A Passion for Democracy*

Elena Panova†

Abstract

Classic voting theories assume that voting choices are dictated by rational preferences over policy alternatives. They justify positive turnout despite small pivot probabilities by a warm glow from the very act of voting. This approach does not allow to model "irrational" behaviors, namely: habitual voting, conformity with a majority, voter ignorance. We propose a dynamic model of a majority vote with asymmetric information accommodating these behaviors. Our approach allows us to analyze the effect of public information on the vote. We find that public information biased in favour of one alternative has an adverse effect on the efficiency of majority outcome.
Key words: voting, asymmetric information, warm glow, cognitive dissonance, self-signaling, habitual voting, conformity with a majority, voter ignorance, public information and voting behavior, inefficient policy persistence.
JEL codes: D03, D71, D72, D82, D83, P16.

*I am grateful to Roland Bénabou, Christian Hellwig, Augustin Landier, Onur Özgür, Wolfgang Pesendorfer, Karin Van der Straeten, Anton Suvorov and especially to Bruno Biais for helpful discussions. I also thank seminar participants at the University of Copenhagen for their attention to my work.
†Academy of National Economy, Vernadskogo prospect 82, Moscow, 119571, Russia; e-mail: e_panova@yahoo.com.

1 Introduction

Why do people vote? How do they vote? For the most part,[1] economic theory proposes independent answers to these questions (see surveys by Aldrich, 1993; Feddersen, 2004; Dhillon and Peralta, 2002; Merlo, 2006; Geys, 2006). An individual vote is not influential. However, a voter is attracted to the ballot box by a warm glow from the very act of voting.[2] The warm glow is due to fulfillment of a civic duty (Riker and Ordeshook, 1968) or else a joy of supporting a most preferred policy alternative (Brennan and Buchanan, 1984).[3] A voter has rational preferences over policy alternatives. If he participates, then he votes so as to maximize the electoral fortunes of his most preferred alternative.

This approach does not accommodate "irrational" patterns observed in voting behavior. One pattern is called *habitual voting*: participation in one election causes nearly a 50 percentage-point increase in the propensity to vote in the next election. This effect is established by both instrumental variables analysis of the American National Election Studies data (Green and Shachar, 2000) and a randomized field experiment (Gerber, Green, and Shachar, 2003).[4] Notably, these studies agree on the size of the effect, emphasizing that the experience in voting is a much stronger predictor of turnout than demographic characteristics such as age or education (Wolfinger and

[1]Ferejohn and Fiorina (1974) is an exception. They assume that a voter minimizes his regret should he fail to provide the decisive support to his most preferred candidate or policy. Thereby, his attention is focused on the situation in which he is pivotal, and turnout paradox is removed.

[2]James Andreoni introduced "warm-glow" as being the pleasure from charitable giving.

[3]These seem to be the most relevant aspects of voter motivation. For a recent statistical analysis of poll data see Carlsson and Johansson-Stenman (2010). For concrete examples, read voter reports on their motivation during the last three US Presidential elections on http://freakonomics.blogs.nytimes.com: "*I always pick up my dog's poop...I enjoy reading about policy and politics and voting is my way of picking a team.*"

[4]Persistence in voting behavior is supported by earlier studies (see references in Gerber, Green and Shachar, 2003). For example, Firebaugh and Chen (1995) find that "disenfranchisement had enduring pernicious effects on Nineteenth Amendment women but not on their postamendment daughters and granddaughters" (the cohort effect).

Rosenstone, 1980).[5] Green and Shachar (2000) suggest a reason may be that "civic participation subtly alters the way that citizens look at themselves" or else that "going to the polls alters positive or negative feelings about engaging in the act of voting itself".

The other pattern is a tendency to *conform with a majority*: Several studies from Bartels (1988) to Cloutier et al. (2010) describe electoral bandwagons.[6] Nadeau et al. (1993) find bandwagon effect in a laboratory experiment. Coleman (2004) proposes a test for conformity in voting behavior based on a positive correlation between the entropies for turnout and voting decision. Using this test, he finds conformity in voting during elections in the US and Western Europe over most of the twentieth century, as well as during recent elections in Eastern Europe and Russia.

The third pattern is called *voter ignorance*. Caplan (2007) distinguishes four major biases in voter beliefs regarding economic policies: underestimation of the market efficiency, underestimation of benefits from international trade, association of prosperity with employment rather than with production, pessimism about overall economic conditions. Numerous polls reveal low factual knowledge about such issues as: distribution of the state budget (see references in Bartlett, 2011), the level of unemployment (Ansolabehere, Meredith and Snowberg, 2009) or term limits (Romano, 2011). Notably, about 70% of participants in the American National Election Studies polls agree that "politics is too complicated." Still, many Americans vote.[7]

We propose a dynamic model of voting with asymmetric information accommodating these "irrational" patterns. Our approach is also relevant in that it allows us to analyze the effect of public information (political news, campaign advertizing) on the vote.[8] We find that public information favoring

[5]Citizens with postgraduate education have no more than 26 percentage points higher propensity to vote than the high-school graduates. 80 year old citizens have about 30 percentage points higher propensity to vote than 20 year old voters.

[6]Sher (2011) isolates bandwagon effect from strategic voting.

[7]The average turnout in the US from 1968 to 2008 is 55.58% in Presidential elections and 46.63% in Congressional elections (U.S. Census Bureau).

[8]Della Vigna and Kaplan (2007) find that Republicans gained votes in US towns which

one particular alternative has an adverse effect on information aggregation, which is an argument for political pluralism.

Our basic game considers two successive majority votes over public policy. In each vote, there are two policy alternatives. The voters have common policy interests. Only a minority of them receives information as to which alternative is superior. A voter's warm glow from participation is equal to his confidence in supporting the superior alternative (akin to Matsusaka, 1995). Maximization of intertemporal warm glow determines both his choices: participation and voting.[9] He remembers his choices but not his confidence in them. This behavioral assumption is due to Bénabou and Tirole (2002). It reflects cognitive dissonance reduction described by a sizable psychological literature (see survey by Harmon-Jones et al., 2009) and observed in voting (Mullainathan and Washington, 2007; Gerber, Huber and Washington, 2009).[10]

If the voters would vote only once, the informed voters would vote for the superior policy and the uninformed voters would abstain. However, this behavior is out of equilibrium in our game with repeated voting: if today the informed voters vote and the uninformed voters abstain, then each uninformed voter would like to deviate and vote in attempt to pool with the informed voters. Recall that his memory is going to retain only his behavior. If he abstains today, he will also abstain tomorrow because he will know that

introduced Conservative Fox News Channel between October 1996 and November 2000. In the randomized field experiment by Gerber, Karlan and Bergan (2006) subscription for a new press outlet increased the probability of voting Democratic in 2005 Virginia gubernatorial election. Enikolopov, Petrova and Zhuravskaya (2011) find that during 1999 parliamentary elections in Russia exposure to news from the only independent TV channel decreased the aggregate vote for the government party and increased the combined vote for major opposition parties. Gordon and Hartman (2011) find significant positive effect of advertising on voting choices in 2000 and 2004 general presidential elections in the US. Finally, in a laboratory experiment by Ladha (2005) the subjects playing the role of committee members rely much on the public signal.

[9] A voter implicitly assumes small pivot probabilities.

[10] Put loosely, an individual who holds interrelated but dissonant elements of knowledge experiences discomfort and changes his cognitions so as to reduce the dissonance. Memory about recent behavior is most resistant to change.

he uninformed. If he behaves as an informed voter today, he will perceive himself an informed voter tomorrow and therefore receive the warm glow from voting.[11] Hence, he votes today in order to enjoy voting tomorrow: he develops a habit to vote.

In the unique symmetric equilibrium[12] we find a high participation by the uninformed voters. This finding comports nicely with the observed voter ignorance. However, the informed voters have an even higher motivation to vote,[13] and their votes are more coherent: they all vote for the superior policy. Therefore, the majority is likely to choose the superior policy. Hence, the voters in the majority receive a positive feedback on their voting choice, which increases their self confidence, hence, their warm glow from the future voting. This is their benefit of conforming with majority choice.

If some public information is available, the uninformed voters rely heavily on this information in their voting choices, but it is done without herding. The reason is that an uninformed voter who differentiates from his peers receives signaling benefits, reminiscent of an investor who bets against a financial bubble. However, the stronger the public information in support of one particular alternative, the more coherent the uninformed votes, hence, the less informative the majority outcome.

Roadmap This paper is organized as follows. Section 2 relates our work to the existing literature. Section 3 presents the basic voting game. Section 4 describes its unique equilibrium. Section 5 presents comparative static analysis with respect to the precision of public information. Section 6 presents a natural extension of the basic game to an overlapping generation

[11] Curiosly, respondents of post- electoral surveys report a higher participation and support to the winners than the actual figures. The reason is that loosers and abstainers repond less to such surveys (see Crow et al. 2010 and references therein).

[12] We focus on symmetric equilibria following Mayerson's argument that identity of every voter in a large election can hardly be assumed a common knowledge. We find the unique equilibrium because the informed voters' dominant strategy is to vote their signal.

[13] This insight is also relevant. For example, Lassen (2005) finds that voter information increases the propensity to vote.

game with an infinite horizon which accommodates policy persistence. Section 7 outlines three main directions for future research. Technical proofs are in the Appendix.

2 Relationship to the literature

We assume that a voter's confidence in his choice at the ballot box increases his immediate payoff from participation. This assumption relates our model to Matsusaka (1995), Degan (2006), and Degan and Merlo (2011). However, they analyze static games with symmetric information, in which the voters have rational heterogenous policy preferences,[14] while we consider a self-signaling game. This is why we find irrationalities in voting which those models do not predict.

Commonality of preferences and informational asymmetry relate our model to Feddersen and Pesendorfer (1996). However, the focuses are complementary: we model participation and voting choices, while they model abstention. Their model is static, and their voters care for the efficiency of the outcome, and not for their private warm glow from participation. A voter conditions his behavior on the situation in which he is pivotal. If he is informed, he votes. If he is uninformed, he abstains, so as not to jam the informed votes.[15] Public information is not influential.

The effect of habitual voting leads us to adaptive models by Bendor, Diermeier and Ting (2003) and Fowler (2006), assuming that, according to a given rule, voting today affects the future propensity to vote. A voter's

[14]Degan (2006) and Degan and Merlo (2011) consider unidimensional policy space. A voter has a sense of civic duty, hence, some warm glow from participation. He knows his "bliss point", but he is uncertain about the locations of two competing policy platforms. His turnout cost is equal to the probability of supporting the platform which is the furthest from his "bliss point". The voters located at the extremes are more confident in their choices than centrally located voters. Therefore, participation among the extreme voters is relatively high.

[15]The uninformed voters participate just enough to offset ideological bias created by partizan voters.

turnout stochastically depends on his propensity to vote. Naturally, the insights are sensitive to the choice of the rule.[16] We model habitual voting without assuming path dependencies: An active voter is likely to receive a positive feedback from his peers, which motivates him to vote again. An abstainer is guaranteed no such feedback, so he continues to abstain.

Voter benefit from conforming with a majority brings us to Callander (2008), Rotemberg (2009) and Shuessler (2000), but mainly to Callander's work.[17] He assumes a benefit from being on the winners' side, creating thereby the social-multiplier effect, hence, the multiplicity of equilibria.[18] In some of them information aggregation is negative. In our game, today's winners benefit from a high confidence in their tomorrow's voting choices, because information aggregation is nonnegative. The higher information aggregation, the higher the winners' signaling benefits.

While different aspects of our model relate it to earlier theories of the vote, the model builds on the literature which is not specific to voting. Our voter engages in self-signaling using imperfection of memory. This mechanizm is due to Bénabou and Tirole (2002). Bénabou and Tirole (2006) and Bénabou (2008, 2009) incorporate it into large games in order to model collective beliefs. In Bénabou (2009), the players simultaneously manipulate the extent to which they remember the initial signal about the underlying state of the world. Then, they act according to the retained information. At the end of the game, a player receives a "material" payoff which depends on his own action and on the other players' actions. A player would like to remain optimistic about this payoff during the game. If he expects the other

[16]Bendor, Diermeier and Ting (2003) use the Bush-Mosteller rule. Fowler (2006) proposes the reinforcement rule with a higher empirical relevance.

[17]Shuessler and Rotemberg also assume complementarities in voting: In Shuessler's model, voting is a way to identify yourself with a group of people voting in the same way. The identification benefit is a $\cap$-shape function of the group's size. In Rotemberg's model, a voter votes in order to aware the like-minded voters that he shares their policy preferences: they care to know and he cares for them.

[18]The social multiplier effect generates a variety of outcomes for similar fundamentals in different contexts (see surveys by: Scheinkman, 2008; Postlewaite, 2010).

players to act blindly and these actions are harmful, then he sticks his own head in the sand so as not to acknowledge miserableness of the approaching reality. Hence, the players may engage themselves in collective illusion. In our game, the players care for their feelings about their actions, and not for their anticipatory feelings. Therefore, they do not create collective illusions.

The adverse effect of public information on the majority outcome leads us to the literature on the social value of public information which is rooted in Morris and Shin (2002). They show that if there are strategic complementarities in players' actions, then public information has an ambiguous welfare effect: on one hand it informs the players, on the other hand it provides them with the incentives to ignore their valuable private information. In our game, the players who use public information have no private information. However, public information creates some coherence in their actions and thereby triggers aggregation of private information by the other players.

3 Basic model

The voters with common values select public policy by a simple majority rule. There are two successive votes, indexed with $t = 1, 2$.[19]

Policy alternatives There are two alternative policies: "0" and "1". The efficient policy is equal to the state variable x_t which is drawn before each vote from the diffuse Bernoulli distribution:

$$\Pr(x_t = j) = \tfrac{1}{2},\ j = 0, 1. \tag{1}$$

For now, we assume that states x_1 and x_2 are not correlated.[20] Policy-winning vote t is denoted with a_t.

[19] Timing of the events is summarized at the end of this section.

[20] Section 6 extends the game to an infinite number of elections with correlated states.

Voter types and signals There is a continuum of voters with a mass of unity, indexed by $i \in [0,1]$. At the start of the game, Nature draws type θ^i by voter i: *informed* ($\theta^i = 1$), with probability α; *uninformed* ($\theta^i = 0$), with probability $1-\alpha$. Most voters are uninformed, that is, $\alpha < \frac{1}{2}$.[21]

Before vote t, voter i receives private signal σ_t^i on the state x_t. If he is informed, his signal is perfect; if he is uninformed, his signal is diffuse:

$$\sigma_t^i = \theta^i x_t + (1-\theta^i) z_t^i, \tag{2}$$

where variable z_t^i is an independent draw from distribution (1).

Voter information and strategies during vote 1 Before vote 1, the voters receive public signal σ of quality q on the state x_1:[22]

$$\Pr(x_1 = 0 \mid \sigma = 0) = \Pr(x_1 = 1 \mid \sigma = 1) = q \geqslant \tfrac{1}{2}. \tag{3}$$

Hence, information set by voter i is

$$\Omega_1^i = \left\{\theta^i, \sigma, \sigma_1^i\right\}. \tag{4}$$

Given information (4), voter i can take one of the following actions: (i) vote for policy "0" ($v_1^i = 0$); (ii) vote for policy "1" ($v_1^i = 1$); (iii) abstain from voting ($v_1^i = \varnothing$). Hence, his pure strategy is mapping

$$v_1(\theta^i, \sigma, \sigma_1^i): \ \{0,1\}^3 \to \{\varnothing, 0, 1\}. \tag{5}$$

Voter information and strategies during vote 2 Voting behavior v_1^i stays in memory by voter i, but not his type θ^i or signals σ and σ_1^i.[23] Everyone can see public policy a_1 chosen by a majority. However, there is no

[21]Recall a sizable evidence of voter ignorance cited in the Introduction.

[22]We assume that the voters receive public signal before vote 1 but not before vote 2. Our insights are robust if the voters receive public signal before each vote. The only difference is that the second public signal decreases participation incentives by the uninformed voters.

[23]Recall references to the literature on cognitive dissonance in Section 1.

direct feedback on its efficiency: state x_1 remains hidden. Hence, information set by voter i during vote 2 is

$$\Omega_2^i = \left\{v_1^i, a_1, \sigma_2^i\right\}, \tag{6}$$

and his pure voting strategy is mapping

$$v_2(v_1^i, a_1, \sigma_2^i): \ \{\varnothing, 0, 1\} \times \{0,1\}^2 \to \{\varnothing, 0, 1\}. \tag{7}$$

Posteriors $\Pr\left(\theta^i = 1 \mid \Omega_2^i\right)$ by voter i are called *self confidence*.

Voter objectives Following classic voting games, we assume that an active voter receives some warm glow from participation. He experiences the warm glow when he votes his private signal, because he expresses his deep-seated opinion.[24] The warm glow is equal to the subjective probability of supporting the efficient policy less that of supporting the inefficient policy, that is, $\Pr(v_t^i = x_t \mid \Omega_t^i) - \Pr(v_t^i = 1 - x_t \mid \Omega_t^i)$, akin to Matsusaka (1995).[25] A voter's payoff is equal to his warm glow less the turnout cost.[26] For now, we assume that the turnout cost is arbitrarily small, taking it null for notational convenience.[27] Hence, date t payoff by voter i is equal to

$$U(v_t^i, \Omega_t^i) = \begin{cases} \Pr(v_t^i = x_t \mid \Omega_t^i) - \Pr(v_t^i = 1 - x_t \mid \Omega_t^i) & \text{if } v_t^i = \sigma_t^i; \\ 0, \text{ otherwise.} \end{cases} \tag{8}$$

[24]This assumption isolates herding on public signal. Nevertheless, we will show that the uninformed voters mostly vote on the public signal.

[25]Our voter benefits from motivated participation. He increases his motivation through reducing cognitive dissonance. Hence, we model action-based motivation behind cognitive-dissonance processes, building on a sizable evidence in Harmon-Jones et al. (2009). The most prominent alternative motivations described therein are increasing self-perception or impression to others. Accordingly, we could assume that the voters maximize their self confidence. Such objectives would naturally create multiple equilibria, each characterized by voting strategy played by the informed voters, including the equilibrium in which the informed voters vote their signals. This is the unique equilibrium of our game.

[26]Given the large size of our voting game, we isolate "instrumental" objectives. They become influential if and only if they are given lexicographic superiority. However, even then we would find an equilibrium in which the majority outcome is the same as the public signal no matter what the state, provided there is a sufficiently strong signal.

[27]The last subsection of Section 5 analyzes the game with a higher turnout cost.

Sequence of events

Nature draws the voters' types. The voters learn their types.

Date 1.

a. Nature draws: state x_1, public signal σ and private signals σ_1^i. The voters receive their signals.

b. Vote 1 takes place.

The voters forget their types and signals.

Date 2.

a. Nature draws state x_2 and private signals σ_2^i. The voters receive their signals.

b. Vote 2 takes place.

4 Equilibrium of the game

This section describes the unique symmetric[28] Perfect Bayesian Equilibrium of the game, hereafter, equilibrium. Note first of all, that by equations (2) and (8) the warm glow experienced by an active voter is equal to his confidence in his voting choice:

$$U(v_t^i, \Omega_t^i) = \Pr\left(\theta^i = 1 \mid \Omega_t^i\right). \tag{9}$$

Now, consider the votes in the reversed order. During vote 2, voter i maximizes his immediate warm glow. He votes his signal if his self confidence is positive, and he abstains from voting otherwise. Formally, by equation (9),

$$v_2^i = \sigma_2^i \text{ if } \Pr\left(\theta^i = 1 \mid \Omega_2^i\right) > 0;\, v_2^i = \varnothing \text{ if } \Pr\left(\theta^i = 1 \mid \Omega_2^i\right) = 0. \tag{10}$$

[28] The agents of the same type with the same signals play the same strategy.

By Bayes rule, self confidence by voter i depends on two signals retained from vote 1: his voting behavior v_1^i and the majority outcome a_1:[29]

$$\Pr\left(\theta^i = 1 \mid \Omega_2^i\right) = \frac{\alpha \Pr\left(v_1^i, a_1 | \theta^i = 1\right)}{\alpha \Pr\left(v_1^i, a_1 | \theta^i = 1\right) + (1-\alpha) \Pr\left(v_1^i, a_1 | \theta^i = 0\right)}. \tag{11}$$

Trivially if voter i is informed, he pools with the informed voters during vote 1:[30]

$$\text{if } \theta^i = 1, \text{ then } v_1^i \in \operatorname{Im}(v_1(1, \sigma, \sigma_1^i)) \text{ , hence } \Pr\left(v_1^i, a_1 \mid \theta^i = 1\right) > 0.$$

Therefore, his self confidence is positive and he votes his signal:

$$\text{if } \theta^i = 1, \text{ then } \Pr\left(\theta^i = 1 \mid \Omega_2^i\right) > 0 \text{ and } v_2^i = \sigma_2^i. \tag{12}$$

Hence, the informed voters (mass α) vote for the efficient policy. The votes by the uninformed voters, if any cast, "cancel out" because their signals have no systematic component. The efficient policy wins:

$$a_2 = x_2. \tag{13}$$

Now, consider vote 1. A voter maximizes his intertemporal warm glow from voting (today and tomorrow). His today's voting behavior affects his self confidence, and thereby, his tomorrow's warm glow. Without accounting for this effect, the informed voters would vote their signals, and the uninformed voters would abstain from voting. However, if all voters behave in this way, an uninformed voter would like to deviate and vote, no matter how: with probability $\frac{1}{2}$ he pools with the informed voters today, thereby winning perfect self confidence, hence, the highest warm glow tomorrow. More generally, under full separation of types, an uninformed voter is tempted to imitate behavior by the informed voters. Therefore, this situation is out of equilibrium.

[29] His new signal σ_2^i is irrelevant because the states x_1 and x_2 are independent.

[30] We use standart notation $\operatorname{Im}(v_1(\theta^i, \sigma_1, \sigma_1^i)) = \left\{v_1^i \mid v_1^i = v_1(\theta^i, \sigma_1, \sigma_1^i)\right\}$.

In equilibrium, there is some pooling of types:

$$\mathrm{Im}(v_1(1,\sigma,\sigma_1^i)) \cap \mathrm{Im}(v_1(0,\sigma,\sigma_1^i)) \neq \varnothing, \tag{14}$$

the informed voters vote their signals:

$$v_1(1,\sigma,\sigma_1^i) = \sigma_1^i, \tag{15}$$

and the uninformed voters participate in voting, at least to some extent. They may bias the outcome towards one of the policies, however, not towards different policies at once. Hence, information aggregation is nonnegative:

$$a_1 = j \text{ in state } x_1 = j \text{ for at least one } j \text{ in set } \{0,1\}. \tag{16}$$

The following three sections describe equilibrium of the game depending on whether information aggregation is: perfect, null or imperfect. The equilibrium is characterized by equations (10), (13), (15) and the voting strategy by the uninformed voters at date 1 which remains to describe.

Notation 1 (the uninformed voters' strategy):

$$v_1(0,\sigma,\sigma_1^i) = \begin{cases} \sigma, \textit{ with probability } p_\sigma; \\ 1-\sigma, \textit{ with probability } p_{1-\sigma}; \\ \varnothing, \textit{ with probability } 1-p_\sigma-p_{1-\sigma}. \end{cases} \tag{17}$$

Informative equilibrium Suppose that information aggregations is perfect:

$$a_1 = x_1. \tag{18}$$

Consider vote 1. Without loss of generality, suppose voter i is uninformed. If he abstains today ($v_1^i = \varnothing$), then his immediate payoff is null; his self confidence remains null:

$$\Pr\left(\theta^i = 1 \mid v_1^i = \varnothing, a_1\right) = 0, \tag{19}$$

and he abstains tomorrow once again ($v_2^i = \varnothing$). If he votes today, no matter how, he pays an arbitrary small turnout cost without receiving any warm glow immediately. However, with probability $\frac{1}{2}$, he pools with the majority, and thereby builds self confidence

$$\Pr\left(\theta^i = 1 \mid v_1^i = j, a_1 = j\right) = \tfrac{\alpha}{\alpha + (1-\alpha) p_j}, \tag{20}$$

which is his tomorrow's warm glow. Hence, abstention is a dominated strategy by the uninformed voters:[31]

$$p_\sigma + p_{1-\sigma} = 1. \tag{21}$$

How do they vote? If they all vote for the same policy, this policy wins no matter what the state. This is generically inefficient: equation (18) is false in one of the states. For equation (18) to be true, the uninformed voters must randomize between voting for different policies. Hence, they must be indifferent between voting for different policies which is true if and only if their expected self confidence does not depend on the way in which they vote:

$$\tfrac{q\alpha}{\alpha + (1-\alpha) p_\sigma} = \tfrac{(1-q)\alpha}{\alpha + (1-\alpha) p_{1-\sigma}}. \tag{22}$$

By equations (21) and (22), the voting probabilities are:

$$p_\sigma = q + \tfrac{\alpha}{1-\alpha}(2q-1) \text{ and } p_{1-\sigma} = 1 - q - \tfrac{\alpha}{1-\alpha}(2q-1) \tag{23}$$

$$\text{if } q \leqslant \tfrac{1}{1+\alpha}; \tag{24}$$

and $p_\sigma = 1$ and $p_{1-\sigma} = 0$ otherwise.

Notably, the uninformed voters tend to vote on the public signal the more, the stronger the signal: $p_\sigma - p_{1-\sigma} = (2q-1)\frac{1+\alpha}{1-\alpha}$. If the public signal is true ($\sigma = x_1$), they increase the margin of victory for the efficient policy. If the public signal is false ($\sigma = 1 - x_1$), they increase the vote margin for the inefficient policy. The efficient policy wins if and only if the public signal is sufficiently weak:[32]

$$q \leqslant \tfrac{2\alpha+1}{2(1+\alpha)}. \tag{25}$$

[31] Naturally, sufficiently high turnout cost creates some abstention: see Section 5.

[32] Note that threshold (25) lies below thershold (24), because $\alpha < \frac{1}{2}$.

Proposition 1 (informative equilibrium) *Suppose that the public signal is sufficiently weak, as described by inequality (25). Then, the unique equilibrium of the game is as follows. During vote 1, the informed voters vote their signals. The uninformed voters play a voting strategy described by a set of equations (23). The efficient policy wins, as described by equation (18). During vote* 2*, the winners of vote* 1 *vote their signals; the losers abstain. Once again, the efficient policy wins.*

Uninformative equilibrium Suppose now that the information aggregation is null. Namely, suppose that the majority outcome coincides with the public signal no matter what the state, that is,

$$a_1 = \sigma \text{ for any } x_1: \tag{26}$$

we will see that these two statements are equivalent. Once again, the uninformed voters participate as described by equation (21), because their self confidence is null if they abstain and it is positive otherwise:

$$\Pr\left(\theta^i = 1 \mid v_1^i = \sigma\right) = \tfrac{\alpha q}{\alpha q + (1-\alpha) p_\sigma}; \tag{27}$$

$$\Pr\left(\theta^i = 1 \mid v_1^i = 1-\sigma\right) = \tfrac{\alpha(1-q)}{\alpha(1-q) + (1-\alpha) p_{1-\sigma}}. \tag{28}$$

Once again, they play a mixed-voting strategy. Indeed, equation (26) allows for only one pure strategy, namely, to vote on the public signal. However, if all the uninformed voters vote on the public signal each of them would like to vote against it in order to build perfect self confidence instead on an imperfect one: $\frac{\alpha q}{\alpha q + 1 - \alpha} < 1$. Playing a mixed strategy requires indifference between the pure strategies. Hence, voting for different policies should deliver the same expected self confidence:

$$\tfrac{\alpha q}{\alpha q + (1-\alpha) p_\sigma} = \tfrac{\alpha(1-q)}{\alpha(1-q) + (1-\alpha) p_{1-\sigma}}. \tag{29}$$

By equations (21) and (29), the voting probabilities are:

$$p_\sigma = q \text{ and } p_{1-\sigma} = 1 - q. \tag{30}$$

Policy σ wins in any state, as described by equation (26) if and only if the public signal is sufficiently strong, namely,

$$q \geqslant \tfrac{1}{2(1-\alpha)}. \tag{31}$$

It remains to show that outcome a_1 is uninformative if and only if it is described by equation (26). Suppose that the outcome is uninformative. Then, self confidence and behavior by the uninformed voters are described by equations (27), (28) and (30). Thus, outcome a_1 is the same as the public signal, at least when the signal is true. In order to be uninformative, it must be equal to the public signal in any state.

Proposition 2 (uninformative equilibrium) *Suppose that the public signal is sufficiently strong, as described by inequality (31). Then, the unique equilibrium of the game is as follows. During vote* 1*, the informed voters vote their signals. The uninformed voters vote on the public signal with probability equal to the signal's quality, as described by set of equations (30). The majority outcome coincides with the public signal. During vote* 2*, the voters vote their signals, and the outcome is efficient.*

Semi-informative equilibrium It remains to analyze the situation in which information aggregation is imperfect. By the informativeness constraint (16), there are two possibilities: (i) Information aggregation is perfect if the public signal is false and imperfect otherwise, that is,

$$a_1 = x_1 \text{ if } x_1 = 1-\sigma; \ \Pr\left(a_1 = x_1 \mid x_1 = \sigma\right) < 1. \tag{32}$$

This can only happen if the uninformed voters tend to vote against the public signal, that is, $p_{1-\sigma} > p_\sigma$. Then, however, each of them would like to deviate and vote on the signal, so as to increase his expected self confidence:

$$\tfrac{\alpha q}{\alpha q+(1-\alpha)p_\sigma} > \tfrac{\alpha(1-q)}{\alpha(1-q)+(1-\alpha)p_{1-\sigma}}.$$

Hence, the outcome (32) cannot be sustained in equilibrium.

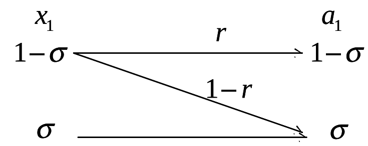

Figure 1: Information aggregation.

(ii) Information aggregation is perfect if the public signal is true and imperfect otherwise, that is,

$$a_1 = x_1 \text{ if } x_1 = \sigma; \ \Pr\left(a_1 = x_1 \mid x_1 = 1-\sigma\right) < 1, \tag{33}$$

as illustrated in Figure 1. Stochastic outcome is due to a close-tie vote when the public signal is false (state $x_1 = 1 - \sigma$):

$$\alpha = (1-\alpha)\left(p_\sigma - p_{1-\sigma}\right). \tag{34}$$

Notation 2 (information aggregation): *Tie-breaking rule*

$$r \ = \Pr\left(a_1 = x_1 \mid x_1 = 1-\sigma\right) \tag{35}$$

measures information aggregation. To create a tie (34), the uninformed voters must play strategy

$$p_\sigma = \tfrac{1}{2(1-\alpha)}, \ \ p_{1-\sigma} = \tfrac{1-2\alpha}{2(1-\alpha)}. \tag{36}$$

The appropriately chosen tie-breaking rule

$$r(q) = \tfrac{1-2q+2\alpha q\left(1+\alpha q-\sqrt{(1+\alpha q)^2-2q}\right)}{(1-q)(1-2q(1-\alpha))} \tag{37}$$

keeps them indifferent between voting for different policies. The stronger the public signal, the easier it is to win by voting on the signal and to lose by voting against it. Increasingly noisy outcome

$$\tfrac{dr(q)}{dq} < 0 \tag{38}$$

guarantees that self confidence built by winning on the side of the signal decreases, and self confidence built by losing on the opposite side increases, so that the above indifference is preserved.

Proposition 3 (semi-informative equilibrium) *Suppose that the public signal is stronger than described by inequality (25), but weaker than described by inequality (31). Then, the unique equilibrium of the game is as follows. During vote* 1*, the informed voters vote their signals. The uninformed voters play a voting strategy described by set of equations (36). The majority outcome is decreasingly informative in the precision of public signal, as described by inequality (38). During vote* 2*, a voter votes his private signal unless he previously voted on the public signal and lost - then, he abstains.*

Note that the intervals of parameter q in propositions 1 to 3 constitute a partition of the parameter space (they are mutually exclusive and completely cover the parameter space).

Corollary *Propositions* 1 *to* 3 *describe the unique equilibrium of the game.*

5 Comparative statics

This section presents a comparative static analysis with respect to the quality of public signal. First, it considers information aggregation, instrumental efficiency and welfare, providing some intuition for their dynamics.

A separate subsection introduces a higher turnout cost and relates each the turnout and the margin of victory to the quality of public signal, so as to accommodate higher turnout in "closer" elections. This correlation is found to be weak but significant by the vast majority of empirical studies.[33]

Information aggregation *Information aggregation decreases (nonstrictly) in the quality of public signal, as illustrated in Figure* 2*-b.*

The intuition behind this insight is transparent: In the lower region (25) the equilibrium is perfectly informative ($r = 1$). The stronger the public signal,

[33] For an account of the established regularities in voting behavior see Blais (2000, 2006). He concludes that 10 percentage-point increase in the vote margin is associated with up to a 2 percentage-point decrease in the turnout. For an example of a dissenting view see Ashworth, Geys and Heyndels (2006). They find a non-monotonic relationship between the vote margin and the turnout.

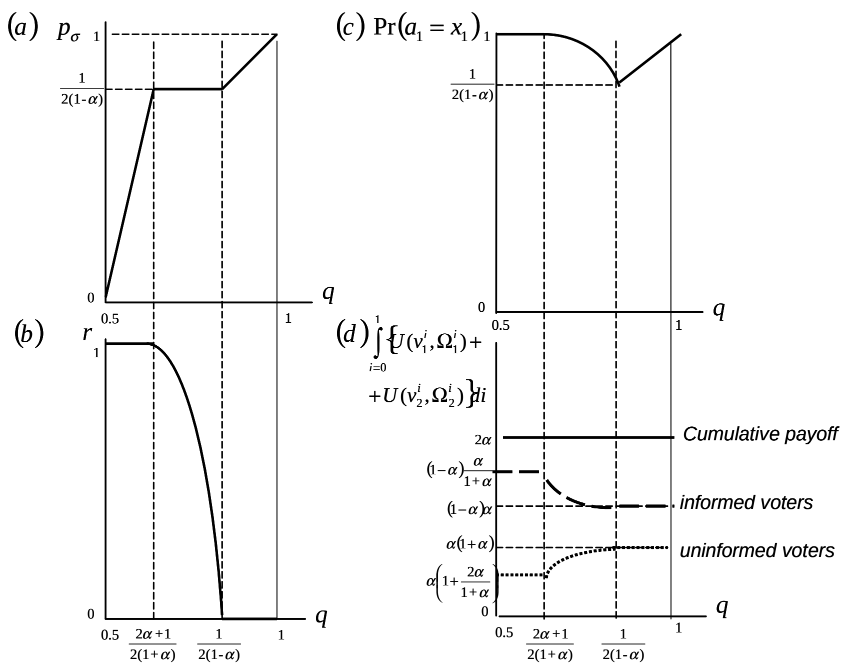

Figure 2: Comparative statics. (a) Vote on public signal; (b) Information aggregation; (c) Instrumental efficiency; (d) Welfare (the warm glow payoff).

the more the uninformed voters vote on it (see Figure 2-a). When the signal is sufficiently strong but false they introduce noise into the majority outcome (in the interim region). The stronger the signal, the stronger the noise until the upper region (31) where outcome becomes completely uninformative.

Instrumental efficiency *The expected efficiency of the majority outcome is* $^{-}\cup$ *shaped in the quality of public signal, as depicted in Figure* 2-*c*.

The flatline in the lower region illustrates that in the informative equilibrium the outcome is efficient. A downward sloping curve in the interim region shows that in the semiinformative equilibrium the outcome is decreasingly efficient. Indeed, it is efficient with probability $q + r(1 - q)$: for sure when the public signal is true, and with probability r otherwise. There are two controversial effects. On the positive side, the stronger the public signal, the more likely it is to be true. On the negative side, the stronger the signal, the higher the efficiency loss when it is false: recall inequality (38). Unfortunately, the negative effect is stronger: $\frac{d}{dq}(q+(1-q)r) < 0$. Upward dynamics in the upper region arise because in the uninformative only the positive is present. Indeed, the outcome is efficient if and only if the public signal is true, which is more likely the higher its quality.

Welfare *The expected payoff by the informed voters decreases (nonstrictly) in the quality of public signal. The opposite is true for expected payoff by the uninformed voters. The welfare is twice proportional to the mass of the informed voters. These patterns are depicted in Figure* 2-*d*.

In our game, a voter cares not for the instrumental efficiency which he cannot affect anyway, but for his private warm glow from participation. The welfare is equal to the warm glow experienced by the all the voters. A voter's warm glow during the first vote is given by his type. His warm glow during the second vote is equal to his self confidence which depends on pooling of types: A higher pooling benefits the uninformed voters at the expense of the informed voters. In the informative equilibrium, the informed voters separate from the uninformed losers. In the semi-informative equilibrium, better public signal helps the uninformed voters to pool more effectively. In the uninformative equilibrium, different types pool completely.

Turnout and "closeness" The uninformed voters have weaker incentives to participate than the informed voters, at least initially. However, when the turnout cost is infinitely small, all the uninformed voters pay for a chance to build a positive self confidence and enjoy voting in the future. When the turnout cost is higher, their participation decision becomes non-trivial.

Notation 3 (turnout cost). *Consider the turnout cost ψ in the interval*

$$\tfrac{1}{4} < \psi < \tfrac{1}{3}: \tag{39}$$

The left limitation guarantees that participation by the uninformed voters is sufficiently low so that the equilibrium is informative no matter how strong the public signal. The right limitation guarantees that their participation is positive, no matter how weak the public signal.[34]

Voter turnout *Voter turnout is $\cup$-shaped in the quality of public signal.*

Variations in the turnout are due to variable participation by the uninformed voters (the informed voters participate uniformly). When the public signal is sufficiently weak, namely,

$$q < \tfrac{1-2\psi}{1-\psi}, \tag{40}$$

the uninformed voters randomize among three feasible voting behaviors: They vote on the public signal with probability

$$p_\sigma = \tfrac{\alpha}{1-\alpha}\tfrac{q-\psi(1+q)}{\psi(1+q)}, \tag{41}$$

[34]When ψ lies above threshold $\frac{1}{3}$, the uninformed voters abstain if $q < \frac{\psi}{1-\psi}$. Otherwise, their support to policy "σ" is described by the least of 1 and the right-hand-side of equation (41), and their support to policy "$1-\sigma$" is described by the most of 0 and the right-hand-side of equation (42). The difference in these voting probabilities remains below thershold $\frac{\alpha}{1-\alpha}$ as q approaches 1 if and only if ψ lies below threshold $\frac{1}{4}$.

they vote against the signal with probability

$$p_{1-\sigma} = \tfrac{\alpha}{1-\alpha}\tfrac{1-q-\psi(2-q)}{\psi(2-q)}, \tag{42}$$

and they abstain with the complementary probability. The stronger the public signal, the weaker their incentives to vote against it: $\frac{dp_{1-\sigma}}{dq} < 0$, and the stronger their incentives to vote on it: $\frac{dp_\sigma}{dq} > 0$. However, if too many uninformed voters vote on the signal, their expected self confidence does not cover the turnout cost. Therefore, increasingly many uninformed voters abstain: $\frac{d(p_{1-\sigma}+p_\sigma)}{dq} < 0$.

When the public signal is stronger than described by inequality (40), the uninformed voters either vote on the signal or they abstain ($p_{1-\sigma} = 0$). They vote on the signal with probability p_σ given by equation (41) and they abstain with the complementary probability. Hence, their turnout increases in q.

Margin of victory *The expected margin of victory increases in the quality of public signal.*

The margin of victory increases in the quality of public signal when the signal is correct, and the opposite is true when the signal is false. The signal is more and more likely to be correct, and the expected margin of victory is higher and higher.

Note that in interval (40) a stronger public signal decreases the turnout and, at the same time, increases the expected margin of victory. Hence, our model with common values accommodates higher turnout in closer elections. Of course, our goal here is to validate our model, and not to debate with natural view attributing this correlation to the conflict of policy interests. Pivotal-voter- and group-based theories all share this view.[35]

[35] These theories model the turnout decision as a trade-off between the expected policy benefit from participation and the turnout cost. In pivotal-voter theories (Ledyard, 1984; Palfrey and Rosenthal, 1983, 1985; Myerson, 1998) pivot probabilities are small, hence, participation is low. Group-based theories accommodate a high turnout by grouping the voters according to their policy interests and assuming that voters in the same group coordinate their turnout.

6 Inefficient policy persistence

Proposition 3 suggest that a possible reason for the inefficient policy persistence[36] is that the status quo is seen as a signal on the appropriate public policy. Put loosely, an uninformed voter believes that a majority has selected the appropriate policy yesterday, and this policy is likely to remain appropriate today. A natural extension of our basic model formalizes this idea.

Consider an overlapping generation game with an infinite horizon. Each generation lives for two periods and plays the basic game. For simplicity, the voters receive no exogenous public information, formally, $q = \frac{1}{2}$. However, they observe the history of public policy. The state variable x_t follows a Markov process:

$$\Pr(x_0 = 0) = \Pr(x_0 = 1) = \tfrac{1}{2}; \tag{43}$$

$$\Pr(x_{t+1} = 0 \mid x_t = 0) = \Pr(x_{t+1} = 1 \mid x_t = 1) = \tau \geqslant \tfrac{1}{2}, \tag{44}$$

where parameter τ measures the persistence of efficient public policy.

Consider vote 1. The informed voters of the first generation vote their signals. The uninformed voters vote for each policy with probability $\frac{1}{2}$. The outcome is efficient ($a_1 = x_1$). Note that it signals the future state x_2:

$$\Pr(x_2 = 0 \mid a_1 = 0) = \Pr(a_1 = 1 \mid x_2 = 1) = \tau. \tag{45}$$

Consider vote 2. By proposition 1, the old winners vote their signals, and the old losers abstain. The votes by the uninformed winners cancel out. The informed winners advance the efficient policy by margin α. The informed young voters vote their signals and replicate this effect. Suppose, the uninformed young voters (mass $1 - \alpha$) believe that there will be no reform anyway. Then, their expected self confidence is given by equations (27)-(28) with q being replaced for τ. Making the same replacement in set of equations (30), we find that they vote for status quo with probability τ and

[36] For examples of inefficient policy persistence, see Coate and Morris (1999), Fernandez and Rodrick (1991). For a survey of relevant theories, see Mitchell and Moro (2006).

for the reform with probability $1-\tau$. They make their beliefs come true if and only if

$$(2\tau-1)(1-\alpha)>2\alpha.$$

Suppose the status quo is maintained in this way until vote t. Once again, the informed voters (old and young) advance the efficient policy by margin 2α. Suppose that the uninformed young voters (mass $1-\alpha$) still believe that the status quo will be maintained no matter what the state. Because the status quo still signals the appropriate policy:

$$\Pr(x_t=a_1 \mid a_1)-\Pr(x_t=1-a_1 \mid a_1)=(2\tau-1)^t,$$

they increase the vote margin for the status quo by $(2\tau-1)^t(1-\alpha)$. The status quo is maintained if and only if

$$(1-\alpha)(2\tau-1)^t>2\alpha. \tag{46}$$

That is, if and only if its prevalence is sufficiently recent, and it is therefore sufficiently strong signal on the appropriate policy. The more persistent the efficient policy, the longer prevails the status quo. Formally, for any τ there exists threshold

$$T=\max\left\{t \mid (1-\alpha)(2\tau-1)^t>2\alpha\right\} \tag{47}$$

such that inequality (46) is true if and only if $t \leqslant T$.

Proposition 4 *Consider an overlapping generation game with an infinite horizon. Each generation lives for two periods and plays the basic game without an exogenous public information ($q=\frac{1}{2}$). The state variable follows a Markov process described by equations (43) and (44). The game has an equilibrium in which the same public policy persists for T successive periods regardless of its efficiency, where T is described by equation (47). In period $T+1$ the efficient policy wins, be it a reform or status quo.*

7 Conclusion

We have proposed a dynamic model accommodating several patterns observed in voting behavior, including "irrational" behaviors. We found that public information influences the vote creating an adverse effect on the majority outcome. We see three main directions for future research:

First, about two thirds of American voters identify themselves with one of the two major parties. Party identification influences their political attitudes and voting behavior (Gerber, Huber and Washington, 2009). This situation is depicted by group-based models. They divide the voters into two competing electoral groups with the opposite policy interests and assume that voters in the same group coordinate their turnout either by following their group leaders (Uhlaner, 1989; Shachar and Nalebuff, 1999; Morton, 1987, 1991) or through adopting group-utilitarian behavior[37] (Feddersen and Sandroni, 2006; Coate and Conlin, 2004; Harsanyi, 1977). These models, however, do not explain how voters identify with their groups. We would like to model group- or partisan identification. Voting for the same alternative may induce such an identification.

Second, a laboratory experiment by Feddersen et al. (2009) shows that voting behavior depends on the size of the election. We would like to analyze small elections in which the voters should care not only for their private warm glow from participation (as in our large game), but also for the outcome which their votes potentially affect. This creates an endogenous cost of uninformed participation, as in Feddersen and Pesendorfer (1996).

Third, we hope that our model of the vote may be useful for formal analysis of other activities involving many participants, such as trading in financial markets or contributing to open-source projects.

[37] In group-utilitarian models, a voter follows a behavioral rule which is optimal for his group if the voters like him follow the same rule.

References

[1] Aldrich, J. H. (1993), "Rational Choice and Turnout," *American Journal of Political Science*, 37:246-278.

[2] Ansolabehere S., Meredith M., and E. Snowberg (2009), "Sociotropic Voting and the Media," *mimeo.*

[3] Ashworth, J., Geys, B. and B. Heyndels (2006), "Everyone likes a winner: An empirical test of the effect of electoral closeness on turnout in a context of expressive voting," *Public Choice*, 128(3):383-405.

[4] Bartels, L. (1988), *Presidential Primaries and the Dynamics of Public Choice*, Princeton University Press.

[5] Bartlett, B. (2011), "Voter Ignorance Threatens Deficit Reduction," *The Fiscal Times*, February 4.

[6] Bénabou, R. (2008), "Ideology," *Journal of the European Economic Association*, 6(2–3):321-352.

[7] Bénabou, R. (2009), "Groupthink: Collective Delusions in Organizations and Markets," *Journal of the European Economic Association*, 6(2–3):321-352.

[8] Bénabou, R. and J. Tirole (2002), "Self Confidence and Personal Motivation," *The Quarterly Journal of Economics*, 117(3):871-915.

[9] Bénabou, R. and J. Tirole (2006), "Belief in a Just World and Redistributive Politics," *The Quarterly Journal of Economics*, 121(2):699-

746.

[10] Bendor, J., Diermeier, D. and M. Ting (2003), "A Behavioral Model of Turnout," *American Political Science Review,* 97(2):261-81.

[11] Blais, A. (2000), *To Vote or Not to Vote: The Merits and Limits of Rational Choice Theory*, Pittsburgh: University of Pittsburgh Press.

[12] Blais, A. (2006), "What affects voter turnout?" *Annual Review of Political Science*, 9:111-25.

[13] Brennan, G. and J. M. Buchanan (1984), "Voter choice: Evaluating political alternatives," *American Behavioral Scientist*, 28(2):185-201.

[14] Callander, S. (2008), "Majority rule when voters like to win," *Games and Economic Behavior*, 64(2):393-420.

[15] Caplan, B. (2007), *The Myth of the Rational Voter: Why Democracies Choose Bad Policies*, Princeton University Press.

[16] Carlsson, F. and O. Johansson-Stenman (2010), "Why Do You Vote and Vote as You Do?" *Kyklos* 63(4):495-516.

[17] Cloutier, E., Guay J.-H. and R. Nadeau (2010), "New Evidence About the Existence of a Bandwagon Effect in the Opinion Formation Process," *International Journal of Public Opinion Research*, 22:230-255.

[18] Coate, S. and M. Conlin (2004), "A Group Rule-Utilitarian Approach to Voter Turnout: Theory and Evidence," *American Economic Review*, 94(5):1476-1504.

[19] Coate, S. and S. Morris (1999), "Policy Persistence," *The American Economic Review,* 89(5):1327-1336.

[20] Coleman, S. (2004), "The Effect of Social Conformity on Collective Voting Behavior," *Political Analysis*, 12:76-96.

[21] Degan, A. (2006), "Policy Positions, Information Acquisition and Turnout," *Scandinavian Journal of Economics*, 108(4):669-682.

[22] Degan, A. and A. Merlo (2011), "A Structural Model of Turnout and Voting in Multiple Elections," *Journal of the European Economic Association*, 9(2):209-245.

[23] Della Vigna, S. and E. Kaplan (2007), "The Fox News Effect: Media Bias and Voting," *Quarterly Journal of Economics*, 122:1187-1234.

[24] Dhillon, A. and S. Peralta (2002), "Economic Theories of Voter Turnout," *Economic Journal*, 112:332-352.

[25] Downs, A. (1957), *An Economic Theory of Democracy*, Harper, New York.

[26] Enikolopov, R., Petrova, M. and E. Zhuravskaya (2011), "Media and Political Persuasion: Evidence from Russia," *American Economic Review*, forthcoming.

[27] Feddersen, T. J. (2004), "Rational Choice Theory and the Paradox of Not Voting," *Journal of Economic Perspectives*, 18(1):99-112.

[28] Feddersen, T. J., Gailmard, S. and A. Sandroni (2009), "Moral Bias

in Large Elections: Theory and Experimental Evidence," *American Political Science Review*, 103:175-192.

[29] Feddersen, T. J. and W. Pesendorfer (1996), "The Swing Voter's Curse," *American Economic Review*, 86(3):408-24.

[30] Feddersen, T. J. and W. Pesendorfer (1997), "Voting Behavior and Information Aggregation in Elections with Private Information," *Econometrica*, 65(5):1029-1058.

[31] Feddersen, T. J. and A. Sandroni (2006), "A Theory of Participation in Elections," *American Economic Review*, 96(4):1271-1282.

[32] Ferejohn, T. J. and M. P. Fiorina (1974), "The Paradox of Not Voting: A Decision-Theoretic Analysis," *American Political Science Review*, 68:525-536.

[33] Fernandez, R. and D. Rodrik (1991), "Resistance to Reform: Status Quo Bias in the Presence of Individual-Specific Uncertainty," *American Economic Review*, 81(5):1146-1155.

[34] Firebaugh, G. and K. Chen (1995), "Vote Turnout of Nineteenth Amendment Women: The Enduring Effect of Disenfranchisement," *The American Journal of Sociology*, 100(4):972-996.

[35] Fowler, J. H. (2006), "Habitual voting and behavioral turnout," *Journal of Politics,* 68:335-344.

[36] Gerber, A. S., Green, D. P. and R. Shachar (2003), "Voting May Be

Habit-Forming: Evidence from a Randomized Field Experiment," *American Journal of Political Science*, 47(3):540–550.

[37] Gerber, A. S., Huber, A. and E. Washington (2009), "Party Affiliation, partizanship, and Political Beliefs: a Field Experiment," *NBER Working Paper* 15365.

[38] Gerber, A. S., Karlan, D. and D. Bergan (2011), "Does the Media Matter? A field experiment measuring the effect of newspapers on voting behavior and political opinions," *American Economic Journal: Applied Economics*, forthcoming.

[39] Geys, B. (2006), "Rational theories of voter turnout: A review," *Political Studies Review,* 4:16-35.

[40] Gordon, B. R. and W. R. Hartmann (2011), "Advertising effects in Presidential Elections," *mimeo.*

[41] Green, D. P. and R. Shachar (2000), " Habit formation and political behavior: Evidence of consuetude in voter turnout," *British Journal of Political Science*, 30:561-573.

[42] Harmon-Jones, E., Amodio, D. M. and C. Harmon-Jones (2009), "Action-Based Model of Dissonance: A Review, Integration, and Expansion of Conceptions of Cognitive Conflict," in *Advances in Experimental Social Psychology*, ed. Mark P. Zanna, Burlington: Academic Press, vol. 41:119-166.

[43] Harsanyi, J. (1977), "Morality and the Theory of Rational Behavior,"

Social Research, 44(4):623-56.

[44] Ladha, K. (1995), "Information pooling through majority-rule voting: Condorcet's jury theorem with correlated votes," *Journal of Economic Behavior & Organization,* 26(3):353-372.

[45] Lassen, D. D. (2005), "The Effect of Information on Voter Turnout: Evidence from a Natural Experiment," *American Journal of Political Science,* 49(1):103-118.

[46] Ledyard, J. O. (1984), "The Pure Theory of LargeTwo-Candidate Elections," *Public Choice*, 44(1):7-41.

[47] Matsusaka, J. G. (1995), "Explaining voter turnout patterns: An information theory," *Public Choice*, 84:91-117.

[48] Merlo, A. (2006): "Whither Political Economy? Theories, Facts, and Issues," in *Advances in Economics and Econometrics, Theory and Applications*: Ninth World Congress of the Econometric Society, ed. by R. Blundell, W. K. Newey, and T. Persson. Cambridge: Cambridge University Press.

[49] Mitchell, M. F. and A. Moro (2006), "Persistent Distortionary Policies with Asymmetric Information," *American Economic Review*, 96(1):387-393.

[50] Morris, S. and H. S. Shin (2002), "The Social Value of Public Information," *The American Economic Review*, 92(4):1521-1534.

[51] Morton, R. B. (1987), "A Group Majority Model of Voting," *Social Choice and Welfare*, 4(2):117-31.

[52] Morton, R. B. (1991), "Groups in Rational Turnout Models," *American Journal of Political Science*, 35(3):758-776.

[53] Mullainathan, S. and E. Washington (2007), "Sticking with Your Vote: Cognitive Dissonance and Political Attitudes," *American Economic Journal: Applied Economics*, 1(1):86-111.

[54] Myerson, R. (1998), "Population Uncertainty and Poisson Games," *International Journal of Game Theory*, 27:375-392.

[55] Nadeau, R., Cloutier, E. and J.-H. Guay (1993), "New Evidence About the Existence of a Bandwagon Effect in the Opinion Formation Process," *International Political Science Review*, 14(2):203-213.

[56] Palfrey, T. R. and H. Rosenthal (1983), "A Strategic Calculus of Voting," *Public Choice*, 41:7-53.

[57] Palfrey, T. R. and H. Rosenthal (1985), "Voter Participation and Strategic Uncertainty," *American Political Science Review*, 79:62-78.

[58] Postlewaite, A. (2010), "Social Norms and Preferences," *Penn Institute for Economic Research Working Papers*, 10-019.

[59] Riker, W. H. and P. C. Ordeshook (1968), "A Theory of the Calculus of Voting," *American Political Science Review*, 62:25-42.

[60] Romano. A. (2011), "How Dumb Are We," *Newsweek*, March 28-April

4 (double) issue.

[61] Rotemberg, J. J. (2009), "Attitude Dependent Altruism, Turnout and Voting," *Public Choice,* 140(1-2):223-244.

[62] Scheinkman, J. A. (2008), *Social Interactions*, The New Palgrave Dictionary of Economics, 2nd ed., S. Durlauf and L. Blume (eds.), Palgrave Macmillan.

[63] Schuessler, A. (2000), "Expressive voting," *Rationality and Society*, 12(1):87-118.

[64] Shachar, R. and B. Nalebuff (1999), "Follow the Leader: Theory and Evidence on Political Participation," *American Economic Review,* 89(3):525-47.

[65] Sher, C.-Y. (2011), "Social Interactions in Voting Behavior: Distinguishing Strategic Voting from a Bandwagon Effect," mimeo, Boston University.

[66] Uhlaner, C. J. (1989), "Rational Turnout: The Neglected Role of Groups," *American Journal of Political Science*, 33(2):390-422.

[67] Wolfinger, R. E. and S. J. Rosenstone (1980), *Who Votes?* New Haven, CT: Yale University Press.

A Appendix

A.1 Proof of proposition 1

By equations (2) and (8), $U(\sigma_t^i, \Omega_t^i) = \Pr\left(\theta^i = 1 \mid \Omega_t^i\right)$, $U(v_t^i, \Omega_t^i) = 0$ for $v_t^i \neq \sigma_t^i$. Therefore, equation (9) is true.

1. Consider vote 2. By equation (9), the voting behavior is described by set of equations (10).

Let us prove statement (12): If $\theta^i = 1$ then v_1^i lies in $\mathrm{Im}(v_1(1, \sigma, \sigma_1^i))$ and so $\Pr\left(v_1^i, a_1 \mid \theta^i = 1\right) > 0$. By equation (11), $\Pr\left(\theta^i = 1 \mid \Omega_2^i\right) > 0$.

By equation (2),

$$\int\limits_{i:\ \theta^i=0,\ \Pr\left(\theta^i=1|\Omega_2^i\right)>0} \sigma_2^i di = \int\limits_{i:\ \theta^i=0,\ \Pr\left(\theta^i=1|\Omega_2^i\right)>0} z_2^i di = 0. \tag{48}$$

Equation (48) and statement (12) imply equation (13).

2. Consider vote 1.

Let us prove statement (14). Suppose, it is false:

$$\mathrm{Im}(v_1(1, \sigma, \sigma_1^i)) \cap \mathrm{Im}(v_1(0, \sigma, \sigma_1^i)) = \varnothing \text{ in either state } x_1. \tag{49}$$

$$\text{Then, } \Pr\left(\theta^i = 1 \mid v_1^i \in \mathrm{Im}(v_1(0, \sigma, \sigma_1^i)), a_1\right) = 0, \tag{50}$$

$$\text{and } \Pr\left(\theta^i = 1 \mid v_1^i \in \mathrm{Im}(v_1(1, \sigma, \sigma_1^i)), a_1\right) = 1. \tag{51}$$

The expected payoff by voter i is equal to:

$$U(v_1^i, \Omega_1^i) + E_{date\ 1} \max_{v_2^i} U(v_2^i, \Omega_2^i) = \begin{cases} \theta^i \sigma_1^i + E_{date\ 1} \Pr\left(\theta^i = 1 \mid v_1^i, a_1\right) & \text{if } v_1^i = \sigma_1^i; \\ -\theta^i \sigma_1^i + E_{date\ 1} \Pr\left(\theta^i = 1 \mid v_1^i, a_1\right) & \text{otherwise.} \end{cases} \tag{52}$$

By equations (50) and (51), maximization of payoff (52) implies that v_1^i lies in $\mathrm{Im}(v_1(1,\sigma,\sigma_1^i))$ for any i, which contradicts to hypothesis (49).

Let us prove equation (15). By statement (14),

$$\Pr\left(\theta^i = 1 \mid v_1^i, a_1\right) < 1. \tag{53}$$

Therefore, if $v_1^i \neq \sigma_1^i$ then

$$U(v_1^i, \left\{1, \sigma, \sigma_1^i\right\}) + \max_{v_2^i} U(v_2^i, \left\{v_1^i, a_1\right\}) = \Pr\left(\theta^i = 1 \mid v_1^i, a_1\right) < 1.$$

At the same time,

$$U(\sigma_1^i, \left\{1, \sigma, \sigma_1^i\right\}) + \max_{v_2^i} U(v_2^i, \left\{\sigma_1^i, a_1, \sigma_2^i\right\}) = 1 + \Pr\left(\theta^i = 1 \mid v_1^i, a_1\right) \geqslant 1.$$

Let us prove equation (21). By Bayes rule, and equations (18) and (15),

$$\Pr\left(\theta^i = 1 \mid v_1^i = \varnothing, a_1\right) = 0, \tag{54}$$

$$\Pr\left(\theta^i = 1 \mid v_1^i = j, a_1 = 1 - j\right) = 0, \tag{55}$$

$$\Pr\left(\theta^i = 1 \mid v_1^i = j, a_1 = j\right) > 0. \tag{56}$$

By equations (54)-(56):

$$U(\varnothing, \{0, \sigma, \sigma_1^i\}) + E_{date\ 1} \max_{v_2^i} U(v_2^i, \{\varnothing, a_1\}) = E_{date\ 1} \Pr\left(\theta^i = 1 \mid \varnothing, a_1\right) = 0,$$
$$U(v_1^i, \{0, \sigma, \sigma_1^i\}) + E_{date\ 1} \max_{v_2^i} U(v_2^i, \{v_1^i, x_1\}) = \tfrac{1}{2} \Pr\left(\theta^i = 1 \mid v_1^i, v_1^i\right) > 0 \text{ for } v_1^i \neq \varnothing.$$

Let us prove inequality (16). Suppose that $a_j \neq x_j$ for both j. Then, both inequalities: $\alpha \leqslant (1-\alpha)(p_j - p_{1-j})$ and $\alpha \leqslant (1-\alpha)(p_{1-j} - p_j)$ must be true. However, the sum of these inequalities is false: $\alpha \leqslant 0$.

Let us prove set of equations (23). Equation (18) implies

$$\alpha \geqslant (1-\alpha) \max \left\{ p_{\sigma} - p_{1-\sigma}, p_{1-\sigma} - p_{\sigma} \right\}. \tag{57}$$

Recall that $\alpha < \frac{1}{2}$. Therefore, inequality (57) requires $0 < p_{\sigma} < 1$. So, the uninformed voters must be indifferent between voting "σ" and "$1-\sigma$". By set of equations (52), this is equivalent to

$$E_{date\ 1} \Pr\left(\theta^{i} = 1 \mid v_1^i = \sigma_1^i, a_1\right) = E_{date\ 1} \Pr\left(\theta^{i} = 1 \mid v_1^i = 1 - \sigma_1^i, a_1\right). \tag{58}$$

Using Bayes rule, we find equation (20). By equations (20) and (55), equations (58) and (22) are equivalent. Set of equations (23) solves the system of equations (21) and (22) for the voting probabilities.

Let us prove inequality (25). By set of equations (23) and inequality (57), equation (18) is true if and only if inequality (36) is met.

A.2 Proof of proposition 2

Using Bayes rule, we find equations (27)-(28). The objective function by voter i is described by set of equations (52). Once again, equation (21) is true. In equilibrium, the uninformed voters must be indifferent between different voting strategies, as described by equation (29). Thereby, we find voting probabilities (30). Given these probabilities, equation (26) is true conditional on inequality (31).

A.3 Proof of proposition 3

1. Suppose the outcome of vote 1 is such as described by equations (33) and (35). By Bayes rule, we find posteriors

$$\Pr\left(\theta^i = 1 \mid v_1^i = 1-\sigma, a_1 = 1-\sigma\right) = \tfrac{\alpha}{\alpha + p_{1-\sigma}(1-\alpha)}; \tag{59}$$

$$\Pr\left(\theta^i = 1 \mid v_1^i = 1-\sigma, a_1 = \sigma\right) = \tfrac{\alpha(1-q)(1-r)}{\alpha(1-q)(1-r) + p_{1-\sigma}(1-\alpha)(q+(1-q)(1-r))}; \tag{60}$$

$$\Pr\left(\theta^i = 1 \mid v_1^i = \sigma, a_1 = \sigma\right) = \tfrac{\alpha q}{\alpha q + p_\sigma(1-\alpha)(q+(1-q)(1-r))}; \tag{61}$$

$$\Pr\left(\theta^i = 1 \mid v_1^i = \sigma, a_1 = 1-\sigma\right) = 0. \tag{62}$$

Once again, voting delivers a positive expected self-confidence, while abstention delivers null self-confidence. Hence, equation (21) is true. By equations (21) and (34), we find voting probabilities (36). Substituting them in equations (59)-(61), we find:

$$\Pr\left(\theta^i = 1 \mid v_1^i = 1-\sigma, a_1 = 1-\sigma\right) = 2\alpha; \tag{63}$$

$$\Pr\left(\theta^i = 1 \mid v_1^i = 1-\sigma, a_1 = \sigma\right) = \tfrac{2\alpha(1-q)(1-r)}{q+(1-q)(1-r)-2\alpha q}; \tag{64}$$

$$\Pr\left(\theta^i = 1 \mid v_1^i = \sigma, a_1 = \sigma\right) = \tfrac{2\alpha q}{2\alpha q + q + (1-q)(1-r)}. \tag{65}$$

2. The uninformed voters should be indifferent between voting for different policies, as described by equation

$$\begin{aligned} &\tfrac{2\alpha q}{2\alpha q+q+(1-q)(1-r)}\left(q+(1-q)(1-r)\right) = \\ &= 2\alpha r(1-q) + \tfrac{2\alpha(1-q)(1-r)}{q+(1-q)(1-r)-2\alpha q}\left(q+(1-q)(1-r)\right). \end{aligned} \tag{66}$$

Using notation

$$x = q + (1-q)(1-r), \tag{67}$$

we rewrite equation (66) as

$$2xq\left(x\left(1-\alpha\right)-2\alpha^2 q\right)=x^2-(2\alpha q)^2, \text{ or, equivalently,}$$

$$x^2\left(2q(1-\alpha)-1\right)-(2\alpha q)^2 x+(2\alpha q)^2=0. \tag{68}$$

and solve it for x. We find the following roots:

$$x_+(q)=\tfrac{2\alpha q}{2q(1-\alpha)-1}\left(\alpha q+\sqrt{\left(\alpha q+1\right)^2-2q}\right) \text{ and} \tag{69}$$

$$x_-(q)=\tfrac{2\alpha q}{2q(1-\alpha)-1}\left(\alpha q-\sqrt{\left(\alpha q+1\right)^2-2q}\right). \tag{70}$$

We are only interested in real roots. Furthermore, they must lie in the interval $(q,1)$, so that r given by equation (67) lies in the interval $(0,1)$.

3. Suppose that inequality (31) is true. Let us prove that both roots (69) and (70) are real, but they lie at least as high as 1, hence, no semi-informative equilibrium.

First, note that discriminant $\left(\alpha q+1\right)^2-2q$ decreases in q:

$$\tfrac{\partial((\alpha q+1)^2-2q)}{\partial q}=2\alpha\left(\alpha q+1\right)-2=2(\alpha\left(\alpha q+1\right)-1)<2(\tfrac{1}{2}\left(\tfrac{1}{2}q+1\right)-1)<0,$$

and it is positive at $q=\frac{1}{2(1-\alpha)}$:

$$\left.\sqrt{\left(\alpha q+1\right)^2-2q}\right|_{q=\frac{1}{2(1-\alpha)}}=\tfrac{\alpha}{2(1-\alpha)}.$$

Therefore, both roots (69) and (70) are real.

Second, by inequality (31),

$$2q\left(1-\alpha\right)-1>0 \text{ and } \alpha q\ >\sqrt{\left(\alpha q+1\right)^2-2q}.$$

Hence, both roots (69) and (70) are positive, root (69) being the highest: $x_+(q) > x_-(q)$.

Let us prove that the smallest root (70) is no lower than 1. Equation (68) is equivalent to

$$F(x,q) = 2q(1-\alpha)x^2 - (2\alpha q)^2 x + (2\alpha q)^2 - x^2 = 0. \tag{71}$$

$$\frac{\partial F(x,q)}{\partial q} = 2x^2(1-\alpha) + (2\alpha)^2\, 2q(1-x) =$$
$$= \tfrac{2}{q}\left(2q(1-\alpha)x^2 - (2\alpha q)^2 x + (2\alpha q)^2 - q(1-\alpha)x^2\right) =$$
$$= \tfrac{2}{q}x^2\left(1 - q(1-\alpha)\right) > 0, \text{ and } \frac{\partial F(x,q)}{\partial x} = 2x\left(2q(1-\alpha)-1\right) - (2\alpha q)^2.$$

By equation (70),

$$\text{By equation (70), } \frac{\partial F(x,q)}{\partial x} = 2\alpha q\left(\alpha q - \sqrt{(\alpha q+1)^2 - 2q}\right) - (2\alpha q)^2 =$$
$$= -2(\alpha q)^2 - 2\alpha q\sqrt{(\alpha q+1)^2 - 2q} < 0.$$

$$\text{By the implicit function theorem, } \frac{dx_-(q)}{dq} = -\frac{\frac{\partial F(x,q)}{\partial q}}{\frac{\partial F(x,q)}{\partial x}} > 0. \tag{72}$$

By inequality (72),

$$x_-(q) \geqslant x_-\left(\tfrac{1}{2(1-\alpha)}\right) = 1.$$

4. Suppose inequality (25) is true. Let us prove that if equation (68) has real roots, then one of them is negative, and the other one lies below q. Hence, no semi-informative equilibrium once again.

Note that for any q below the upper threshold (31),

$$2q\,(1-\alpha) - 1 < 0 \text{ and } \alpha q < \sqrt{(\alpha q+1)^2 - 2q}. \tag{73}$$

Therefore $x_+(q) < 0$ and $x_-(q) > 0$. By inequality (72),

$$x_-(q) \leqslant x_-\left(\tfrac{2\alpha+1}{2(1+\alpha)}\right) = q. \tag{74}$$

5. It remains to consider the interim interval

$$\tfrac{2\alpha+1}{2(1+\alpha)} < q < \tfrac{1}{2(1-\alpha)}. \tag{75}$$

By inequalities (73), equation (68) has the unique positive root (70). By equation (67),

$$r(q) = \tfrac{1-x_-(q)}{1-q}, \tag{76}$$

which is equivalent to equation (37).

5.1. Let us prove inequality (38). By equation (76), it is equivalent to

$$\tfrac{dx_-(q)}{dq} > \tfrac{1-x_-(q)}{1-q}. \tag{77}$$

By the implicit function theorem (recall equation (68)),

$$\tfrac{dx_-(q)}{dq} = -\tfrac{x_-^2(q)(1-\alpha)+(2\alpha)^2(1-x_-(q))q}{(2q(1-\alpha)-1)x_-(q)-2(\alpha q)^2} = \tfrac{x_-^2(q)(1-\alpha)+(2\alpha)^2(1-x_-(q))q}{2\alpha q\sqrt{(\alpha q+1)^2-2q}}. \tag{78}$$

Thereby, inequality (77) is equivalent to

$$x_-^2(q)(1-q)(1-\alpha) > (1-x_-(q))\left(\sqrt{(\alpha q+1)^2-2q}-2\alpha(1-q)\right)2\alpha q. \tag{79}$$

According to the second inequality in set (73), inequality (79) follows from

$$x_-^2(q)(1-q)(1-\alpha) > (1-x_-(q))\,(1-2\alpha(1-q))\,2\alpha q. \tag{80}$$

By the first inequality in set (75) and inequality (72), inequality (74) is inverted. Therefore, inequality (80) follows from inequality

$$q^2(1-q)(1-\alpha) > (1-q)\,(1-2\alpha(1-q))\,2\alpha q,$$

which is equivalent to the first inequality in set (75).

5.2. By inequalities (38) and (75), tie-breaking rule given by equation (37) lies in set $(0,1)$:

$$r\left(\tfrac{2\alpha+1}{2(1+\alpha)}\right)=1;\ \lim_{q\longrightarrow\frac{1}{2(1-\alpha)}} r\left(q\right)=0.$$

A.4 Comparative statics

Information aggregation Follows from propositions 1-3.

Instrumental efficiency Straightforward algebra shows that

$$\tfrac{d}{dq}\left(q+(1-q)r\right)<0.$$

Welfare 1. Consider the informative equilibrium described by Proposition 1. We use equations (20) and (23) to find the voters' expected payoffs. The uninformed voters who vote for policy σ receive payoff $\frac{\alpha}{q(1+\alpha)}$ with probability q. The uninformed voters who vote for policy $1-\sigma$ receive a higher payoff $\frac{\alpha}{(1-q)(1+\alpha)}$ with a lower probability $1-q$. Either way, the common expected payoff is equal to:

$$E\left(U(v_1^i,\Omega_1^i)+U(v_2^i,\Omega_2^i)\mid\theta^i=0\right)=E\left(U(v_2^i,\Omega_2^i)\mid\theta^i=0\right)=\tfrac{\alpha}{1+\alpha}. \qquad (81)$$

Payoff by the informed voters depends on whether the public signal is true or false. It is equal to $1+\frac{\alpha}{q(1+\alpha)}$ if the signal is true, and to $1+\frac{\alpha}{(1-q)(1+\alpha)}$ if the signal is false. The expected payoff by the informed voters is equal to

$$E\left(U(v_1^i,\Omega_1^i)+U(v_2^i,\Omega_2^i)\mid\theta^i=1\right)=1+E\left(U(v_2^i,\Omega_2^i)\mid\theta^i=1\right)=1+\tfrac{2\alpha}{1+\alpha}. \qquad (82)$$

2. Consider the uninformed equilibrium described by Proposition 2. By equations (27), (28) and (30), all voters receive payoff α during vote 2. The informed voters however, also receive payoff 1 during vote 1. Hence,

$$E\left(U(v_1^i, \Omega_1^i) + U(v_2^i, \Omega_2^i) \mid \theta^i = 0\right) = E\left(U(v_2^i, \Omega_2^i) \mid \theta^i = 0\right) = \alpha; \tag{83}$$

$$E\left(U(v_1^i, \Omega_1^i) + U(v_2^i, \Omega_2^i) \mid \theta^i = 1\right) = 1 + E\left(U(v_2^i, \Omega_2^i) \mid \theta^i = 1\right) = 1 + \alpha. \tag{84}$$

The uninformed voters benefit from pooling (compare equations (81) and (83)). The informed voters lose (compare equations (82) and (84)).

3. Consider the semi-informative equilibrium described by Proposition 3. By equations (37) and (63)-(65), the uninformed voters receive payoff

$$E\left(U(v_1^i, \Omega_1^i) + U(v_2^i, \Omega_2^i) \mid \theta^i = 0\right) = \alpha\left(1 + \alpha q - \sqrt{(\alpha q + 1)^2 - 2q}\right), \tag{85}$$

which is increasing in q:

$$\frac{\partial}{\partial q}\left(E\left(U(v_1^i, \Omega_1^i) + U(v_2^i, \Omega_2^i) \mid \theta^i = 0\right)\right) = \alpha\left(\alpha + \frac{\left(1-\alpha-\alpha^2 q\right)\sqrt{(\alpha q+1)^2 - 2q}}{(\alpha q+1)^2 - 2q}\right) > 0.$$

The informed voters receive payoff

$$E\left(U(v_1^i, \Omega_1^i) + U(v_2^i, \Omega_2^i) \mid \theta^i = 1\right) = 1 + \alpha + (1-\alpha)\left(\sqrt{(\alpha q + 1)^2 - 2q} - \alpha q\right), \tag{86}$$

which is decreasing in q:

$$\frac{\partial}{\partial q}\left(E\left(U(v_1^i, \Omega_1^i) + U(v_2^i, \Omega_2^i) \mid \theta^i = 1\right)\right) = -(1-\alpha)\left(\alpha + \frac{\left(1-\alpha-\alpha^2 q\right)\sqrt{(\alpha q+1)^2 - 2q}}{(\alpha q+1)^2 - 2q}\right) > 0.$$

4. By equations (81)-(85) and (86), the common payoff is equal to $\alpha E\left(U(v_1^i,\Omega_1^i)+U(v_2^i,\Omega_2^i)\mid\theta^i=1\right)+$

$$+(1-\alpha)E\left(U(v_1^i,\Omega_1^i)+U(v_2^i,\Omega_2^i)\mid\theta^i=0\right)=2\alpha. \tag{87}$$

Voter turnout and vote margin Suppose that equilibrium is informative. Then, self confidence is described by equations (20) and (55).

Notations: Let

$$V(p_\sigma,q)=q\left(\tfrac{\alpha}{\alpha+(1-\alpha)p_\sigma}-\psi\right)-\psi$$

be the expected payoff by an uninformed voter who votes for policy σ, and $V(p_{1-\sigma},1-q)$ be the expected payoff by an uninformed voter who votes for policy $1-\sigma$.

1. Suppose $\psi\geqslant\frac{1}{2}$. Then, $V(p_\sigma,q)<V(0,1)<0$ and $V(p_{1-\sigma},1-q)<V(0,\frac{1}{2})<0$. Therefore, $p_\sigma=p_{1-\sigma}=0$. Hence, if $\psi\geqslant\frac{1}{2}$, the uninformed voters abstain. This is consistent with outcome $a_1=x_1$.

2. Suppose $\frac{1}{3}\leqslant\psi<\frac{1}{2}$. By inequalities

$$V(p_{1-\sigma},1-q)<V(0,\tfrac{1}{2})<0, \tag{88}$$

$p_{1-\sigma}=0$.

If $V(p_\sigma,q)>0$ then $p_\sigma=1$. However, if $p_\sigma=1$ then $a_1\neq x_1$: a contradiction.

If $V(p_\sigma,q)<0$ then $p_\sigma^i=0$; $V(0,q)<0$ if and only if $q<\frac{\psi}{1-\psi}$.

$V(0,\frac{\psi}{1-\psi})=0$, $dV(0,q)/dq>0$, therefore, $V(0,q)>0$ for any $q>\frac{\psi}{1-\psi}$. Hence, it must be $V(p_\sigma,q)=0$, which is equivalently to equation (41). Note that

$$\frac{dp_\sigma}{dq} = \frac{1}{\psi(1+q)^2} > 0. \tag{89}$$

To summarize, the uninformed voters do not vote contrary to the public signal, that is, $p_{1-\sigma} = 0$. If $q < \frac{\psi}{1-\psi}$, they do not vote according to the public signal either, that is, $p_\sigma = 0$. If $q \geqslant \frac{\psi}{1-\psi}$, they support policy σ the more, the stronger the public signal, as described by equation (41) and inequality (89). However, the efficient policy wins at a positive margin even if the public signal is false:

$$\frac{\alpha}{1-\alpha}\frac{q-\psi(1+q)}{\psi(1+q)} < \frac{\alpha}{1-\alpha} \text{ for } \psi \geqslant \frac{1}{4}, \tag{90}$$

hence for $\psi \geqslant \frac{1}{3}$.

3. Suppose $\frac{1}{4} \leqslant \psi < \frac{1}{3}$.

3.1. Suppose $q \geqslant \frac{1-2\psi}{1-\psi}$. Then, inequality (88) is true, and so $p_{1-\sigma} = 0$. By step 2, p_σ is given by equation (41). It increases in q (inequality (89)), but lies below threshold $\frac{\alpha}{1-\alpha}$ for any q (by inequality (90)).

3.2. Suppose $q < \frac{1-2\psi}{1-\psi}$.

3.2.1. Let us prove by contradiction that there is some abstention, that is, $p_\sigma + p_{1-\sigma} < 1$. Suppose equation (21) is true. Then, both inequalities

$$q\left(\frac{\alpha}{\alpha+(1-\alpha)p_\sigma} - \psi\right) \geqslant \psi \text{ and } (1-q)\left(\frac{\alpha}{\alpha+(1-\alpha)(1-p_\sigma)} - \psi\right) \geqslant \psi \tag{91}$$

must be true, where p_σ is given by equation

$$q\left(\frac{\alpha}{\alpha+(1-\alpha)p_\sigma} - \psi\right) - (1-q)\left(\frac{\alpha}{\alpha+(1-\alpha)(1-p_\sigma)} - \psi\right) = 0, \tag{92}$$

guaranteeing the uninformed voters' indifference between voting "σ" and "$1-\sigma$".[38] Adding up inequalities (91), we find inequality

$$\tfrac{q\alpha}{\alpha+(1-\alpha)p_\sigma} + \tfrac{(1-q)\alpha}{\alpha+(1-\alpha)(1-p_\sigma)} \geqslant 3\psi. \tag{93}$$

By equation (92), inequality (93) is equivalent to

$$\tfrac{(1-q)\alpha}{\alpha+(1-\alpha)(1-p_\sigma)} \geqslant \psi(2-q). \tag{94}$$

Comparing equations (22) and (92), we find that $p_{1-\sigma}$ lies higher than that in the set of equations (23), that is,

$$p_{1-\sigma} \geqslant 1-q-\tfrac{\alpha}{1-\alpha}(2q-1). \text{ Therefore,} \tag{95}$$

$$\tfrac{(1-q)\alpha}{\alpha+(1-\alpha)(1-p_\sigma)} \leqslant \tfrac{\alpha}{1+\alpha}. \tag{96}$$

By inequalities (94) and (96),

$$\tfrac{\alpha}{1+\alpha} \geqslant \psi(2-q). \tag{97}$$

However, $\frac{\alpha}{1+\alpha} < \frac{1}{3}$, because $\alpha < \frac{1}{2}$. At the same time, $\psi(2-q) < \frac{\psi}{1-\psi}$ for any $q < \frac{1-2\psi}{1-\psi}$ and $\frac{\psi}{1-\psi} \geqslant \frac{1}{3}$ for any $\psi \geqslant \frac{1}{4}$. Hence, inequality (97) is false: a contradiction.

3.2.2. $V(0,1-q) > 0$ for any $q < \frac{1-2\psi}{1-\psi}$. By step 3.2.1, the equilibrium is characterized by equations:

$$V(p_{1-\sigma}, 1-q) = V(p_\sigma, q) = 0.$$

[38] Recall that if the uninformed voters play a pure voting strategy the equilibrium is uninformative.

Hence, p_σ is given by equation (41), and $p_{1-\sigma}$ is given by equation (42). Note that these voting probabilities are consistent with outcome $a_1 = x_1$:

$$p_\sigma - p_{1-\sigma} = \tfrac{\alpha}{1-\alpha}\tfrac{1}{\psi}\tfrac{2q-1}{(1+q)(2-q)} \leqslant \tfrac{\alpha}{1-\alpha}$$

if and only if

$$q \leqslant \tfrac{\sqrt{4+9\psi^2}+\psi-2}{2\psi}, \tag{98}$$

Straightforward algebra shows that $\frac{\sqrt{4+9\psi^2}+\psi-2}{2\psi} > \frac{1-2\psi}{1-\psi}$ for any $\psi \geqslant \frac{1}{2} - \frac{\sqrt{3}}{6}$, hence for any $\psi \geqslant \frac{1}{4}$.

Note that the uninformed voters' turnout is decreasing in the quality of public signal:

$$\tfrac{\partial}{\partial q}\left(p_\sigma + p_{1-\sigma}\right) = \tfrac{\alpha}{1-\alpha}\tfrac{1}{\psi}\left(\tfrac{1}{(1+q)^2} - \tfrac{1}{(2-q)^2}\right) < 0.$$

4. Suppose $\psi < \frac{1}{4}$. Let us show that there exists q such that the informed equilibrium is not supported. Consider $q \geqslant \frac{1-2\psi}{1-\psi}$. By step 2, $p_{1-\sigma} = 0$. If $p_\sigma = 1$ then $a_1 = \sigma$, which is generically different from x_1. If $p_\sigma < 1$ then p_σ is given by equation (41). Hence, $p_\sigma > \frac{\alpha}{1-\alpha}$ for any $q \geqslant \frac{2\psi}{1-2\psi}$. Hence, the informative equilibrium is not supported.

5. Let us show that the expected margin of victory

$$MV = q(\alpha + (1-\alpha)(p_\sigma - p_{1-\sigma})) + (1-q)(\alpha - (1-\alpha)(p_\sigma - p_{1-\sigma})), \tag{99}$$

increases in the quality of public signal. Let us rewrite equation (99) as

$$MV = \alpha + (1-\alpha)(p_\sigma - p_{1-\sigma})(2q-1). \tag{100}$$

Consider q outside interval (40). By equations $p_{1-\sigma} = 0$ and (100)

$$MV = \alpha + (1-\alpha)\, p_\sigma \,(2q-1)\,. \tag{101}$$

By inequality (89), MV increases in q. Now, consider q inside interval (40). By equations (41), (42) and (100),

$$MV = \alpha \left(1 + \tfrac{(2q-1)^2}{\psi(1+q)(2-q)}\right).$$

Straightforward algebra shows that

$$\tfrac{\partial MV}{\partial q} = \tfrac{\alpha(2q-1)\left((1-q)^2+q^2+4\right)}{\psi(1+q)^2(2-q)^2} > 0.$$

A.5 Proof of proposition 4

See the main text.