Taxation, transfer income and stock market participation*

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Abstract

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We study a redistributive tax system that taxes investment profits and redistributes them in such a way that relatively rich agents are net contributors to relatively poor agents. Our closed form solution allows us to draw two main conclusions. First, even though the redistribution mechanism seeks to reduce the disparity in the distribution of wealth among agents, wealth levels are not harmonized despite ongoing transfers from richer to poorer agents. Specifically, when the level of poorer agents' transfer income is inversely related to their wealth levels, poorer agents have an incentive to reduce their wealth levels to increase the level of future transfer income. Second, since transfer income is subject to stock market risk poorer agents optimally reduce their exposure to equity. In particular, a redistributive tax system can thus contribute to explaining the low empirically documented equity exposures and stock market participation rates of poorer agents.

JEL Classification Codes: G11, E21, H24
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1 Introduction

Transfer of income from relatively rich to relatively poor individuals through a redistributive tax system is a widespread phenomenon throughout many countries in the world. For example, the U.S. government has spent more than 1.2 trillion U.S. dollars on wealth transfers in 2009 via income security and social security.[1] Even though investment profits are subject to taxation in most countries around the world, and despite the empirically documented evidence (Dai et al. (2008), Sialm (2009)), surprisingly little is known about the general equilibrium impact of a redistributive tax system on the distribution of wealth, consumption, asset prices and optimal investment strategies. A notable exception is the work of Sialm (2006) who studies general equilibrium effects of stochastic tax rates assuming a representative agent.

In our work, we explicitly allow for agent heterogeneity. In particular, we take into account that agents may differ by their financial endowments. We develop a stylized model of an exchange economy – in the tradition of Lucas (1978) – where agents can trade a risk-free asset that comes in zero net supply and a risky stock that represents a claim on aggregate consumption in the economy. The government taxes investment profits and immediately redistributes them in a way that seeks to reduce the disparity in the distribution of wealth among agents. I.e., in contrast to partial equilibrium models with taxation going back to Domar and Musgrave (1944), the overall risk in the economy is not reduced. Instead it is redistributed together with the transfer income. The closed form solution of our model shows that a redistributive tax system affects optimal consumption and investment strategies of poorer agents that are net recipients of transfer payments and richer agents, the net payers, in different ways. Specifically, we show that the dependency of tax revenues on the evolution of the stock market implies that transfer income is subject to stock market risk. As a consequence, poorer agents that are the net recipients of transfer income optimally reduce their exposure to stocks.

Furthermore, we demonstrate that even though the redistributive tax system is implemented in an attempt to reduce the disparity in the distribution of wealth among the agents, wealth levels are not necessarily harmonized despite ongoing transfers from richer to poorer agents. On the contrary, the spread between wealth levels of richer and poorer agents can actually widen when relatively poor agents have an incentive to consume and thereby remain relatively poor in order to increase the level of future transfer income.

Our work contributes to two strands of literature. First, we contribute to the asset pricing literature by showing how a redistributive tax system affects asset prices and optimal consumption-investment strategies in general equilibrium.

It has long been known (Brennan and Kraus (1978), Merton (1971), Rubinstein (1974)) that when CRRA investors have identical preferences it is straightforward to get from individual preferences to a representative investor, the pricing kernel and the market equilibrium. In particular, a Pareto

[1] This corresponds to 34.57% of total federal outlays.

optimal sharing rule is linear; it requires the amount of risk borne by any individual investor to be proportional to his wealth level. I.e., agents do not trade risk-free bonds with each other. We show that with a redistributive tax system agents still seek to establish a linear sharing rule. However, this requires richer agents to increase their exposure to risky stocks and poorer agents to decrease it. Simultaneously, the bond market plays an active role in establishing the linear sharing rule.

Our work also contributes to the growing literature that seeks to explain the low empirically observed household equity exposures and stock market participation rates. It has long been observed that stock market participation historically has been far from universal (Blume and Friend (1974), Mankiw and Zeldes (1991), King and Leape (1998)). According to the 2007 Survey of Consumer Finances (SCF), only 51.1% of U.S. families have stock holdings in direct or indirect form, even though theoretical research concludes that households should usually hold stocks to earn the equity premium and to diversify risks. The same fraction when only direct stock holding is accounted for is 17.9%. At the same time, 91.0% of the 10% of households with the highest income have stock holdings in direct or indirect form, whereas stock mar0ket participation drops to only 13.6% for the 20% of households with the lowest income. The same numbers for direct stock holdings show a drop from 47.5% to 5.5%. Although empirical research documents that participation is increasing with wealth, age and education (Mankiw and Zeldes (1991), Haliassos and Bertaut (1995), Bertaut (1998), Guiso et al. (2003), Calvet et al. (2007), Christiansen et al. (2008), Calvet et al. (2009a,b)), varies greatly across countries and has increased recently (Guiso et al. (2003), Giannetti and Yrjö (2010)), the overall impression is that participation is still low and at odds with the predictions from commonly applied asset allocation and asset pricing models (Campbell (2006)).

There are two main strands of literature trying to explain the low empirically observed equity exposure and stock market participation rates. A first strand of literature bases its arguments on agent-specific characteristics, including loss aversion (Ang et al. (2005)), lack of trust in financial markets (Guiso et al. (2008)), low experienced stock returns (Malmendier and Nagel (2011)), narrow framing (Barberis et al. (2006)), intelligence (Grinblatt et al. (2011)), financial literacy (van Rooij et al. (2011)), marital status and children (Love (2010)), historical portfolio decisions (Alessie et al. (2004)), internet access (Bogan (2008)), and political preferences (Kaustia and Torstila (2011)).

Another strand bases its arguments on market frictions, including differences between risk-free return and borrowing rate (Davis et al. (2006), Becker and Shabani (2010)), liquidity constraints (Haliassos and Michaelides (2003)), lacking diversification of labor income risk and its correlation with stock returns (Haliassos and Bertaut (1995), Mankiw and Zeldes (1991), Vissing-Jørgensen (2002), Guo (2004)), and stock market entry costs (Abel (2001), Allen and Gale (1994), Gomes and Michaelides (2005, 2008)).[2]

[2]Approaches that do not fall into one of these two categories include model uncertainty (Dow and Werlang (1992), Epstein and Schneider (2007)), background risk correlated with the stock market (Heaton and Lucas

We contribute to this line of research by showing that redistributive features of many tax systems found around the world can help explaining the low empirical equity exposures and stock market participation rates of poorer agents.

Overall, our contribution to the literature is three-fold. First, to the best of our knowledge, we are the first to solve a general equilibrium model with a redistributive tax system and heterogeneous agents. Second, we show that when tax revenues depend on the evolution of the economy and the stock market, poorer agents optimally reduce their exposure to stocks. Third, we show that even when the government implements the transfer mechanism attempting to reduce the disparity in the distribution of wealth among the agents, this objective is not necessarily attained despite ongoing transfers from richer to poorer agents. I.e., even in the long run poorer agents with initial wealth levels below average might optimally keep their wealth levels below their initial endowments in order to increase the level of future transfer income.

The paper is organized as follows. Section 2 introduces our model. In section 3 we present its closed form solution, and section 4 shows numerical examples and demonstrates how the optimal solution is quantitatively affected by the values of the chosen input variables. Section 5 shows both analytical and numerical results from a taxation system that allows for the separation of wealth transfer from the transfer of stock market risk. Section 6 generalizes some of the results in a setting with agents that are heterogenous with respect to their levels of risk aversion; section 7 concludes.

2 The model

We consider a model for an exchange economy with n agents and a financial market on which two assets can be traded. First, agents can trade a locally risk-free asset paying a pre-tax return of r_t from time t to $t+1$. This asset comes in zero net supply. I.e., if a subset of agents want to hold a positive fraction of their wealth in the risk-free asset, the market equilibrium has to bring about an interest rate that makes the remaining agents willing to issue such a risk-free asset. Second, agents can trade a risky stock that represents the ownership to aggregate consumption. Risk is modeled by assuming that aggregate consumption is the fruit/dividend from a binomial Lucas tree (Lucas (1978)). The initial dividend at time $t=0$ is normalized to $D_0=1$. The dividend growth G from time t to $t+1$ depends on the evolution of the economy. We assume there is a 50% probability each for a boom and a bust in the economy. The corresponding dividend growths are denoted by G^+ and G^-. Hence, the market is dynamically complete.

The risky stock comes in net supply normalized to one unit. The initial endowments of the agents are denoted by $\alpha_{0-,j}>0,\ j=1,2,\ldots,n$, with $\sum_{j=1}^{n}\alpha_{0-,j}=1$. In the following, we assume that the agents are indexed in ascending order relative to their initial shares of wealth $\alpha_{0-,j}$; i.e., agent 1 has the lowest initial endowment, and agent n has the highest.

(2000), Benzoni et al. (2007)) and changes in correlation of equity, income and consumption over the life cycle (Constantinides et al. (2002)).

2.1 The redistributive tax system

We consider a tax system where investment returns are subject to taxation at the common rate $\tau \in [0,1)$.[3] By taxing investment returns the tax system collects tax revenues of

$$\tau \left(P_t + D_t - P_{t-1}\right), \qquad t = 1, 2, \ldots, N \tag{1}$$

where P_t is the price of the risky stock at time t and D_t is its dividend payment at time t. Since the net supply of the risk-free asset is zero, the tax revenues only depend on the evolution of the price and the dividend of the risky stock. We assume that the model is parameterized in such a way that $P_t + D_t > P_{t-1}$, i.e. that tax revenues are non-negative.

We further assume that the government redistributes the tax revenue among the agents in an attempt to reduce economic disparities among the agents. Assuming that only part of the tax revenue is used to finance transfer income and the remaining part is used to finance a supply of public good, as, e.g., in Sialm (2006), affects our results quantitatively, but not qualitatively. We therefore disregard this possible dual role of the government.

We require that the redistributive tax mechanism fulfills three conditions. First, it has to fully distribute the collected tax revenues, i.e. the government neither builds up wealth nor debt. This is the government budget constraint in this paper. Given that the agents are the same in all periods, government debt (wealth) would never be considered net wealth (debt) for the individuals in the economy. Any outstanding position for the government must be settled by the same individuals within the time horizon of the model, cf. the reasoning in Barro's seminal work (Barro (1974)). Second, the redistribution of tax revenues must increase the wealth level for the relatively poor agents and decrease the wealth level for relatively rich agents.[4] Third, the ascending ordering of the agents with respect to their wealth levels before taking transfers into account must remain unchanged after taking transfers into account. A simple rule that fulfills these conditions is to redistribute the total tax revenue equally among the agents. We therefore use this rule in the following.

2.2 The optimization problem

We assume that our agents are expected present discounted utility maximizers with time-additive CRRA preferences, and that they only differ by their initial endowments.[5] In order to study dynamic effects, we allow for multiple periods and assume that the investment horizon of our agents is N periods, such that the agents dynamically have to make consumption and investment

[3] In section 5 we allow for a more general taxation mechanism. We also considered a consumption tax as, e.g., used in Sialm (2006). However, a consumption tax only leads to economic effects equivalent to an upfront redistribution of wealth. These results are therefore not presented in the paper, but they are available from the authors upon request.

[4] To be precise, the relatively poorer agents are those with an initial ownership of aggregate wealth less than $1/n$.

[5] In section 6 we also allow for heterogeneity in the agents' degree of risk aversion.

decisions at time $t = 0, 1, \ldots N-1$ and consume their remaining wealth at time $t = N$. The following notation is used:

- ρ denotes the agents' common utility discount factor.
- $\alpha_{t,j}$ denotes the number of units of the risky stock held by agent j from time t to $t+1$.
- $\beta_{t,j}$ is the number of units of the risk-free asset held by agent j from time t to $t+1$.
- $C_{t,j}$ is agent j's consumption at time t.
- r_t ($R_t \equiv 1 + r_t$) is the net (gross) risk-free rate from time t to $t+1$.
- $\widehat{r}_t$ ($\widehat{R}_t \equiv 1 + \widehat{r}_t$) is the net (gross) risk-free rate after tax from time t to $t+1$.
- $W_{t,j}$ is agent j's wealth level at time t before consumption.

Having introduced the notation we can now state agent j's optimization problem, where we have omitted the subscript j for notational simplicity:

$$\max_{\{\{C_t\}_{t=0}^{t=N}, \{\alpha_t, \beta_t\}_{t=0}^{t=N-1}\}} \frac{C_0^{1-\gamma}}{1-\gamma} + \sum_{t=1}^{N} \rho^t \mathbb{E}_0 \left[\frac{C_t^{1-\gamma}}{1-\gamma} \right] \tag{2}$$

s.t.

$$C_t = W_t - \alpha_t P_t - \beta_t \qquad t = 0, 1, \ldots, N \tag{3}$$

$$\begin{aligned} W_t = {} & \left[\alpha_{t-1}(1-\tau) + \frac{\tau}{n} \right] (P_t + D_t) + \beta_{t-1} \widehat{R}_{t-1} + \\ & \tau \left(\alpha_{t-1} - \frac{1}{n} \right) P_{t-1}, \;\; t = 1, 2, \ldots, N \end{aligned} \tag{4}$$

$$W_0 = (P_0 + D_0)\, \alpha_{0-} \tag{5}$$

with $\alpha_N = \beta_N \equiv 0$.

Equation (3) is the investor's budget constraint and equation (4) describes the dynamic evolution of wealth. They are equalities between random variables reflecting the binomial filtration.

3 Consumption and investment in general equilibrium

Having introduced our model, we next turn to a demonstration of how a redistributive tax system affects the agents' optimal consumption and portfolio policies.

If the tax rate is zero, there is no redistribution of wealth and any agent's optimal exposure to the risky stock will be equal to his entering exposure to the risky asset at any point in time. Similarly, there will be no holding of risk-free assets by anyone. After the introduction of taxation and redistribution of tax revenues, this is no longer true. The redistribution mechanism implies a dynamic transfer of wealth with an endogenously determined wealth distribution, including the capitalized values of future transfers. Furthermore, the redistribution mechanism also implies a transfer of stock market risk. This causes the relatively poor agents – the net recipients of wealth transfers – to invest less in the stock market, because their transfer income is already subject to imputed stock market risk.

The general equilibrium model can be solved in closed form. We state the solution as Theorem 1.

Theorem 1. *The general equilibrium solution to the optimization problems for n agents differing only in their initial endowments, as stated in equations (2)-(5), is given as follows:*

1. *The allocation of market risk is in accordance with a linear sharing rule relative to the wealth distribution after tax.*

2. *The martingale measure is uniquely determined and independent of the initial distribution of wealth as well as the tax rate τ. The risk neutral probabilities, with q denoting the probability for a boom in the economy, are given as*

$$q = \frac{(G^{+})^{-\gamma}}{(G^{+})^{-\gamma} + (G^{-})^{-\gamma}}, \; 1 - q = \frac{(G^{-})^{-\gamma}}{(G^{+})^{-\gamma} + (G^{-})^{-\gamma}} \tag{6}$$

3. *The discount factor after tax, $\widehat{R}$, is constant and independent of the tax rate τ:*

$$\widehat{R} = \frac{2}{\rho} \frac{1}{(G^{+})^{-\gamma} + (G^{-})^{-\gamma}} \tag{7}$$

   The risk-free rate of interest before tax, r, is also constant. However, it depends on the tax rate τ:

$$r = \frac{\widehat{R} - 1}{1 - \tau} = \frac{\widehat{r}}{1 - \tau} \tag{8}$$

4. *The asset prices are given by*

$$P_{N-1} = (1 + r)^{-1} \mathbb{E}^{Q}_{N-1} \left[D_N \right] = (1 + r)^{-1} D_{N-1} \mathbb{E}^{Q}_{N-1} \left[G^{\pm} \right] \tag{9}$$

$$P_t = (1 + r)^{-1} \mathbb{E}^{Q}_{t} \left[P_{t+1} + D_{t+1} \right] \tag{10}$$

   The risk premium is constant for a given tax rate τ and takes on the value:

$$R \left[\frac{1}{2} \frac{G^{+} + G^{-}}{qG^{+} + (1 - q)G^{-}} - 1 \right] = R \frac{\mathbb{E}^{P}_{0} \left[G \right] - \mathbb{E}^{Q}_{0} \left[G \right]}{\mathbb{E}^{Q}_{1} \left[G \right]} \tag{11}$$

5. *The equity exposure is given by the following dynamic relations with X_t being a sequence*

of time-dependent but state-independent coefficients X_t:

$$\alpha_t = \frac{1}{n} + X_t\left(\alpha_{t-1} - \frac{1}{n}\right),\ t = 1, 2, \ldots, N-1 \tag{12}$$

$$X_t = \frac{\widehat{R}(P_t + D_t)}{RP_t + \left[\prod_{j=t+1}^{N-1} X_j\right]\widehat{R}D_t},\ t = 1, 2, \ldots, N-1, \tag{13}$$

$$\alpha_0 = \frac{1}{n} + \frac{1}{1-\tau}\frac{(P_0 + D_0)\,\widehat{R}}{RP_0 + \widehat{R}D_0\left[\prod_{j=1}^{N-1} X_j\right]}\left(\alpha_{0-} - \frac{1}{n}\right) \tag{14}$$

Provided that the risk-free rate of interest as well as the tax rate are both positive it is the case that $X_t \in (0,1)\ \forall t$. Furthermore, the deviance enlarged, i.e. $|\alpha_0 - 1/n| > |\alpha_{0-} - 1/n|$, when the choice of the initial exposure α_0 is made.

6. *The consumption policy is given by a constant share of aggregate output:*

$$\frac{C_t}{D_t} = \alpha_{N-1}(1-\tau) + \frac{\tau}{n} = \left(\alpha_{N-1} - \frac{1}{n}\right)(1-\tau) + \frac{1}{n}, \quad t = 0, 1, \ldots, N \tag{15}$$

7. *The risk-free asset plays a role in order to establish a linear sharing rule. The position in the risk-free asset is determined by*

$$\beta_t = \widehat{R}^{-1}\tau P_t\left(\frac{1}{n} - \alpha_t\right), \quad t = 0, 1, \ldots, N-1 \tag{16}$$

8. *The share of wealth invested, $\alpha_t + \frac{\beta_t}{P_t}$, can be expressed as*

$$\alpha_t + \frac{\beta_t}{P_t} = \alpha_t\left[1 - \frac{\tau}{\widehat{R}}\right] + \frac{\tau}{\widehat{R}}\frac{1}{n} \tag{17}$$

For relatively poor (rich) investors the share of wealth invested is increasing (decreasing) over time.

9. *The level of the received net transfer payment for each individual at time t is*

$$\tau\left(\frac{1}{n} - \alpha_{t-1}\right)\left(P_t + D_t - P_{t-1}\frac{R}{\widehat{R}}\right) \tag{18}$$

There is a fixed relation, independent of time and state, between the net transfer payments received in the boom and the bust states, respectively. The ratio is given by

$$\frac{\tau\left(\frac{1}{n} - \alpha_{t-1}\right)\left(P_t^+ + D_t^+ - P_{t-1}\frac{R}{\widehat{R}}\right)}{\tau\left(\frac{1}{n} - \alpha_{t-1}\right)\left(P_t^- + D_t^- - P_{t-1}\frac{R}{\widehat{R}}\right)} = \frac{\widehat{R}G^+ - E_{t-1}^Q[G]}{\widehat{R}G^- - E_{t-1}^Q[G]} = \frac{\widehat{R}G^+ - E_0^Q[G]}{\widehat{R}G^- - E_0^Q[G]} \tag{19}$$

Proof The details of the derivations are found in Appendix A. ∎

The model without taxation (i.e. $\tau = 0$) is a benchmark model in asset pricing theory with a number of well known properties, such as the linear sharing rule for consumption, linear asset demand functions and straightforward aggregation of agents into a "representative agent" with preferences corresponding to those of the agents in the economy. In this benchmark model we know, cf., e.g., Brennan and Kraus (1978), Merton (1971) and Aase (2002), that no agent takes any position in the bond market and that the allocation α_t is constant and equal to the initial distribution α_{0-}. In other words, no dynamic trading takes place. Additionally, the martingale measure is determined by the marginal utilities of the representative agent.

In our model with a redistributive tax system some of these key properties are reproduced: The linear sharing rule for consumption (items 1 and 6), the martingale measure (item 2) and the asset pricing equations and the risk premium (item 4). The equity premium takes the same value at all points in time and all states considered, reflecting that the aggregate risk in the dividend growth is identical at each and every point in time and in each and every state. Hence, the P-expected values of the growth rates are identical at each and every point in time and in each and every state. The premium to be paid for bearing risk is independent of the wealth distribution. This is reflected in the Q-expected values of the growth rates, which are also identical at each and every point in time and in each and every state. The risk-free rate of interest is also being reproduced, but in an after tax setting (item 3).[6]

However, the redistributive tax system also implies important changes. First, the first term in equation (4) contains the linear exposure to the market portfolio, including the effect from the transfer payment. This term shows that the tax mechanism reduces the volatility in the agent's exposure to market risk stemming from his direct investment in the market portfolio ($\alpha_{t-1}(1-\tau)(P_t+D_t)$). It also shows that the transfer mechanism involves a simultaneous transfer of wealth and market risk ($(\tau/n)(P_t+D_t)$). This transfer significantly affects consumption levels and portfolio decisions.

Second, the relatively poor agents with initial endowment below $1/n$ are net recipients of transfers and reach a constant consumption share (C_t/D_t) in excess of their initial share of wealth α_{0-}, which corresponds to their share of wealth after taking their endogenously determined future transfers into account. As time goes by they also know that the capitalized value of their future net receipts from transfers decreases, which requires them to save and increase their share of wealth invested, $\alpha_t + \frac{\beta_t}{P_t}$ (Theorem 1, item 8). To keep their consumption share at the desired level over the entire investment horizon they need to keep their savings on a sufficient level. On the other hand, by saving they also diminish their future net receipts – saving is costly in this respect, but necessary in order to smooth out the relative consumption pattern over time.

Even though $X_t \in (0,1)$ implies a convergence of wealth towards an equal distribution, this convergence is rather slow for reasonable parameter choices. I.e., after the initial adjustment of the stock market position from α_{0-} to α_0, according to equation (14), the future levels of α_t only

[6] This is because the taxation mechanism is neutral, which implies that the discounting mechanism is affected, but the martingale measure remains unchanged when taxes are introduced. For further details, see Jensen (2009).

move very slowly towards $1/n$. Hence, in addition to affecting consumption levels via wealth transfers, the redistributive tax system also significantly affects the asset demand functions (Theorem 1, items 5 and 7). Theorem 1, item 6 shows that the Pareto optimal solution requires consumption shares to be constant over time, which requires a linear exposure of the agents to stock market risk. Equation (4) shows that such a linear exposure to stock market risk is not attained if agents were to disregard the bond market and choose an exposure to the risky asset corresponding to their shares of wealth. Specifically, the second and third summand in equation (4) bring agents away from the Pareto optimal linear sharing rule. However, both of these terms are predictable. That is, agents can neutralize the risk transfer by actively using the bond market in such a way that these two terms vanish (Theorem 1, item 7). I.e., with our redistributive tax system asset demand functions are no longer linear but become affine functions with time-varying coefficients instead of linear functions with constant coefficients. The optimal portfolio strategies now involve dynamic trading in both the bond and the stock market, reflecting that the agents take into account the redistribution of stock market risk as well as the capitalized value of future transfer payments when solving for optimal dynamic consumption-investment strategies.

Although the coefficient X_t in front of $(\alpha_{t-1} - 1/n)$ in equation (12) is less than one, these X_t values can be very close to one when the remaining investment horizon is long. Even though the redistribution mechanism implies a convergence of equity holdings and wealth towards an equal distribution, it is an interesting feature of our model that this equal distribution of wealth is not attained. In the following subsection we show a numerical example in graphical form and discuss this issue in more detail.

4 Numerical examples

After presenting the closed form solution to our model, we next turn to illustrating the impact of a redistributive tax system on optimal consumption-investment strategies, as well as the evolution of wealth and transfer income over the investment horizon. Throughout our numerical examples, we restrict ourselves to a setting with $n = 2$ agents, which allows us to depict our results in graphical form. Because of the well-known aggregation properties of the CRRA utility function (Merton (1971), Rubinstein (1974), Brennan and Kraus (1978)), this setup can be interpreted as a setting with two groups of agents.

4.1 Base case

We choose an investment horizon of $N = 60$ periods which enables us to demonstrate long-run effects of the redistributive tax mechanism. The growth of aggregate consumption is calibrated to the real empirical annual estimates for U.S. consumption in Lettau and Ludvigson (2005), i.e. $G^+ = 1.0315$ and $G^- = 1.0087$. I.e., throughout our numerical examples, one period corresponds to one year. The level of risk aversion, the subjective utility discount factor and the tax rate are set to $\gamma = 5$, $\rho = 0.96$, and $\tau = 20\%$.[7] We assume that the poor agent 1 is initially endowed with a claim on $\alpha_{0-,1} = 10\%$ of aggregate consumption; this does not include the present discounted value of future transfer payments. We summarize this set of parameters in Table 1 and refer to it as our base case parameter choice in the following.

Parameter	Description	Value
n	Number of agents	2
N	Investment horizon	60
$\alpha_{0-,1}$	Agent 1's initial endowment	10%
γ	Degree of risk aversion	5
ρ	Utility discount factor	0.96
τ	Tax rate	20%
G^+	Dividend growth boom	1.0315
G^-	Dividend growth bust	1.0087

Table 1: **Parameter values for the base case**

In Figure 1 we depict the impact of the length of the remaining investment horizon on agent 1's exposure to equity ($\alpha_{t,1}$, upper left graph), his share of wealth invested ($\alpha_{t,1} + \frac{\beta_{t,1}}{P_t}$, upper right graph), his consumption share (lower left graph), and his net transfer income in percent of the present value of the dividend (lower right graph).

The upper left panel in Figure 1 shows that the relatively poor agent 1 with an initial claim of $\alpha_{0-,1} = 10\%$ of aggregate consumption actually starts out by not participating in the stock market when the remaining investment horizon is the full 60 periods. This is because his relatively low wealth levels implies that he will receive a significant amount of transfer income in the following periods. Since this transfer income is subject to stock market risk and already meets the agent's risk-appetite, it is optimal for him not to participate in the stock market at all. Again, our numerical example shows that a redistributive tax system can help explain the low empirically observed equity exposures and stock market rates of poorer agents.

As the remaining investment horizon decreases, agent 1 slowly starts increasing his wealth level as can be seen from the upper right graph. Simultaneously, he increases his exposure to the risky asset as a higher wealth level implies a lower future level of transfer income and thereby a lower level of imputed stock market risk. However, when the remaining investment horizon is long, both the stock market position and agent 1's wealth remain at a low level. In fact, agent 1's wealth level is below his initial wealth level when the remaining investment horizon is sufficiently long. This is heavily driven by agent 1's incentive to smooth his consumption share over time. The lower left graph shows that agent 1's consumption share is constant at about 18% of aggregate consumption. I.e., the agent sells off from his initial endowment of stocks in order to finance his high consumption rate. However, the agent does not only sell stocks for financing consumption, but also for purchasing the long position in bonds required by Theorem 1, item 7,

[7] We allow for other tax rates in section 4.3 and a different tax mechanism in section 5.

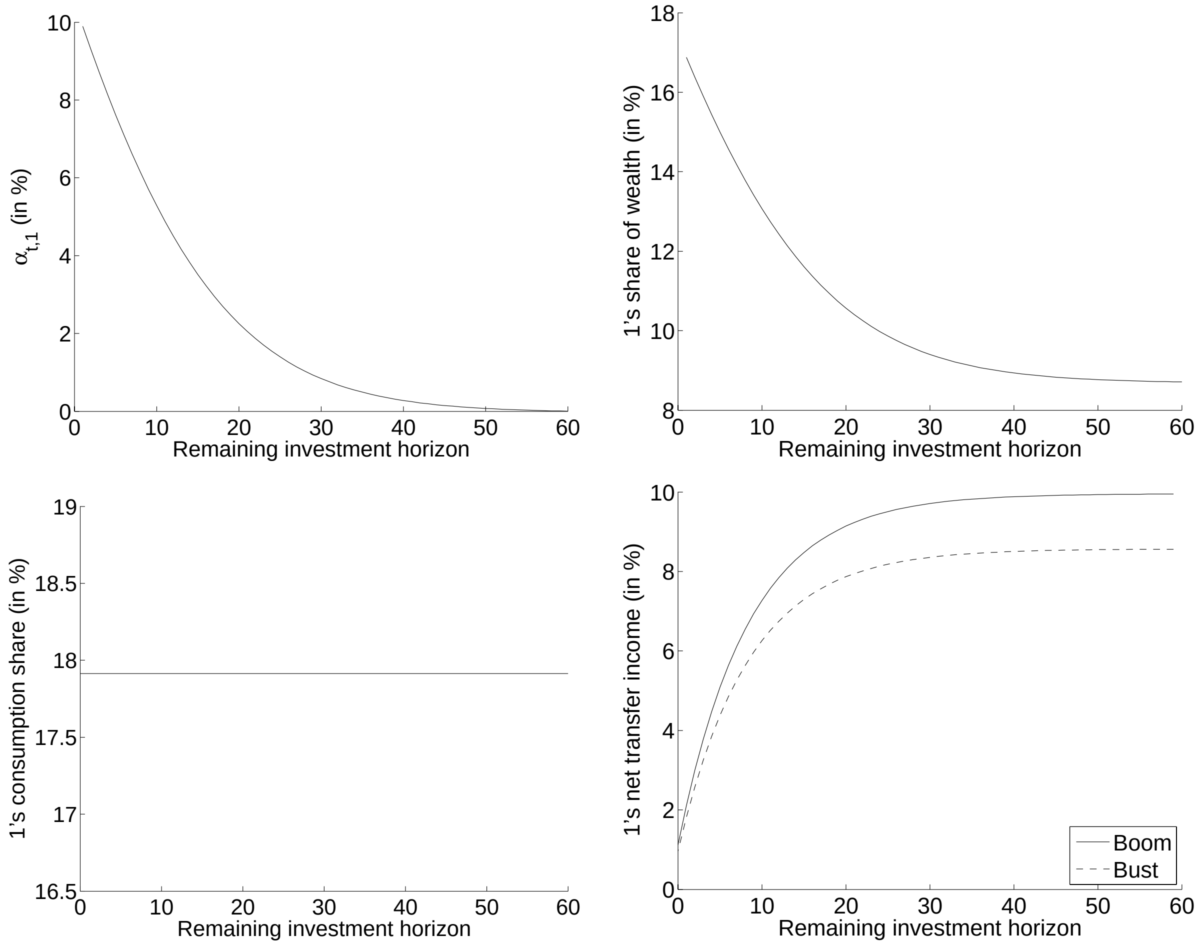

Figure 1: **Base case dynamics:** This figure shows the base case behavior over time for agent 1's equity holdings (upper left graph), share of wealth invested (upper right graph), consumption share (lower left graph), and net transfer income in percent of aggregate consumption (lower right graph) in our base case parameter setting. That is, we consider a setting with $n = 2$ agents, a tax rate of $\tau = 20\%$, an investment horizon of $N = 60$ periods and assume that the relatively poor agent 1 initially has a claim on $\alpha_{0-,1} = 10\%$ of aggregate consumption in the economy.

to smooth his consumption share.

Reducing his initial wealth level for financing a higher consumption level has the important side-effect for agent 1 that it increases the level of future transfer income. I.e., agent 1 has a strong incentive to choose a high consumption level as this simultaneously provides him with high utility from present consumption and also increases his future transfer payments.

The graph in the upper right hand panel does not include the capitalized value of future transfers, which is decreasing over time. The lower right panel shows the level of these transfers, expressed as fractions of aggregate consumption, in both the boom state and the bust state. As shown in Theorem 1, item 9, the level of transfer payments in case of a boom in the economy is a fixed multiple of the level of transfer payments in case of a bust. In our numerical example this multiple is 1.19.

The lower right graph shows that agent 1's transfer income in percent of aggregate present consumption is below his optimal consumption share irrespective of the length of the remaining investment horizon and irrespective of whether the economy was experiencing a boom or a bust in the previous period. Consequently, agent 1 needs savings to attain his desired consumption share.

When the remaining investment horizon is very long, the agent uses most of his transfer income for consumption and only uses a small fraction for saving to avoid reducing his future transfer income too much and to keep his consumption rate at the desired level. As the length of the investment horizon decreases, the agent's wealth level grows at a faster rate since agent 1 earns profits from his investments and thus can use a higher share of his transfer income for saving. Even though this increase in wealth is dampened by the decrease in agent 1's transfer income (lower right graph), both his wealth level and his exposure to equity increase most when the remaining investment horizon is short. However, his exposure to the risky asset always remains below his entering exposure of $\alpha_{0-,1} = 10\%$.

Another interesting implication of our model relates to the evolution of wealth. Even though the redistributive tax system redistributes tax revenues in an attempt to reduce the disparity in the distribution of wealth, this goal is never attained. On the contrary, there is a tendency that "poorer agents remain poor". Even though Theorem 1, item 5 indicates that the poorer agent's exposure to the risky asset and thereby also his wealth level increases as the length of the remaining investment horizon decreases, this does not imply that an equal distribution of wealth is ultimately attained. This is because the X_t values are very close to 1 when the length of the investment horizon is long. I.e., the relatively poor agent 1 i) maintains a low direct stock market position significantly below his initial endowment $\alpha_{0-,1}$, and ii) uses most of the received transfer income for consumption when the length of the remaining investment horizon is long. Especially, the second effect is so strong that the distribution of wealth does not, even for a long investment horizon, converge to an equal distribution of wealth.

4.2 Length of investment horizon and consumption

Our results in Theorem 1, item 6, and the lower left panel of Figure 1 show that consumption shares are time- and state-independent for a given length of the investment horizon. I.e., given the length of the investment horizon N, the optimal consumption shares do not depend on the length of the remaining investment horizon. However, the consumption shares vary with the length of the investment horizon N. This is because the length of the investment horizon affects the present value of the future transfer income. We depict the quantitative impact of the length of the investment horizon N on agent 1's optimal consumption share in Figure 2.

The results in Figure 2 show that the poorer agent 1's consumption share increases with the length of the investment horizon. However, it increases at a decreasing rate. Both these effects

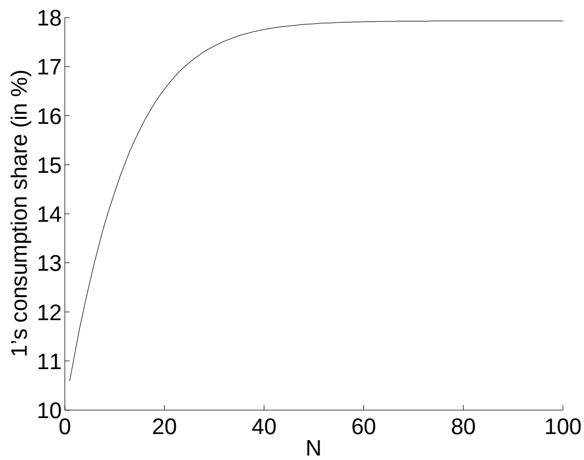

Figure 2: **Impact of length of investment horizon:** This figure shows the impact of the investment horizon N on agent 1's consumption share in our base case parameter setting. That is, we consider a setting with $n=2$ agents, a tax rate of $\tau=20\%$, and assume that the relatively poor agent 1 initially has a claim on $\alpha_{0-,1}=10\%$ of aggregate consumption in the economy.

are driven by the impact of the length of the investment horizon on the present discounted value of future transfer income at time $t = 0$. Theorem 1, item 6, shows that consumption shares are time- and state-independent for a given length of the investment horizon. I.e., as already argued above, the agents agree about the asset prices and the consumption-investment strategies in the sense that these bring about a solution that allows them to attain the consumption shares they have agreed upon at time $t = 0$. The longer the length of the investment horizon, the higher the present value of future transfer payments. As a consequence, the poorer agent's consumption share increases as the length of the investment horizon does. However, the fact that future transfer income is discounted implies that the impact of extending the length of the investment horizon on the present value of future transfer income decreases as the length of the investment horizon increases. As a consequence, the poorer agent's consumption share increases at a decreasing rate.

Throughout the remainder of this section, we demonstrate how changes in our assumptions impact our results. We demonstrate how the level of the tax rate, the initial distribution of wealth and different tax mechanisms affect our results. In particular, we demonstrate that our two key findings – low equity holdings of poor agents and "poorer agents remaining poor" are robust to various variations of our assumptions. ,

4.3 Level of the tax rate

In this section we study the impact of the level of the tax rate τ on agent 1's optimal consumption-investment strategy over the investment horizon. To visualize the effect of the tax rate τ on agent 1's optimal consumption-investment strategy we vary the tax rate between $\tau=0\%$ and $\tau=50\%$ in our base case parameter setting. Similar to Figure 1, we show in Figure 3 agent 1's optimal exposure to equity in the upper left graph, his share of wealth invested in the upper right, his consumption level in the lower left and his net transfer income as a fraction of aggregate consumption in the lower right graph. Given that agent 1's net transfer income in case of a boom in the economy is a constant multiple of it in case of an economic bust, cf. Theorem 1, item 8, we improve the readability of that graph by only showing the level of agent 1's transfer income for a boom throughout the backdating period.

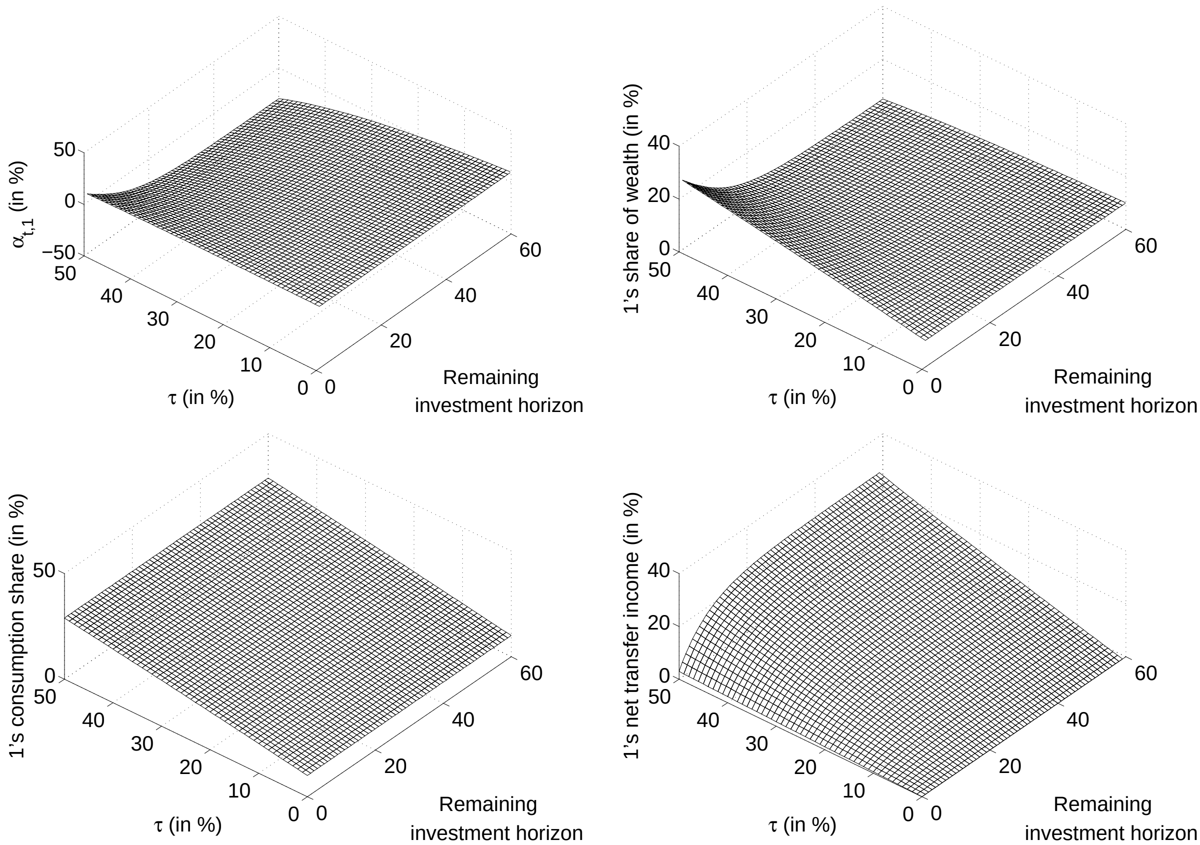

Figure 3: **Impact of tax rate:** This figure shows the impact of the tax-rate τ for agent 1's equity holdings (upper left graph), his share of wealth invested (upper right graph), his consumption share (lower left graph), and his net transfer income in percent of aggregate consumption when the economy was booming throughout the previous period (lower right graph) in our base case parameter setting. That is, we consider a setting with $n=2$ agents, an investment horizon of $N=60$ periods and assume that the relatively poor agent 1 initially has a claim on $\alpha_{0-,1}=10\%$ of aggregate consumption in the economy.

In line with the well known key properties of the model without taxation and redistribution, Figure 3 shows that for a tax rate of $\tau=0\%$, agent 1's exposure to the stock coincides with his entering exposure of $\alpha_{0-,1}=10\%$. Furthermore, his share of wealth and his consumption share also remain at a constant level of 10%.

The upper two graphs show that an increased level of the tax rate increases the extent to which agent 1 changes his exposure to stocks, reflecting the increased level of imputed stock market risk. For a remaining investment horizon of 60 periods agent 1's optimal exposure to the stock drops from 10% at a tax rate of 0% to -30% at a tax rate of 50%. That is, agent 1 partly finances his present consumption by shortening the risky stock and repurchasing it using the anticipated future transfer income. Likewise, the evolution of agent 1's share of wealth is most affected for high tax rates. Whereas it is constant at 10% for a tax rate of 0%, it increases from 4.8% to 26.3% at at tax rate of 50%. This reflects that the agent's implicit wealth from the capitalized value of future transfers is significant and increasing in both the tax rate and the length of the horizon. However, this capitalized value is depreciating as the length of the horizon shortens, and the agent needs to save in line with this depreciation to maintain his high consumption share constant.

As can be seen from Theorem 1, item 8, the level of the net transfer payment for each individual is:

$$\tau\left(\frac{1}{n}-\alpha_{t-1}\right)\left(P_t+D_t-P_{t-1}\frac{R}{\widehat{R}}\right) \tag{20}$$

This relation is close to being linear in the tax rate τ as shown in the lower right graph. In addition to the obvious and direct linear influence of the tax rate τ as the first term in (20), the effects on the remaining two terms contribute to making the level of net transfer payments progressively increasing in the tax rate τ. However, these effects are very small.

According to Theorem 1, item 6, the consumption share is given by (15) as

$$\left(\alpha_{N-1}-\frac{1}{n}\right)(1-\tau)+\frac{1}{n} \tag{21}$$

This expression depends in a direct linear manner on the tax rate τ and in an indirect manner through α_{N-1}. Again, the effect on α_{N-1} is very small as is also visible from the upper left graph.

Overall, our results in this section demonstrate that even though variations in the tax rates affect our results qualitatively, our two key findings that poorer agents choose lower equity exposures and that "poorer agents remain poor" are robust to varying the tax rate.

4.4 Initial distribution of wealth

The initial distribution of wealth to the agents affects to which extent they are net recipients or net payers to the redistributive tax system. In this section we vary agent 1's initial exposure $\alpha_{0-,1}$ to the stock to illustrate its quantitative impact on his consumption-investment strategies as well as the evolution of his wealth and transfer income over the life cycle.

In Figure 4 we study the impact of agent 1's initial share, $\alpha_{0-,1}$ of the risky stock on his exposure to the risky stock (upper left graph), his share of wealth (upper right graph), his consumption

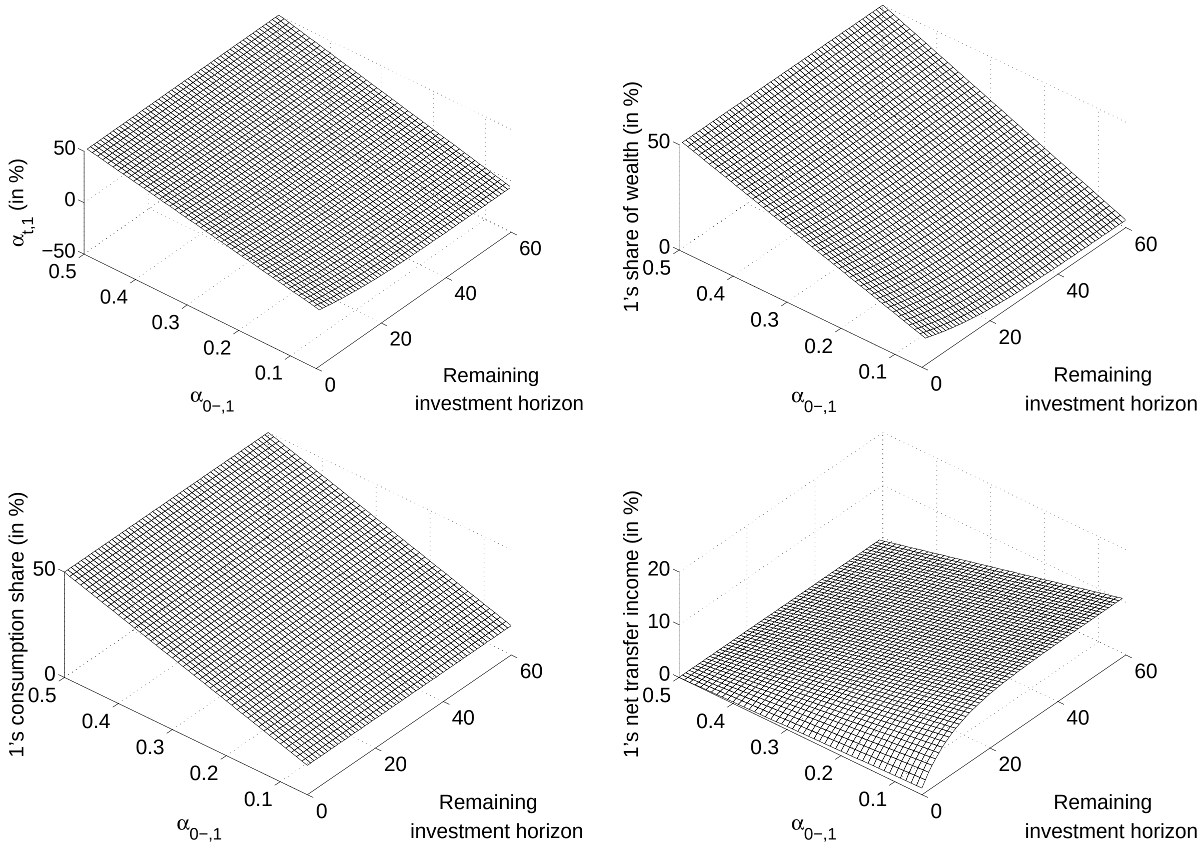

Figure 4: **Impact of initial wealth:** This figure shows the impact of agent 1's initial claim $\alpha_{0-,1}$ on aggregate consumption in the economy on his equity holding (upper left graph), his share of wealth invested (upper right graph), his consumption share (lower left graph), and his net transfer income in percent of aggregate consumption when the economy was booming throughout the previous period (lower right graph) in our base case parameter setting. That is, we consider a setting with $n=2$ agents, an investment horizon of $N=60$ periods, and a tax rate of $\tau=20\%$.

share (lower left graph), and his net transfer income expressed as a share of aggregate consumption in the economy (lower right graph) in our base case parameter setting. I.e., we stick to the setting with $n=2$ agents, an investment horizon of $N=60$ periods, and a tax rate of $\tau=20\%$.

In line with economic intuition, our results in Figure 4 show that an increase in agent 1's initial endowment $\alpha_{0-,1}$ results in an increase of his equity holdings $\alpha_{t,1}$, an increase in his share of wealth invested, an increase in his consumption share and a decrease in the transfer income he receives. However, it seems worth noting that $\alpha_{t,1}$ increases at a faster rate than $\alpha_{0-,1}$, reflecting that with increasing initial endowment the net transfer income received by agent 1 decreases. Since this does not only decrease the net wealth transfer to agent 1 but simultaneously decreases the imputed stock market risk, agent 1 optimally increases his equity holdings $\alpha_{t,1}$ at a faster rate than the initial endowment $\alpha_{0-,1}$ does in order to be endowed with the same exposure to stock market risk after accounting for transfer income. Likewise, agent 1's optimal consumption share and his share of wealth increase at a weaker rate than $\alpha_{0-,1}$, reflecting the decrease in his transfer income.

Even though variations in agent 1's initial endowment $\alpha_{0-,1}$ affect our results quantitatively, our two key findings that poorer agents choose lower equity exposures and that "poorer agents remain poor" are robust to varying agent 1's initial endowment.

5 Different tax rates on risk-free rate and equity premium

In many tax codes around the world capital gains are subject to a lower tax rate than earned interest. In this section, we allow for different tax rates, τ_r and τ_e, on the risk-free rate and the equity premium, respectively. We restrict ourselves to a neutral tax system to avoid tax arbitrage and to eliminate dispositions that are solely made in order to avoid tax payments. In a neutral and linear tax system, the (imputed) risk-free return, $P_{t-1}r_{t-1}$, is taxed at the rate τ_r, whereas the realized risk premium after correction for the imputed risk-free return, $P_t+D_t-(1+r_{t-1})P_{t-1}$, is taxed at another tax rate τ_e.[8] Hence, the evolution of an agent's wealth before consumption is given by

$$\begin{aligned} W_t = \left(\alpha_{t-1}\left(1-\tau_e\right)+\frac{\tau_e}{n}\right)\left(P_t+D_t\right)+\beta_{t-1}\widehat{R}_{t-1}+ \\ P_{t-1}\left(\alpha_{t-1}-\frac{1}{n}\right)\left(\tau_e+r_{t-1}(\tau_e-\tau_r)\right) \end{aligned} \tag{22}$$

Specifically, the key difference compared to previous results is that the tax mechanism now disentangles the transfer of wealth and the transfer of imputed stock market risk. The latter is related solely to the tax rate τ_e. The former is related to both of the tax parameters.

In the special case with no transfer of risk, but only transfer of wealth, i.e. $\tau_e = 0$, the wealth dynamics becomes

$$W_t = \alpha_{t-1}\left(P_t+D_t\right)+\beta_{t-1}\widehat{R}_{t-1}-P_{t-1}\left(\alpha_{t-1}-\frac{1}{n}\right)r_{t-1}\tau_r \tag{23}$$

Observe that the last term now has the opposite sign of what was the case previously with a uniform tax rate τ for capital gains. Poorer agents will now take short positions in the bond market. This is so because, as net recipients of transfer income, they need to lever up their positions to reach the desired exposure to stock market risk.

The necessary modifications to Theorem 1 can now be stated for the case with different tax rates, τ_r and τ_e, for the (imputed) time value and the risk premium, respectively.

[8] A neutral tax system is the benchmark for a tax system that eliminates tax arbitrage opportunities in the tax code. Some key references to neutral taxation systems in the public economics literature are, e.g., Samuelson (1964) and the retrospective tax system described in Auerbach (1991). For a short overview see, e.g., Harberger (2008). The characteristics of such tax systems from a finance perspective have recently been described in detail in Jensen (2009), where the taxation of imputed risk-free returns is called "taxation due to passage of time".

Theorem 2. *The general equilibrium solution to the optimization problems for agents that only differ by their initial endowments and who face different tax rates, τ_r on risk-free returns and τ_e on risk premia, respectively, is identical to the equilibrium in Theorem 1 for the martingale measure and the interest rate after tax. The following modifications apply:*

1. *The interest rate before tax is*
$$r = \frac{\widehat{R} - 1}{1 - \tau_r} = \frac{\widehat{r}}{1 - \tau_r} \tag{24}$$

2. *The equity exposure is given by*
$$\alpha_t = \frac{1}{n} + X_t \left(\alpha_{t-1} - \frac{1}{n} \right), \ t = 1, 2, \ldots, N-1 \tag{25}$$
$$X_t = \frac{\widehat{R}(P_t + D_t)}{RP_t + \left[\prod_{j=t+1}^{N-1} X_j \right] \widehat{R} D_t}, \ t = 1, 2, \ldots, N-1 \tag{26}$$
$$\alpha_0 = \frac{1}{n} + \frac{1}{1 - \tau_e} \frac{(P_0 + D_0)\, \widehat{R}}{RP_0 + \widehat{R} D_0 \left[\prod_{j=1}^{N-1} X_j \right]} \left(\alpha_{0-} - \frac{1}{n} \right) \tag{27}$$
Provided that the risk-free rate of interest as well as the tax rate τ_r are both positive it is the case that $X_t \in (0,1)\ \forall t$. Furthermore, provided that the tax rate τ_e is sufficiently large, the deviance $|\,\alpha_0 - 1/n\,|$ is enlarged in comparison with $|\,\alpha_{0-} - 1/n\,|$, when the choice of the initial exposure α_0 is made. A sufficient condition for this is that $R\tau_e - r\tau_r > 0$.

3. *The consumption policy is given by a constant share of aggregate output:*
$$\frac{C_t}{D_t} = \alpha_{N-1} (1 - \tau_e) + \frac{\tau_e}{n} = \left(\alpha_{N-1} - \frac{1}{n} \right) (1 - \tau_e) + \frac{1}{n}, \quad t = 0, 1, \ldots, N \tag{28}$$

4. *The risk-free asset plays a role in order to establish the linear sharing rule. The position in the risk-free asset is determined by*
$$\beta_t = \left(\widehat{R} \right)^{-1} (R\tau_e - r\tau_r) \left(\frac{1}{n} - \alpha_t \right) P_t, \quad t = 0, 1, \ldots, N-1 \tag{29}$$

5. *The level of the received net transfer payment for each individual at time t is*
$$\left(\frac{1}{n} - \alpha_{t-1} \right) \left[\tau_e (P_t + D_t) - P_{t-1} \frac{R}{\widehat{R}} (R\tau_e - r\tau_r) \right] \tag{30}$$
There is a fixed relation, independent of time and state, between the net transfer payments received in the boom and the bust states, respectively. The ratio is given by
$$\frac{\widehat{R} G^+ - E_{t-1}^Q[G](R\tau_e - r\tau_r)}{\widehat{R} G^- - E_{t-1}^Q[G](R\tau_e - r\tau_r)} = \frac{\widehat{R} G^+ - E_0^Q[G](R\tau_e - r\tau_r)}{\widehat{R} G^- - E_0^Q[G](R\tau_e - r\tau_r)} \tag{31}$$

Proof The details of the derivations are found in Appendix A. ∎

Theorem 2 shows that allowing for different tax rates on the risk-free rate and the risk premium affects our key findings relatively little. It remains the case that the optimal consumption shares are constant through time and states (Theorem 2, item 3). Likewise, the risk-free asset allows the agents to establish a linear sharing rule (Theorem 2, item 4) and the ratio of the net transfer payment in case of an economic boom and bust remains a time- and state-independent constant.

Different tax rates on the risk-free rate and the risk premium, respectively, allows us to disentangle the wealth transfer from the transfer of stock market risk. We illustrate the impact of varying the tax rate τ_e on the risk premium between 0% and 50% while keeping the tax rate on the risk-free rate as well as other parameters fixed at our base case parameter choices of $\tau_r = 20\%$, $n = 2$, $N = 60$, and $\alpha_{0-,1} = 10\%$. Figure 5 shows how the level of the tax rate τ_e affects the relatively poor agent 1's exposure to equity (upper left graph), his share of wealth invested (upper right graph), his consumption share (lower left graph), and the transfer income received in percent of aggregate consumption (lower right graph).

Compared to our results in Figure 3 where we varied a common tax rate $\tau = \tau_r = \tau_e$ three key differences become apparent. First, agent 1's share of wealth invested does not vary with τ_e, once the initial adjustment of α_{0-} to α_0 has been made. Second, his consumption share does not vary with τ_e, and third, his net transfer income is less sensitive to changes in τ_e than in τ. Whereas the latter result is due to the fact that the tax basis that τ_e applies to is smaller than the basis τ applies to, the former two findings are closely related to the fact that the market value of the risk premium is zero. This implies that the agents are able to adjust their shares of the risk premium and unwind the effect of its taxation in order to obtain their desired linear exposure to market risk independent of the level of the tax rate τ_e. If $(\alpha_{t-1}, \beta_{t-1})$ is the optimal investment strategy at time $t-1$ for a given agent and a given level of τ_e, the same exposure can be obtained for any different level $\widetilde{\tau}_e$ by choosing portfolio weights $(\widetilde{\alpha}, \widetilde{\beta})$ as follows:

$$\widehat{\alpha}_{t-1} = \left[\alpha_{t-1}\frac{1-\tau_e}{1-\widehat{\tau}_e} + \frac{\tau_e - \widehat{\tau}_e}{n(1-\widehat{\tau}_e)}\right] \tag{32}$$

$$\widehat{\beta}_{t-1} = \beta_{t-1} + P_{t-1}\left(\frac{1}{n} - \alpha_{t-1}\right)\frac{\widehat{\tau}_e - \tau_e}{1-\widehat{\tau}_e} \tag{33}$$

Hence, the evolution of wealth and the consumption share does not depend on τ_e. In order to achieve the optimal allocation of risk between the two agents shown in Figure 5, agent 1 must decrease his exposure to equity as the level of imputed stock market risk increases, i.e. as the tax rate τ_e increases. As can be seen from the upper left graph, for very high levels of τ_e this might require holding a short position in the risky asset. That is, also in a tax system with different tax rates on the risk-free rate and the equity risk premium poorer agents tend to hold lower equity exposures. Likewise, the relatively poor agents tend to remain relatively poor.

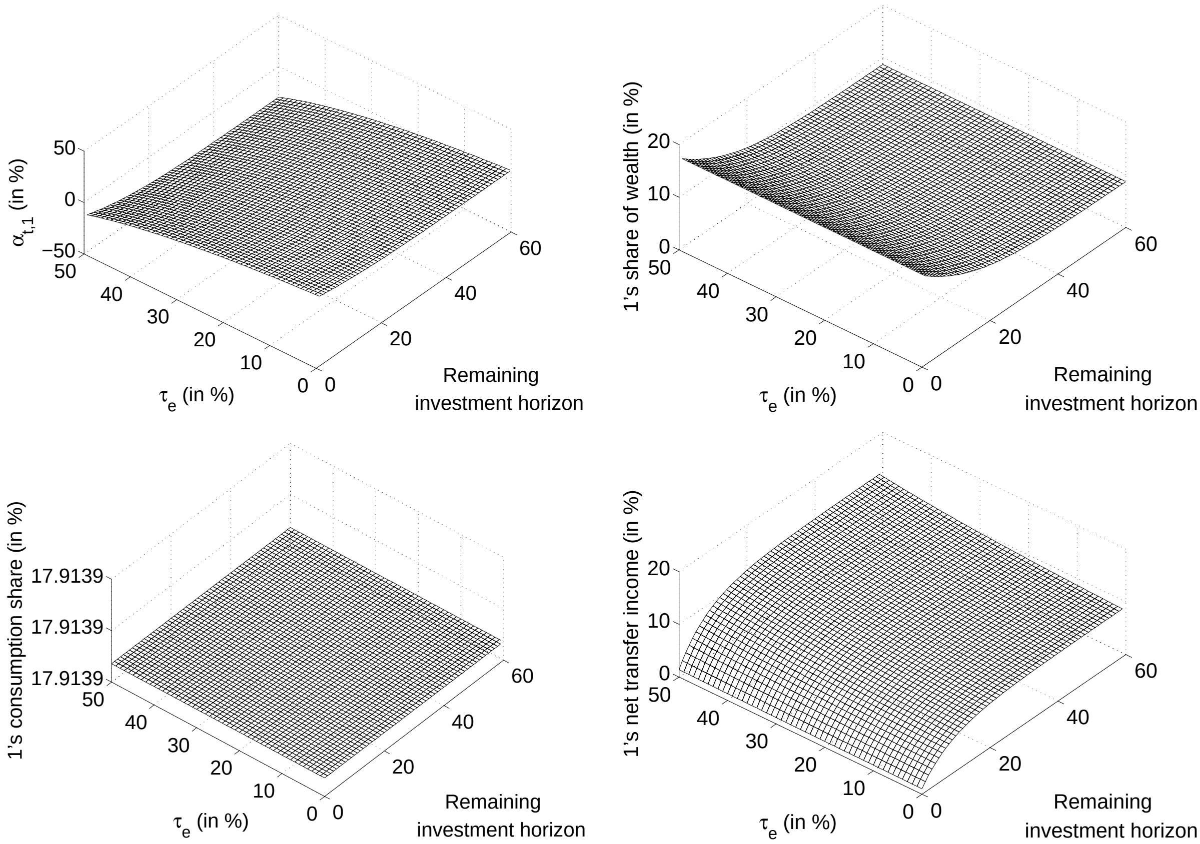

Figure 5: **Impact of different tax rates:** This figure illustrates the impact of different tax rates on the risk-free rate and the equity premium. It shows the impact of the tax rate τ_e on the equity premium for agent 1's equity holdings (upper left graph), his share of wealth invested (upper right graph), his consumption share (lower left graph), and his net transfer income in percent of aggregate consumption when the economy was booming throughout the previous period (lower right graph) in our base case parameter setting. That is, we consider a setting with $n = 2$ agents, and an investment horizon of $N = 60$ periods. The tax rate on the risk-free rate is set to its base case parameter value of $\tau_r = 20\%$.

6 Agents with different levels of risk aversion

An agent's risk aversion is one of the key determinants driving the relation between the demand for risky and risk-free assets. However, it is a well-established fact that allowing for different levels of risk aversion greatly increases the complexity of general equilibrium asset pricing models.

There is a known – although yet limited – literature on Pareto optimal sharing rules when agents are heterogenous with respect to their degree of risk aversion. The seminal paper and modeling framework in this area is due to Dumas (1989). Other subsequent papers along these lines are Benninga and Mayshar (2000), Bhamra and Uppal (2010), Cvitanić and Malamud (2010), Cvitanić et al. (2011), Franke et al. (1998), Vasicek (2005), Wang (1996) and Weinbaum (2009).

The analytical solutions in the previous sections relied on some key relations that no longer hold.

First, when agents differ in their attitude towards risk, they no longer agree ex ante about the "correct" level of the after-tax risk-free rate, i.e. equation (7) no longer holds. Second, agents no longer seek to smooth consumption shares through time and different states. Instead, the relatively more risk averse agents seek to attain higher consumption shares in bust states and in turn accept lower consumption shares in boom states and vice versa for the relatively less risk averse agents. In other words the Pareto optimal sharing rules are no longer linear and the role of the risk-free asset is no longer to establish such linear sharing rules; i.e., equation (16) no longer holds. Instead, the after-tax risk-free rate as well as the martingale measure becomes time- and state-dependent reflecting the varying wealth distribution across the agents.

We know that as a necessary condition for an optimal solution the first-order partial derivatives of the Lagrangian with respect to the decision variables α_t, β_t have to be equal to zero for each agent in general equilibrium. Since the optimization problems are concave and the constraints are linear, a unique solution exists, and a solution fulfilling the necessary condition is guaranteed to be optimal. For a problem with two agents and N periods this leaves us with $4 \cdot \left(2^N - 1\right)$ first-order conditions that have to be solved for the $4 \cdot \left(2^N - 1\right)$ decision variables, asset prices, and risk-free rates simultaneously. In this section, we therefore restrict ourselves to studying a setting with $N = 10$ periods which is already numerically quite challenging as it requires us to solve a system of more than 4,000 nonlinear equations simultaneously for the more than 4,000 unknowns.[9]

In Figure 6 we illustrate the impact of varying agent 1's level of risk aversion between $\gamma_1 = 2$ and $\gamma_1 = 8$, which is in the range of values considered reasonable by Mehra and Prescott (1985). The other parameters are set to our base case parameter values, i.e. $n = 2$, $\tau = 20\%$, $\alpha_{0-,1} = 10\%$, and $\gamma_2 = 5$.

In contrast to the settings studied in the previous sections, the evolution of wealth, consumption shares and equity holdings becomes state dependent. This state dependence also complicates the graphical illustration of our results. In Figure 6 we therefore only depict results for time $t = 0$, and for time $t = 9$ in the best and worst possible state of the economy. Focusing on the latter two gives us an upper and a lower bound on the attainable consumption shares, shares of wealth and equity exposures.

In line with economic intuition, agent 1's exposure to equity increases as his level of risk aversion decreases. At time $t = 9$ his holdings in the risky asset are larger when he is the less risk averse and lower when he is the more risk averse agent. This is because when agent 1 is the less risk averse agent he is bearing a higher share of risk, implying that his wealth level is highest

[9] When solving the model numerically we make use of the fact that some of the variables can be eliminated by simple substitution. The optimal consumption strategy is not an argument in the numerical optimization as it directly follows from the budget equation (3). This is also why the number of variables in Appendix A, where we count the number of decision variables including the optimal consumption decisions as well as the Lagrange multipliers, differs from the number of arguments reported in this section.

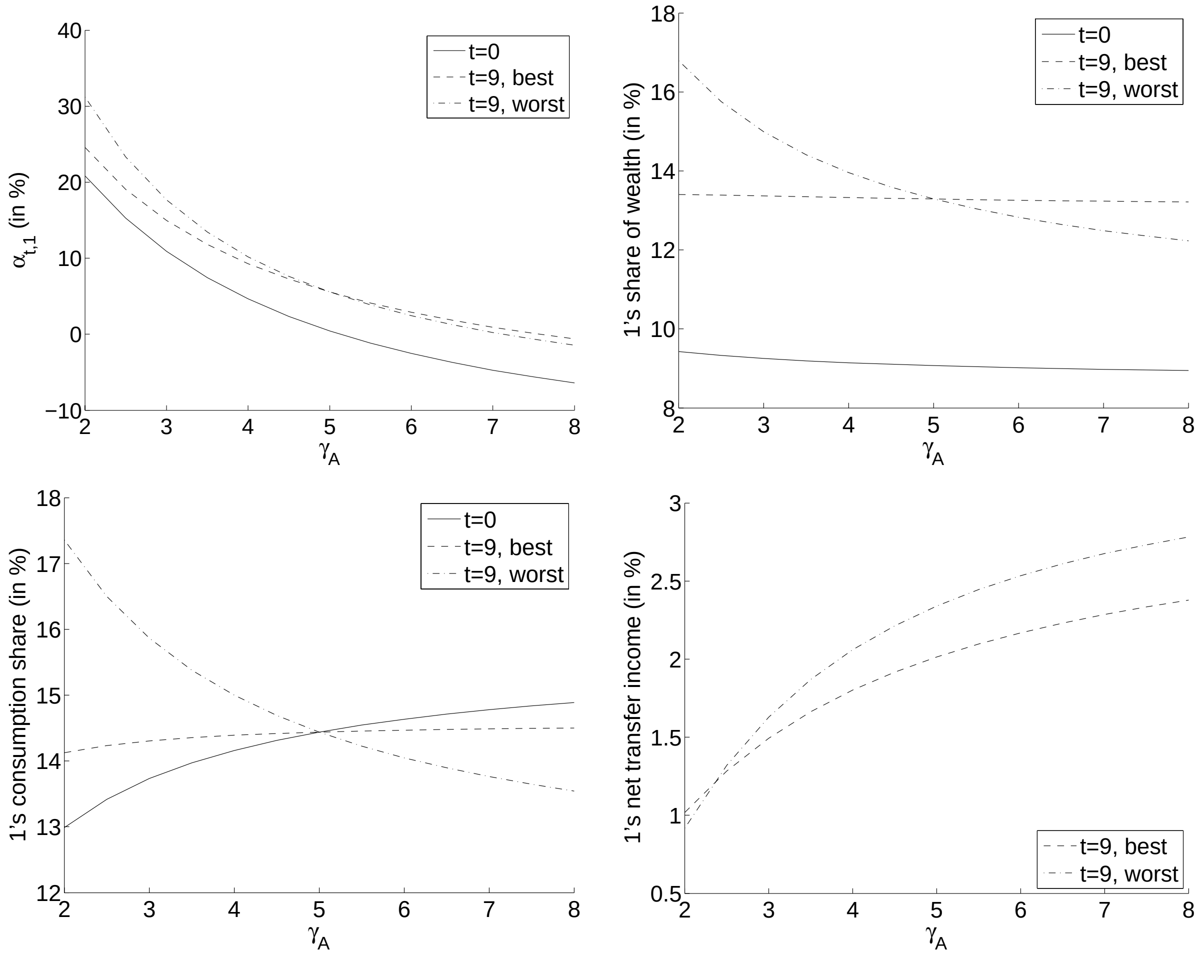

Figure 6: **Different levels of risk aversion:** This Figure shows the poorer agent 1's equity holdings (upper left graph), his share of wealth (upper right graph), his consumption share (lower left graph), and the net transfer income in percent of the present dividend payment (lower right graph) as a function of his degree of risk aversion. The richer agent's level of risk aversion is set to our base case parameter choice of $\gamma_2 = 5$. As in our base case parameter setting, the tax rate and the poorer agent's initial claim on aggregate consumption are set to $\tau = 20\%$ and $\alpha_{0-,1} = 10\%$. The length of the investment horizon is set to $N = 10$ periods. The solid lines shows results at time $t = 0$, the dashed and dash-dotted lines for the best and worst possible states of the economy at time $t = 9$.

after economic booms. With the difference between the wealth levels being large enough, it can happen that agent 1's transfer income in the bust state exceeds that in the boom state (lower right graph). Similar to our results in previous periods we see that agent 1 increases his savings (upper right graph) and exposure to equity (upper left graph) to be able to finance his desired consumption level.

The lower left graph, depicting agent 1's consumption share does not only show that agent 1's consumption share in the best state exceeds that in the worst state when agent 1 is the less risk averse agent. It also indicates that the evolution of agent 1's consumption share becomes time dependent. Specifically, agent 1's consumption share at time $t = 0$ exceeds his consumption share at time $t = 9$ when agent 1 is the more risk averse agent. This reflects agent 1's incentive to limit the growth of his wealth level to avoid sharply reducing his future transfer income. It is driven through two main channels.

First, it is affected by the evolution of the risk-free rate over time. When agents differ in their level of risk aversion, the level of the risk-free rate is no longer determined by Theorem 1, item 3. Instead, it depends on the shares of wealth invested by the agents. As the wealth distribution changes in favor of the relatively most risk averse agent, the risk free rate of interest decreases.

Second, for our agents with CRRA preferences, the elasticity of intertemporal substitution is inversely related to the degree of risk aversion. I.e., when agent 1's level of risk aversion increases, his elasticity of intertemporal substitution decreases, implying that he reacts stronger to the decrease in the interest rate over time. Consequently, when agent 1 is the relatively more risk averse agent, he has a stronger preference for present than future consumption, which is why his consumption share decreases over time. Simultaneously, this implies that the level of his future net transfer income increases.

Empirically, Guiso and Paiella (2008) document that poorer agents tend to exhibit higher levels of risk aversion than richer agents. Such a dynamic development may well be in accordance with a Pareto optimal intertemporal situation when agents are heterogenous with respect to their degrees of risk aversion, because the least risk averse agents tend to receive a disproportionate share of the aggregate risk premium. In the same vein, recent theoretical papers have analyzed a phenomenon known as "relative extinction". This means that some investors – typically the least risk averse investors – systematically drive out other investors in a dynamic framework and asymptotically end up owning the entire economy; see, e.g., Cvitanić and Malamud (2010) and references therein. However, although the distribution of wealth empirically shows tendencies towards being more and more unequal, the tax policy in many countries tend to provide some sort of redistribution mechanism that acts against such a development becoming too extreme.

In our model, this has two important implications in general equilibrium. First, compared to a setting where agents have identical degrees of risk aversion, the level of net transfer payments should increase. Second, the higher level of risk aversion combined with the higher imputed level of stock market risk should further decrease poorer agents' incentives to hold long positions in the stock market. I.e., allowing for heterogeneity in agents' risk aversion can quantitatively enhance our key findings that poorer agents hold less equity and that "poorer agents remain poor".

7 Conclusion

In this paper, we document how a redistributive tax system that taxes interest, dividends and other profits from investments and redistributes tax revenues in an attempt to reduce the disparity in the distribution of wealth among agents affects optimal consumption-investment strategies as well as the distribution of wealth in general equilibrium.

Our model allows us to contribute to the literature along two dimensions. First, we show that even if the government implements the transfer mechanism attempting to reduce the disparity in the distribution of wealth among the agents, this objective is not necessarily attained despite ongoing transfers from richer to poorer agents. This is so because poorer agents with below-average wealth levels have an incentive to use their transfer income to finance present consumption. That is, they optimally intentionally keep their wealth at low levels to avoid reducing their future transfer income.

Second, we show that the dependency of the evolution of tax revenues on the evolution of the stock market implies that the level of poorer agents' transfer income depends on the evolution of the stock market. That is, poorer agents are already endowed with stock market risk via their transfer income. As a consequence, they optimally investment less into stocks than they would do in the absence of a redistributive tax system. Our work thus also contributes to help explaining the empirically documented puzzle that poorer agents participate in the stock market less frequently than richer agents.

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A Appendix

A.1 Proof of Theorem 1

We first restate the optimization problem for an agent, as given in equations (2)-(5), and associate Lagrange multipliers λ_t to the constraints.

$$\max_{\{\{C_t\}_{t=0}^{t=N},\{\alpha_t,\beta_t\}_{t=0}^{t=N-1}\}} \frac{C_0^{1-\gamma}}{1-\gamma} + \sum_{t=1}^{N} \rho^t \mathbb{E}_0 \left[\frac{C_t^{1-\gamma}}{1-\gamma}\right] \tag{A.1}$$

s.t.

$$\begin{aligned} C_t = & \left[\alpha_{t-1}(1-\tau) + \frac{\tau}{n}\right](P_t + D_t) + \beta_{t-1}\widehat{R}_{t-1} + \tau\left(\alpha_{t-1} - \frac{1}{n}\right)P_{t-1} \\ & - \alpha_t P_t - \beta_t \qquad\qquad [\lambda_t], \quad t = 0, 1, \ldots, N \end{aligned} \tag{A.2}$$

$$C_0 = (P_0 + D_0)\,\alpha_{0-} - \alpha_0 P_0 - \beta_0 \qquad [\lambda_0] \tag{A.3}$$

$$\tag{A.4}$$

where $\alpha_N = \beta_N = 0$.

The Lagrangian is as follows, where $< \cdot, \cdot >$ is the scalar product over the relevant states:

$$\begin{aligned} & \frac{C_0^{1-\gamma}}{1-\gamma} + \sum_{t=1}^{N} \rho^t \mathbb{E}_0 \left[\frac{C_t^{1-\gamma}}{1-\gamma}\right] - \lambda_0 \left[C_0 - (P_0 + D_0)\,\alpha_{0-} + \alpha_0 P_0 + \beta_0\right] \\ & - \sum_{t=1}^{N-1} \Big[< \lambda_t, C_t - \left[\alpha_{t-1}(1-\tau) + \frac{\tau}{n}\right](P_t + D_t) > \\ & \quad - < \lambda_t, \beta_{t-1}\widehat{R}_{t-1} + \tau\left(\alpha_{t-1} - \frac{1}{n}\right)P_{t-1} - \alpha_t P_t - \beta_t > \Big] \\ & \quad - < \lambda_N, C_N - \left[\alpha_{N-1}(1-\tau) + \frac{\tau}{n}\right] D_N - \tau\left(\alpha_{N-1} - \frac{1}{n}\right)P_{N-1} > \\ & \quad - < \lambda_N, \beta_{N-1}\widehat{R}_{N-1} > \end{aligned} \tag{A.5}$$

The number of decision variables for each agent is $2^{N+2}-3$. The number of Lagrangian variables for each agent is $2^{N+1}-1$, a total of $3 \cdot 2^{N+1}-4$. There is a first-order condition matching each of these variables. Additionally, there are $2^{N+1}-2$ market variables in the form of endogenously determined asset prices and interest rates. There is a market clearing condition (or "adding up constraint") matching each of these variables. Given that the optimization problems are concave and the constraints are linear, a unique solution exists, and it is sufficient to find a candidate solution fulfilling the first-order conditions.

The first-order conditions are:

$$\left(\frac{\rho}{2}\right)^t C_t^{-\gamma} = \lambda_t, \quad t = 0, 1, \dots, N \tag{A.6}$$

$$\frac{1}{2}\lambda_t P_t = \mathbb{E}_t \left[\lambda_{t+1} \left[(1-\tau)(P_{t+1} + D_{t+1}) + P_t \tau\right]\right], \ t = 0, 1, \dots, N-1 \tag{A.7}$$

$$\frac{1}{2}\lambda_t = \mathbb{E}_t \left[\lambda_{t+1}\right] \widehat{R}_t, \quad t = 0, 1, \dots, N-1 \tag{A.8}$$

First, observe that the first-order conditions are homogeneous in the following sense. If a given solutions $(\{C_t\}_{t=0}^{t=N}; \{\lambda_t\}_{t=0}^{t=N})$ satisfies the conditions, then

$$(k\{C_t\}_{t=0}^{t=N}; k^{-\gamma}\{\lambda_t\}_{t=0}^{t=N})$$

also satisfies the conditions. This also justifies that the choice of $D_0 = 1$ is an immaterial normalization.

The difference between the agents is their endowment α_{0-}, which to some extent is distorted by the transfer mechanism. However, **given** the magnitude of this distortion they will behave proportional to each other as far as the consumption pattern goes and any Pareto optimal sharing rule is linear in aggregate consumption. This is the classical foundation for the aggregation of such agents into a representative CRRA agent with the same coefficient of risk aversion γ. We can derive these classical results and show that they are valid also in our framework.

Combining (A.7) with (A.8) we have the equations for the asset price P_t:

$$\begin{aligned} P_t &= \mathbb{E}_t \left[\frac{2\lambda_{t+1}}{\lambda_t} \left[(1-\tau)(P_{t+1} + D_{t+1}) + P_t \tau\right]\right] \\ &= \widehat{R}_t^{-1} \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\mathbb{E}_t \left[\lambda_{t+1}\right]} \left[(1-\tau)(P_{t+1} + D_{t+1}) + P_t \tau\right]\right] \end{aligned} \tag{A.9}$$

with $P_N = 0$. By moving around terms this can also be written in a pre-tax version as

$$P_t = R_t^{-1} \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\mathbb{E}_t \left[\lambda_{t+1}\right]} (P_{t+1} + D_{t+1})\right] \tag{A.10}$$

In equilibrium the pricing kernel is the ratio of the marginal utilities of consumption, which is also what successive ratios of the first-order conditions (A.6) show. This holds for all agents, which leads to the relations

$$\frac{2\lambda_{t+1}}{\lambda_t} = \rho \left(\frac{C_{t+1,j}}{C_{t,j}}\right)^{-\gamma} = \rho \left(\frac{C_{t+1,k}}{C_{t,k}}\right)^{-\gamma} \ \Rightarrow \ \frac{C_{t+1,j}}{C_{t,j}} = \frac{C_{t+1,k}}{C_{t,k}} = \frac{D_{t+1}}{D_t} \tag{A.11}$$

We conclude from here that all agents must have the same share of aggregate consumption in each of the states in all periods. Hence, when the agents have the same level of risk aversion consumption is in accordance with a linear sharing rule. The risk neutral probabilities follow directly from this. These derivations prove items 1 and 2 in Theorem 1.

We can then also conclude that the term $\beta_{t-1}\widehat{R}_{t-1} + P_{t-1}\tau\left(\alpha_{t-1} - \frac{1}{n}\right)$ in the budget equations must vanish:

$$\beta_{t-1}\widehat{R}_{t-1} + P_{t-1}\tau\left(\alpha_{t-1} - \frac{1}{n}\right) = 0 \tag{A.12}$$

This is so because it is a constant relative to the filtration at time t.[10] This means that with the wealth transfer mechanism, despite the identical coefficients of risk aversion γ, there is now a need for a bond market in order to enable the two agents to arrive at a Pareto optimal solution. Hence, we can find the bond market position in terms of the stock market position as

$$\beta_t = \widehat{R}_t^{-1}\tau\left(\frac{1}{n} - \alpha_t\right)P_t \tag{A.13}$$

This proves item 7 in Theorem 1.

Furthermore, in the standard asset pricing model with a representative CRRA agent the interest rate (before tax) is a constant with the given assumptions of identical growth rates of aggregate consumption at any node in the binomial tree. It is the expected value of the pricing kernel. In the present setting the relevant discount factor is after tax, and from (A.8) we observe that the discount factor after tax is indeed a constant due to the homogeneity property. Denoting the stochastic growth factors in aggregate consumption by $G^{\pm}$, we then know that in equilibrium it holds that

$$\widehat{R}_t^{-1} = \frac{\rho}{2}\left[\left(G^{+}\right)^{-\gamma} + \left(G^{-}\right)^{-\gamma}\right] \tag{A.14}$$

Since the same growth factors are assumed for all periods we conclude that the interest rate after tax is constant over time as well as states. We denote this constant by $\widehat{R}$.

Then we also know that the interest rate before tax, r, is a constant:

$$r = \frac{\widehat{R} - 1}{1-\tau} = \frac{\widehat{R} - 1}{1-\tau} = \frac{\widehat{r}}{1-\tau} \tag{A.15}$$

Although a constant, the rate of interest will depend on the tax rate τ. As a function of the interest rate after tax, $\widehat{r}$, it shows a mirrored hyperbolic shape as the rate of tax τ increases. This proves item 3 in Theorem 1.

Using the basic definition of the equity risk premium we can after some rewriting arrive at the expression as formulated in Theorem 1:

[10] Also called a predictable random variable, relative to the binomial filtration.

$$\frac{E_t^P[P_{t+1}+D_{t+1}]}{P_t} - R = \frac{E_t^P[P_{t+1}+D_{t+1}]}{R^{-1}E_t^Q[P_{t+1}+D_{t+1}]} - R \tag{A.16}$$

$$= R\left[\frac{E_t^P[P_{t+1}+D_{t+1}]}{E_t^Q[P_{t+1}+D_{t+1}]} - 1\right] = R\left[\frac{E_t^P[P_{t+1}+D_{t+1}] - E_t^Q[P_{t+1}+D_{t+1}]}{E_t^Q[P_{t+1}+D_{t+1}]}\right] \tag{A.17}$$

$$= R\left[\frac{E_t^P[G] - E_t^Q[G]}{E_t^Q[G]}\right] \tag{A.18}$$

The transition from (A.17) to (A.18) relies on the proportionality property that can be read off from (A.10). This proves item 4 in Theorem 1.

Looking at the budget equation for the last period we see that the consumption policy can be expressed as

$$C_N = \left(\alpha_{N-1}(1-\tau) + \frac{\tau}{n}\right) D_N \Leftrightarrow \frac{C_N}{D_N} = \alpha_{N-1}(1-\tau) + \frac{\tau}{n} \tag{A.19}$$

From (A.11) we also know that when the agents have identical risk aversion coefficients γ, consumption shares are constant. Hence, we know that

$$\frac{C_t}{D_t} = \left(\alpha_{N-1}(1-\tau) + \frac{\tau}{n}\right) \tag{A.20}$$

Combining (A.20) with (A.2) and (A.13) we arrive at a dynamic relation for the optimal equity holdings:

$$\begin{aligned}\left(\alpha_{N-1}(1-\tau) + \frac{\tau}{n}\right) D_t = &\left(\alpha_{t-1}(1-\tau) + \frac{\tau}{n}\right)(P_t + D_t) \\ &- \widehat{R}^{-1}\frac{\tau}{n}P_t + \alpha_t P_t\left(\widehat{R}^{-1}\tau - 1\right)\end{aligned} \tag{A.21}$$

where we have substituted β_t on the right hand side. By elementary algebraic manipulations this is equivalent to

$$\widetilde{\alpha}_{N-1}D_t = \widetilde{\alpha}_{t-1}(P_t + D_t) - \frac{R}{\widehat{R}}\widetilde{\alpha}_t P_t \tag{A.22}$$

where

$$\widetilde{\alpha}_t \equiv \alpha_t - \frac{1}{n} \tag{A.23}$$

Consider first $t = N-1$. Then (A.22) becomes

$$\widetilde{\alpha}_{N-1}\left[\widehat{R}D_{N-1} + RP_{N-1}\right] = \widetilde{\alpha}_{N-2}\left[\widehat{R}(P_{N-1} + D_{N-1})\right] \quad \Leftrightarrow$$

$$\widetilde{\alpha}_{N-1} = \frac{\widehat{R}(P_{N-1} + D_{N-1})}{\widehat{R}D_{N-1} + RP_{N-1}}\widetilde{\alpha}_{N-2} = X_{N-1}\widetilde{\alpha}_{N-2} \tag{A.24}$$

Next consider $t = N-2$:

$$\widetilde{\alpha}_{N-2}X_{N-1}D_{N-2}\widehat{R} = \widetilde{\alpha}_{N-3}\widehat{R}\left(P_{N-2} + D_{N-2}\right) - \widetilde{\alpha}_{N-2}RP_{N-2} \quad \Leftrightarrow \tag{A.25}$$

$$\widetilde{\alpha}_{N-2} = \frac{\widehat{R}\left(P_{N-2} + D_{N-2}\right)}{\widehat{R}X_{N-1}D_{N-2} + RP_{N-2}}\widetilde{\alpha}_{N-3} = X_{N-2}\widetilde{\alpha}_{N-3} \tag{A.26}$$

Iterating backwards through time provides a backward induction proof of item 5, equations (12)-(13), except for the claim that $X_t \in (0,1)$ for $t = 1, 2, \ldots, N-1$. This is done below.

To prove the initial relation (14) we continue in the same manner and observe that

$$C_0 = \left(\left(\alpha_{N-1} - \frac{1}{n}\right)(1-\tau) + \frac{1}{n}\right)D_0 = (P_0 + D_0)\alpha_{0-} - \alpha_0 P_0 - \beta_0 \quad \Leftrightarrow \tag{A.27}$$

$$\prod_{j=1}^{N-1} X_j \cdot \widetilde{\alpha}_0\left(1-\tau\right)D_0 = (P_0 + D_0)\widetilde{\alpha}_{0-} - \widetilde{\alpha}_0 P_0 - \beta_0 \tag{A.28}$$

Using the same substitution for β_0 as was used for β_t, $t = 1, 2, \ldots, N-1$, results in relation (14).

To complete the proof of item 5 we need to show that $X_t \in (0,1)$ for $t = 1, 2, \ldots, N-1$. We do this by backward induction. We first express the relations in terms of the price-dividend ratio $PD_t \equiv \frac{P_t}{D_t}$:

$$X_t = \frac{\widehat{R}(P_t + D_t)}{RP_t + \widehat{R}D_t\left[\prod_{j=t+1}^{N-1} NX_j\right]} = \frac{1 + PD_t}{\frac{R}{\widehat{R}}PD_t + \left[\prod_{j=t+1}^{N-1} X_j\right]} \tag{A.29}$$

Assume now that as the induction hypothesis that

$$\prod_{j=t+1}^{N-1} X_j > 1 - \frac{r\tau}{\widehat{R}}PD_t \tag{A.30}$$

which is trivially true for $t = N-1$ when $\tau > 0$.[11] Then

$$\prod_{j=t}^{N-1} X_j = X_t \prod_{j=t+1}^{N-1} X_j = \frac{1+PD_t}{\frac{R}{\widehat{R}}PD_t + \left[\prod_{j=t+1}^{N-1} X_j\right]} \prod_{j=t+1}^{N-1} X_j >$$

$$\frac{1+PD_t}{\frac{R}{\widehat{R}}PD_t + 1 - \frac{r\tau}{\widehat{R}}PD_t}\left[1 - \frac{r\tau}{\widehat{R}}PD_t\right] = 1 - \frac{r\tau}{\widehat{R}}PD_t \tag{A.31}$$

The price-dividend ratio P_t/D_t decreases over time due to the horizon effect. A stringent proof of this is slightly involved and given below. However, assuming this to be the case it follows that

$$\prod_{j=t}^{N-1} X_j > 1 - \frac{r\tau}{\widehat{R}}PD_t \;\Rightarrow\; \prod_{j=t}^{N-1} X_j > 1 - \frac{r\tau}{\widehat{R}}PD_{t-1} \;\Rightarrow$$

$$X_{t-1} = \frac{1+PD_{t-1}}{\frac{R}{\widehat{R}}PD_{t-1} + \left[\prod_{j=t}^{N-1} X_j\right]} < \frac{1+PD_{t-1}}{\frac{R}{\widehat{R}}PD_{t-1} + 1 - \frac{r\tau}{\widehat{R}}PD_{t-1}} = 1 \tag{A.32}$$

To complete the proof we need to show that $PD_t < PD_{t-1}$, which was essential for the argument leading from (A.31) to (A.32). This is also done by backward induction.

For $t = N$ the claim is trivial. Assume now that $PD_t > PD_{t+1}$. This is equivalent to

$$\frac{P_t}{D_t} - \frac{P_{t+1}}{D_{t+1}} > 0 \quad\Leftrightarrow\quad \frac{D_{t+1}}{D_t} - \frac{P_{t+1}}{P_t} > 0 \quad\Leftrightarrow\quad G_{t+1} > \frac{P_{t+1}}{P_t}$$

where G_{t+1} is the realized growth factor from time t to time $t+1$. Note that this relation is an equality between two outcomes of a binomial random variable as seen from time t. This goes for some of the following relations as well.

To proceed backwards we first note the following simple relation:

$$G_t E_{t-1}^Q[D_t] = D_t E_{t-1}^Q[G_t] \tag{A.33}$$

There is a time argument connected to the growth factor G; but since the sequence of growth factors are mutually independent and identically distributed (an iid sequence) there is no effect of using conditional expected values instead of unconditional expected values. When appropriate we simply drop this time argument.

[11] Recall the mathematical convention that the product (sum) over an empty set is one (zero).

Next, we write the preceding difference between the price-dividend ratios and make use of (A.33):

$$
\begin{aligned}
PD_{t-1} - PD_t &= \frac{1}{RD_t}\left[G_t E^Q_{t-1}\left[P_t + D_t\right] - E^Q_t\left[P_{t+1} + D_{t+1}\right]\right] \\
&= \frac{1}{RD_t}\left[G_t E^Q_{t-1}\left[P_t\right] - E^Q_t\left[P_{t+1}\right]\right]
\end{aligned}
\tag{A.34}
$$

There are two versions of the right hand side of (A.34) with the same numerical value, namely given a boom and a bust in the economy from time $t-1$ to t. Hence, the expression on the left hand side of (A.34) is positive if and only if the sum of these two expressions is positive. In terms of notation we look from a given state at time $t-1$ and use, e.g., P_t^+ as the value of P_t in the following "boom" state at time t and P_{t+1}^{-+} as the value of P_{t+1} in the "boom" state following a "bust" state at time t.

Writing out the sum in detail and using the fact that G_t is an iid sequence we have

$$
\begin{aligned}
&\frac{1}{RD_t}\left[\left(G_t^+ + G_t^-\right)\left[qP_t^+ + (1-q)P_t^-\right] - \left[qP_{t+1}^{++} + (1-q)P_{t+1}^{+-}\right] - \left[qP_{t+1}^{-+} + (1-q)P_{t+1}^{--}\right]\right] > \\
&\frac{1}{RD_t}\left[\left(G^+ + G^-\right)\left[qP_t^+ + (1-q)P_t^-\right] - qG^+\left[P_t^+ + P_t^-\right] - (1-q)G^-\left[P_t^+ + P_t^-\right]\right] = \\
&\frac{1}{RD_t}\left[1-2q\right]\left[G^+P_t^- - G^-P_t^+\right]
\end{aligned}
\tag{A.35}
$$

Given a positive risk premium we know that $q < 1/2$. Hence, to prove that the last expression is non-negative it is sufficient to look at the last term:

$$
G^+P_t^- - G^-P_t^+ = P_t^- \frac{D_{t+1}^{-+}}{D_t^-} - P_t^+ \frac{D_{t+1}^{+-}}{D_t^+} = D_{t+1}^{-+}\frac{P_t^-}{D_t^-} - D_{t+1}^{+-}\frac{P_t^+}{D_t^+} = 0 \tag{A.36}
$$

because the price-dividend ratio is the same in all states at time t and

$$
D_{t+1}^{-+} = D_{t-1}G^-G^+ = D_{t-1}G^+G^- = D_{t-1}^{+-}
$$

In total, this proves the claim that $PD_t > PD_{t+1}$ for all t by backward induction.

Finally, the claim that $\mid \alpha_0 - 1/n \mid > \mid \alpha_{0-} - 1/n \mid$ for $\tau > 0$ is equivalent to the claim that the coefficient in front of $\alpha_{0-} - 1/n$ in (14) is larger than one. Since both $R(1-\tau) < \widehat{R}$ and $(1-\tau)\left[\prod_{j=1}^{N-1} X_j\right] < 1$ we also have that

$$
(1-\tau)\left[\widehat{R}D_0\left[\prod_{j=1}^{N-1} X_j\right] + P_0R\right] < \left(P_0 + D_0\right)\widehat{R} \tag{A.37}
$$

which is equivalent to

$$
\frac{1}{1-\tau}\frac{\left(P_0 + D_0\right)\widehat{R}}{\widehat{R}D_0\left[\prod_{j=1}^{N-1} X_j\right] + P_0R} > 1 \tag{A.38}
$$

and the claim follows.

This proves item 5 of Theorem 1. To complete the proof we finally verify the relation in item 8. The net taxes paid by a given agent on the stock market position is

$$\tau\left(\alpha_{t-1}-\frac{1}{n}\right)\left(P_t+D_t-P_{t-1}\right) \tag{A.39}$$

For the bond market position there is no net tax revenue to redistribute. A given agent pays the amount $\tau r \beta_{t-1}$. Substituting from (16) this becomes

$$\tau r \frac{\tau}{\widehat{R}} P_{t-1}\left(\frac{1}{n}-\alpha_{t-1}\right) \tag{A.40}$$

Adding these two together results in the expression in (18). To proceed we consider the ratio the net transfer received in case of a boom of a bust from time $t-1$ to t:

$$\begin{aligned}
\frac{\tau\left(\frac{1}{n}-\alpha_{t-1}\right)\left(P_t^{+}+D_t^{+}-P_{t-1}\frac{R}{\widehat{R}}\right)}{\tau\left(\frac{1}{n}-\alpha_{t-1}\right)\left(P_t^{-}+D_t^{-}-P_{t-1}\frac{R}{\widehat{R}}\right)} &= \frac{P_t^{+}+D_t^{+}-\frac{1}{\widehat{R}}\mathbb{E}_{t-1}^{Q}\left[P_t+D_t\right]}{P_t^{-}+D_t^{-}-\frac{1}{\widehat{R}}\mathbb{E}_{t-1}^{Q}\left[P_t+D_t\right]} && \text{(A.41)}\\
&= \frac{D_t^{+}\left(PD_t+1\right)-\frac{1}{\widehat{R}}\mathbb{E}_{t-1}^{Q}\left[D_t\left(PD_t+1\right)\right]}{D_t^{-}\left(PD_t+1\right)-\frac{1}{\widehat{R}}\mathbb{E}_{t-1}^{Q}\left[D_t\left(PD_t+1\right)\right]} && \text{(A.42)}\\
&= \frac{\widehat{R}G^{+}-\mathbb{E}_{t-1}^{Q}\left[G\right]}{\widehat{R}G^{-}-\mathbb{E}_{t-1}^{Q}\left[G\right]} = \frac{\widehat{R}G^{+}-\mathbb{E}_{0}^{Q}\left[G\right]}{\widehat{R}G^{-}-\mathbb{E}_{0}^{Q}\left[G\right]} && \text{(A.43)}
\end{aligned}$$

which is a (time- and state-independent) constant.

$$\begin{aligned}
\alpha_t+\frac{\beta_t}{P_t}-\left(\alpha_{t-1}+\frac{\beta_{t-1}}{P_t}\right) &= \alpha_t+\widehat{R}^{-1}\tau\left(\frac{1}{n}-\alpha_t\right)-\left(\alpha_{t-1}-\widehat{R}^{-1}\tau\left(\frac{1}{n}-\alpha_{t-1}\right)\right) && \text{(A.44)}\\
&= \left(\alpha_t-\alpha_{t-1}\right)\left(1-\frac{\tau}{\widehat{R}}\right) && \text{(A.45)}\\
&= \left(\frac{1}{n}+X_t\left(\alpha_{t-1}-\frac{1}{n}\right)-\alpha_{t-1}\right)\left(1-\frac{\tau}{\widehat{R}}\right) && \text{(A.46)}\\
&= \left(\alpha_{t-1}-\frac{1}{n}\right)\left(X_t-1\right)\left(1-\frac{\tau}{\widehat{R}}\right) && \text{(A.47)}
\end{aligned}$$

This completes the proof of Theorem 1. ∎

A.2 Proof of theorem 2

Almost all the derivations and conclusions from Theorem 1 go through. The changes in the first-order conditions concern the asset pricing equations:

$$\frac{1}{2}\lambda_t P_t = \mathbb{E}_t\left[\lambda_{t+1}\left[(1-\tau_e)(P_{t+1}+D_{t+1}) + P_t\left(R\tau_e - r\tau_r\right)\right]\right], \; t = 0, 1, \ldots, N-1 \tag{A.48}$$

$$\frac{1}{2}\lambda_t = \mathbb{E}_t\left[\lambda_{t+1}\right]\widehat{R} \tag{A.49}$$

Combining (A.48) with (A.49) we have the equations for the asset price P_t:

$$P_t = \mathbb{E}_t\left[\frac{2\lambda_{t+1}}{\lambda_t}\left[(1-\tau_e)(P_{t+1}+D_{t+1}) + P_t\left(R\tau_e - r\tau_r\right)\right]\right] \tag{A.50}$$

$$= \widehat{R}^{-1}\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\mathbb{E}_t\left[\lambda_{t+1}\right]}\left[(1-\tau_e)(P_{t+1}+D_{t+1}) + P_t\left(R\tau_e - r\tau_r\right)\right]\right] \tag{A.51}$$

The same reasoning as for Theorem 1 applies as far as $\widehat{R}$ and the martingale measure goes. By moving around terms, (A.51) can be written in its pre-tax version as

$$P_t = R^{-1}\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\mathbb{E}_t\left[\lambda_{t+1}\right]}(P_{t+1}+D_{t+1})\right] \tag{A.52}$$

As for the discounting, only τ_r applies. Variations in τ_e have no effect on the discount factors. The dynamics of the stock positions are derived fully analogous to the derivations in theorem 1 above. For the bond position, both tax rates are essential.

As for item 2 we need to check when the coefficient in front of $|\alpha_{0-} - \frac{1}{n}|$ is larger than one. This is so when

$$\begin{aligned}
(P_0 + D_0)\,\widehat{R} >& (1-\tau_e)\left[RP_0 + \widehat{R}D_0\left[\prod_{j=1}^{N-1} X_j\right]\right] \quad \Leftrightarrow \\
P_0\left[\widehat{R} - R(1-\tau_e)\right] >& \widehat{R}D_0\left[(1-\tau_e)\prod_{j=1}^{N-1} X_j - 1\right] \quad \Leftrightarrow \\
P_0\left[R\tau_e - r\tau_r\right] >& \widehat{R}D_0\left[(1-\tau_e)\prod_{j=1}^{N-1} X_j - 1\right]
\end{aligned} \tag{A.53}$$

Clearly, a sufficient condition is that $R\tau_e - r\tau_r > 0$, because then the lhs of A.53 is positive, whereas the rhs is negative.

As for item 5 we first calculate the tax revenue in the economy as

$$\left(P_t + D_t - RP_{t-1}\right)\tau_e + P_{t-1}r\tau_r = \left(P_t + D_t\right)\tau_e - P_{t-1}\left(R\tau_e - r\tau_r\right) \tag{A.54}$$

and the tax payment of a given agent as

$$\alpha_{t-1}\left[\left(P_t+D_t\right)\tau_e-P_{t-1}\left(R\tau_e-r\tau_r\right)\right]+\beta_{t-1}r\tau_r \tag{A.55}$$

$$=\alpha_{t-1}\left[\left(P_t+D_t\right)\tau_e-P_{t-1}\left(R\tau_e-r\tau_r\right)\right]+\left(\widehat{R}\right)^{-1}\left(R\tau_e-r\tau_r\right)\left(\frac{1}{n}-\alpha_{t-1}\right)P_{t-1}r\tau_r \tag{A.56}$$

The net transfer payment to this agent is thus given by

$$\left(\frac{1}{n}-\alpha_{t-1}\right)\left[\left(P_t+D_t\right)\tau_e-P_{t-1}\left(R\tau_e-r\tau_r\right)\right]+\left(\widehat{R}\right)^{-1}\left(R\tau_e-r\tau_r\right)\left(\frac{1}{n}-\alpha_{t-1}\right)P_{t-1}r\tau_r$$
$$=\left(\frac{1}{n}-\alpha_{t-1}\right)\left[\left(P_t+D_t\right)\tau_e-P_{t-1}\left(R\tau_e-r\tau_r\right)\frac{R}{\widehat{R}}\right] \tag{A.57}$$

The relation between the net transfer payments in the boom and the bust state is thus given by

$$\begin{aligned}&\frac{\left(\frac{1}{n}-\alpha_{t-1}\right)\left[\left(P_t^{+}+D_t^{+}\right)\tau_e+P_{t-1}\left(R\tau_e-r\tau_r\right)\frac{R}{\widehat{R}}\right]}{\left(\frac{1}{n}-\alpha_{t-1}\right)\left[\left(P_t^{-}+D_t^{-}\right)\tau_e+P_{t-1}\left(R\tau_e-r\tau_r\right)\frac{R}{\widehat{R}}\right]}\\ =&\frac{D_t^{+}\left(PD_t+1\right)\tau_e+\frac{1}{\widehat{R}}\mathbb{E}_{t-1}^{Q}\left[D_t\left(PD_t+1\right)\right]\left(R\tau_e-r\tau_r\right)}{D_t^{-}\left(PD_t+1\right)\tau_e+\frac{1}{\widehat{R}}\mathbb{E}_{t-1}^{Q}\left[D_t\left(PD_t+1\right)\right]\left(R\tau_e-r\tau_r\right)}\\ =&\frac{G^{+}\tau_e+\frac{1}{\widehat{R}}\mathbb{E}_{t-1}^{Q}\left[G\right]\left(R\tau_e-r\tau_r\right)}{G^{-}\tau_e+\frac{1}{\widehat{R}}\mathbb{E}_{t-1}^{Q}\left[G\right]\left(R\tau_e-r\tau_r\right)}=\frac{G^{+}\tau_e+\frac{1}{\widehat{R}}\mathbb{E}_{0}^{Q}\left[G\right]\left(R\tau_e-r\tau_r\right)}{G^{-}\tau_e+\frac{1}{\widehat{R}}\mathbb{E}_{0}^{Q}\left[G\right]\left(R\tau_e-r\tau_r\right)}\end{aligned} \tag{A.58}$$

This completes the proof of theorem 2. ∎