Wealth and Volatility

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Figure 1 documents a strong negative relation in the United States between wealth (household net worth, from the Federal Reserve Flow of Funds, as a fraction of GDP) and aggregate volatility, measured as standard deviation of real GDP growth rate. Periods when net worth is high, reflecting high prices for housing and/or stocks, tend to be periods of low volatility in aggregate output, employment and consumption. Conversely, periods in which asset values are low tend to be periods of high macroeconomic volatility. The 1970s and the late 2000s were periods of low asset values and high volatility. The early 1960s and the Great Moderation of the 1980s and 1990s were periods of high asset values and low volatility.

Motivated by these facts, we develop a simple theoretical model that links asset values to the size of business cycle fluctuations. Our set-up is a micro-founded dynamic equilibrium model that contains elements of a traditional Keynesian framework in which economic fluctuations are driven by fluctuations in household optimism or pessimism. The novel feature is that the range of the fluctuations due to "animal spirits" (and hence the level of volatility) depends crucially on the value of wealth in the economy. When wealth is high "animal spirits" do not cause fluctuations: the economy has a unique equilibrium and it behaves neoclassically. When wealth is low, the economy is vulnerable to confidence shocks because fundamentals do not necessarily pin down a unique equilibrium. In this case, demand management is potentially an effective policy instrument.

In our economy competitive firms operate a linear technology that transforms labor into a perishable consumption good. In addition to non-durable consumption, households enjoy utility from durable housing, which is in fixed supply. Households comprise many members, each of which is willing to work at any positive wage. However, labor markets do not necessarily clear due to a search friction which prevents firms from attracting unemployed workers by offering a low wage: this friction is important because it allows for the possibility of equilibrium unemployment. Households must commit to consumption orders for their members before jobs are allocated. Firms then hire sufficient workers to satisfy those orders. Unemployed agents must finance their consumption orders either through home equity or through expensive borrowing. Given expensive credit, the fact that consumption is committed prior to realization of employment status increases the sensitivity of demand to perceived unemployment risk.

Our first result is to show that this environment allows for multiple equilibria in which households can collectively co-ordinate on a range of expectations about unemployment, each of which turns out to be self-fulfilling. In particular, the model typically has two steady states. In the optimistic steady state, households expect low unemployment, are not too concerned about credit constraints, and set consumption demand high. Facing high demand, firms employ a large fraction of workers, and the expectation of low unemployment is rationalized. In the pessimistic steady state, households expect high unemployment. Because they do not want to commit to high consumption given high idiosyncratic unemployment risk and costly credit, they set consumption demand low. Facing low demand, firms hire few workers, and unemployment is in fact high, as expected.

Home values in the model reflect both the fundamental flow utility from home ownership and the liquidity value of being able to finance consumption out of home equity in the event of unemployment. Because this liquidity value is tied to the level of unemployment, house prices themselves are indeterminate, and like the unemployment rate, can fluctuate in response to changes in expectations. However, the range of house prices that is observed in equilibrium is bounded from below by the presence of a fringe group of households that does not face unemployment risk, and that establishes a lower bound on housing demand.

A key feature of the model is that precautionary savings in housing on the part of households offers a way to self-insure against unemployment risk. The less wealth a household has, the more reliant the household will be on costly credit in the event of unemployment. Thus the lower is household wealth, the more sensitive is consumption demand to the expected unemployment rate. This increased sensitivity of demand to expectations increases the range of unemployment rates that can be supported in a rational expectations equilibrium. To see this consider the extreme case in which households have no wealth and they are maximally pessimistic (i.e. they expect 100% unemployment) they will set consumption to 0, and so 100% unemployment can be an equilibrium. But if households have positive wealth they will still order positive consumption even if they expect 100% unemployment. Firms must then hire a positive fraction of workers to fill these orders and so 100% unemployment is not an equilibrium.

This means that in times when the price of assets (and net worth) is low the economy is potentially subject to large fluctuations in economic activity driven by fluctuations in households "animal-spirits", while in times when the price of asset is high, the economy becomes less sensitive to these sunspot-like shocks. For sufficiently high asset prices, households never need to resort to costly credit, and full employment is the only possible equilibrium. Thus the model suggests an explanation for why wealth and volatility are so strongly positively correlated at the aggregate level.

To make things concrete, the Great Moderation was a time in which US house and stock prices were very high by historical standards relative to US GDP. We argue that high household wealth levels in this period meant that the economy was robust in the sense that it was not subject to large recessions induced by declines in confidence. However, the sharp declines in house and stock prices between mid 2007 and mid 2009 left the economy fragile, in the sense that demand became much more sensitive to expectations.

Of course, fluctuations in consumer confidence are only one source of business cycles, and over a longer history economic cycles in the United States likely have a number of causes above and beyond fluctuations in animal spirits. However, we find a confidence-type shock quite appealing as one force underlying the Great Recession, in part because there is little evidence of a negative shock to labor productivity being operative over this period.

We are not the first to argue for a link between asset values and volatility, but our mechanism reverses the usual direction of causation. Others (see Lettau et al. 2008) have pointed out that higher aggregate risk should drive up the risk premium on risky assets relative to safe assets. Lower prices for risky assets like housing and equity then just reflect higher expected future returns on these assets. In our model, asset prices are the primitive, and the level of asset prices determines the possible range of equilibrium output fluctuations, i.e. volatility.

The model has policy implications. We evaluate two specific policies. The first is a lump-sum unemployment benefit, financed by a proportional income tax. This policy makes unemployment less painful, and thereby reduces the sensitivity of demand to the expected unemployment rate. A sufficiently generous benefit rules out sunspot-driven fluctuations and ensures full employment. The second policy we consumer is government consumption financed by lump-sum taxation, in the spirit of the 2009 stimulus plan. This policy also makes aggregate (private plus public) demand less sensitive to expectations, and thereby rules out equilibria with very high unemployment rates. However, taxation also reduces asset values, which makes it harder to self-insure against unemployment risk.

1 Model

There are two goods in the economy: a perishable consumption good, produced by a continuum of identical competitive firms using labor, and an durable asset, which is in fixed supply and which we label housing. There are two types of households in the model, and a continuum of identical households of each type. These types share common preferences, but differ with respect to the risk they face: income for the first type is risky, while income for the second is not.

Each household of the first "risky" type contains a continuum of measure one of individuals. The measure of firms is equal to the measure of risky households. Thus we can envision a representative firm interacting with mass one members of a representative risky household. The price of the consumption good is normalized to one in each period. The quantity of housing is normalized to one. The riskless type of household is measure zero, but its presence will establish a floor for asset prices. The economy is closed.

Let s_t denote the current state of the economy, and s^t denote the history up to date t. In each period, households of the risky type send out members to buy consumption and to look for jobs. Employment opportunities are randomly allocated across individuals, and the consumption order must be specified before this allocation is realized. Thus, the optimal strategy is to send each member out with the same order $c(s^t)$ and an equal fraction $h(s^{t-1})$ of the assets the household carries in the period. The fraction $1 - u(s^t)$ of household members who find a job are paid a wage $w(s^t)$ and use wage income and asset holdings to clear their consumption orders. The fraction $u(s^t)$ who are unemployed pay for as much of their consumption order $c(s^t)$ as they can given assets on hand, and borrow to pay the rest at a penalty rate. These penalties are rebated to the households as lump-sum transfers denoted $T(s^t)$. At the end of the period the household regroups and pools resources, which determines the quantity of the asset carried into the next period $h(s^t)$.

At the start of each period t households observe s_t, update s^t, and assign probabilities to future sequences $\{s_\tau\}_{\tau=t+1}^{\infty}$. We assume that all households form the same expectations.

Preferences for a household (exploiting the fact that each household member enjoys the same consumption level) are given by

$$\sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t) u(c(s^t), h(s^{t-1}))$$

The household budget constraint for a risky household has the form:

$$
\begin{aligned}
c(s^t) + p(s^t)(h(s^t) - h(s^{t-1})) &\leq (1 - u(s^t))w(s^t) - \frac{\psi}{2\bar{z}} u(s^t) \min\left\{\left(p(s^t)h(s^{t-1}) - d - c(s^t)\right), 0\right\}^2 + T(s^t) \\
c(s^t), h(s^t) &\geq 0
\end{aligned}
$$

The left hand side of the budget constraint captures consumption and the cost of net asset purchases. The first term on the right hand side if household earnings, while the second is the cost of penalties for unemployed workers who use credit to pay for consumption. Note that this cost is quadratic and only applies to the fraction $u(s^t)$ of workers who are unemployed, and only if the household sets the consumption order above the value of home equity. Home equity is the value of housing owned by the household, $p(s^t)h(s^{t-1})$, minus an amount d that we think of as mortgage debt. Note that $h(s^{t-1})$ was effectively chosen in the previous period. In the current period, given aggregate variables $u(s^t)$, $w(s^t)$, $p(s^t)$ and $T(s^t)$, the choice for $c(s^t)$ implicitly defines the quantity of wealth carried into the next period $h(s^t)$.

The analogous budget constraint for the riskless household is identical, except that unemployment and transfers for this type are equal to zero.

1.1 Household's problem

Consider the problem for the type that faces unemployment risk. Let $\mu(s^t)$ be the multiplier on the budget constraint at history s^t. The first order conditions with respect to $c(s^t)$ and $h(s^t)$ are:

$$
\beta^t \pi(s^t) u_c(s^t) + \mu(s^t) \left[-1 + \frac{\psi}{\bar{z}} u(s^t) \min\left\{\left(p(s^t)h(s^{t-1}) - d - c(s^t)\right), 0\right\}\right] = 0 \tag{1}
$$

$$
\begin{aligned}
0 = & -\mu(s^t)p(s^t) + \kappa(s^t) + \\
& \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \left(\beta^{t+1} u_h(s^{t+1}) + \mu(s^{t+1}) \left[p(s^{t+1}) - \frac{\psi}{\bar{z}} u(s^{t+1}) p(s^{t+1}) \min\left\{\left(p(s^{t+1})h(s^t) - d - c(s^{t+1})\right), 0\right\}\right]\right)
\end{aligned}
$$

where $\kappa(s^t)$ is the multiplier on the the non-negativity constraint for housing.

Substituting (1) into (**??**)

$$
\frac{u_c(s^t)p(s^t)}{1 - \frac{\psi}{\bar{z}} u(s^t) \min\left\{\left(p(s^t)h(s^{t-1}) - d - c(s^t)\right), 0\right\}} \leq \beta E_{|s^t} \left[u_h(s^{t+1}) + u_c(s^{t+1})p(s^{t+1})\right] \tag{2}
$$

This looks like a standard inter-temporal first order condition for a consumption-savings problem, except the denominator of the left hand side indicates an additional motivation for saving when the borrowing constraint is binding: saving one additional unit of the asset is really cheaper than the price $p(s^t)$ because reducing current consumption reduces the expected penality cost of borrowing.

The analogous first order condition for the type that does not face unemployment risk is

$$u_{\hat{c}}(s^t)p(s^t) \leq \beta E_{|s^t}\left[u_{\hat{h}}(s^{t+1}) + u_{\hat{c}}(s^{t+1})p(s^{t+1})\right]$$

where hats denote allocations for this type.

1.2 Production and Labor Markets

Each representative firm produces according the following linear technology:

$$y(s^t) = z(s^t)n(s^t)$$

where $n(s^t)$ is the mass of workers employed by the representative firm. In equilibrium $u(s^t) = 1 - n(s^t)$. We now describe how equilibrium employment is determined.

Households first observe the aggregate state s_t and then give workers instructions about what wages to accept, i.e. they specify a reservation wage $w^*(s^t)$. Firms and workers meet in a decentralized labor market. A unit mass of workers meets each firm, where these meetings occur in a random sequence throughout the period. Firms take as given the wage $w^*(s^t)$, the price at which they can sell output (normalized to one), and decides whether or not to hire each successive worker it meets. When a firm hires a worker it produces and sells the resulting output immediately, as long as the aggregate order $c(s^t)$ has not yet been filled.

The optimal strategy for the firm in this environment is to employ a worker if and only if (i) the worker's reservation wage $w^*(s^t)$ is less than or equal to the worker's marginal product $z(s^t)$, and (ii) cumulative aggregate output in the period is less than the aggregate order $c(s^t)$, so that there is a market for additional output. Understanding the firms' incentives, a representative household will optimally assign its members a reservation wage $w^*(s^t) = z(s^t)$. Recall that a lower reservation wage does not increase the probability that a given household member will find a job, while a higher reservation wage would guarantee non-employment.

We assume that $z(s^t)$ follows a first order Markov process, with mean $\bar{z} = 1$.

1.3 Equilibrium

In some versions of the model, labor productivity z_t will be a sufficient statistic for all aggregate variables. In other versions, fluctuations in demand and house prices will exert independent influences on economic outcomes. Thus we define the current aggregate state of the economy $s_t = (z_t, c_t, p_t)$. The distribution of housing wealth does not appear because we will focus on equilibria in which all housing is owned by the risky household type, and in which each risky household is representative. (In Section XX we will consider an example in which the distribution of wealth between types is allowed to vary). A symmetric equilibrium in this model is a process for s_t and associated decision rules and prices $n(s_t)$, $u(s_t)$, $w(s_t)$, $h(s_t)$, $T(s_t)$ that satisfy, for all t and for all s_t :

1. $$w(s^t) = w^*(s^t) = z(s^t)$$

2. $$n(s^t) = 1 - u(s^t)$$

3. $$h(s^t) = 1$$

4. $$c(s^t) = z(s^t)(1 - u(s^t))$$

5. $$T(s^t) = \frac{\psi}{2\bar{z}} u(s^t) \min\left\{\left(p(s^t) - d - c(s^t)\right), 0\right\}^2$$

6. $$u_c(s^t)p(s^t)\frac{1}{1 - \frac{\psi}{\bar{z}} u(s^t) \min\left\{\left(p(s^t) - d - c(s^t)\right), 0\right\}} \geq \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t)\left[u_h(s^{t+1}) + u_c(s^{t+1})p(s^{t+1})\right]$$

7. $$u_{\hat{c}}(s^t)p(s^t) \geq \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t)\left[u_{\hat{h}}(s^{t+1}) + u_{\hat{c}}(s^{t+1})p(s^{t+1})\right]$$

1.4 Discussion

In a symmetric equilibrium, each firm employs sufficient workers to satisfy demand: $n(s^t) = c(s^t)/z(s^t)$. Thus, in this environment, the consumption order $c(s^t)$ determines employment $n(s^t)$ and unemployment $u(s^t) = 1 - n(s^t)$. If orders fall short of potential output, i.e., if $c(s^t) < z(s^t)$, then labor supply will exceed labor demand, in the sense that all measure 1 of workers in each household are willing to work at any positive wage, while employment is determined by labor demand $n(s^t) = c(s^t)/z(s^t) < 1$.

In this environment, firms are implicitly allowed to compete on price, which drives real wages to the value of a worker's marginal product and ensures firms are on their labor demand curve. However, when a potential match is formed, firms and workers are not able to negotiate wages. No single atomistic household has an incentive to choose a lower reservation wage, becuase a lower wage will not increase the probability of its members forming successful matches. Thus unemployment does not exert downward pressure on wages, breaking the standard Walrasian adjustment process that ultimately equates labor demand and labor supply in models with frictionless labor markets.

The economic logic for the quadratic credit cost is that in the event that an unemployed worker is forced to borrow, he will exhaust cheap sources of credit first before turning to more expensive sources - thus the marginal cost of credit should be increasing in the amount borrowed. For our purposes a quadratic cost function is particularly tractable, but any exponent larger than unity delivers qualitatively similar results.[1]

1.5 Preferences

We will assume the utility function is of the following separable quasi-linear form

$$u(c, h) = \frac{c^{1-\gamma}}{1-\gamma} + \phi h$$

Given this utility function coupled with $\hat{h}(s^t) = 0$ for all s^t, the inter-temporal first order condition for the riskless type simplifies to

$$p(s^t) \geq \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \left[\frac{\phi + z(s^{t+1})^{-\gamma} p(s^{t+1})}{z(s^t)^{-\gamma}} \right] \tag{3}$$

[1] A linear cost function would introduce a kink in the agent's budget set.

Thus the presence of this type puts a floor under house prices. Given $h(s^t) = 1$ the inter-temporal FOC for the risky type must hold with equality.[2]

2 Steady States

In steady state, the expression for the asset price floor above simplifies to

$$p \geq \underline{p} = \frac{\beta}{1-\beta}\phi$$

Steady state consumption is given by

$$c = 1 - u$$

Thus a sufficient condition for the borrowing constraint to not bind at any unemployment rate in steady state is

$$\underline{p} - d \geq 1$$

which is satisfied if

$$\phi \geq \bar{\phi} = \frac{1-\beta}{\beta}(1+d)$$

2.1 Case in which the borrowing constraint does not bind

Claim: If $\phi \geq \bar{\phi}$, the only possible steady state is $p = \underline{p}$, $u = 0$.

Proof: The steady state version of the risky type's intertemporal first order condition is

$$\begin{aligned} \frac{p}{(1-u)^\gamma} &= \beta\phi + \beta\frac{p}{(1-u)^\gamma} \\ p &= \frac{\beta\phi}{(1-\beta)}(1-u)^\gamma \end{aligned} \tag{4}$$

Given the price floor

$$p \geq \underline{p} = \frac{\beta}{1-\beta}\phi$$

the only possible steady state is $p = \underline{p}$, $u = 0$.

Note that this uniqueness result hinges on the presence of the riskless household type. Without this type, there would be a continuum of steady states with unemployment rates between zero and one, with each unemployment rate corresponding to a different steady state asset price as given by

[2] Note also that with $h(s^t) = 1$ the inter-temporal first order condition would be identical if preferences were given by $u(c,h) = \frac{c^{1-\gamma}}{1-\gamma} + \phi\frac{h^{1-\gamma}}{1-\gamma}$.

eq. 4 (see Farmer 2010). The presence of the riskless type puts a floor on the asset price, which effectively establishes a floor for steady state demand.

2.2 Case in which the borrowing constraint binds

Now consider the case $\phi < \bar{\phi}$, so that demand for housing by the riskless type is not strong enough to drive house prices to a level at which borrowing is never necessary.

Now steady states are solutions (c, u, p) to the following equations

$$\frac{pc^{-\gamma}}{(1 - \psi u \min\{(p - d - c), 0\})} = \beta\left[\phi + pc^{-\gamma}\right]$$

$$c = 1 - u$$

$$p \geq \underline{p} = \frac{\beta}{1-\beta}\phi$$

Suppose there exists a steady state in which $u > 0$.

Claim: Any such steady state must feature costly credit: $u > 0 \Longrightarrow c > p - d$.

Proof: Suppose, contrary to the claim, that $c \leq p - d$. Then the price that solves the intertemporal FOC would be

$$p = p_F(u) = \frac{\beta\phi}{1-\beta}(1-u)^{\gamma} < \underline{p}$$

where $p_F(u)$ is the "fundamental" steady state price given u. But $p < \underline{p}$ contradicts $p \geq \underline{p}$ which must hold in any steady state.

The logic for this result is that in any steady state with positive unemployment, households facing risk have lower expected income than households who do not. For the risky households to be nonetheless willing to pay more for housing, it must be that housing has a liqudity value, which in turn implies that the borrowing constraint must bind.

2.3 Condition for existence of a steady state with positive unemployment

Suppose preferences are logarithim $(\gamma = 1)$.

Claim: There exists a steady state with positive unemployment if and only if

$$\phi < \bar{\phi} \text{ and } \psi \geq \bar{\psi} = \frac{(1-\beta)^2}{(1-\beta)(1+d) - \beta\phi}$$

Proof: To be Added

The logic for the threshold $\bar{\psi}$ is as follows. Suppose we start with $p = \underline{p}$ and $u = 0$, and consider how the steady state asset price changes in response to a marginal increase in unemployment. On the one hand, higher unemployment reduces expected income, reducing fundamental housing demand and the fundamental component of the price $p_F(u)$. On the other hand, increasing unemployment raises the liquidity value for housing. At $\psi = \bar{\psi}$ the two effects exactly offset, and a marginal increase in unemployment is consistent with the same steady state asset price. For $\psi > \bar{\psi}$, the liquidity effect dominates, and a marginal increase in unemployment necessitates an increase in the steady state asset price: i.e. $\frac{\partial p}{\partial u} > 0$ at $p = \underline{p}$ and $u = 0$. For higher unemployment rates, the fundamental effect must eventually dominate, and by continuity, there must be at least one additional equilibrium at $p = \underline{p}$ and $u > 0$.

2.3.1 Simple Example with $\phi = 0$

For $\phi = 0$, the set of solutions to these equations is particularly simple. This is a model in which housing offers no utility, and no financial return, and is only held because it offers liquidity and reduces borrowing costs. Steady state consumption demand is given by

$$c = \frac{\rho}{\psi u} + p - d$$

where $\rho = \frac{1-\beta}{\beta}$ is the rate of time preference. Thus steady state demand is increasing in housing wealth p, and decreasing in unemployment risk u. Note that positive wealth establishes a positive floor on consumption demand, even if the expected unemployment rate is 100%. Note also that if households are impatient ($\rho > 0$) then steady state demand becomes arbitrarily large as $u \to 0$. As the unemployment risk u is reduced, a higher steady state borrowing level for unemployed workers $(c - (p - d))$ is required for agents to maintain constant consumption over time.

In any steady state, the inter-temporal motive to borrow plus the precautionary motive to save add up to a constant desired level of asset holdings equal to p. Equivalently steady state demand must equal steady state supply:

$$\frac{\rho}{\psi u} + p - d = 1 - u$$

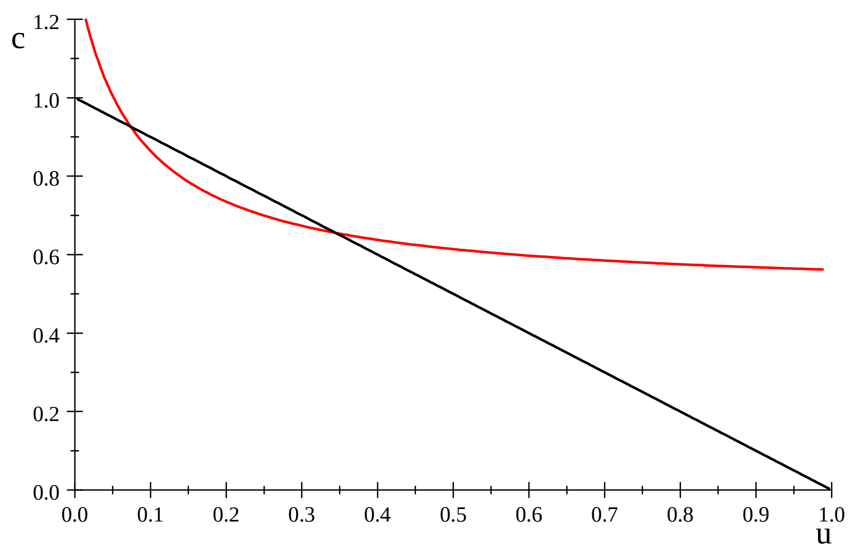
This is a quadratic equation with potentially two interior solutions:

$$u = \frac{1}{2}\left((1 - (p - d)) \pm \sqrt{(1 - (p - d))^2 - 4\frac{\rho}{\psi}}\right)$$

This equation has two interior solutions as long as $\psi > \bar{\psi} = \frac{1-\beta}{1+d}$. At both steady state unemployment rates, the household chooses to set $c = 1 - u$. Note that wealth to expected income differs across the two steady states. In both steady states, wealth is the same, and equal to p, while expected income differs. In the high unemployment steady state, one can interpret the higher steady state wealth to income ratio as reflecting a greater precautionary demand for saving in the face a greater unemployment risk.

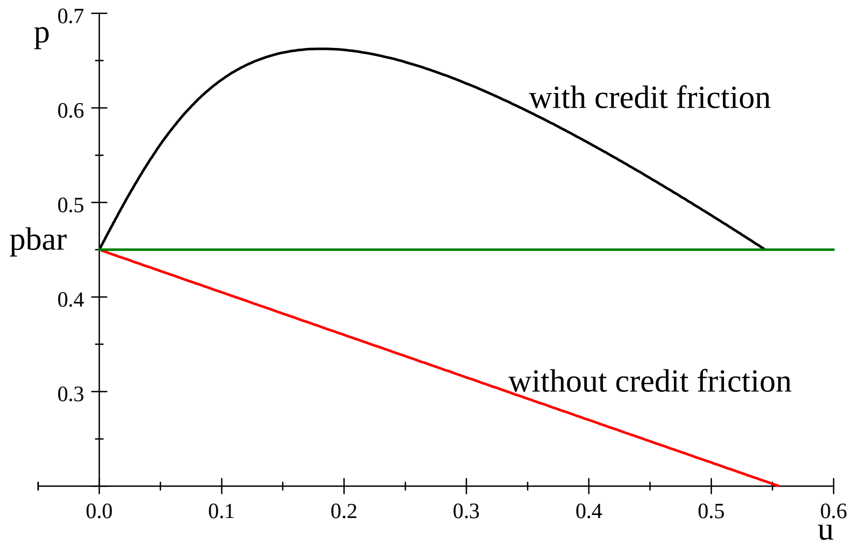
2.3.2 General case with $\phi > 0$

For $\phi > 0$ solving for steady state unemployment rates in closed-form is slightly more complicated, because the equation defining steady states is now a cubic in u. However, it is straightforward to characterize the set of steady states numerically. We now plot the set of steady states for $\psi = 1$ $\gamma = 1$ $\beta = 0.9$ $\phi = 0.05$ and $d = 0.1$. Note that this parameter configuration satisfies $\phi < \bar{\phi} = 0.1$ and $\psi > \bar{\psi} = 0.15$, so there are steady states in which $u > 0$ and $p > \bar{p} = 0.45$. The plot below shows the equations defining steady state consumption demand and supply given $p = 0.6$.



It is clear that there are two steady states here, one with low, and one with higher unemployment. In the low unemployment equilibrium wealth is low relative to consumption, but the household does not want to increase saving because unemployment risk is low. In the high unemployment equilibrium, unemployment risk is high, but the household does not want to increase saving because wealth is high relative to consumption, so borrowing is limited.

The plot above shows the set of steady states for a particular price p. The next plot shows the set of steady states for all $p \geq \underline{p}$.



The green line here is $p = \bar{p}$, and the hump-shaped black line shows unemployment rates that satisfy the inter-temporal first order condition and market clearing at prices $p \geq 0$, i.e. solutions to

$$\frac{p(1-u)^{-\gamma}}{(1-\psi u(p-d-(1-u)))} = \beta\left[\phi + p(1-u)^{-\gamma}\right] \tag{5}$$

with $\gamma = 1$. It is clear that for all p such that a steady state exists, there are two steady state unemployment rates given this particular parameter configuration. The distance between the two equilibria is decreasing in p.

The red line in the picture shows the price the household would be willing to pay absent the liquidity value for housing, i.e. the solutions to

$$\begin{aligned} pc^{-\gamma} &= \beta\left[\phi + pc^{-\gamma}\right] \\ c &= 1-u \end{aligned}$$

Thus for a given steady state unemployment rate, the red line shows the "fundamental" asset value $p_F(u)$, while the vertical distance between the black and red lines measures the "liquidity" value of housing (which is zero when $u = 0$).

Comparing the two steady states for a particular p, the low unemployment equilibrium is one in which the fundamental share of house value is high relative to the liquidity value, while in the high unemployment steady state the opposite is true.

3 Dynamics

We now want to consider dynamics. Suppose the constraint is always binding. The inter-temporal FOC is (using the resource constraint to substitute out consumption)

$$\frac{\left[z(s^t)(1-u(s^t))\right]^{-\gamma} p(s^t)}{1-\psi u(s^t)\left[p(s^t)-d-z(s^t)(1-u(s^t))\right]} = \beta\phi + \beta E_{|s^t}\left[p(s^{t+1})\left[z(s^{t+1})(1-u(s^{t+1}))\right]^{-\gamma}\right]$$

Suppose we assume, for a second, that $z(s^t)$ and $p(s^t)$ are constant at steady state values. Then the equation simplifies to

$$\frac{\left(1-u(s^t)\right)^{-\gamma}}{1-\psi u(s^t)\left[p-d-(1-u(s^t))\right]} = \beta\frac{\phi}{p} + \beta E_{|s^t}\left[\left(1-u(s^{t+1})\right)^{-\gamma}\right]$$

Let $x(s^t) = \left(1-u(s^t)\right)^{-\gamma}$, so $u(s^t) = 1 - x(s^t)^{-\frac{1}{\gamma}}$. Then we can rewrite this as

$$E_{|s^t}\left[x(s^{t+1})\right] = \frac{x(s^t)}{\beta\left[1-\psi\left(1-x(s^t)^{-\frac{1}{\gamma}}\right)\left(p-d-x(s^t)^{-\frac{1}{\gamma}}\right)\right]} - \frac{\phi}{p}$$

or, more compactly, as

$$E_{|s^t}\left[x(s^{t+1})\right] = F(x(s^t))$$

Suppose we now introduce a "sunspot", v_t, where v_{t+1} (part of the date $t+1$ state, s_{t+1}) is symmetrically distributed with mean zero with a support $[-\varepsilon_{t+1}, +\varepsilon_{t+1}]$ defined by $\varepsilon_{t+1} = G(x(s^t))$, and where

$$x(s^{t+1}) = E_{|s^t}\left[x(s^{t+1})\right] + v_{t+1}(s_{t+1})$$

The assumption that v_{t+1} has mean zero is a pre-requisite for rational expectations. The assumption that the distribution for v_{t+1} is symmetric could be relaxed.

Given this specification for a sunspot shock, we can re-write the inter-temporal FOC as

$$x(s^{t+1}) = F(x(s^t)) + v_{t+1}(s_{t+1})$$

Thus the F function tells us the expected part of $x(s^{t+1})$ while the sunspot v_{t+1} tells us how to revise expectations given the sunspot.

Absent any fundamental shocks (shocks to productivity or prices) we can define equilibrium recursively, with the current state being x_t, and the shock being v_{t+1}. (The current sunspot shock v_t is redundant as a state because x_t is a sufficient statistic for current unemployment, and v_t does not help forecast v_{t+1} given that the sunspot shock is iid over time.)

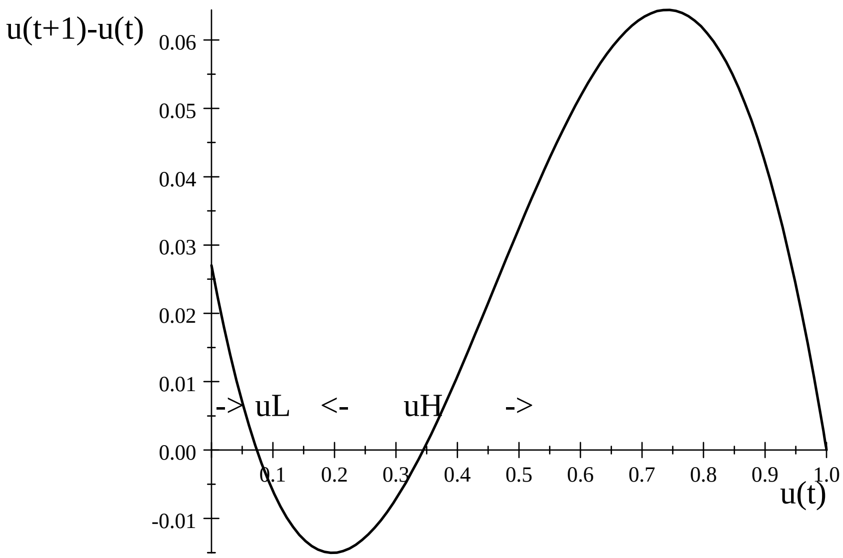
3.1 No sunspot shocks

Suppose first that $G(x(s^t)) = \varepsilon_{t+1} = 0$, so that $x(s^{t+1}) = E_{|s^t}\left[x(s^{t+1})\right]$.

It is straightforward to compute the dynamics for x_t given an initial x_0, and thus to plot the corresponding dynamics for unemployment.

$$\Delta u = u' - u = 1 - \left(\frac{(1-u)^{-\gamma}}{\beta\left[1 - \psi u\left(p - d - (1-u)\right)\right]} - \frac{\phi}{p}\right)^{-\frac{1}{\gamma}} - u$$

The following picture plots Δu as a function of u, assuming $p = 0.6$:



Let u_L and u_H denote the low and high unemployment steady states. The plot indicates that the low unemployment steady state is locally dynamically stable: if unemployment starts out below u_L, unemployment will rise, while if it starts above u_L (but below u_H) unemployment will fall. Because this steady state is dynamically stable, we can introduce sunspot shocks that will generate fluctuations in the neighborhood of u_L.

The high unemployment steady state is not stable. If unemployment starts above u_H, it will increase towards maximum unemployment, in expected terms. Note that any such paths are not equilibria, because in the limit they imply that households will end up with zero income and consumption, which cannot be optimal given positive wealth.

Note that costly credit is being used at each point along any equilibrium path for unemployment corresponding to different initial unemployment rates $u_0 \in [0, u_H]$. The logic is as follows. First, we have argued that the costly credit is used in all steady states with positive unemployment, and is thus being used at the high unemployment steady state u_H. Second, given that $(1-u_H) > p-d$, the credit constranit must also be binding at all lower unemployment rates, since these correspond to higher consumption levels.

3.2 Introducing Sunspots

When we introduce sunspots, a range of paths become possible. For a particular current x_t,

$$x_{t+1} \in [F(x_t) - G(x_t), F(x_t) + G(x_t)]$$

The bounds $G(x_t)$ have to be such that $x_{t+1} \leq (1-u_H)^{-\gamma}$ and $x_{t+1} \geq 0$. The reason x_{t+1} can never exceed $(1-u_H)^{-\gamma}$ is that such realizations would make explosive paths possible (for example, $v_{t+\tau} = 0$ for all $\tau \geq 2$.).

Figure *Recession_Simulation* shows the dynamics for unemployment assuming a sunspot shock at date 0.

In the period of the sunspot shock, unemployment jumps, and then subsequently unemployment gradually declines towards u_L. How should these dynamics be interpreted? In the period of the shock, households suddenly become much more pessimistic than they had been in the previous period. Fearing high unemployment, they expect higher borrowing costs, and cut consumption orders to reduce credit costs. In aggregate this decline in demand does indeed translate into higher unemployment.

Note that in response to a negative sunspot shock, households cut back consumption, even though they expect the recession to be temporary, and expect positive income and consumption growth looking forward. Moreover, they have not experienced any loss in wealth. The reason they nonetheless cut consumption is that higher unemployment risk generates a stronger precautionary motive to save, and this precautionary motive is so strong that asset prices would actually be driven up absent positive expected consumption growth and an associated inter-temporal motive to dis-save. As the economy converges towards the low unemployment steady state, the precautionary motive declines, and so does expected income and consumption growth.

3.3 Volatility in productivity and prices

It is straightforward to introduce volatility in p and z. Suppose p and z follow first order Markov processes, with the joint process such that for all possible $(p(s^t), z(s^t))$, the riskless household type does not want to buy houses, i.e. 3 is satisfied. In the next period we draw $p(s^{t+1})$, $z(s^{t+1})$ and a mean zero sunspot shock $v(s^{t+1}) = (1-u(s^{t+1}))^{-\gamma} - E\left[(1-u(s^{t+1}))^{-\gamma}\right]$ that is uncorrelated with the draws $(p(s^{t+1}), z(s^{t+1}))$. The support for the sunspot shock must be such that for all possible shocks, unemployment is locally stable.

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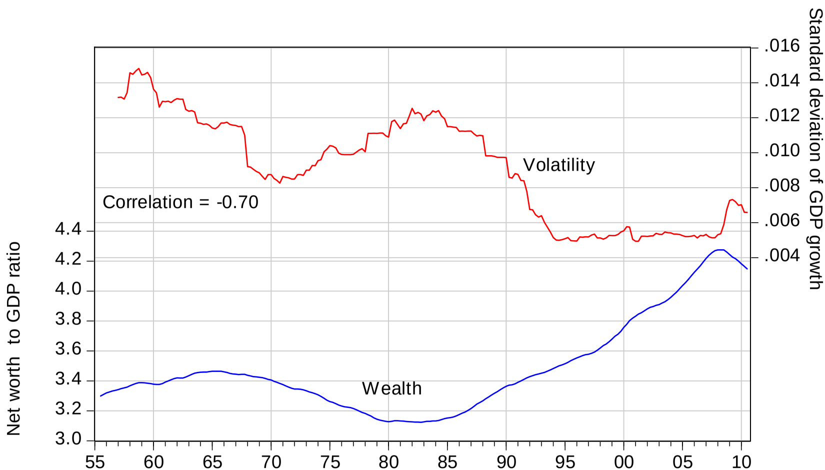
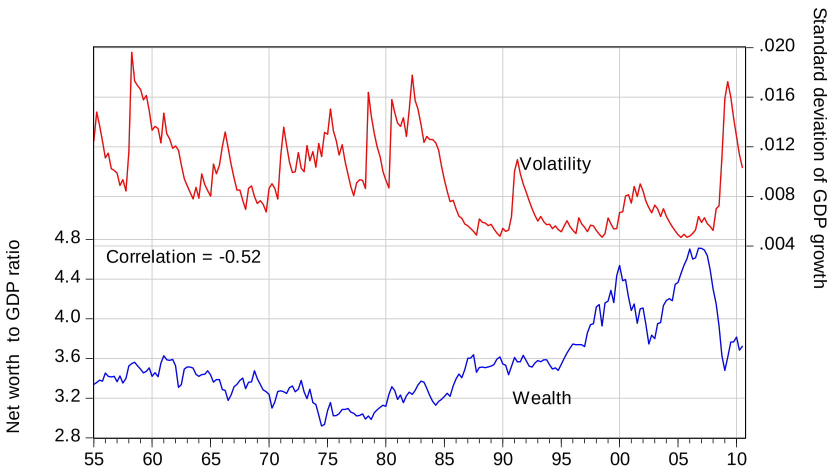
Wealth & GDP Volatility: rolling window correlation



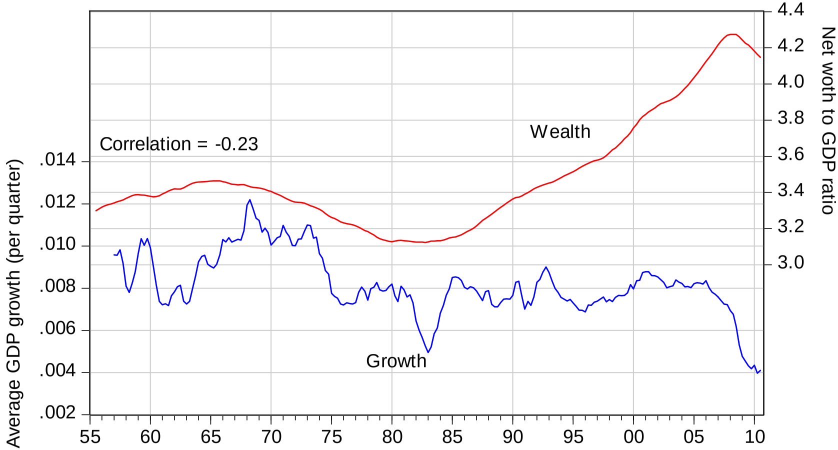
Note: Standard deviation of quarterly GDP growth are computed over 10 years rolling windows.
Observations for net worth to GDP ratio are average over the same windows

Wealth & GDP Volatility: instantaneous correlation



Note: Standard deviation of quarterly GDP growth are computed using GARCH(1,1).

Wealth & GDP Growth



Note: Average quarterly GDP growth are computed over 10 years rolling windows.
Observations for net worth to GDP ratio are average over the same windows

Micro Evidence from the CEX

- Households aged 25-60, who report consumption and income in the 2nd and 5th quarterly interviews

  $\Rightarrow$ can compute annual consumption and income growth for the same households

- Split sample in two based on household net worth to income ratio in the 5th interview

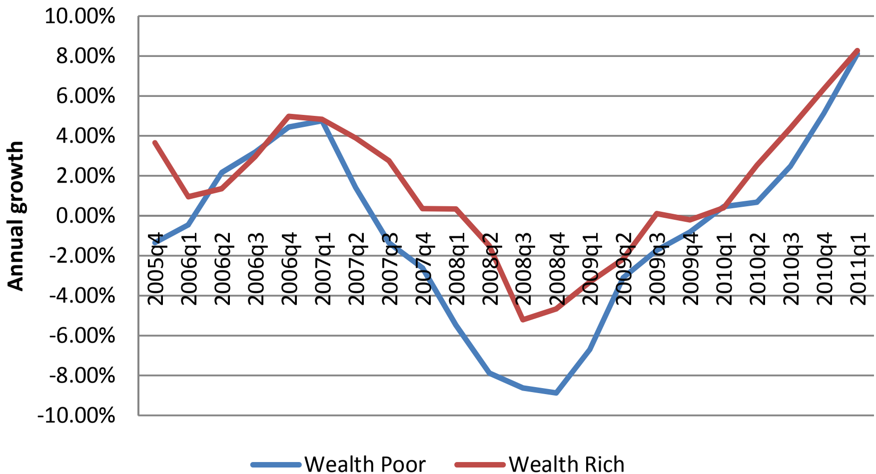
- Compare consumption growth for the wealth rich versus the wealth poor during the Great Recession

Summary statistics (2007.1 -2010.1)

	Wealth Rich	Wealth Poor
Sample	3,522	3,527
Age	47	41
Family size	2.9	2.9
% Male	47	48
% College	52	38
Income after tax	$79,659	$60,315
Consumption	$62,943	$47,394
Wealth	$280,414	$2,060

Log Changes in Ave. Total Consumption:

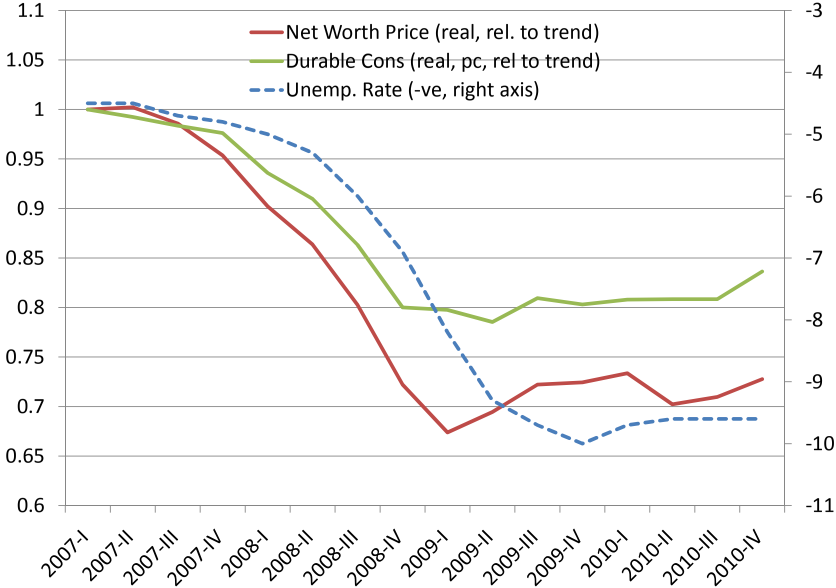
Wealth rich v/s wealth poor



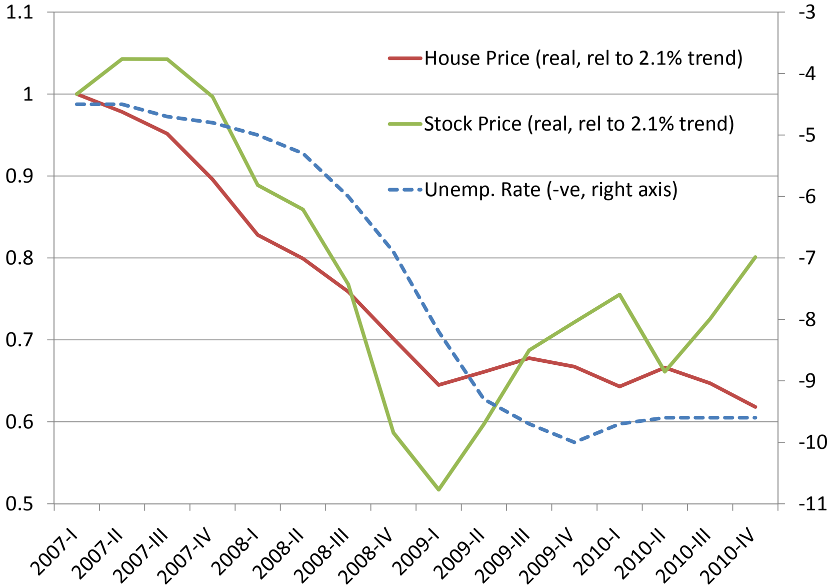
Features of the Great Recession

1. Large fall in asset values
2. Sharp decline in consumer spending, especially durables
3. Sharp rise in unemployment, labor productivity strong
4. Slow recovery

Great Recession



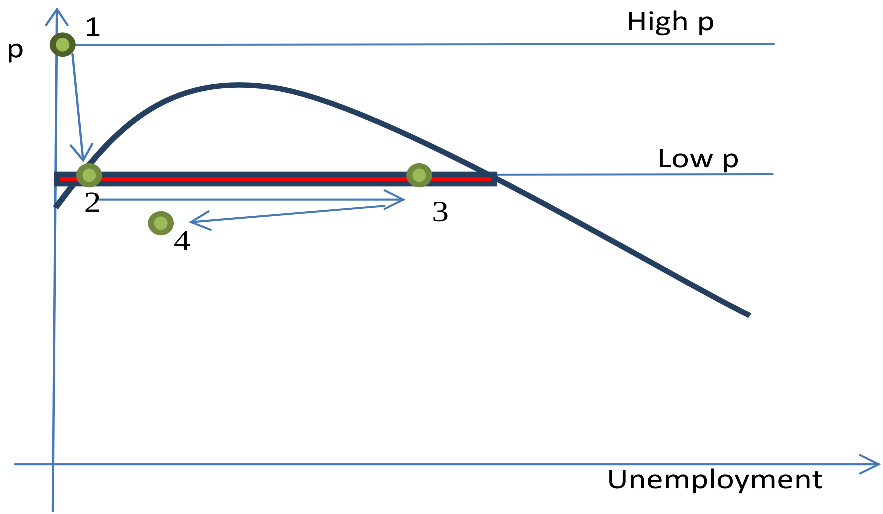
Asset Prices



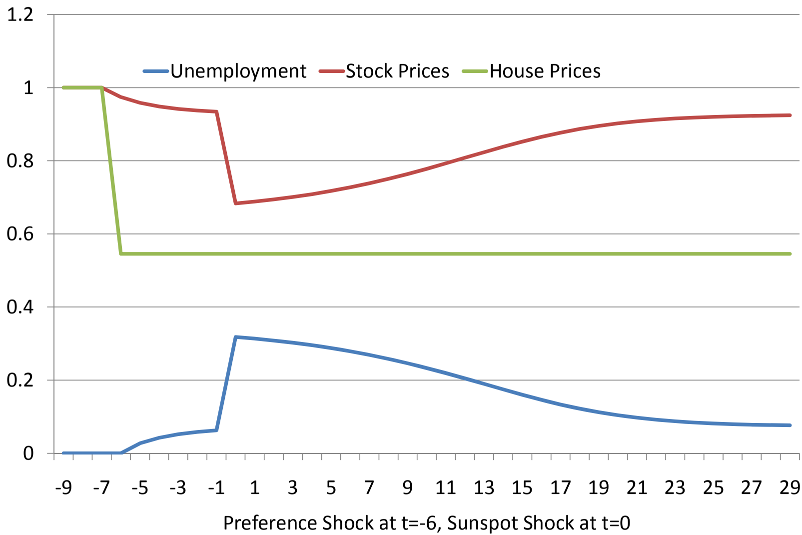
Using the model to capture the Great Recession

1. Fall in demand for housing (fall in ϕ) reduces $\underline{p}$ so that economy becomes fragile
2. Sunspot (Lehman Brothers?) triggers jump in unemployment
3. Slow recovery to low unemployment steady state

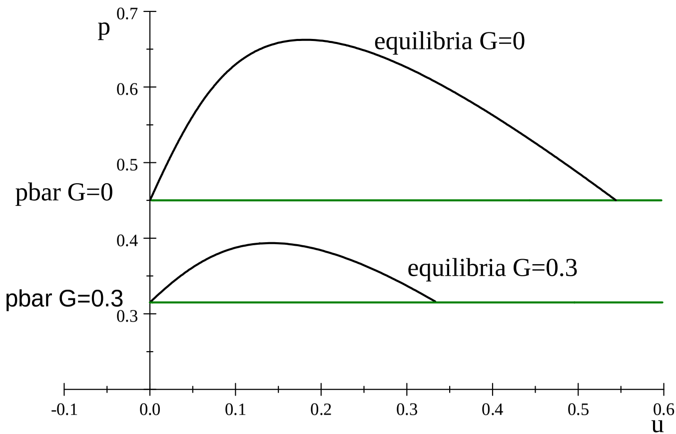
Graphically



Model Can Produce A Great Recession



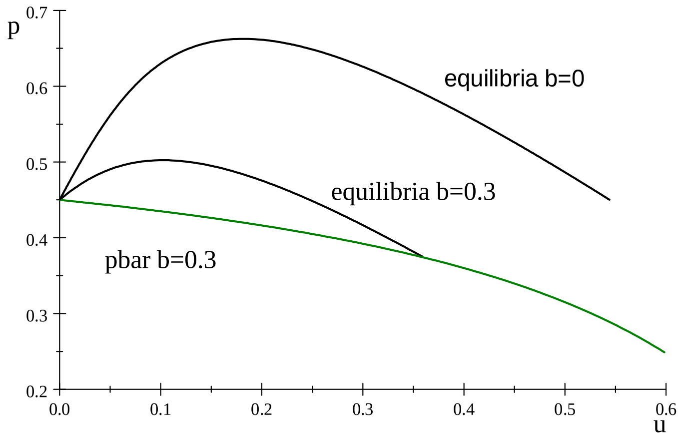
Policy 1: Tax and Spend



Policy 1: Review

- Reduces elasticity of aggregate demand to expectations
- Also reduces asset values
- Narrows range of equilibrium unemployment, but need very high G to rule out equilibria with unemployment
- Welfare implications will depend on utility from G

Policy 2: Unemployment benefit b financed by proportional tax τ on earnings



Policy 2: Review

- Policy reduces need for costly credit $\Rightarrow$ shrinks range of possible unemployment rates

- Unique full employment equilibrium if

$$b \geq \frac{\psi\left((d+1) + \frac{\beta}{(\beta-1)}\phi\right) + (\beta-1)}{(\beta-1)+\psi}$$

- ... which implies $b \geq 0.61$ in our numerical example

Conclusions

- Model in which macroeconomic stability threatened by low asset values or tight credit markets
- Great Recession: Decline in home values + costly credit left economy vulnerable to wave of pessimism
- Macro evidence of a link between level of wealth and aggregate volatility
- Micro evidence that low wealth households reduced consumption most sharply