A Macroeconomic Framework for Quantifying Systemic Risk*

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Abstract

Systemic risk arises when shocks lead to states where a disruption in financial intermediation adversely affects the economy and feeds back into further disrupting financial intermediation. We present a macroeconomic model with a financial intermediary sector subject to an equity capital constraint. The novel aspect of our analysis is that the model produces a stochastic steady state distribution for the economy, in which only some of the states correspond to systemic risk states. The model allows us to examine the transition from "normal" states to systemic risk states. We calibrate our model and use it to match the systemic risk apparent during the 2007/2008 financial crisis. We also use the model to compute the conditional probabilities of arriving at a systemic risk state, such as 2007/2008. Finally, we show how the model can be used to conduct a macroeconomic "stress test" linking a stress scenario to the probability of systemic risk states.

JEL Codes: G12, G2, E44

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1 Introduction

It is widely understood that a disruption in financial intermediation, triggered by losses on housing-related investments, has played a central role in the recent economic crisis. Figure 2 plots the market value of equity for the financial intermediary sector, along with a credit spread, investment, and a land price index. All variables have been normalized to one in 2007Q2. The figure illustrates the close relation between reductions in the value of financial intermediary equity, rising spreads, and falling land prices and aggregate investment.

In the wake of the crisis, understanding *systemic risk*, i.e., the risk of a disruption in financial intermediation with adverse effects for the real economy (see, e.g., Bernanke, 2009, Brunnermeier, Gorton and Krishnamurthy, 2010), has been a priority for both academics and policy-makers. The objective of this paper is to develop a macroeconomic model within which systemic risk can be quantified. We embed a financial intermediary sector within a simple real business cycle model. Equity capital constraints in the intermediation sector affect asset prices, real investment, and output. Moreover, since the tightness of constraints depend endogenously on expected future output, there is a two-way feedback between financial intermediation and real activity. These aspects of the model are by now familiar from the macroeconomics literature on financial frictions (see, e.g., Bernanke, Gertler, and Gilchrist, 1999, Kiyotaki and Moore, 1997, Gertler and Kiyotaki, 2010).

The principal innovation of the paper relative to much of the prior literature is that we model an occasionally binding constraint. We think this is a necessary methodological step in order to study systemic risk because systemwide financial disruptions are rare, and in most cases we are interested in understanding the transition of the economy from non-systemic states into systemic states. The model's equilibrium is a stochastic steady state distribution for the economy, in which "systemic risk" states correspond to only some of the possible realizations of the state variables. Moreover, in any given state, agents anticipate that shocks may realize that lead to constraints tightening, triggering systemic risk. As the economy moves closer to a systemic risk state, these anticipation effects cause banks to reduce lending and hence investment falls even though capital constraints are not binding. Relative to other papers in the literature (e.g., Bernanke, Gertler, and Gilchrist, 1999, Kiyotaki and Moore, 1997), our approach enables us to study the global dynamics of the system, not just the dynamics around a steady state. In this regard, our paper is closest methodologically to Mendoza (2010), He and Krishnamurthy (2011a,b), and Brunnermeier and Sannikov (2012), which we discuss further below.

We calibrate our model to replicate a systemic crisis, as in 2008. A significant challenge in quantifying the model is that crises are rare so that there is little data on which to calibrate the model. Our approach is to calibrate the model to match data during a downturn ("distress") in which the anticipation of a possible systemic crisis can affect behavior so that financial friction effects are present, but are not acute. We then use the non-linear structure imposed by the theoretical model to extrapolate to a more extreme crisis.

The first result of this calibration is that our model is able to quantitatively match the asymmetry present in the data between distress and non-distress periods, even though the calibration targets are neutral (i.e. unconditional moments). In particular, the simulated model matches the conditional covariances between growth in intermediary equity and Sharpe ratios, aggregate investment, consumption, and land prices, across both distress and non-distress periods. Central to this result is our modeling of a housing sector whereby the price of land is affected by the intermediary capital constraint, and where land prices themselves affect intermediary balance sheets. We assume that land is in fixed supply while physical capital is subject to adjustment costs. When the equity capital constraint tightens, land prices fall sharply, while the price of physical capital only falls slightly. In particular, we find that the amplification mechanism in our model is substantially through the feedback between the value of intermediary equity and land prices, and it is this amplification mechanism that helps to match the asymmetry in the data.

The second result of the analysis is in simulating a crisis to match patterns from 2007 to 2009. We choose a sequence of underlying shocks to match behavior of intermediary equity from 2007 to 2009. Given this sequence, we then compute the equilibrium values of the Sharpe ratio, aggregate investment and land prices. The analysis shows that the model's equity capital constraint drives a quantitatively significant amplification mechanism. That is, the size of the asset price declines produced by the model are much larger than the size of the underlying shocks we consider. In addition, the analysis shows that focusing only on shocks to intermediary equity results in an equilibrium that matches the behavior of aggregate investment, the Sharpe ratio, and land prices. This analysis lends further weight to explanations of the 2007-2009 crisis that emphasize shocks to the financial intermediary sector.

The third result of the analysis regards the likelihood of a systemic crisis. Our model allows us to compute the likelihood of a crisis, given an initial condition. We find that the odds of hitting the crisis states over the next 2 years, based on an initial condition chosen to match 2007Q2, is 1.12%. When we expand the horizon these probabilities rise to 2.62% (5 years) and 10.05% (10 years). While these numbers are small, it should be noted that most financial market indicators in early 2007, such as credit spreads or the VIX, were low and did not anticipate the severity of the crisis that followed. That is, without the benefit of hindsight, in both the model and data the probability of the 2007-2009 crisis is low. However, with the benefit of hindsight, it is now widely understood that the financial sector had embedded leverage through off-balance sheet activities, for example, which meant that true leverage was higher than the measured leverage based on balance sheets. In our baseline calibration, financial sector leverage is 3. To illustrate how "hidden" leverage higher than 3 could increase crisis probabilities, we conduct a simple experiment where we suppose that shocks translate to a larger than anticipated effect on bank balance sheets. In particular, we assume that agents' decisions rules, equilibrium prices and asset returns are all based on an aggregate intermediary leverage of 3, but that actually shocks impact intermediary balance sheets with a leverage that is 50% higher (i.e. leverage of 4.5). We find that the probability of the crisis over the next 2 years rises from from 1.12% to 60%, and for 5 years it rises from 9.12% to 85%. This computation suggests how stressing the financial system, because of the non-linearity in the model, can have a large impact on crisis probabilities.

In a similar vein, the model allows us to ask how a stress scenario, similar to the Federal Reserve's stress test, increases the probability of systemic risk. The key economics that our model captures that cannot be captured in a scenario-type analysis like the Fed's stress tests is the endogenous feedback of the economy to the stress scenario. That is, conditional on a scenario triggering a significant reduction in the equity capital of financial firms, it is likely that the endogenous response of the economy will lead to a further loss on assets and further reduction in equity capital. We illustrate through an example how to compute the probability of systemic risk based on a hypothetical stress test.

The papers that are most similar to ours are Mendoza (2010) and Brunnermeier and Sannikov (2010). Both papers develop stochastic and non-linear financial frictions models to study financial crises. Mendoza is interested in modeling and calibrating crises, or sudden stops, in emerging markets. From a technical standpoint, Mendoza relies on numerical techniques to solve his model, while we develop a homogeneous model with unidimensional state variable whose equilibrium behavior can be fully characterized by a system of ordinary differential equations. Our approach is thus complementary to his. Brunnermeier and Sannikov also take the differential equation approach of our paper. Their model illustrates the non-linearities in crises by showing that behavior deep in crises regions is substantially different than that in normal periods and underscores the importance of studying global dynamics and solving non-linear models. In particular, their model delivers a steady state distribution in which the economy can have high occupation time in systemic risk states. While our model is somewhat different than theirs, the principal difference relative to their paper is that we aim to quantitatively match the non-linearities in the data, thus providing a model that can be used to quantify systemic risk. Finally, both Mendoza and Brunnermeier-Sannikov study models with an exogenous interest rate, while the interest rate is endogenous in our model.

The model we employ is closely related to our past work in He and Krishnamurthy (2012a,b). He and Krishnamurthy (2012a) develop a model integrating the intermediary sector into a general equilibrium asset pricing model. The intermediary sector is modeled based on a moral hazard problem, akin to Holmstrom and Tirole (1997), and optimal contracts between intermediaries and households are derived.[1] Asset prices are also derived analytically. He and Krishnamurthy (2012b) assume the form of intermediation contracts derived in He and Krishnamurthy (2012a), but enrich the model so that it can be realistically calibrated to match asset market phenomena during the mortgage market financial crisis of 2007 to 2009. In the present paper, we also assume the structure of intermediation in reduced form. The main innovation relative to our prior work is that the present model allows for a real investment margin with capital accumulation and lending, and includes a housing price channel whereby losses on housing investments affect intermediary balance sheets. Thus the current paper speaks to not only effects on asset prices but also real effects on economic activity.

The paper is also related to the literature on systemic risk measurement. The majority of this literature motivates and builds statistical measures of systemic risk extracted from asset market data. Papers include Hartmann, Straetmans and De Vries (2005), Huang, Zhou, and Zhu (2010), Acharya, Pedersen, Philippon, and Richardson (2010), Adrian and Brunnermeier (2010), Billio, Getmansky, Lo, and Pelizzon (2010), and Giglio (2011). Our line of inquiry is different from this literature in that we build a macroeconomic model to understand how economic variables relate to systemic risk. Acharya, Pedersen, Philippon, and Richardson (2010) is closest to our paper in this regard, although the model used in that paper is a static model that is not suited to a quantification exercise. It is ultimately important that our model-based approach meets the data-oriented approaches.

The paper is laid out as follows. Section 2 describes the model. Section 3 goes through the steps of how we solve the model. Section 4 presents our choice of parameters for the calibration. Sections 5, 6, and 7 present the results from our model. Figures and an appendix with further details on the model solution are at the end of the paper.

2 Model

Time is continuous and indexed by t. The economy has two types of capital: productive capital K_t and housing capital H. We assume that housing is in fixed supply and normalize $H \equiv 1$. We denote by P_t the price of a unit of housing, and q_t the price of a unit of capital; both will be endogenously determined in equilibrium. The numeraire is the consumption good. There are three types of agents: equity households, debt households, and bankers.

We begin by describing the production technology and the household sector. These elements of the model are a slight variant on a standard stochastic growth model. We then describe bankers

[1]Our paper belongs to a larger literature, which has been growing given the recent crisis, on the macro effects of disruptions to financial intermediation. Papers most closely related to our work include Adrian and Shin (2010), Ashcraft, Garleanu and Pedersen (2010), Gertler and Kiyotaki (2010), Kiley and Sim (2011), Myerson (2012), Rampini and Viswanathan (2011), Bigio (2012), Adrian and Boyarchenko (2012), He and Kondor (2012), and Dewachter and Wouters (2012).

and intermediaries, which are the non-standard elements of the model. We assume that all of the housing and capital stock are owned by intermediaries that are run by bankers. Intermediaries also fund new investments. Households are assumed to not be able to directly own the housing and capital stock. Instead, the intermediaries raise equity and debt from households and use these funds to purchase housing and capital. The key assumption we make is that intermediaries face an equity capital constraint. The diagram below presents the main pieces of the model, which we explain in detail over the next sections.

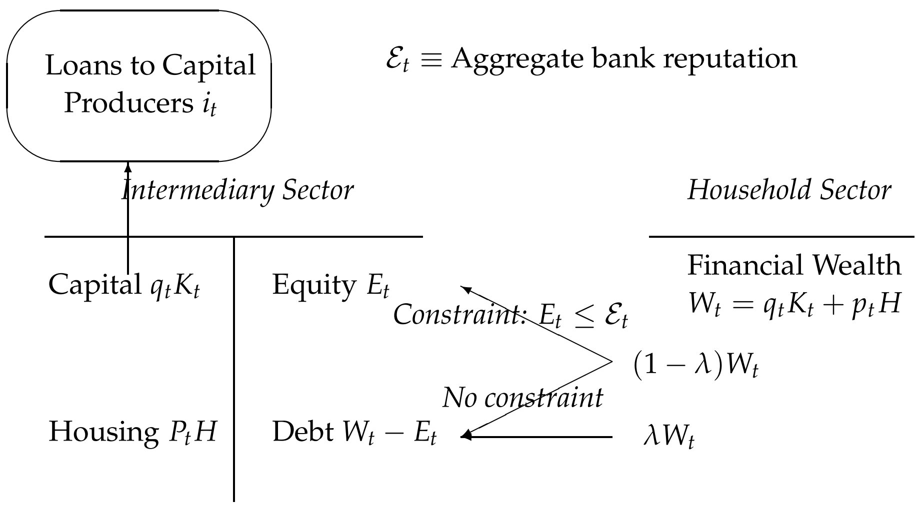

Figure 1: Model Schematic

2.1 Production and Households

There is an "AK" production technology that generates per-period output Y_t:

$$Y_t = AK_t, \tag{1}$$

where A is a positive constant. The evolution of capital is given by:

$$\frac{dK_t}{K_t} = i_t dt - \delta dt + \sigma dZ_t. \tag{2}$$

The term i_t is the amount of new capital installed at date t. Capital depreciates by δdt, where δ is constant. The last term σdZ_t is a capital quality shock, following Gertler and Kiyotaki (2010). For example, K_t can be thought of as the effective quality/efficiency of capital rather than the amount of capital outstanding. The capital quality shock is a simple device to introduce an exogenous source of variation in the value of capital. Note that the price of capital q_t and the price of housing P_t are endogenous. Thus, we will be interested in understanding how the exogenous capital quality shock translates into endogenous shocks to asset prices. Finally, the shock σdZ_t is the only source of uncertainty in the model ($\{Z_t\}$ is a standard Brownian motion, while σ is a positive constant).

Commonly, RBC models introduce shocks to the productivity parameter A rather than the quality shocks we have introduced. Introducing shocks to A will add another state variable and greatly complicate solutions to the model. We assume shocks directly in the evolution of the capital stock, K_t, because capital will be one of the state variables in the solution. But, note that a shock to A and the direct shock to $\frac{dK_t}{K_t}$ will work similarly. That is imagine a model with A shocks and consider a -10% drop in A. In this case Y_t falls by 10% and, for a fixed price/dividend ratio, the drop in the dividend on capital will lead to a -10% return to owners of capital. Now consider the shock we model as a direct -10% shock to $\frac{dK_t}{K_t}$. The shock also leads output to fall by 10%. Owners of capital "lose" 10% of their capital so that, for a fixed price/dividend ratio, they experience a -10% return to capital. These aspects thus appear similar across the two ways of modeling the shock. The main difference will be in the price of capital, q. With a shock to A, we would expect that q will fall through a direct effect of approximately 10% (ignoring the general equilibrium effects), while with the shock to $\frac{dK_t}{K_t}$, there is no direct effect on q (only general equilibrium effects cause q to fall).

We assume adjustment costs so that installing $i_t K_t$ new units of capital costs $\Phi(i_t, K_t)$ units of consumption goods where,

$$\Phi(i_t, K_t) = i_t K_t + \frac{\kappa}{2} (i_t - \delta)^2 K_t.$$

That is, the adjustment costs are assumed to be quadratic in net investment.

There is a unit measure of households. Each household enters period t with financial wealth W_t. It consumes out of this wealth, allocates resources to real investment, and invests the wealth in the equity and debt of financial intermediaries. The utility of the household is of the Cobb-Douglas form,[2]

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t} \left(c_t^y\right)^{1-\phi} \left(c_t^h\right)^{\phi} dt\right],$$

where the constant ρ is the discount rate, c_t^y is consumption of the output good, c_t^h is consumption of housing services, and ϕ is the expenditure share on housing. Then, given the preferences, the optimal consumption rule must satisfy:

$$\frac{c_t^y}{c_t^h} = \frac{1-\phi}{\phi} D_t, \tag{3}$$

where D_t is the endogenous rental rate on housing to be determined in equilibrium. In equilibrium, ϕ affects the relative market value of the housing sector to the goods producing sector.

[2] Our modeling approach can handle richer specifications of the household's utility function. We have investigated versions in the power (i.e. CRRA) family. Details are available upon request. Also, note that there is curvature in the preferences through the superscript $1-\phi$. With two goods, the intratemporal elasticity of substitution between the goods enters the household's Euler equation. For our two-good model, it is easy to show that Euler equation resembles a one-good Euler equation but where the intertemporal elasticity of substitution is $1/\phi$. See (8) below. Piazzesi, Schneider and Tuzel (2007) clarify how risk over the composition of consumption in a two-goods setting with housing and a non-durable consumption good enters into the Euler equation.

2.2 Bankers, Equity Capital, and the Flow-Performance Relationship

We assume that all productive capital and housing stock can only be owned directly by "financial intermediaries." There is a continuum of competitive intermediaries. The intermediaries are owned by households, but run by bankers who have the know-how to manage investments. These bankers make all investment decisions of the intermediary. That is, we assume that there is a separation between the ownership and control of the intermediary. Households invest their wealth of W_t in the equity and debt of the intermediary sector, who then directly own the capital/housing stock and fund new investments.

At time t, a given banker has "reputation" of ϵ_t. Faced with such a banker, we assume that equity-households are willing to invest up to ϵ_t to own the equity of the intermediary. Any remaining funds raised by the intermediary are in the form of short-term (from t to $t + dt$) debt financing. Equity can only be raised from equity-households, while debt can be raised from either equity or debt households (see the Schematic in Figure 1).

Denote the realized profit-rate on the intermediary's assets (i.e. holdings of capital and housing) from t to $t + dt$, net of any debt repayments, as $d\tilde{R}_t$. This is the return on the shareholder's equity of the intermediary. The profit is stochastic and depends on shocks at time $t + dt$. Then, we assume that the reputation of the banker making that intermediary's investment decisions evolves as,

$$\frac{d\epsilon_t}{\epsilon_t} = m d\tilde{R}_t, \tag{4}$$

where $m > 0$ is a constant. Poor investment returns reduce ϵ_t and thus reduce the maximum amount of equity a given intermediary can raise going forward.

Equation (4) is a contemporaneous relationship between the flows into an intermediary and the performance of the intermediary. This sort of flow-performance relationship is a well documented empirical regularity among mutual funds (see Warther, 1995, or Chevalier and Ellison, 1997), for which there is substantial data on returns and equity inflows/outflows.[3] The flow-performance relationship has also been documented for hedge funds (Getmansky, 2012) and private equity funds (Kaplan and Schoar, 2005). We are making a natural assumption that this relation holds broadly across the financial intermediary sector. The leading explanation for the flow-performance relationship is based on investors' learning the skill of the fund manager (Berk and Green, 2004). Although we do not model learning, this type of explanation is our motivation for equation (4). That is, one can give a rational underpinning for a loss of equity capital of an

[3] Warther (1995) documents a positive contemporaneous correlation between aggregate monthly flows into stock funds and stock returns over a sample from 1984 to 1992. His baseline estimate is that a 5.7% stock return is associated with a 1% contemporaneous unexpected inflow into funds. He also shows that flows are AR(1) with parameter of 0.6, so that the cumulative effect on inflows due to a 1% increase in stock returns is $\frac{1}{5.7} \times \frac{1}{1-0.6} = 0.43\%$. In terms of (4), consider a 1% stock return, which increases assets in a fund by 1%, and further generates cumulative new inflows of 0.43%, so that total assets rise by 1.43%. This means that $m = 1.43$.

intermediary following bad past returns. The thorny issue for such an explanation is that indexing or benchmarking the returns of one manager to another manager, which is typically optimal in a learning setting, can substantially reduce aggregate effects. We note that this type of indexation issue arises in many macroeconomic models, including those of collateral constraints (see Krishnamurthy, 2003).

The finance literature has explored the effects of the flow-performance relationship on asset prices in limits-to-arbitrage models. An equation like (4) underlies the influential analysis of Shleifer and Vishny (2004). More recently, Vayanos and Woolley (2012) have studied such a model to explain the momentum effect in stock returns. In this paper, we consider the macroeconomic implications of the flow-performance relationship.

Reputation and skill of a banker are one way to think about how past returns may affect household's willingness to invest in intermediaries. But there are other ways, such as moral hazard or adverse selection, in which past returns of the intermediary reduces net worth and thereby reduces households' willingness to invest in intermediaries. In He and Krishnamurthy (2012a) we consider a setting where bankers have preferences over consumption and households write incentive contracts with bankers to manage intermediaries. In that setting, the bankers' net worth plays the same role as ϵ_t in our current setting and we find that bankers' equity capital constraint is similarly a function of their past performance. Moreover, from a macroeconomic standpoint all of the model's dynamics are driven by the equity capital constraint.[4]

We assume that a banker makes investment decisions to maximize his future reputation. Bankers do not consume goods (a feature which is convenient when clearing the goods market). A given banker may die at any date at a Poisson rate of η. Thus, a banker makes investment decisions to maximize,

$$\mathbb{E}\left[\int_0^\infty e^{-\eta t}\ln \epsilon_t dt\right].$$

Given the log form objective function and equation (4), it is easy to show that the time t decision of the banker is chosen to maximize,

$$\mathbb{E}_t[d\tilde{R}_t] - \frac{m}{2} Var_t[d\tilde{R}_t]. \tag{5}$$

The constant m thus parameterizes the "risk aversion" of the banker.

To summarize, a given intermediary can raise at most ϵ_t of equity capital. If the intermediary's investments perform poorly, then ϵ_t falls going forward, and the equity capital constraint tightens. The banker in charge of the intermediary chooses the intermediary's investments to maximize the

[4]The modeling leads to two changes relative to He and Krishnamurthy (2012a,b). First, we do not have to keep track of the bankers' consumption decisions which simplifies the model's analysis somewhat. More substantively, in our previous work we find that, in crisis states, the interest rate diverges to negative infinity. In the present modeling, the interest rate is determined purely by the household's Euler equation, which leads to a better behaved interest rate.

mean excess return on equity of the intermediary minus a penalty for variance multiplied by the "risk aversion" m.

2.3 Aggregate Intermediary Capital

Consider now the aggregate intermediary sector. We denote by $\mathcal{E}_t$ the maximum equity capital that can be raised by this sector, which is just the aggregate version of individual banker's reputation ϵ. The maximum equity capital $\mathcal{E}_t$ will be the key state variable in our analysis, and its dynamics are given by,

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = md\tilde{R}_t - \eta dt + d\psi_t. \tag{6}$$

The first term here reflects that all intermediaries are identical, so that the aggregate stock of intermediary reputation evolves with the return on the intermediaries' equity.[5] The second-term, $-\eta dt$, captures exit of bankers who die at the rate η. Exit is important to include; otherwise, $d\mathcal{E}_t/\mathcal{E}_t$ will have a strictly positive drift in equilibrium, which makes the model non stationary. In other words, without exit, intermediary capital will grow and the capital constraint will not bind. The last term, $d\psi_t \geq 0$ reflects entry. We describe this term more fully below when describing the boundary conditions for the economy. In particular, we will assume that entry occurs when the aggregate intermediary sector has sufficiently low capital, because the incentives to enter are high in these states.

2.4 Capital Goods Producers

Capital goods producers, owned by households, undertake real investment. As with the capital stock and the housing stock, we assume that capital goods must be sold to the intermediary sector. Thus, q_t, based on the intermediary sector's valuation of capital also drives investment. Given q_t, i_t is chosen to solve,

$$\max_{i_t} \; q_t i_t K_t - \Phi(i_t, K_t) \;\Rightarrow\; i_t = \delta + \frac{q_t - 1}{\kappa}. \tag{7}$$

Recall that $\Phi(i_t, K_t)$ reflects a quadratic cost function on investment net of depreciation.

2.5 Household Members and Portfolio Choices

We make assumptions so that a minimum of λW_t of the household's wealth is invested in the debt of intermediaries. We may think of this as reflecting household demand for liquid transaction balances in banks, although we do not formally model a transaction demand. The exogenous constant λ is useful to calibrate the leverage of the intermediary sector, but is not crucial for the qualitative properties of the model.

[5] The model can accommodate heterogeneity in reputations, say ϵ_t^i where i indexes the intermediary. Because the optimal decision rules of a banker are linear in ϵ_t^i, we can aggregate across bankers and summarize the behavior of the aggregate intermediary sector with the average reputation, which is equivalent to $\mathcal{E}_t$.

The modeling is as follows. Each household is comprised of two members, an "equity household" and a "debt household." In each period, W_t is split between the household members as $1-\lambda$ fraction to the equity household and λ fraction to the debt household. We assume that the debt household can only invest in intermediary debt, while the equity household can invest in either debt or equity. Thus households collectively invest in at least λW_t of intermediary debt. The household members individually make financial investment decisions. The investments pay off at period $t+dt$, at which point the members of the household pool their wealth again to give wealth of W_{t+dt}. The modeling device of using the representative family follows Lucas (1990).

Collectively, equity households invest their allocated wealth of $(1-\lambda)\,W_t$ into the intermediaries subject to the restriction that, given the stock of banker reputations, they do not purchase more than $\mathcal{E}_t$ of intermediary equity. When $\mathcal{E}_t > W_t(1-\lambda)$ so that the intermediaries reputation is sufficient to absorb the households' maximum equity investment, we say that the capital constraint is not binding. But when $\mathcal{E}_t < W_t(1-\lambda)$ so that the capital constraint is binding, the equity household restricts its equity investment and places any remaining wealth in bonds. In the case where the capital constraint does not bind, it turns out to be optimal – since equity offers a sufficiently high risk-adjusted return – for the equity households to purchase $(1-\lambda)W_t$ of equity in the intermediary sector. We verify the latter statement when solving the model. Let,

$$E_t \equiv \min\left(\mathcal{E}_t, W_t(1-\lambda)\right)$$

be the amount of equity capital raised by the intermediary sector. The households' portfolio share in intermediary equity, paying return $d\tilde{R}_t$, is thus, $\frac{E_t}{W_t}$.

The debt household simply invests his portion λW_t into the riskless bond. The household budget constraint implies that the amount of debt purchased by the combined household is equal to $W_t - E_t$.

2.6 Riskless Interest Rate

Denote the interest rate on the short-term bond as r_t. Given our Brownian setting with continuous sample paths, the short-term debt is riskless.[6] The household's Euler equation can then be used to derive the interest rate . With two-goods, the Euler equation is a bit more involved than the usual one (as discussed in footnote 2). Consider at the margin a household that cuts its consumption of the output good today (the envelope theorem allows us to evaluate all of the consumption reduction in terms of the output good), investing this in the riskless bond to finance more consumption tomorrow. The marginal utility of consumption of the output good is $e^{-\rho t}(1-\phi)\left(c_t^y\right)^{-\phi}\left(c_t^h\right)^{\phi}$

[6]Note that we place no restriction on the raising of debt financing by the intermediary. Debt is riskless and is always over-collateralized so that a debt constraint would not make sense in our setting. It is clear in practice that there are times in which debt or margin constraints are also quite important. Our model sheds light on the effects of limited equity capital (e.g., limited bank capital) and its effects on intermediation.

which equals $e^{-\rho t}(1-\phi)\left(c_t^y\right)^{-\phi}$ since in equilibrium $c_t^h = H \equiv 1$. Thus, the equilibrium interest rate r_t satisfies:

$$r_t = \rho + \phi \mathbb{E}_t \left[\frac{dc_t^y}{c_t^y}\right] - \frac{\phi(\phi+1)}{2} Var_t \left[\frac{dc_t^y}{c_t^y}\right]. \tag{8}$$

2.7 Intermediary Portfolio Choice

Each intermediary chooses how much debt and equity financing to raise from households, subject to the capital/reputation constraint, and then makes a portfolio choice decision to own housing and capital. The return on purchasing one unit of housing is,

$$dR_t^h = \frac{dP_t + D_t dt}{P_t}, \tag{9}$$

where P_t is the pricing of housing, and D_t is the equilibrium rental rate given in (3). Let us define the risk premium on housing as $\pi_t^h \equiv \mathbb{E}_t[dR_t^h]/dt - r_t$. That is, by definition the risk premium is the expected return on housing in excess of the riskless rate. Then,

$$dR_t^h = (\pi_t^h + r_t)dt + \sigma_t^h dZ_t.$$

Here, the volatility of investment in housing is σ_t^h, and from (9), σ_t^h is equal to the volatility of dP_t/P_t.

For capital, if the intermediary buys one unit of capital at price q_t, the capital is worth q_{t+dt} next period and pays a dividend equal to Adt. However, the capital depreciates at the rate δ and is subject to the capital quality shocks σdZ_t. Thus, the return on capital investment, accounting for the Ito quadratic variation term, is as follows:

$$dR_t^k = \frac{dq_t + Adt}{q_t} - \delta dt + \sigma dZ_t + \left[\frac{dq_t}{q_t}, \sigma dZ_t\right]. \tag{10}$$

We can also define the risk premium and risk on capital investment suitably so that,

$$dR_t^k = (\pi_t^k + r_t)dt + \sigma_t^k dZ_t.$$

We use the following notation in describing an intermediary's portfolio choice problem. Define α_t^k (α_t^h) as the ratio of an intermediary's investment in capital (housing) to the equity raised by an intermediary. Here, our convention is that when the sum of αs exceed one, the intermediary is shorting the bond (i.e., raising debt) from households. For example, if $\alpha_t^k = \alpha_t^h = 1$, then an intermediary that has one dollar of equity capital will be borrowing one dollar of debt (i.e. $1 - \alpha_t^k - \alpha_t^h = -1$) to invest one dollar each in housing and capital. The intermediary's return on equity is,

$$d\tilde{R}_t = \alpha_t^k dR_t^k + \alpha_t^h dR_t^h + (1 - \alpha_t^k - \alpha_t^h) r_t dt. \tag{11}$$

From the assumed objective in (6), a banker solves,

$$\max_{\alpha_t^k, \alpha_t^h} \mathbb{E}_t[d\tilde{R}_t] - \frac{m}{2} Var_t[d\tilde{R}_t]. \tag{12}$$

The optimality conditions are,

$$\frac{\pi_t^k}{\sigma_t^k} = \frac{\pi_t^h}{\sigma_t^h} = m\left(\alpha_t^k \sigma_t^k + \alpha_t^h \sigma_t^h\right). \tag{13}$$

The Sharpe ratio is defined to be the risk premium on an investment divided by its risk (π/σ). Optimality requires that the intermediary choose portfolio shares so that the Sharpe ratio on each asset is equalized. Additionally, the Sharpe ratio is equal to the riskiness of the intermediary portfolio, $\alpha_t^k \sigma_t^k + \alpha_t^h \sigma_t^h$, times the "risk aversion" of m. This latter relation is analogous to the CAPM. If the intermediary sector bears more risk in its portfolio, and/or has a higher m, the equilibrium Sharpe ratio will rise.

2.8 Market Clearing and Equilibrium

1. In the goods market, the total output must go towards consumption and real investment (where we use capital C to indicate aggregate consumption)

$$Y_t = C_t^y + \Phi(i_t, K_t). \tag{14}$$

   Note again that bankers do not consume and hence do not enter this market clearing condition.

2. The housing rental market clears so that

$$C_t^h = H \equiv 1. \tag{15}$$

3. The intermediary sector holds the entire capital and housing stock. The intermediary sector raises total equity financing of $E_t = \min(\mathcal{E}_t, W_t(1-\lambda))$. Its portfolio share into capital and housing are α_t^k and α_t^h.[7] The total value of capital in the economy is $q_t K_t$, while the total value of housing is P_t. Thus, market clearing for housing and capital are:

$$\alpha_t^k E_t = K_t q_t \text{ and } \alpha_t^h E_t = P_t. \tag{16}$$

   These expressions pin down the equilibrium values of the portfolio shares, α_t^k and α_t^h.

4. The total financial wealth of the household sector is equal to the value of the capital and housing stock:

$$W_t = K_t q_t + P_t.$$

[7] Keep in mind that while we use the language "portfolio share" as is common in the portfolio choice literature, the shares are typically larger than one because in equilibrium the intermediaries borrow from households.

An equilibrium of this economy consists of prices, (P_t, q_t, D_t, r_t), and decisions, $(c_t^y, c_t^h, i_t, \alpha_t^k, \alpha_t^h)$. Given prices, the decisions are optimally chosen, as described by (3), (7), (8) and (12). Given the decisions, the markets clear at these prices.

3 Model Solution

We derive a Markov equilibrium where the state variables are K_t and $\mathcal{E}_t$. That is, we look for an equilibrium where all the price and decision variables can be written as functions of these two state variables. Given homogeneity features of the economy, we can simplify this further. We look for price functions of the form $P_t = p(e_t)K_t$ and $q_t = q(e_t)$ where

$$e_t \equiv \frac{\mathcal{E}_t}{K_t}.$$

Therefore, K_t scales the economy while e_t describes the equity capital constraint of the intermediary sector. The equity capital constraint, e_t, evolves stochastically. The appendix goes through the algebra detailing the solution. We show how to go from the intermediary optimality conditions, (13), to a system of ODEs for $p(e)$ and $q(e)$.

The solution of the model revolves around equation (13) which is the optimality condition for an intermediary. The equation states that the required Sharpe ratio demanded by an intermediary to own housing and capital is linear in the total risk borne by that intermediary, $m\left(\alpha_t^k \sigma_t^k + \alpha_t^h \sigma_t^h\right)$. If intermediaries hold more risky portfolios, which can happen if α_t^k and α_t^h are high, and/or if σ_t^h and σ_t^k are high, they will require a higher Sharpe ratio to fund a marginal investment.

Equilibrium conditions pin down the αs (portfolio shares) and the σs (volatilities). Consider first the αs as they are the more important factor. The variable α_t^k is the ratio of the intermediary's investment in capital to the amount of equity it raises. Market clearing dictates that the numerator of this ratio must be equal to $q_t K_t$ across the entire intermediary sector, while the denominator is the equity capital raised by the intermediary sector, E_t (see (16)).

Before studying the effect of the constraint, it useful to consider the economy without a reputation/equity constraint. Then, the household sector would invest $(1-\lambda)W_t$ in equity and λW_t in debt. That is, from the standpoint of households and given the desire for some debt investment on the part of households, the optimal equity/debt mix that households would choose is $(1-\lambda)W_t$ of equity and λW_t of debt. In this case, α_t^k is equal to $\frac{q_t K_t}{(1-\lambda)W_t}$. Moreover, because $W_t = K_t(q_t + p_t)$, i.e., the aggregate wealth is approximately proportional to the value of the capital stock, this ratio is near constant. A negative shock that reduces K_t also reduces W_t proportionately with no effects on α_t^k. A similar logic applies to α_t^h. This suggests that the equilibrium Sharpe ratio would be nearly constant if there was no equity capital constraint. While we have not considered the σ's in this argument, because they are endogenous objects that depend on the equilibrium price functions P_t and q_t, they turn out to be near constant as well without a capital constraint. Thus, without the capital constraint, shocks to K_t just scale the entire economy up or down, with investment, consumption, and asset prices moving in proportion to the capital shock.

3.1 Capital Constraint, Amplification, and Anticipation Effects

Now consider the effect of the capital constraint. If $\mathcal{E}_t < W_t(1-\lambda)$, then the intermediary sector only raises $E_t = \mathcal{E}_t$ of equity. In this case, α_t^k and α_t^h must be higher than without capital constraint. In turn, the equilibrium Sharpe ratios demanded by the intermediary sector must rise relative to the case without capital constraint. In this state, consider the effect of negative shock. Such a shock reduces W_t, but reduces $E_t = \mathcal{E}_t$ more through two channels. First, since the intermediary sector is levered (i.e. in equilibrium the sum of αs are larger than one simply because some households only purchase debt), the return on equity is a multiple of the underlying return on the intermediary sector's assets. Second, we parameterize the model so that the speed in the flow-performance relationship, m, is larger than one, which implies that $\mathcal{E}_t$ moves more one-for-one with the return on equity (see (4)). Thus negative shocks are amplified and cause the equilibrium αs to rise when the capital constraint binds. The higher αs imply a higher Sharpe ratio on capital and housing investment, which in turn implies that the price of capital and housing must be lower in order to deliver the higher expected returns implied by the higher Sharpe ratios. This means in turn that the capital constraint is tighter, further reducing equity capital. This effect also amplifies negative shocks. There is a further amplification mechanism: since the price of housing and capital are more sensitive to aggregate equity capital when such capital is low, the equilibrium volatility (i.e, σs) of housing and capital are higher, further increasing Sharpe ratios and feeding through to asset prices and the equity capital constraint. All of these effects reduce investment, because investment depends on q_t which is lower in the presence of the equity capital constraint.

Next consider how the economy can transit from a state where the equity capital constraint does not bind to one where the constraint binds. Even when the constraint is not active, returns realized by the intermediaries affect the reputation stock $\mathcal{E}_t$, as in equation (4). If there is a series of negative shocks causing low returns, $\mathcal{E}_t$ falls, and as described above, the fall is larger than the fall in W_t. Thus, a series of negative shocks can cause $\mathcal{E}_t$ to fall below $W_t(1-\lambda)$, leading to a binding capital constraint.

Last consider how the effect of an anticipated constraint may affect equilibrium in states where the constraint is not binding. Asset prices are the discounted presented value of future dividends. As the economy moves closer to the constraint binding, the discount rates (i.e. required expected returns) rise which causes asset prices to fall. That is asset prices fall to anticipate the possibility that the constraint may bind in the future. Through this channel, the equilibrium is affected by $\mathcal{E}_t$ even in cases where it is larger than $W_t(1-\lambda)$. This is an anticipation effect that emerges from solving for the global dynamics of the model.

The anticipation effect is in important in empirically verifying the model. A significant challenge in identifying any crisis model is that crises are rare so that there is little data on which to calibrate the model. Our approach is to calibrate the model to match data during a downturn ("distress") in which the anticipation of a possible systemic crisis can affect behavior so that financial friction effects are present, but are not acute. We then use the non-linear structure imposed by the theoretical model to extrapolate to a more extreme crisis.

3.2 Boundary Conditions

The ODEs are solved numerically subject to two boundary conditions. First, the upper boundary is characterized by the economy with $e \to \infty$ so that the capital constraint never binds. We derive exact pricing expressions for the economy with no capital constraint and impose these as the upper boundary. The Appendix provides details.

The lower boundary condition is as follows. We assume that new bankers enter the market when the Sharpe ratio reaches γ, which is an exogenous parameter in the model. This captures the idea that the value of entry is high when the Sharpe ratio of the economy is high. Entry alters the evolution of the state variable e. In particular, the entry point $\underline{e}$ is endogenous and is a reflecting barrier. We assume that entry increases the aggregate intermediary reputation (and therefore the aggregate intermediary equity capital), but requires some physical capital. We assume that paying $\beta > 0$ units of capital increases $\mathcal{E}_t$ by one unit. Since the entry point is a reflecting barrier it must be that the price of a unit of capital, $q(e)$, and the price of a unit of housing, $p(e)K$, have zero derivative with respect to e at the barrier (if not, an investor can make unbounded profits by betting on an almost sure increase/decrease in the asset price). Hence we have that $q'(\underline{e}) = 0$. For the housing price, imposing that pK has zero derivative implies the lower boundary condition $p'(\underline{e}) = \frac{p(\underline{e})\beta}{1+\underline{e}\beta} > 0$. The derivative is positive because K falls at the entry boundary, since entry uses up capital, and hence p must rise in order to keep pK constant. See the Appendix for the exact argument and derivation.

4 Calibration

The parameters, ρ (household time preference), δ (depreciation), and κ (adjustment cost) are relatively standard. We use conventional values for these parameters (see Table 1). Note that since our model is set in continuous time, the values in Table 1 correspond to annual values rather than the typical quarterly values one sees in discrete time DSGE parameterizations.

The most important parameter in the model is σ which governs the exogenous uncertainty in this model. Increasing σ increases the volatility of all quantities and prices in the model. We choose $\sigma = 4\%$ as our baseline, and show how changing σ affects results. The baseline generates

Table 1: Parameters

Panel A: Intermediation			
	Parameter	Choice	Target
m	Performance sensitivity	2	Average Sharpe ratio
λ	Debt ratio	0.67	Average intermediary leverage
η	Banker exit rate	17%	Good model dynamics
γ	Entry barrier	6.5	Highest Sharpe ratio
β	Entry cost	2.34	Land price volatility
Panel B: Technology			
σ	Capital quality shock	4%	Consumption volatility
δ	Depreciation rate	10%	Literature
κ	Adjustment cost	3	Literature
A	Productivity	0.148	Investment-to-capital ratio
Panel C: Other			
ρ	Time discount rate	3%	Literature
ϕ	Housing share	0.5	Housing-to-wealth ratio

a volatility of investment growth in the model of 4.97% and a volatility of consumption growth of 2.21%. In the data, the volatility of investment growth from 1973 to 2010 is 7.78% while the volatility of consumption growth is 2.17%. We have chosen a σ value that is too low for investment but matches consumption. We will also present results for a variation with higher σ.

The main intermediation parameters are m and λ. The parameter m governs the "risk aversion" of the banker. As we vary m, the Sharpe ratio in the model changes proportionately (see (13)). The choice of $m = 2$ gives an average Sharpe ratio from the model of 43%, which is in the range of typical asset pricing calibrations. If we look to the flow-performance relationship for mutual funds as a guide, the results of Warther (1995) imply a value of $m = 1.43$ (see footnote 3). The parameter λ is equal to the financial intermediary sector's debt/assets ratio when the capital constraint does not bind. We choose $\lambda = 0.67$, which translates to financial leverage ($\equiv$ assets/equity) of 3. The main challenge in choosing λ is that it represents the leverage across the entire and heterogenous sophisticated intermediary sector, encompassing commercial banks, investment banks, hedge funds, and venture capital/private equity funds. For example, commercial and investment banks have debt/assets ratios of 80% or higher. Ang, Gorovyy and van Inwegen (2011) report average hedge fund leverage of 2.1 (or debt/assets of 49%), with considerable variation across strategies. Our choice of 2/3 for λ is a simple attempt to represent leverage across this entire sector.[8]

The entry boundary condition (i.e. lower boundary) is determined by γ and β. We set $\gamma = 6.5$, so that new entry occurs when the Sharpe ratio is 650%. Based on movements in credit spreads, as measured by Gilchrist and Zakrajsek (2010)'s excess bond premium (see the data description in

[8]As another benchmark, Gertler and Kiyotaki (2011) target a leverage ratio of 4.

Section 6.1), we compute that Sharpe ratio of corporate bonds during the 2008 crisis was roughly 15 times the average. Since in our simulation the Sharpe ratio is around 43%, we set the highest Sharpe ratio to be 650%. Although a high entry threshold is crucial for our model, the exact choice of γ is less important because the probability of reaching the entry boundary is almost zero. Our choice is principally motivated by setting γ sufficiently high that it does not affect the model's dynamics in the main part of the distribution. The value of β is far more important because it determines the slope of the land price function at the entry boundary, and therefore the slope all through the capital constrained region. The volatility of land prices is closely related to the slope of the price function (see equation (18)). In the data, the empirical volatility of land price growth from 1975 to 2009 is 14.47%. The choice of $\beta = 2.34$ produces unconditional land price volatility of 14.93%. [9]

We set η (the bankers' death rate) equal to 17% in our baseline. It is hard to pin down η based on data on the U.S. economy. Our choice is rather dictated by targeting "good" model dynamics. The choice of η is important for our results because it shifts the center of the steady state distribution of intermediary equity capital. For example, if η is very small, the steady state distribution places little weight on being a crisis region. If η is too large, the model is always in a crisis region. We have chosen η so that the drift of intermediary reputation $(\mathcal{E})$ is slightly positive in the unconstrained region (2% on average in the simulation). By targeting the drift near zero, we allow the probability of a crisis to be driven primarily by the volatility of the economy rather than a contrived death rate parameter.[10] When we vary parameters we also vary η so as to keep the average value of e across the simulation to be the same.

We set $\phi = 0.5$. The parameter ϕ governs the dividend on housing which in turns drives the total value of the housing assets relative to wealth. From Flow of Funds data, Table B100, the total wealth of the household sector in 2005 is 64tn. Of this wealth, real estate accounts for 25tn, or 39%. In our simulation, the choice of $\phi = 0.5$ yields that the mean ratio $\frac{p}{p+q}$ is 37%.

Finally, we set $A = 0.1485$ to target the average investment to capital ratio in the data. From 1973 to 2010, this average is 11% in the data and 10% in the simulation.

[9] This choice of β leads to a slope of $p'(\underline{e}) = 0.415$ at the endogenous entry point $\underline{e}$. Also note that it is tautological within our model that at the entry barrier the household sector is willing to pay exactly βK units of capital to boost wealth (i.e. P and q) by increasing e. That is, the value of β cannot be independently pinned down from this sort of computation.

[10] The value of η ends up being critical for governing the probability of a crisis since it essentially shifts the steady state distribution. An alternative way to parameterize η would be based on the historical probability of financial crises. This approach would allow us to understand how different types of shocks or changes in parameters change the probability of a crisis.

5 Results

5.1 Price and Policy Functions: "Anticipation" Effects

Figures 4 and 5 plot the price and policy functions for the baseline parameterization and a variation with a higher σ. Consider the baseline in Figure 4 first. The X-axis in all of the graphs is $e = \frac{\mathcal{E}}{K}$. The equity capital constraint binds for points to the left of 0.44. The lower right panel graphs plots the steady-state distribution of the intermediary equity state variable. Most of the weight is on the part of the state space where the capital constraint does not bind. That is, a systemic crisis, defined as periods where the capital constraints bind, are rare in the model.

The top row, third panel is the Sharpe ratio. The Sharpe ratio is about 37.6% in the unconstrained region and rises rapidly upon entering the constrained region. The interest rate (second row, left panel) also falls sharply when the economy enters the constrained region. Both effects reflect the endogenous increase in "risk aversion" of the intermediary sector during a systemic crisis.

The first two panels on the top row are $p(e)$ (housing price divided by capital stock) and $q(e)$. Both price functions are increasing in equity capital as one would expect. It is worth noting that going from right-to-left, prices fall before entering the capital constrained region. This occurs through *anticipation effects*. As the economy moves closer to the constraint, the likelihood of falling into the constrained region rises and this affects asset prices immediately. Moreover, note that if the model had no capital constraint, these price functions $p(e)$ and $q(e)$ would be flat lines. The crisis-states, even though unlikely, affect equilibrium across the entire state space. This result is similar to results from the rare-disasters literature (Rietz, 1988, Barro, 2006).

The first panel in the bottom row graphs the investment policy function. Since investment is driven by $q(e)$, investment also falls before the intermediary sector is constrained. In the next section, we present simulated moments from the model in states where $e < 2.14$ to states where $e > 2.14$ (we label these "distress" and "non-distress" periods). The distress events in the simulation are predominantly ones where the capital constraint is not binding. Yet the effect of the capital constraint is present through the anticipated effects. We compare these conditional moments to ones from U.S. data to gauge how well our model captures the effects of a potential crisis. As noted earlier, this is our approach to calibrating a model of a financial crisis based on data when there is only one realization of a financial crisis.

Comparing the first two panels for $p(e)$ and $q(e)$, the main difference is that the range of variation for q is considerably smaller than that for p. This is because housing is in fixed supply while physical capital is subject to adjustment costs. With the $\kappa = 3$ parameterization, the adjustment costs are sufficiently small that capital prices do not vary much. It may be possible to arrive at higher volatility in q if we consider higher adjustment costs or flow adjustment costs as in Gertler and Kiyotaki (2010). As noted earlier, q will also vary more if we allowed for shocks to A instead of directly shocking K_t.[11] The graph illustrates that the aggregate asset price volatility in the economy is substantially driven by housing volatility. The middle and right panel of the second row are for return volatility of q and p. Housing volatility is much higher than q volatility. Note also that the actual price of housing is equal to p times K, and since K is also volatile, housing prices are more volatile than just p.

The second panel in the bottom row graphs the consumption policy function. Investment-to-capital falls as q falls. The resource constraint implies that $C/K + \Phi(I, K)/K = A$. Thus, consumption-to-capital rises as the constraint becomes tighter. Note that aggregate consumption depends on this policy function and the dynamics of capital. In the constrained region, capital falls so that while C/K rises, K falls, and the net effect on aggregate consumption depends on parameters. For our baseline parameters, consumption growth in the non-distress region averages 0.08% while it is -0.19% in the distress region.

Figure 5 plots the baseline plus a variation with higher sigma ($\sigma = 4.5\%$). The results are intuitive. With higher exogenous volatility, Sharpe ratios, return volatility and risk premia are higher (the Sharpe ratio rises in the unconstrained region from 0.37 to 0.44, but given the range of variation in the Sharpe ratio, it hard to make this out in the figure). Thus asset prices and investment are lower.

5.2 Model Nonlinearity

An important feature of the model, apparent in the figures, is its nonlinearity. A reduction to intermediary equity, conditional on a low current value of intermediary equity, has a larger effect on the economy than the same size shock, conditional on a high value of intermediary equity. Figure 6 illustrates this feature. We study the effect of -2% shock in σdZ_t, so that the fundamental shock leads capital to fall exogenously by 2%. We consider the effect of this shock in a "crisis" state ($e = 0.44$, which is the boundary of the constrained region) and a "normal" state ($e = 20.44$). We trace out the effect on investment (first panel), the Sharpe ratio (second panel), and the price of land (third panel). Because the stochastic economy of the model is always subject to shocks, these impulse response functions are slightly different relative to usual impulse response functions.[12] First, we compute the benchmark path of these variables without any shocks (but still subject to the endogenous drift of the state variable in our model). Second, we compute the "shocked"

[11] In investigating the model, we have also found that increasing the intertemporal elasticity of substitution (IES) for the household increases the range of variation of q. This appears to be through an effect on the interest rate. In the current calibration, interest rates fall dramatically the constrained region, which through a discount rate effect supports the value of q. Dampening this effect by increasing the IES increase the range of variation of q. This observation also suggests to us that introducing nominal frictions that bound the interest rate from falling below zero will increase the range of variation of q.

[12] For more on the difference between impulse responses in non-linear models with a stochastic steady state and those in linear models with a non-stochastic steady state see Koop, Pesaran, and Potter (1996).

path of these variables given this initial shock but no further shocks (i.e. $dZ_t = 0$ after the initial shock). We then calculate the (log) difference between the path with the shock and the benchmark path without any shocks. Therefore, the effect illustrated in Figure 6 should be thought of as the marginal effect of the shock on the mean path for the variables plotted.

The solid line in the first panel indicates that investment falls by a little over 0.02 on incidence of the shock, and this effect continues out to 8 quarters. The effect is a little larger than 0.02 because there is some amplification in the model. The effect does not revert back to zero because K_t is permanently lower by about 2% and our economy scales with K_t. The dashed line is the effect of the same shock in the crisis state. Now investment falls by 2.85%, with the effect dying out over three quarters. The second panel shows that the Sharpe ratio is completely unaffected by the shock in the normal region, while it rises by 50% (roughly doubling) in the crisis region. The effect also dies out after three quarters. The last panel plots the price of land. Land prices fall by a little over 4% in the normal region indicating some amplification even when the constraint is not active, while it falls by 15% in the crisis region indicating the significant amplification in the crisis region.

6 Matching Nonlinearity in Data

Guided by the nonlinearity present in the model, we first ask if such nonlinearity is present in historical data, and second, we ask how well our model can quantitatively match the nonlinearity in the data.

6.1 Data

We compute covariances in growth rates of intermediary equity, investment, consumption, the price of land, as well as the level of a credit risk spread, using quarterly data from 1973Q1 to 2010Q4 (except for the land price where our series begins 1975Q1). We sample the data quarterly but compute annual log changes in the series. We focus on annual growth rates because there are slow adjustment mechanisms in practice (e.g., flow adjustment costs to investment) that our model abstracts from. We thus sample at a frequency where these adjustment mechanisms play out fully. The intermediary equity measure is the sum across all financial firms (banks, broker-dealers, insurance and real estate) of their stock price times the number of shares from the CRSP database.[13,14] The consumption and investment data are from NIPA. Consumption is non-housing services and nondurable goods. Investment is business investment in soft-

[13]Muir (2011) shows that this measure is useful for predicting aggregate stock returns as well as economic activity. Moreover, intermediary equity is a priced factor in the cross-section of stock returns.

[14]We have also considered an alternative equity measure based only on banks and broker-dealers and the results are quite similar to the ones we report.

ware, equipment, structures, and residential investment. We have also considered an investment category that includes durable goods, since such purchases are likely to be credit sensitive and hence affected by the intermediary frictions we study. This broader investment measure has lower volatility, but higher covariance with intermediary equity. Land price data is from the Lincoln Institute (http://www.lincolninst.edu/subcenters/land-values/price-and-quantity.asp), where we use *LAND_PI* series based on Case-Shiller-Weiss. These measures are expressed in per-capita terms and adjusted for inflation using the GDP deflator. The credit risk spread is drawn from Gilchrist and Zakrajsek (2010). There is a large literature showing that credit spreads (e.g., the commercial paper to Treasury bill spread) are a leading indicator for economic activity (see Philippon (2010) for a recent contribution). Credit spreads have two components: expected default and an economic risk premium that lenders charge for bearing default risk. In an important recent paper, Gilchrist and Zakrajsek (2010) show that the spread's forecasting power stems primarily from variation in the risk premium component (the "excess bond premium"). The authors also show that the risk premium is closely related to measures of financial intermediary health. Our model has predictions for the link between intermediary equity and the risk premium demanded by intermediaries, while being silent on default.[15] We convert the Gilchrist and Zakrajsek's risk premium into a Sharpe ratio by scaling by the risk of bond returns, as the Sharpe ratio is the natural measure of risk-bearing capacity in our model.[16] The Sharpe ratio is labeled EB in the table.

6.2 Conditional Moments

Table 2 presents covariances depending on whether or not the economy is in a "distress" period. (Annual growth rates are centered around the quarter classified as distress). Table 3 lists the distress classification. Ideally, we would like to split the data based on observations of $\mathcal{E}_t$. However, $\mathcal{E}_t$ is not directly observable in data. Instead, the model suggests that there is a one-to-one link between the Sharpe ratio and $\mathcal{E}_t$. Thus, we consider as distress periods the highest one-third of realizations of the EB Sharpe ratio, but requiring that the distress or non-distress periods span at least two contiguous quarters. In choosing the distress/non-distress classification, we face the tradeoff that if we raise cutoff to define distress (say, worst 10% of observations), then the data is more reflective of the crisis effects suggested by the model but we have too little data on which

[15]There is no default in the equilibrium of the model. Of course, one can easily price a defaultable corporate bond given the intermediary pricing kernel, where default is chosen to match observables such as the correlation with output. We do not view having default in the equilibrium of the model as a drawback of our approach.

[16] Suppose that the yield on a corporate bond is y^c, the yield on the riskless bond is y^r and the default rate on the bond is $E[d]$. The expected return on the bond is $y^c - y^r - E[d]$, which is the counterpart to the excess bond premium of Gilchrist and Zakrajsek (2010). To compute the Sharpe ratio on this investment, we need to divide by the riskiness of the corporate bond investment. Plausibly, the risk is proportional to $E[d]$ (for example, if default is modeled as the realization of Poisson process, this approximation is exact). Thus the ratio $\frac{y^c - y^r - E[d]}{E[d]}$ is proportional to the Sharpe ratio on the investment, and this is how we construct the Sharpe ratio.

to compute meaningful statistics. After experimenting with the data, we have settled on the one-third/two-thirds split.

The distress periods roughly correspond to NBER recession dates, with one exception. We classify distress periods in 1985Q4-1987Q3, 1988Q4-1990Q1, and again 1992Q3-1993Q2. The NBER recession over this period is in 1990 to 1991. The S&L crisis and falling real estate prices in the late 80s put pressure on banks which appears to result in a high EB and hence leads us to classify these other periods as distress.

Table 2: Covariances in Data

The table presents standard deviations and covariances for intermediary equity growth (Eq), investment growth (I), consumption growth (C), land price growth (PL), and Sharpe ratio (EB). Suppose quarter t is classified as a distress quarter. We compute growth rates as annual changes in log value from $t-2$ to $t+2$. The Sharpe ratio is the value at t. The first column is using the distress classification of Table 1. The second uses NBER recession dates, from Table 1. The third uses these recession dates, plus two adjoining quarters at the start and end of the recession. The last is based on the distress dates from Table 1 but drops the last period (the recent crisis).

	EB	NBER Recession	NBER+/-2Qs	EB, Drop Crisis
Panel A: Distress Periods				
vol(Eq)	31.48	32.40	31.78	22.19
vol(I)	8.05	8.79	7.44	4.56
vol(C)	1.71	1.54	1.59	0.95
vol(PL)	21.24	23.34	21.07	7.91
vol(EB)	60.14	93.59	74.57	28.69
cov(Eq, I)	1.31	1.08	0.84	0.37
cov(Eq, C)	0.25	0.16	0.13	0.04
cov(Eq, PL)	4.06	5.61	4.39	-0.63
cov(Eq, EB)	-6.81	-10.89	-7.57	-2.12
Panel B: Non-distress Periods				
vol(Eq)	17.54	19.42	17.11	17.26
vol(I)	6.61	5.97	4.91	6.60
vol(C)	1.28	0.98	0.91	1.28
vol(PL)	9.79	10.00	8.46	9.34
vol(EB)	12.72	30.93	30.42	12.78
cov(Eq, I)	0.07	0.09	-0.06	0.03
cov(Eq, C)	0.03	0.01	0.01	0.03
cov(Eq, PL)	0.12	0.07	-0.31	-0.01
cov(Eq, EB)	-0.14	-0.81	-0.78	-0.19

Table 2 shows that there is an asymmetry in the covariances across the distress and non-distress periods, qualitatively consistent with the model. There is almost no relation between equity and the other variables in the non-distress periods, while the variables are closely related in the distress periods. Volatilities are much higher in the distress periods than the non-distress periods. The table also presents results for alternative classifications of the distress periods. All of the classifications display the pattern of asymmetry so that our results are not driven by an arbitrary

Table 3: Distress Classification

Distress Periods	NBER Recessions
1974Q3 - 1975Q4	11/73 - 3/75
1982Q3 - 1982Q4	7/81 - 11/82
1985Q4 - 1987Q3	
1988Q4 - 1990Q1	
	7/90 - 3/91
1992Q4 - 1993Q2	
2001Q2 - 2003Q1	3/01 - 11/01
2007Q3 - 2009Q3	12/07 - 6/09

classification of distress. The only column that looks different is the last one where we drop the recent crisis. For this case, most of the covariances in the distress period drop in half, as one would expect. In addition, the land price volatility drops substantially while the covariance goes to zero. This is because it is only the recent crisis which involve losses on real estate investments and financial intermediaries.

6.3 Simulated Conditional Moments

We compare the results from simulating the model to quarterly data from 1973 to 2010, as presented in Table 2. When simulating the model we follow the one-third/two-thirds procedure as when computing moments in historical data and label distress as the worst one-third of the sample realizations. Importantly therefore our definitions are consistent and comparable across both model and data. From Figure 4, points to the left of 2.1 are classified as distressed.

We simulate the model, quarterly, for 2000 years. To minimize the impact of the initial condition, we first simulate the economy for 2000 years, and then record data from the economy for the next 2000 years. We then compute sample moments and the probability of distress region accordingly. We run the simulation 5000 times and report the sample average.

Table 4 provides numbers from the data and the simulation. When reading these numbers it is important to keep in mind that our calibration targets are neutral and we have not explicitly targeted the asymmetry across distress and non-distress periods. Thus one criterion for the success of our work is whether the non-linearity imposed by the theoretical structural of the model can match the asymmetry in the data.

In the data, the covariance between equity and investment is 1.31% in distress and 0.07% in non-distress. In the simulation, these numbers are 0.9% and 0.3%. The model also comes close to matching the asymmetry in land price volatility and covariance with land prices and equity. In the data, the volatility numbers are 21.24% and 9.79%; while the corresponding land price volatilities from the model are 22.1% and 9.8%. Recall that our parameters (particularly β) are chosen to

Table 4: Model Simulation and Data

The table presents standard deviations and covariances for intermediary equity growth (Eq), investment growth (I), consumption growth (C), land price growth (PL), and Sharpe ratio (EB). Growth rates are computed as annual changes in log value from t to $t+1$. The Sharpe ratio is the value at $t+1$. The column labeled data are the statistics for the period 1973 to 2010 (except for land prices, where our series begins in 1975). The Sharpe ratio is constructed from the excess bond premium, and other variables are standard and defined in the text. The next four columns are from the model, reflecting different parameter choices. Numbers are presented conditional on being in the distress period or non-distress period. For the data, the classification of the periods follows Table 1. For the model simulation, the distress period is defined as the 33% worst realizations of the Sharpe ratio.

	Data	Baseline	$\sigma = 4.5\%$	$\phi = 0$	$m = 1.8$	$\lambda = 0.5$
Panel A: Distress Periods						
vol(Eq)	31.48%	31.2	34.8	23.3	25.4	19.2
vol(I)	8.05%	5.4	6.2	4.7	4.9	4.8
vol(C)	1.71%	1.8	2.1	2.6	2.1	2.1
vol(PL)	21.24%	22.1	28.8		11.2	14.7
vol(EB)	60.14%	71.1	83.0	31.3	46.2	41.2
cov(Eq, I)	1.31%	0.9	1.1	0.7	0.7	0.5
cov(Eq, C)	0.25%	0.0	-0.1	0.1	0.1	0.1
cov(Eq, PL)	4.06%	5.6	8.4		2.0	2.4
cov(Eq, EB)	-6.81%	-13.0	-17.6	-2.7	-5.9	-3.9
Panel B: Non-distress Periods						
vol(Eq)	17.54%	6.4	7.4	4.0	5.1	5.2
vol(I)	6.61%	4.8	5.7	4.3	4.4	4.3
vol(C)	1.28%	2.4	2.5	3.8	3.1	3.1
vol(PL)	9.79%	9.8	12.1		6.3	6.5
vol(EB)	12.72%	8.7	10.5	0.2	4.4	3.7
cov(Eq, I)	0.07%	0.3	0.4	0.2	0.2	0.2
cov(Eq, C)	0.03%	0.1	0.2	0.2	0.2	0.2
cov(Eq, PL)	0.12%	0.6	0.9		0.3	0.3
cov(Eq, EB)	-0.14%	-0.2	-0.3	0.0	0.0	0.0

match the unconditional volatility of 14.47% in the data. Therefore, matching the asymmetry across distress and non-distress periods should be considered as a success of the model. The land-equity covariances in the data are 4.06% and 0.12%; while in the model, they are 5.6% and 0.6%. The model is also quite close in matching the asymmetry patterns in the Sharpe ratio, although asymmetry in the covariance with intermediary equity is too high in the model.

The model misses substantially in a few dimensions. First equity volatility is too low relative to the data in non-distress periods. Part of the explanation here is that there are likely shocks in the non-distress periods that our one-shock model is missing. The model also produces investment volatility that is uniformly too low while being close on consumption volatility. A significant part of this is because the total output volatility of the economy is roughly constant at σ so that matching consumption volatility necessarily means missing on investment volatility. This feature of the economy also drives the result that consumption volatility in the non-distress period is higher than that in the distress periods, contrary to the data. This happens in the model because if investment becomes more volatile, holding output volatility fixed, consumption must become less volatile. It seems clear that more work needs to be done in order to better match both investment and consumption dynamics. For example, introducing endogenous labor supply can lead output volatility to differ significantly from σ.

The last four columns in the table consider variations where in different ways we change the volatility of the economy. In each of these variations, we ensure that the mean of the steady-state distribution of the state variable is the same as in the baseline. We do this by altering η. Thus, the variations should be thought of as delivering a mean-preserving spread around the baseline.

The variation with σ raised to 4.5% from 4% increases the volatility of most variables considerably. The increase is larger in the distress period than the non-distress period which should be expected given the non-linearity in the model. An interesting point from this case is that the volatility of investment rises more than the volatility of consumption. This comparison makes clear that the main effect of the constraint we have introduced is on investment. Increasing σ raises the effects of the non-linear constraint and particularly affects investment.

The variation with $\phi = 0$ is interesting in that it reveals the workings of the model. When $\phi = 0$, land drops out of the model (we introduce curvature in household preferences to keep the EIS of the household equal to 2, so that the Euler equation determining interest rates is not altered by changing ϕ). From Figure 6 note that land price volatility rises in the constrained region while the volatility of q remains roughly constant. Thus, when land is removed from the economy, the volatility of intermediary equity in the constrained region falls from 31.2% to 23.3%. The intermediary pricing kernel is far less volatile which in turn greatly reduces the non-linearity in the model. Because land is in fixed supply, reduced demand for assets in the constrained region causes land prices to fall. Physical capital is subject to adjustment costs so that reduced demand both reduces quantity and price. This distinction is what drives the high volatility of land relative to physical capital in our baseline. And eliminating land thereby reduces the non-linear effects produced by the model.

The variation where we reduce m to 1.8 effectively reduces volatility in most parameters. Here again we see a different effect on investment and consumption, as consumption volatility rises while investment volatility falls. Reducing m reduces the strength of the intermediation friction which explains these effects.

The last column in the table considers a variation with a lower λ. Reducing the leverage of intermediaries in the unconstrained region reduces the effect amount of risk borne by intermediary equity and thus reduces risk premia and the intermediation effects of our model. That is, this variation is qualitatively similar to the effect of reducing m.

7 Systemic Risk

Our analysis in the previous section focused on distress periods which are ones where the capital constraint does not bind, but where the anticipation of the constraint binding in the future affects asset prices and decisions. These events in the model and the data are akin to a recession rather than a systemic crisis. We now turn our attention to the crisis in our model, which we define as the states where the capital constraint binds. We compute probabilities that the economy can transit into these states, thus measuring "systemic risk."

7.1 Measures in Systemic States

Table 5 reports the values of prices and policies in a given state, focusing in on a systemic event when the intermediary capital constraints binds. At the mean Sharpe ratio of the model the capital constraint does not bind. Column (1) provides numbers for the mean Sharpe ratio, the unconditional probability that the Sharpe ratio will exceed the mean, the ratio of intermediary equity to capital, housing values, q, the investment rate, interest rate and consumption growth. These latter numbers are computed at the state with the mean Sharpe ratio. Columns (2), (3), and (4) report the same measures at states with higher Sharpe ratios. The equity capital constraint binds for $E/K < 0.44$, so that the constraint binds at each of these higher values. The numbers illustrate how housing prices and investment fall non-linearly as the constraint tightens. The fall in investment is driven by the fall in q. Quantitatively, our model produces a "credit crunch" that reduces q and investment from 10.20% of K at the mean of the distribution to 9.32% of K in an extreme crisis state.

The last two rows give the interest rate and consumption growth. The real interest rate is negative in the systemic states. If our model had a monetary side, the analysis could bring in zero-lower-bound considerations which have been the subject of a large literature recently (see, e.g., Christiano, Eichenbaum and Rebelo, 2010). The interest rate is negative largely because expected consumption growth is negative in the model's crisis.

7.2 Simulation of 2007-2009 Crisis

We next use our model to attempt to replicate the crisis of 2008, as reflected in Figure 2. To do so, we need to pick an initial condition in terms of e and a sequence of shocks that can reflect events in 2007-2009.

We assume that the economy in 2007Q2 is at $e = 2.14$ which is the threshold we have used earlier in classifying distress and non-distress states. In our classification from Table 1, 2007Q3 is the first quarter of the recent crisis that is above the distress threshold. Recall that the distress/non-

Table 5: Systemic States

The table compares values of asset prices and macroeconomic aggregates at different points in a crisis, indexed by the Sharpe ratio. The first column of data are the numbers at the mean Sharpe ratio. The rest of the columns present data at a given multiple of that mean Sharpe ratio.

	(1)	(2)	(3)	(4)
	At mean e	X4	X8	X16
Sharpe ratio	0.37	1.48	2.97	5.94
Prob Sharpe being higher	69.74%	0.49	0.15	0.05
Equity (E/K)	0.56	0.29	0.18	0.09
Housing (P/K)	0.69	0.29	0.25	0.22
Capital (q)	1.01	0.98	0.98	0.98
Investment (I/K)	10.20%	9.41	9.35	9.32
Interest rate	3.17%	-0.75	-6.03	-14.12
Consumption growth	0.38%	-7.50	-18.06	-34.18

distress classification is based on the behavior of the Gilchrist-Zakrajsek EB Sharpe Ratio. We classify the highest 33% of the realizations of this ratio as the distress regions. Thus we are using information from the EB to pin down the initial condition.

Starting from the $e = 2.14$ state, we impose a sequence of exogenous quarterly shocks, $\sigma(Z_{t+0.25} - Z_t)$ to the capital dynamics equation (2). These shocks are in units of percentage change in capital. From 2007Q3 to 2009Q4, these shocks are $(-3.1\%, -5.5\%, -3.0\%, -1.4\%, -0.8\%, -2.2\%, -2.3\%, -2.2\%, -1.0\%, -1.0\%)$ which totals about -19% (geometric sum). We compute the values of all endogenous variables, intermediary equity, land prices, investment, and the Sharpe ratio, after each shock. The shocks are chosen so that the endogenous model value of intermediary equity matches the data for intermediary equity from the crisis as reflected in Figure 2.

By matching the intermediary equity data, our model focuses on shocks in the world that most directly affect the intermediary sector. Note also that a given shock, say the -3.1% first shock does not only reflect losses by banks for 2007Q3, but also reflects losses anticipated by investors over the future, which is then impounded in the current market value of equity. That is, as the world is evolving over 2007Q3 to 2009Q4, investors receive information that cause them to anticipate losses to the intermediary sector which then immediately reduces the market value of the intermediary sector. We pick the shock in a given quarter to match the reduction in the market value of the intermediary equity over that quarter.

Figure 3 plots the values of the endogenous variables from the model simulation at each quarter (all variables are normalized to one in 2007Q2). The exogenous shocks total 19% while intermediary equity and land prices fall by about 70% in the trough of the model. Thus, there is clearly an amplification of shocks. The equity capital constraint comes to bind after the first three shocks, totaling 11.2%, and corresponding to 2008Q2. The Sharpe ratio rises dramatically after 2008Q1.

Also, note that from that point on, the shocks are smaller but the response of the endogenous variables is larger, reflecting the non-linearity of the model.

Figure 3 should be compared to Figure 2. It is apparent that the model can replicate the crisis with a sequence of shocks that can plausibly reflect current and anticipated losses on bank mortgage investments. In addition, the analysis shows that focusing only on shocks to intermediary equity results in an equilibrium that matches the behavior of aggregate investment, the Sharpe ratio, and land prices. This result suggests that an intermediary-capital-based mechanism, as in our model, can be a successful explanation for the macroeconomic patterns from 2007 to 2009.

7.3 Probability of Systemic Crisis and Leverage

We use our model to compute the probability of falling into a systemic crisis. Consider first the sequence of shocks as in Figure 3 that leads the capital constraint to bind in 2008Q1. We ask, what is the probability of the capital constraint binding any time over the next T years, given the initial condition of being on the distress boundary ($e_{distress} = 2.14$). These probabilities are 0.12% for 1 year, 1.12% for 2 years, 9.12% for 5 years, and 20.73% for 10 years. These results suggest that the crisis even in early 2007 is unlikely over the next 2 years. Indeed, a number of financial market measures (e.g., spreads, VIX) were near normal levels prior to the summer of 2007 and offered little advance warning of the crisis that followed. That is, without the benefit of hindsight, in both the model and data the probability of the 2007-2009 crisis is low.

However, as many observers have pointed out, it is clear with the benefit of hindsight that there was a great deal of leverage "hidden" in the system. For example, many were unaware of the size of the structured investment vehicles (SIVs) that commercial banks had sponsored and the extent to which these assets were a call on bank's liquidity and capital. As Acharya, Schnabl and Suarez (2010) have documented, much of the assets in SIVs came back onto bank balance sheets causing their leverage to rise. Likewise, hedge funds and broker/dealers were carrying high leverage in the repo market, but this was not apparent to observers given the opacity of the repo market. As Gorton and Metrick (2011) have argued, this high leverage was a significant factor in the crisis. However, in early 2007, this hidden leverage was not apparent to financial markets, and is perhaps one reason why financial market indicators did not signal a crisis.

In our baseline calibration, financial sector leverage is 3. We consider a simple comparative static to see how accounting for the hidden leverage in the system may change the probability of the crisis. Recall that the return on equity produced by an intermediary is,

$$d\tilde{R}_t = \alpha_t^k dR_t^k + \alpha_t^h dR_t^h + (1 - \alpha_t^k - \alpha_t^h) r_t dt.$$

where,

$$\alpha_t^k = \frac{1}{1-\lambda} \frac{q_t K_t}{W_t} \quad \text{and,} \quad \alpha_t^k = \frac{1}{1-\lambda} \frac{P_t}{W_t},$$

when the capital constraint does not bind. The leverage parameter, λ, enters by affecting the αs and thus the exposure of intermediary equity to returns on housing and capital.

We replace $\lambda = 0.67$ with $\widehat{\lambda} = 0.77$ in these expressions. This increases the leverage of the intermediaries, causing the αs to rise. We assume however that this increase in leverage is "hidden," in the sense that agents continue to make decisions assuming that $\lambda = 0.67$ so that the equilibrium decisions rules, prices, and returns correspond to the baseline calibration. But when returns are realized, the hidden leverage leads to a larger than expected effect (i.e. $\widehat{\lambda} = 0.77$) on the return to intermediary equity. Thus our experiment is trying to hold fixed agents decisions rules and equilibrium prices and returns, and only allowing these returns to have a levered effect on the dynamics of intermediary equity. With the higher leverage, one can expect that shocks will be amplified and thus the crisis state will be more likely. We compute exactly how much more likely by simulating the model. The appendix describes the simulation procedure in detail.

With $\widehat{\lambda} = 0.77$, the effective intermediary leverage in the constrained region rises from 3 to 4.5 ($= 1/(1 - 0.77)$). While this leverage may still not appear large, consider that we are applying this to the broad intermediary sector encompassing banks, hedge funds, and private equity funds. The probability of the crisis over the next year rises from 0.12% to 35%, while for 2 years it rises from 1.12% to 60%, and for 5 years it rises from 9.12% to 85%. This computation suggests how stressing the financial system, because of the non-linearity in the model, can have a large impact on crisis probabilities. However, this computation is subject to an important caveat: the rise in leverage is out-of-equilibrium and one should expect that if all agents understand that leverage is higher and the economy as a result is more volatile, then they endogenously will reduce their risk-taking and this will also change the probability of the crisis.[17]

7.4 Stress Tests

The fact that financial market indicators offered a poor signal of the crisis has led regulators to emphasize "stress testing" as a tool to uncover vulnerabilities in the financial system. That is, if a stress test reveals that a number of financial intermediaries will face a capital shortfall in a given scenario, then the likelihood of a crisis should be higher. But to quantify this effect, one needs a model. This section explains how to use our model to compute crisis probabilities from stress test information.

Central bank stress tests map a scenario into a loss to equity holders. For example, a stress-test

[17] We can recompute the entire equilibrium assuming that $\lambda = 0.77$. In this case, the higher leverage implies a higher Sharpe ratio and lower average asset values. The higher average Sharpe ratio leads intermediaries on average to deliver higher return on equity which in turn means that the steady state distribution of the economy shifts in a manner that places less weight on crisis states. In addition, because asset values are on average lower, the transition to the crisis is not as sharp. For our purposes, this higher average leverage case is also not of interest. What we would ideally like to compute is a model in which leverage is thought to be low prior to 2007, and then is suddenly seen to be much higher. The hidden leverage computation is a crude representation of this case.

may assess how much equity capital a given bank will lose in the event that loss rates on mortgage loans double. To use our model we need to convert from the equity loss to a dZ_t shock. Consider the following example to illustrate how this may work. Suppose that the stress test reveals that in a given scenario the return on equity of financial firms is -30%. Note that the stress test by its nature is a partial equilibrium experiment as it does not account for the endogenous feedback of the stress scenario on the macroeconomy. We can likewise do a partial equilibrium experiment in our model and ask what shock is needed to produce a -30% return on equity. The answer is a shock of $\sigma dZ_t = -10\%$. This shock will lead K_t to fall by 10%, causing cash flows on capital to fall by 10% and the rental income on housing to fall by 10%. Since leverage in the model is 3, this -10% loss on assets leads to a -30% loss on equity. We can feed this shock into our model over a quarter, $\sigma(Z_{t+0.25} - Z_t) = -10\%$, given some initial condition. The shock will trigger an endogenous feedback that causes P_t and q_t to fall further. The fall in q_t will further reduce K_t over time as investment is reduced, and all of this will lead the return on equity to exceed -30%.[18] We can compute in our model the fixed point of this feedback to evaluate the effect of the shock on e. We find in this case that starting from the 2007Q2 initial condition, the -10% shock causes e to fall from 2.14 to below 0.44. That is, such a shock will immediately trigger the systemic states!

The model allows further flexibility. In practice, the stress test scenarios considered by the Fed were over six quarters rather than over one quarter. In terms of the model, we can consider simulating a series of negative shocks over six quarters in terms of six changes in $\sigma(Z_{t+0.25} - Z_t)$. Over the longer horizon, bank profits will buffer the effect of the negative shock. Table 6 below gives the probability of a crisis within the next 2 years for a number of different six-quarter scenarios. The scenarios are chosen based on feeding in shocks equally over six quarters to match a given return on equity (left hand column). As one can see, even in the six quarter case, the -30% return on equity leads to a crisis.

Table 6: Probability of Crisis and Return on Equity

The table gives the probability of a crisis within the next 2 years for a number of different six-quarter scenarios. The scenarios are chosen based on feeding in shocks over six quarters to match a given return on equity (left hand column).

Return on Equity	6 QTR Shocks	Prob(Crisis within 2 years)
-2%	-1.52%	1.53 %
-5	-3.11	2.80
-10	-5.67	7.37
-20	-10.41	36.78
-30	-13.06	100.00

[18] Brunnermeier, Gorton and Krishnamurthy (2011) argue that it is important to account for the endogenous feedback of the economy to the stress scenario because crises are non-linear. Our model offers a clear illustration of their point.

Our analysis in this section should be viewed as illustrative. We can consider other shocks, scenarios, and initial conditions. The model can be used by mapping these scenarios into the dynamics of the state variable e_t which is the key to understanding crisis probabilities. The right panel of Figure 7 plots the probability of the crisis over the next 2, 5, and 10 years, given an initial condition of e. The left panel plots the probability of falling into the distress region from an initial condition of e.

8 Conclusion

We presented a fully stochastic model of a systemic crisis in which the main friction is an equity capital constraint on the intermediary sector. We first showed that the model offers a good quantitative representation of the U.S. economy. In particular, the model is able to replicate behavior in non-distress periods, distress periods, and extreme systemic crisis, quantitatively matching the nonlinearities that distinguish patterns across these states. We then used the model to evaluate and quantify systemic risk, defined as the probability of reaching a state where capital constraints bind across the financial sector. We showed how the model can be used to evaluate the macroeconomic impact of a stress scenario on the systemic risk probability.

References

Acharya, V., L. Pedersen, T. Philippon, and M. Richardson, 2010, "Measuring Systemic Risk," NYU Working Paper

Acharya, V., P. Schnabl, and G. Suarez, 2012, "Securitization without Risk Transfer," *Journal of Financial Economics*, forthcoming.

Adrian, Tobias and Boyarchenko, Nina, 2012, "Intermediary Leverage Cycles and Financial Stability," FRB of New York Staff Report No. 56.

Adrian, T. and H. Shin (2010), Liquidity and leverage. *Journal of Financial Intermediation*, 19, p.418-437.

Adrian, Tobias and Markus Brunnermeier, 2008, "CoVaR," working paper, Federal Reserve Bank of New York.

Ashcraft, Adam, Nicolae Garleanu and Lasse Pedersen, 2010, "Two Monetary Tools: Interest Rates and Haircuts," NBER Macroannual 2010.

Berk, J. and R. Green, 2004, "Mutual Fund Flows and Performance in Rational Markets", *Journal of Political Economy* 112, 1269–1295.

Bernanke, Ben, 2009, Financial Reform to Address Systemic Risk, Speech at the Council on Foreign Relations.

Bernanke, Ben, Mark Gertler, and Simon Gilchrist, 1999, "The Financial Accelerator in a Quantitative Business Cycle Framework," *Handbook of Macroeconomics*, John Taylor and Michael Woodford editors.

Barro, Robert, 2006, "Rare disasters and asset markets in the twentieth century," *The Quarterly Journal of Economics,* 121(3), pp. 823-866.

Bigio, Saki, Financial Risk Capacity, working paper, NYU.

Billio, M., M. Getmansky, A. Lo, and L. Pelizzon, 2010, "Measuring Systemic Risk in the Finance and Insurance Sectors," MIT Working Paper.

Brunnermeier, Markus and Yuliy Sannikov, 2010, "A Macroeconomic Model with a Financial Sector," mimeo, Princeton University.

Brunnermeier, Markus, Arvind Krishnamurthy, and Gary Gorton, 2011, "Risk Topography," in Acemoglu, D., and Woodford, M. (Eds.), *NBER Macro Annual 2011*.

Chevalier, J., and G. Ellison, 1997, "Risk Taking by Mutual Funds as a Response to Incentives," *Journal of Political Economy* 105, 1167–1200.

Christiano, Larry, Martin Eichenbaum, and Sergio Rebelo, 2010, "When is the Government Spending Multiplier Large?", *Journal of Political Economy*.

Dewachter, Hans and Raf Wouters, 2012, "Endogenous risk in a DSGE model with capital-constrained Financial intermediariees," NBB working paper.

Gertler, Mark, and Kiyotaki, Nobuhiro, 2010, "Financial Intermediation and Credit Policy in Business Cycle Analysis." In Friedman, B., and Woodford, M. (Eds.), *Handbook of Monetary Economics*. Elsevier, Amsterdam, Netherlands.

Getmansky, Mila, 2012, The Life Cycle of Hedge Funds: Fund Flows, Size, Competition, and Performance *Quarterly Journal of Financial Economics*

Kaplan, S. and A. Schoar, 2005, "Private Equity Performance: Returns, Persistence and Capital Flows", *The Journal of Finance* 60, 1791–1823.

Kiyotaki, Nobuhiro and John Moore, 1997, "Credit Cycles," *Journal of Political Economy*.

Giglio, S., 2011, "Credit Default Swap Spreads and Systemic Financial Risk," working paper, University of Chicago.

Gorton, Gary and Andrew Metrick, 2012, "Securitized Banking and the Run on Repo," *Journal of Financial Economics*, forthcoming.

Gray, Dale F., Robert C. Merton, and Zvi Bodie, 2008, "New Framework for Measuring and Managing Macrofinancial Risk and Financial Stability," Working Paper, Harvard Business School.

Gilchrist, Simon, and Egon Zakrajsek, 2010, "Credit Spreads and Business Cycle Fluctuations," *American Economic Review*.

Hartmann, P, S. Straetmans and C. de Vries, 2005, "Banking System Stability: A Cross- Atlantic Perspective"

He, Zhiguo and Arvind Krishnamurthy, 2012a. "A Model of Capital and Crises," *Review of Economic Studies* 79(2): pp. 735-777.

He, Zhiguo and Arvind Krishnamurthy, 2012b. "Intermediary Asset Pricing," *American Economic Review*, forthcoming.

He, Zhiguo and Peter Kondor, 2012. "Inefficient Investment Waves," Working paper, University of Chicago.

Holmstrom, Bengt and Jean Tirole, 1997, "Financial Intermediation, Loanable Funds and the Real Sector," *Quarterly Journal of Economics*.

Huang, Xin, Hao Zhou, and Haibin Zhu, 2009, "A Framework for Assessing the Systemic Risk of Major Financial Institutions," *Journal of Banking and Finance*, 33(11) 2036 - 2049

Kiley, Michael, and Jae Sim, 2011, Financial Capital and the Macroeconomy: A Quantitative Framework, Federal Reserve Board Working Paper 2011-27.

Koop, G., Pesaran, H., Potter, S., 1996. Impulse Response Analysis in Nonlinear Multivariate Models. *Journal of Econometrics* 74, 119-148.

Lui, Zheng, Pengfei Wang and Tao Zha, 2011, "Land-Price Dynamics and Macroeconomic Fluctuations," NBER working paper #17045.

Lucas, Robert E, 1990, "Liquidity and Interest Rates," *Journal of Economic Theory.*

Mendoza, Enrique, 2010, "Sudden Stops, Financial Crises and Leverage," *American Economic Review*

Myerson, Roger, 2012, "A Model of Moral-Hazard Credit Cycles," *Journal of Political Economy*, forthcoming.

Muir, Tyler, 2012, "Financial Crises, Risk Premia, and the Term Structure of Risky Assets," working paper, Northwestern University.

Piazzesi, Monika, Martin Schneider and Selale Tuzel, 2007, "Housing, Consumption, and Asset Pricing", *Journal of Financial Economics* 83, pp. 531-569.

Philippon, Thomas, 2010, "The Bond Market's Q," *Quarterly Journal of Economics*.

Rampini, Adriano and S. Viswanathan, "Financial Intermediary Capital," working paper, Duke University.

Rietz, Thomas, 1988, "The equity risk premium: A solution," *Journal of Monetary Economics,* 2(1):117-131.

Shleifer, A. and R. Vishny, 1997, "The Limits of Arbitrage,' *Journal of Finance*, 52, 35-55.

Vayanos, Dimitris, and Paul Woolley, 2012, An Institutional Theory of Momentum and Reversal, working paper, London School of Economics.

Warther, Vincent, 1995, Aggregate mutual fund flows and security returns, *Journal of Financial Economics* 39, 209-236

A Derivation of ODE

A.1 Asset returns and Intermediary Optimality

We write the evolution of e_t in equilibrium as

$$de_t = \mu_e dt + \sigma_e dZ_t,$$

The functions μ_e and σ_e are state-dependent drift and volatility to be solved in equilibrium.

The terms in equation (13) can be expressed in terms of the state variables of the model. Consider the risk and return terms on each investment. We can use the rental market clearing condition $C_t^h = H = 1$ to solve for the housing rental rate D_t:

$$D_t = \frac{\phi}{1-\phi} C_t^y = \frac{\phi}{1-\phi} K_t (A - i_t - \frac{\kappa}{2}(i_t - \delta)^2),$$

where we have used the goods market clearing condition in the second equality. Note that i_t, as given in (7), is only a function of $q(e_t)$. Thus, D_t can be expressed as a function of K_t and e_t.

Given the conjecture $P_t = p(e_t)K_t$, we use Ito's lemma to write the return on housing as,

$$\begin{aligned} dR_t^h &= \frac{dP_t + D_t dt}{P_t} = \frac{K_t dp_t + p_t dK_t + [dp_t, dK_t] + D_t dt}{p_t K_t} \\ &= \left[\frac{p'(e)(\mu_e + \sigma\sigma_e) + \frac{1}{2} p''(e) \sigma_e^2 + \frac{\phi}{1-\phi}\left(A - i_t - \frac{\kappa}{2} i_t^2\right)}{p(e)} + i_t - \delta \right] dt + \sigma_t^h dZ_t, \end{aligned} \tag{17}$$

where the volatility of housing returns is,

$$\sigma_t^h = \sigma + \sigma_e \frac{p'(e)}{p(e)}.$$

The return volatility has two terms: the first term is the exogenous capital quality shock and the second term is the endogenous price volatility due to the dependence of housing prices on the intermediary reputation e (which is equal to equity capital, when the constraint binds). In addition, when e is low, prices are more sensitive to e (i.e. $p'(e)$ is high), which further increases volatility.

Similarly, for capital, we can expand (10):

$$dR_t^k = \left[-\delta + \frac{(\mu_e + \sigma\sigma_e) q'(e) + \frac{1}{2}\sigma_e^2 q''(e) + A}{q(e)} \right] dt + \sigma_t^k dZ_t,$$

with the volatility of capital returns,

$$\sigma_t^k = \sigma + \sigma_e \frac{q'(e)}{q(e)}$$

The volatility of capital has the same terms as that of housing. However, when we solve the model, we will see that $q'(e)$ is far smaller than $p'(e)$ which indicates that the endogenous component of volatility is small for capital.

The supply of housing and capital via the market clearing condition (16) pins down α_t^k and α_t^h. We substitute these market clearing portfolio shares to find an expression for the equilibrium volatility of the intermediary's portfolio,

$$\alpha_t^k \sigma_t^k + \alpha_t^h \sigma_t^h = \frac{K_t}{E_t} \left(\sigma_e (q' + p') + \sigma(p+q) \right). \tag{18}$$

From the intermediary optimality condition (13), we note that:

$$\frac{\pi_t^k}{\sigma_t^k} = \frac{\pi_t^h}{\sigma_t^h} = m \frac{K_t}{E_t} \left(\sigma_e (q' + p') + \sigma(p+q) \right) \equiv \text{Sharpe ratio.} \tag{19}$$

When K_t/E_t is high, which happens when intermediary equity is low, the Sharpe ratio is high. In addition, we have noted earlier that p' is high when E_t is low, which further raises the Sharpe ratio.

We expand (19) to find a pair of second-order ODEs. For capital:

$$(\mu_e + \sigma\sigma_e) q' + \frac{1}{2}\sigma_e^2 q'' + A - (\delta + r_t) q = m \left(\sigma q + \sigma_e q' \right) \frac{K_t}{E_t} \left(\sigma_e (q' + p') + \sigma(p+q) \right); \tag{20}$$

and for housing:

$$\begin{aligned} & (\mu_e + \sigma\sigma_e) p' + \frac{1}{2}\sigma_e^2 p'' + \frac{\phi}{1-\phi} \left(A - i_t - \frac{\kappa}{2}(i_t - \delta)^2 \right) - (\delta + r_t - i_t) p \\ = \; & m \left(\sigma p + \sigma_e p' \right) \frac{K_t}{E_t} \left(\sigma_e (q' + p') + \sigma(p+q) \right). \end{aligned} \tag{21}$$

A.2 Dynamics of State Variables

We derive equations for μ_e and σ_e which describe the dynamics of the capital constraint. Applying Ito's lemma to $\mathcal{E}_t = e_t K_t$, and substituting for dK_t from (2), we find:

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = \frac{K_t de_t + e_t dK_t + \sigma_e \sigma K dt}{e_t K_t} = \frac{\mu_e + \sigma_e \sigma + e\,(i_t - \delta)}{e} dt + \frac{\sigma_e + e\sigma}{e} dZ_t. \tag{22}$$

We can also write the equity capital dynamics directly in terms of intermediary returns and exit, from (6). When the economy is not at at a boundary (hence $d\psi = 0$), equity dynamics are given by,

$$\begin{aligned} \frac{d\mathcal{E}_t}{\mathcal{E}_t} &= m\alpha_t^k \left(dR_t^k - r_t \right) + m\alpha_t^h \left(dR_t^h - r_t \right) + (m r_t - \eta) dt \\ &= m\alpha_t^k \left(\pi_t^k dt + \sigma_t^k dZ_t \right) + m\alpha_t^h \left(\pi_t^h dt + \sigma_t^h dZ_t \right) + (m r_t - \eta) dt. \end{aligned}$$

We use (13) relating equilibrium expected returns and volatilities to rewrite this expression as,

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = m^2 \left(\alpha_t^k \sigma_t^k + \alpha_t^h \sigma_t^h\right)^2 dt + m \left(\alpha_t^k \sigma_t^k + \alpha_t^h \sigma_t^h\right) dZ_t + (mr_t - \eta)dt \tag{23}$$

where the portfolio volatility term is given in (18). We match drift and volatility in both equations (22) and (23), to find expressions for μ_e and σ_e. Matching volatilities, we have,

$$m\frac{K_t}{E_t}\left(\sigma_e(q' + p') + \sigma(p+q)\right) = \frac{\sigma_e}{e} + \sigma$$

while matching drifts, we have,

$$\left(m\frac{K_t}{E_t}\left(\sigma_e(q' + p') + \sigma(p+q)\right)\right)^2 + mr_t - \eta = \frac{\mu_e + \sigma_e\sigma + e\left(i_t - \delta\right)}{e}.$$

Because

$$\frac{E_t}{K_t} = \frac{\max\left(\mathcal{E}_t, (1-\lambda)\, W_t\right)}{K_t} = \max\left(e_t, (1-\lambda)\left(p\left(e\right) + q\left(e\right)\right)\right),$$

these equations can be rewritten to solve for μ_e and σ_e in terms of e, $p\left(e\right), q\left(e\right)$,and their derivatives.

A.3 Interest Rate

Based on the household consumption Euler equation, we can derive the interest rate r_t. Since

$$C_t^y = Y_t - i_t K_t - \frac{\kappa K_t}{2}(i_t - \delta)^2 = \left(A + \delta - \frac{q_t - 1}{\kappa} - \frac{\left(q_t - 1\right)^2}{2\kappa}\right) K_t,$$

we can derive $\mathbb{E}_t\left[dC_t^y / C_t^y\right]$ and $Var_t\left[dC_t^y / C_t^y\right]$ in terms of $q\left(e\right)$ (and its derivatives), along with μ_e and σ_e. Then using (8) it is immediate to derive r_t in these terms as well.

A.4 The System of ODEs

Here we give the expressions of ODEs, expecially write the second-order terms p'' and q'' in terms of lower order terms. For simplicity, we ingore the argument for $p\left(e\right)$, $q\left(e\right)$ and their derivateives. Let

$$\begin{aligned} x\left(e\right) &= A - \delta - \widehat{\imath}\left(e\right) - \frac{\kappa\left[\widehat{\imath}\left(e\right)\right]^2}{2}, w\left(e\right) \equiv p\left(e\right) + q\left(e\right), F\left(e\right) \equiv \frac{w\left(e\right)}{e} - m\theta\left(e\right) w'\left(e\right), \\ \text{and } G\left(e\right) &\equiv x\left(e\right)\kappa F\left(e\right) + qq' m\left(1 - \theta\left(e\right)\right) w, \end{aligned}$$

where

$$\theta\left(e\right) \equiv \max\left[\frac{w\left(e\right)}{e}, \frac{1}{1-\lambda}\right] \text{ and } \widehat{\imath}\left(e\right) \equiv \frac{q\left(e\right) - 1}{\kappa}.$$

We have

$$\sigma_e = \frac{ew\left(e\right)\sigma\left(m\theta\left(e\right) - 1\right)}{w\left(e\right) - em\theta\left(e\right) w'\left(e\right)}.$$

Define

$$
\begin{aligned}
a_{11} &\equiv p'\left(\frac{x(e)\kappa}{G(e)}\left(-m(1-\theta(e))\phi\left(\frac{\frac{1}{2}q\sigma_e^2}{x(e)\kappa}\right)w(e)+m\theta(e)\frac{1}{2}\sigma_e^2\right)\right)+\frac{p\phi}{G(e)}\left(qq'm\theta(e)\frac{1}{2}\sigma_e^2+\frac{F(e)}{2}q\sigma_e^2\right),\\
a_{12} &\equiv p'\left(\frac{x(e)\kappa}{G(e)}m\theta(e)\frac{1}{2}\sigma_e^2\right)+\frac{1}{2}\sigma_e^2+\frac{p\phi}{G(e)}\left(qq'm\theta(e)\frac{1}{2}\sigma_e^2\right),\\
a_{21} &\equiv q'\left(\frac{x(e)\kappa}{G(e)}\left(\left[-m(1-\theta(e))\phi\frac{q\sigma_e^2}{2x(e)\kappa}\right]w(e)+\frac{1}{2}m\theta(e)\sigma_e^2\right)\right)+\frac{1}{2}\sigma_e^2+\frac{q\phi}{G(e)}\left(qq'm\theta(e)\frac{1}{2}\sigma_e^2+\frac{F(e)}{2}q\sigma_e^2\right)\\
a_{22} &\equiv q'\frac{x(e)\kappa}{G(e)}m\theta(e)\frac{1}{2}\sigma_e^2+\frac{q^2\phi}{G(e)}q'm\theta(e)\frac{1}{2}\sigma_e^2,
\end{aligned}
$$

and

$$
\begin{aligned}
b_1 \equiv\; & (p\sigma+p'\sigma_e)\sigma m\theta(e)\frac{w(e)-ew'(e)}{eF(e)}-\frac{p'x(e)\kappa}{G(e)}\left(\begin{array}{c}\left[m(1-\theta(e))\left(\rho+\phi\left(\widehat{i}-\frac{(q')^2\sigma_e^2}{2x(e)\kappa}\right)-\frac{\phi(1+\phi)\left[x(e)\sigma-\frac{qq'\sigma_e}{\kappa}\right]^2}{2x(e)^2}\right)-\widehat{i}-\eta\right]w(e)\\
+m\theta(e)\left[\frac{A-\delta}{\phi}+\left(p-\frac{1-\phi}{\phi}\right)\widehat{i}-\frac{1-\phi}{\phi}\frac{\kappa}{2}\widehat{i}^2+\delta(1-q(e))\right]\end{array}\right)\\
&-\frac{1-\phi}{\phi}x(e)-p\widehat{i}+\frac{p}{G(e)}\left(\begin{array}{c}\left(\rho+\phi\widehat{i}\right)x(e)\kappa F(e)-\phi\frac{F(e)}{2}(q')^2\sigma_e^2-\frac{\phi(1+\phi)}{2}\frac{F(e)\kappa\left[x(e)\sigma-\frac{qq'\sigma_e}{\kappa}\right]^2}{c^y}\\
-\phi qq'\left(\left[-\widehat{i}-\eta\right]w(e)+m\theta(e)\left[\frac{A-\delta}{\phi}+\left(p-\frac{(1-\phi)}{\phi}\right)\widehat{i}-\frac{1-\phi}{\phi}\frac{\kappa}{2}\widehat{i}^2+\delta(1-q(e))\right]\right)\end{array}\right),\\
b_2 \equiv\; & (\sigma_e q'+q\sigma)\sigma m\theta(e)\frac{w(e)-ew'(e)}{eF(e)}-\frac{q'x(e)\kappa}{G(e)}\left(\begin{array}{c}\left[m(1-\theta(e))\left(\rho+\phi\left(\widehat{i}-\frac{(q')^2\sigma_e^2}{2x(e)\kappa}\right)-\frac{\phi(1+\phi)\left[x(e)\sigma-\frac{qq'\sigma_e}{\kappa}\right]^2}{2x(e)^2}\right)-\widehat{i}-\eta\right]w(e)\\
+m\theta(e)\left[\frac{A-\delta}{\phi}+\left(p-\frac{1-\phi}{\phi}\right)\widehat{i}-\frac{1-\phi}{\phi}\frac{\kappa}{2}\widehat{i}^2+\delta(1-q(e))\right]\end{array}\right)\\
&-A+q\delta+\frac{q}{G(e)}\left(\begin{array}{c}\left(\rho+\widehat{i}\right)x(e)\kappa F(e)-\frac{F(e)}{2}(q')^2\sigma_e^2-\frac{F(e)\kappa\left[x(e)\sigma-\frac{qq'\sigma_e}{\kappa}\right]^2}{x(e)}\\
-qq'\left(\left[-\widehat{i}-\eta\right]w+m\theta(e)\left[\frac{A-\delta}{1-\phi}+\left(p-\frac{\phi}{1-\phi}\right)\widehat{i}-\frac{1-\phi}{\phi}\frac{\kappa}{2}\widehat{i}^2+\delta(1-q)\right]\right)\end{array}\right).
\end{aligned}
$$

Then the second-order terms can be solved as

$$
\begin{bmatrix} q'' \\ p'' \end{bmatrix}=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}=\frac{1}{a_{11}a_{22}-a_{12}a_{21}}\begin{bmatrix} a_{22}b_1-a_{12}b_2 \\ -a_{21}b_1+a_{11}b_2 \end{bmatrix}.
$$

A.5 Boundary Conditions and Numerical Methods

A.5.1 When $e\to\infty$ without capital constraint

When $e\to\infty$, we have q and p as constants. Let $\widehat{i}=\frac{q-1}{\kappa}$, and since

$$
C_t^y=\left(A-\delta-\widehat{i}-\frac{\kappa\widehat{i}^2}{2}\right)K_t,
$$

we have $dC_t^y/C_t^y=dK_t/K_t=\widehat{i}dt+\sigma dZ_t$. As a result, both assets have the same return volatility $\sigma_R^k=\sigma_R^h=\sigma$, and the interest rate is

$$
r=\rho+\phi\widehat{i}-\frac{\phi(1+\phi)}{2}\sigma^2.
$$

Because the intermediary's portfolio weight $\theta = \frac{1}{1-\lambda}$, the banker's pricing kernel is $\sigma m \theta(e) = \frac{m\sigma}{1-\lambda}$. Therefore

$$\frac{\mu_R^k - r}{\sigma_R^k} = \frac{m\sigma}{1-\lambda} \Rightarrow \mu_R^k = \frac{m\sigma^2}{1-\lambda} + \rho + \phi\hat{i} - \frac{\phi(1+\phi)}{2}\sigma^2 = \rho + \phi\hat{i} + \frac{2m - \phi(1+\phi)(1-\lambda)}{2(1-\phi)}\sigma^2.$$

Because $\mu_R^k = -\delta + \frac{A}{q}$ by definition, we can solve for

$$q = \frac{A}{\rho + \delta + \phi\hat{i} + \frac{2m-\phi(1+\phi)(1-\lambda)}{2(1-\lambda)}\sigma^2}.$$

Because $\hat{i} = \frac{q-1}{\kappa}$, plugging in the above equation we can solve for

$$q = \frac{-\left(\rho + \delta + \frac{2m-\phi(1+\phi)(1-\lambda)}{2(1-\lambda)}\sigma^2 - \frac{\phi}{\kappa}\right) + \sqrt{\left(\rho + \delta + \frac{2m-\phi(1+\phi)(1-\lambda)}{2(1-\lambda)}\sigma^2 - \frac{\phi}{\kappa}\right)^2 + \frac{4A\phi}{\kappa}}}{\frac{2\phi}{\kappa}} \tag{24}$$

which gives the value of q and $\hat{i}$ when $e = \infty$.

Now we solve for p. Using $\frac{\mu_R^h - r}{\sigma_R^h} = \frac{m\sigma}{1-\lambda}$ we know that $\mu_R^h = \rho + \phi\hat{i} + \frac{2m-\phi(1+\phi)(1-\lambda)}{2(1-\lambda)}\sigma^2$. Since $\frac{\frac{\phi}{1-\phi}\left(A-\delta-\hat{i}-\frac{\kappa\hat{i}^2}{2}\right)}{p} + \hat{i} = \mu_R^h$ by definition, we have

$$p = \frac{\frac{(1-\phi)}{\phi}\left(A - \delta - \hat{i} - \frac{\kappa\hat{i}^2}{2}\right)}{\rho + (\gamma - 1)\hat{i} + \frac{2m-\phi(1+\phi)(1-\lambda)}{2(1-\lambda)}\sigma^2}. \tag{25}$$

Numerically, instead of (24) and (24) we impose the slope conditions $p'(\infty) = q'(\infty) = 0$ which gives more stable solutions.

A.5.2 Lower entry barrier

Consider the boundary condition at $\underline{e}$ which is a reflecting barrier due to linear technology of entry. More specifically, at the entry boundary $\underline{e}$, we have

$$dE = m\theta(e_t)\left[dR_t^{agg} - r_t dt\right] E_t dt + dU_t$$

where dU_t reflects E_t at $\underline{e}K$. Heuristically, suppose that at $\underline{E} = \underline{e}K$, a negative shock ϵ sends E to $\underline{e}K - \epsilon$ which is below $\underline{e}K$. Then immediately there will be βx unit of physical capital to be converted into x units of E, so that the new level $\hat{E} = \underline{e}K - \epsilon + x = \underline{e}\hat{K} = \underline{e}(K - \beta x)$. This implies that the amount of capital to be converted to E is $x = \frac{\epsilon}{1+\underline{e}\beta} > 0$, and the new capital is $\hat{K} = K - \beta x = K - \beta\frac{\epsilon}{1+\underline{e}\beta}$.

Now we give the boundary conditions for p and q. First, although entry reduces physical capital K, since q is measured as per unit of K, the price should not change during entry. Therefore we must have $q'(\underline{e}) = 0$. For scaled housing price p, there will be a non-zero slope. Intuitively, entry lowers the aggregate physical capital K, hence future equilibrium consumption as well as future equilibrium housing rents are lower, translating to a lower P directly. Formally, right after the negative shock described above, the housing price is $p\left(\frac{E}{K}\right)K$ can be rewritten as $p\left(\frac{E}{K}\right)K = p\left(\underline{e} - \frac{\epsilon}{K}\right)K$, which must equal the housing price $p\left(\underline{e}\right)\widehat{K} = p\left(\underline{e}\right)\left(K - \beta\frac{\epsilon}{1+\underline{e}\beta}\right)$ right after the adjustment (otherwise there will be an arbitrage). Hence,

$$p\left(\underline{e}\right)\left(K - \beta\frac{\epsilon}{1+\underline{e}\beta}\right) = p\left(\underline{e} - \frac{\epsilon}{K}\right)K = p\left(\underline{e}\right)K - p'\left(\underline{e}\right)\epsilon \Rightarrow p'\left(\underline{e}\right) = \frac{p\left(\underline{e}\right)\beta}{1+\underline{e}\beta} > 0$$

where we have used the fact that ϵ can be arbitrarily small in the continuous-time limit.

Define $\xi \equiv \frac{p(\underline{e})\beta}{1+\underline{e}\beta}$. In numerical solution instead of imposing β, we directly impose the following boundary conditions for equilibrium pricing functions

$$p'\left(\underline{e}\right) = \xi \text{ and } q'\left(\underline{e}\right) = 0. \tag{26}$$

We will treat ξ as our primitive parameter, calibrated to match land price volatility.

A.5.3 Numerical method

Given (26), the following results is useful. We know that at $\underline{e}$ the equilibrium Sharpe ratio is (recall $w\left(e\right) = p\left(e\right) + q\left(e\right)$)

$$\gamma = \sigma m\theta\left(\underline{e}\right)\frac{w\left(\underline{e}\right) - \underline{e}w'\left(\underline{e}\right)}{w\left(\underline{e}\right) - \underline{e}m\theta\left(\underline{e}\right)w'\left(\underline{e}\right)} = \sigma m\frac{w\left(\underline{e}\right)}{\underline{e}}\frac{w\left(\underline{e}\right) - \underline{e}w'\xi}{w\left(\underline{e}\right) - \underline{e}m\frac{w(\underline{e})}{\underline{e}}\xi} = \sigma m\frac{w\left(\underline{e}\right) - \underline{e}\xi}{\underline{e}\left(1 - m\xi\right)}$$

which implies that

$$p\left(\underline{e}\right) + q\left(\underline{e}\right) = w\left(\underline{e}\right) = \frac{\gamma\underline{e}\left(1 - m\xi\right)}{\sigma m} + \underline{e}\xi. \tag{27}$$

Based on (27) numerically we use the following 2-layer loops to solve the ODE system in (A.4) with endogenous entry boundary $\underline{e}$.

1. In the inner loop, we fix $\underline{e}$. Consider different trials of $q\left(\underline{e}\right)$; given $q\left(\underline{e}\right)$, we can get $p\left(\underline{e}\right) = \frac{\gamma\underline{e}(1-m\xi)}{\sigma m} + \underline{e}\xi - q\left(\underline{e}\right)$. Then based on the four boundary conditions

$$p\left(\underline{e}\right), q\left(\underline{e}\right), p'\left(\infty\right) = q'\left(\infty\right) = 0,$$

   we can solve this 2-equation ODE system with boundary conditions using the *Matlab* builtin ODE solver bvp4c. We then search for the right $q\left(\underline{e}\right)$ so that $p'\left(\underline{e}\right) - q'\left(\underline{e}\right) = \xi$ holds.

2. In the outer loop, we search for appropriate $\underline{e}$. For each trial of $\underline{e}$, we take the inner loop, and keep searching until $q'\left(\underline{e}\right) = 0$.

B Derivation for hidden leverage case

The dynamics of the state variable are, $de_t = \mu_e dt + \sigma_e dZ_t$. We recompute μ_e and σ_e based on the higher leverage. The reputation dynamics are:

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = m\alpha_t^k \left(\pi_t^k dt + \sigma_t^k dZ_t\right) + m\alpha_t^h \left(\pi_t^h dt + \sigma_t^h dZ_t\right) + (mr_t - \eta)dt,$$

where $\alpha_t^k = \frac{1}{1-\widehat{\lambda}} \frac{q_t K_t}{W_t}$ and $\alpha_t^k = \frac{1}{1-\widehat{\lambda}} \frac{P_t}{W_t}$ are larger than the baseline equilibrium portfolio shares to reflect the higher leverage based on $\widehat{\lambda}$. For illustration here we focus on the case where capital constraint is not binding and the leverage is simply the intermeidary leverage is simply $\frac{1}{1-\widehat{\lambda}}$. When the capital constraint is binding, the leverage is determined by $\frac{W}{\mathcal{E}_t}$ as the baseline model.

We assume that the interest rate (r_t) and ex-ante risk premia (π_t^k, π_t^h) are the functions of e_t that solve the model based on λ rather than $\widehat{\lambda}$. That is we hold expected returns and interest rates fixed in the experiment. Recall that,

$$\sigma_t^h = \sigma + \sigma_e \frac{p'(e)}{p(e)} \quad \text{and,} \quad \sigma_t^k = \sigma + \sigma_e \frac{q'(e)}{q(e)}.$$

We also assume that the price functions, $p(e)$ and $q(e)$, solve the model based on λ rather than $\widehat{\lambda}$. We account for the fact that higher leverage implies a more volatile σ_e which in turn means that σ_t^h and σ_t^k rises. That is, a given shock dZ_t causes e_t to fall which feeds back into a further fall in asset prices and a larger fall in e_t. It is essential to account for this amplification since it is the non-linearity of the model. Thus,

$$\begin{aligned}
\frac{d\mathcal{E}_t}{\mathcal{E}_t} &= m\alpha_t^k \left(\pi_t^k dt + \left(\sigma + \sigma_e \frac{q'(e)}{q(e)}\right) dZ_t\right) + m\alpha_t^h \left(\pi_t^h dt + \left(\sigma + \sigma_e \frac{p'(e)}{p(e)}\right) dZ_t\right) + (mr_t - \eta)dt \\
&= m\left(\alpha_t^k \pi_t^k + \alpha_t^h \pi_t^h + r_t - \eta\right) dt + m\frac{1}{1-\widehat{\lambda}} \left[\sigma + \frac{q'(e) + q'(e)}{p(e) + q(e)} \sigma_e\right] dZ_t \qquad (28)
\end{aligned}$$

where the second equality uses the fact that $\alpha_t^k = \frac{1}{1-\widehat{\lambda}} \frac{q_t K_t}{W_t}$, $\alpha_t^k = \frac{1}{1-\widehat{\lambda}} \frac{P_t}{W_t}$, and $W_t = K_t\left(p(e) + q(e)\right)$. From (22), we can also write,

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = \frac{\mu_e + \sigma_e \sigma + e\left(i_t - \delta\right)}{e} dt + \frac{\sigma_e + e\sigma}{e} dZ_t. \qquad (29)$$

By matching (28) and (29), we can solve for μ_e and σ_e with hidden leverage. For instance, for σ_e, we have

$$m\frac{1}{1-\widehat{\lambda}} \left[\sigma + \frac{q'(e) + q'(e)}{p(e) + q(e)} \sigma_e\right] = \frac{\sigma_e}{e} + \sigma \Rightarrow \sigma_e = \sigma \frac{m\frac{1}{1-\widehat{\lambda}} - 1}{\frac{1}{e} - m\frac{1}{1-\widehat{\lambda}} \frac{q'(e)+q'(e)}{p(e)+q(e)}}$$

Increasing λ to $\widehat{\lambda}$ increases the numerator and decreases the denominator. In particular, σ_e rises more than one for one with the increase in the leverage $\frac{1}{1-\widehat{\lambda}}$, which is 1.5 times of the leverage in the base model. Moreover, this amplification effect is stronger when the economy is closer to crisis. We find that the σ_e rises by around 1.5 times relative to the baseline in the first quarter of the simulation, but rises by about 15 times at the point in the simulation when the capital constraint binds.

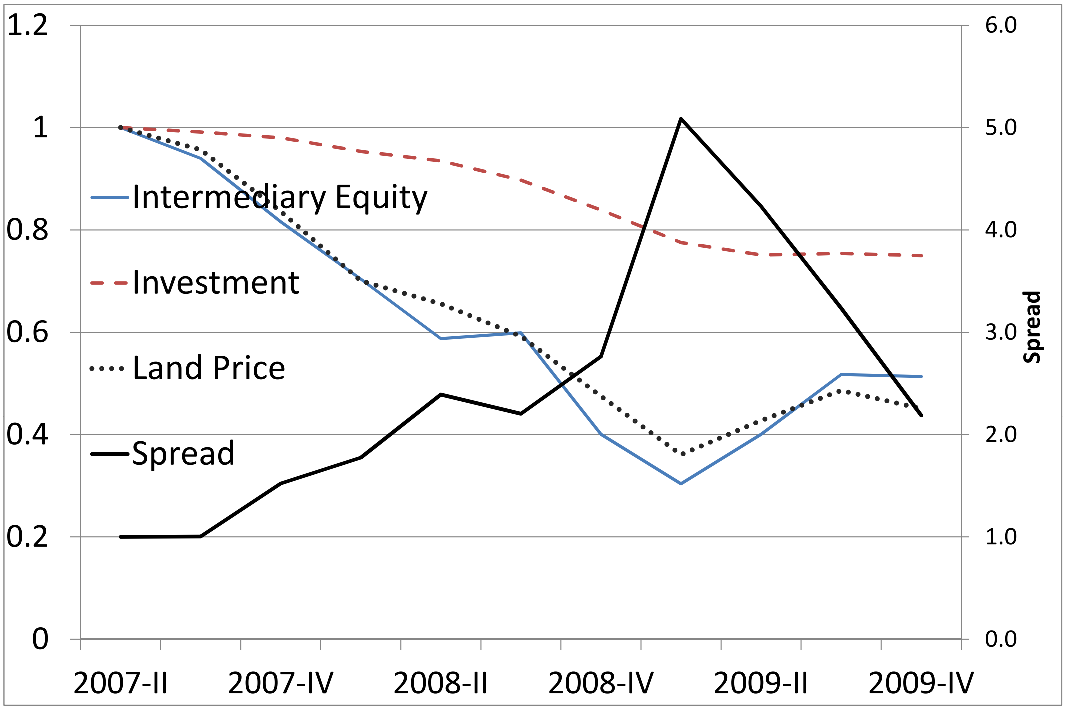

Figure 2: Data from 2007 to 2009. Intermediary equity, investment, land price index are on left-axis. Excess bond premium (labeled spread) is on right-axis. Variables are scaled by their initial values in 2007Q2.

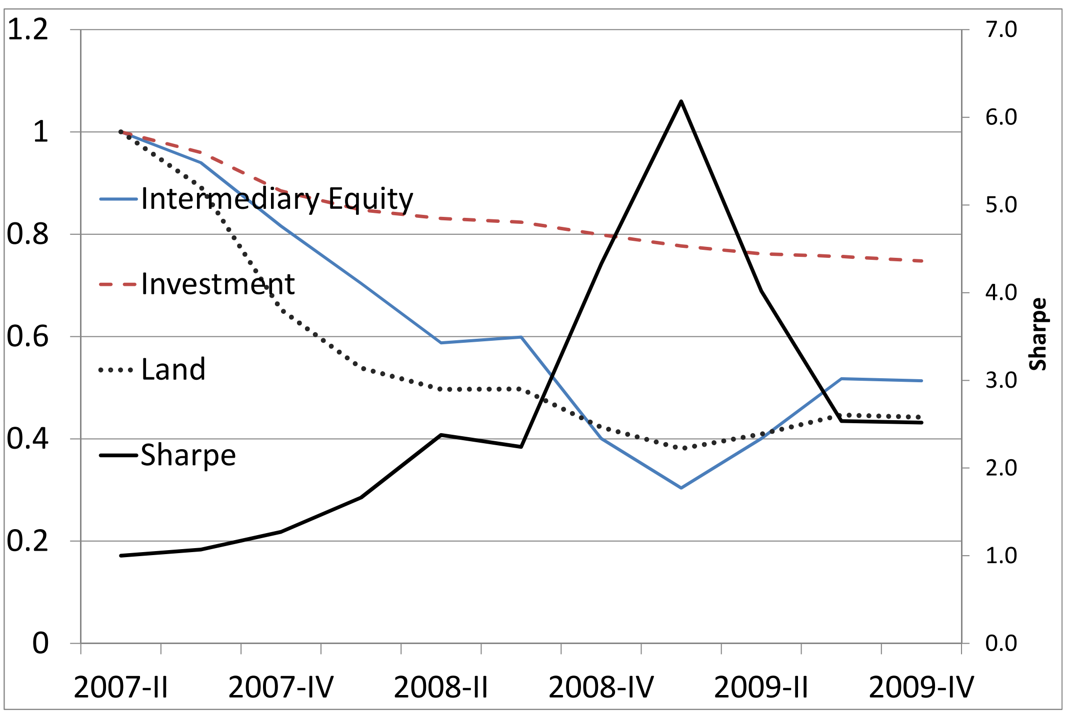

Figure 3: Model simulation matching data from 2007 to 2009. Intermediary equity, investment, land price index are on left-axis. Sharpe ratio is on right-axis. Variables are scaled by their initial values in 2007Q2.

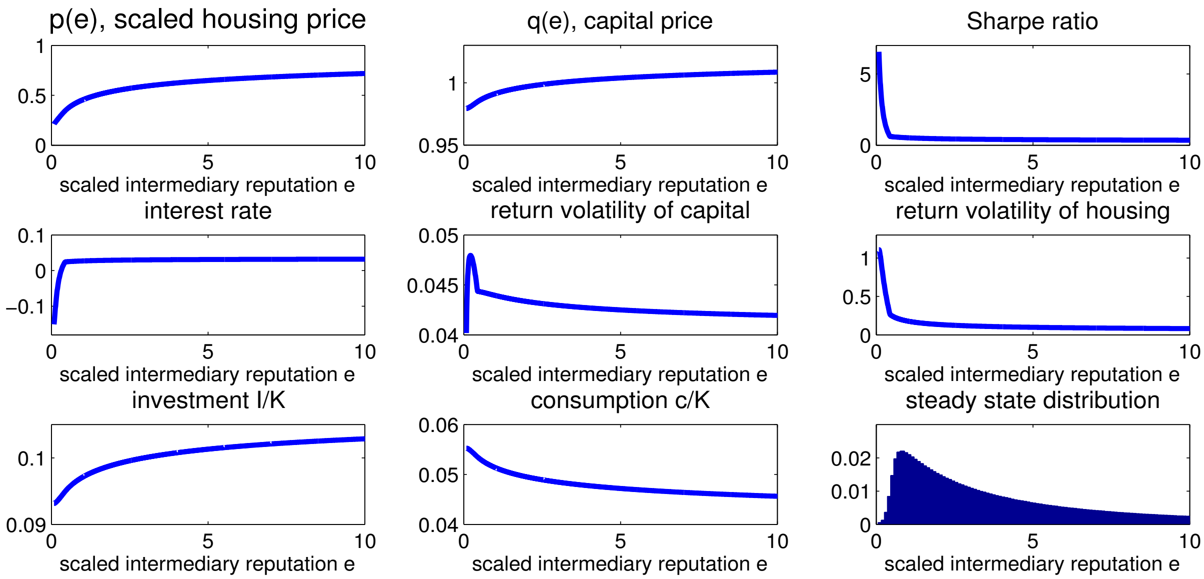

Figure 4: Price and Policy Functions for Baseline Parameters

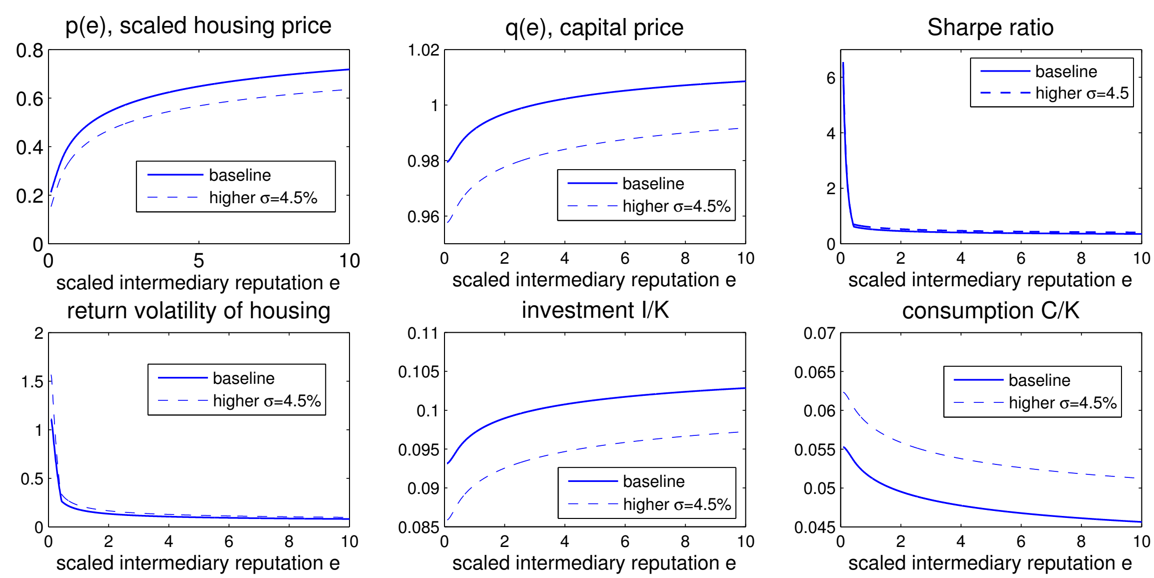

Figure 5: Price and Policy Functions for $\sigma = 4.5\%$ Case

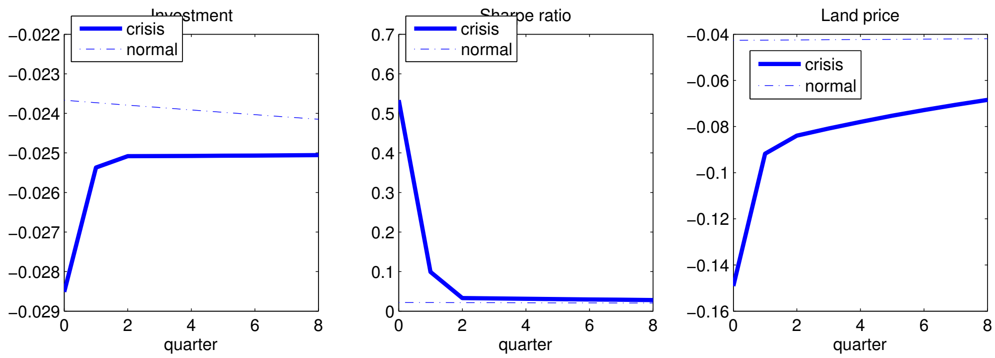

Figure 6: Effect of a -2% shock on investment, Sharpe ratio, and land prices, conditional on crisis and normal states

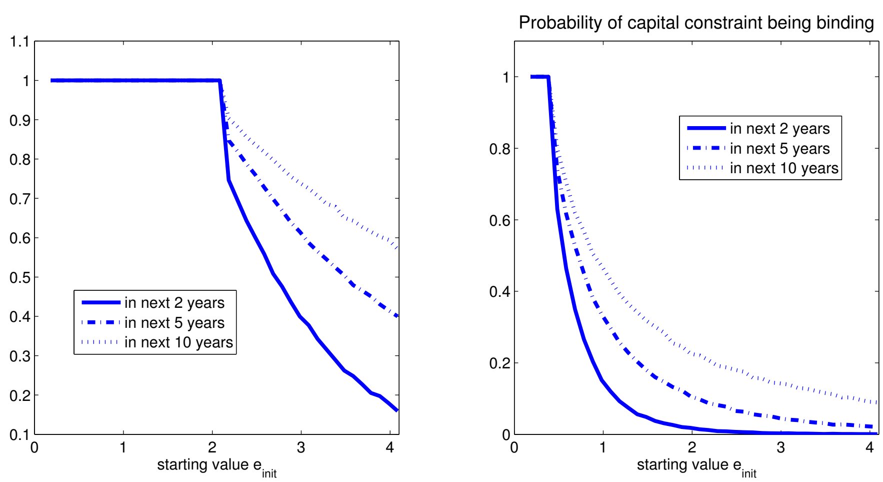

Figure 7: The figure graphs the probabilities of entering distress states (left panel) and states where the capital constraint binds (right panel), anytime over the next 2, 5 and 10 years, conditional on a given value of the state variable e.