.40	0.45	0.50	
u	0.059	0.065	0.071	unemployment
m	0.100	0.1407	0.1500	annual mobility
v/u	0.6206	0.5695	0.5028	the vacancy-unemployment ratio
y_{\min}	0.9062	0.9505	0.9615	minimum productivity, $1 + \hat{x}_1$
\bar{y}	1.0775	1.0933	1.0957	average productivity
\bar{w}	1.0008	1.0199	1.0267	the average wage
γ	n/a	-0.0047	-0.0102	the moral hazard effect
π	n/a	-0.0724	-0.1560	the consumption effect

Table 5: Mobility-Inducing Benefits

<i>Variables</i>	<i>Benefit levels</i>			<i>Description</i>
	0.40	0.45	0.50	
u	0.0558	0.0652	0.0711	unemployment
m	0	0.1394	0.1494	annual mobility
v/u	0.5469	0.5686	0.5023	the vacancy-unemployment ratio
y_{\min}	0.8	0.9492	0.9608	the lowest productivity level, $1 + \hat{x}_1$
\bar{y}	1.000	1.0929	1.0955	per-worker output /productivity
\bar{w}	0.9278	1.0196	1.0266	the average wage
γ	n/a	-0.0034	-0.0088	the moral hazard effect
π	n/a	-0.0519	-0.1348	the consumption effect

Table 5. They show that annual mobility increases from 0 to 14 percent as the benefit level increases from 0.4 to 0.45. What is more remarkable is that economy-wide productivity increases by almost 10 percent. Thus, a moderate change in unemployment benefits can have a large positive impact on both productivity and mobility.

7 Conclusion

We construct a search-matching model with sectoral mobility and analyze the provision of unemployment benefits. Unemployment in this environment poses an additional risk

because the unemployed workers are subject to idiosyncratic productivity shocks that affect future wage prospects. Unemployment benefits increase mobility, which reduces the risks of idiosyncratic productivity shocks the unemployed face. Our results show that unemployment benefits can have a substantial impact on productivity through higher mobility. Ignoring costly sectoral mobility leads to a downward bias in the impact of benefits on unemployment, relative to a standard one-sector model. The optimal replacement ratio decreases with the costs of moving, and the welfare gains decrease from 0.6% when moving is costless to 0.4% when moving is prohibitively costly.

Our model and results have implications beyond the direct relationship between unemployment and mobility. In the model, an increase in moving costs is observationally equivalent to a decrease in productivity. Since mobility is driven by idiosyncratic productivity shocks, higher mobility implies higher average productivity. Thus, higher moving costs imply lower average productivity. Interpreted in this manner, our model potentially has implications for how unemployment benefits should respond to business cycles, an issue that has received some attention recently (see for example [Mitman and Rabinovich \(2011\)](#)). Specifically, our analysis suggests that in response to a productivity shock (in our model an increase in moving costs) unemployment benefits should decrease.

The 2007-2009 recession in the U.S. and its aftermath have brought increased attention to both the provision of unemployment insurance and to sectoral reallocation. The period is characterized by (1) unprecedented extensions of the duration of UI benefits, and (2) a coexistence of a persistently high unemployment rate and an increase in vacancies. The latter raises the question to what extent job seekers' skills do not correspond to the requirements of jobs they search for. The former raises the question to what extent the extensions of UI benefits limit the reallocation of the unemployed to their best-match vacancies. The model in this paper offers a fitting laboratory to study such phenomena, which represents an interesting direction for future research.

A Analytical Results

Here we prove a set of claims made in the text.

A.1 Wages, Queue Length and Productivity

Proof of Proposition 1. Recall that f is a strictly decreasing function of $q_{w,x,i}$. Moreover, for the value of search to exceed the value of unemployment, it must be that $S_i(x) > \log(b)/(1 - \beta)$. Thus, the right-hand side of equation (18) strictly decreases with $q_{w,x,i}$, while the left-hand side increases. Thus, for each (x, i) there is a unique queue length, $q_{w,x,i}$, which we denote by $q_i(x)$. \square

Proof of Corollary 1. See Proof of Proposition 1. \square

Proof of Corollary 2. Rewrite equation (15) using the uniqueness result,

$$\log(w_{x,i}) = C(b) + \left(1 + \frac{1 - \beta}{\beta f(q_{x,i})}\right) K_i(x), \quad (\text{A.1})$$

where

$$C(b) = \frac{A \log(b)}{1 - \beta} - \beta \lambda \bar{U}_i \quad (\text{A.2})$$

and

$$K_i(x) = A \left(S_i(x) - \frac{\log(b)}{1 - \beta} \right). \quad (\text{A.3})$$

Note that the term $C(b)$ is common across different productivity levels and different sectors. It is important to keep this in mind in the analysis below. On the other hand, using equation (18),

$$K_i(x) = \frac{r}{w_{x,i} q_{x,i}}, \quad (\text{A.4})$$

where $r = \frac{\eta^k A}{(1-\eta)(1-\beta)}$. Inserting equation (A.4) for $K_i(x)$ and then equation (19) for w into

equation (A.1), respectively, one can get

$$\log\left(y_i(x) - \frac{kA}{\beta\alpha(q_{x,i})}\right) = C(b) + \left(\frac{r}{q_{x,i}} + \frac{r(1-\beta)}{\beta\alpha(q_{x,i}(x))}\right) \left(y_i(x) - \frac{kA}{\beta\alpha(q_{x,i})}\right)^{-1}. \quad (\text{A.5})$$

Without loss of generality let $i = 1$. Also, notice that the left-hand side of equation A.5 increases with $q_{x,i}$, while the right-hand side decreases with $q_{x,i}$. Since $y_1(x) = 1 + x$, an increase in x raises the left-hand side of equation A.5 while lowering its right-hand side. Therefore, the equilibrium queue length $q_{x,1}$ decreases with x . Using the symmetric production function, it can be seen that the equilibrium queue length $q_{x,0}$ increases with x . Therefore, an increase in productivity $y_i(x)$ lowers the queue length. \square

Proof of Corollary 3. Combining equations (A.1) and (A.4), one can write

$$\log(w_{x,i}) - \left(\frac{1}{q_{x,i}} + \frac{1-\beta}{\beta\alpha(q_{x,i})}\right) \frac{r}{w_{x,i}} = C(b). \quad (\text{A.6})$$

Recall that α is a strictly increasing function. Therefore, the left hand side of equation (A.6) is an increasing function of $q_{x,i}$. Thus, $w_{x,i}$ and $q_{x,i}$ are negatively related between different values of x . Therefore, since the queue length $q_{x,i}$ decreases with productivity, the wage $w_{x,i}$ increases with productivity. \square

Proof of Corollaries 4 and 5. Combine equation (4) with Corollaries 2 and 3. \square

A.2 Impact of Benefits

In Section 5.2.2 we discuss the effects of within sector frictions on the average duration of unemployment. The following two results summarize the effects of unemployment benefits *in* a particular sector.

Proposition A.1 (Queue length and benefits). *Benefits raise the queue length at each*

productivity level.

Proof. Recall that the left-hand side of equation A.5 increases with $q_{x,i}$, while the right-hand side decreases with $q_{x,i}$. Moreover, the benefit level b affects the right-hand side through the term $C(b)$. Specifically, an increase in b raises $C(b)$, since the first term on the right-hand side of equation (A.2), $A \log(b)/(1 - \beta)$, dominates the second term $\beta \lambda \bar{U}_i$. Therefore, the benefit level will also raise the equilibrium queue length $q_{x,i}$ for each pair (x, i) . \square

Proposition A.2 (Wage and benefits). *Benefits raise the wage at each productivity level.*

Proof. The productivity specific wage increases with the queue length (see equation (19)). Then using Corollary 4, it can be seen that the wage $w_{i,x}$ increases with the benefit level b for each pair (i, x) . \square

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