Credit Crises, Precautionary Savings and the Liquidity Trap*

Veronica Guerrieri
University of Chicago and NBER

Guido Lorenzoni
MIT and NBER

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Abstract

We use a model *à la* Bewly-Huggett-Ayagari to explore the effects of a credit crunch on consumer spending. Households borrow and lend to smooth idiosyncratic income shocks facing an exogenous borrowing constraint. We look at the economy response after an unexpected permananent tightening of this constraint. The interest rate drops sharply in the short run and then adjusts to a lower steady state level. This is due to the fact that after the shock a large fraction of agents is far below their target holdings of precautionary savings and this generates a large temporary positive shock to net lending. We then look at the effects on output. Here two opposing forces are present, as households can deleverage in two ways: by consuming less and by working more. We show that under a reasonable parametrization the effect on consumer spending dominates and precautionary behavior generates a recession. If we add nominal rigidities two things happen: (i) the demand-side dominates output dynamics, and (ii) there is a lower bound on the interest rate adjustment. These two elements tend to amplify the recession caused by the credit tightening.

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1 Introduction

How does an economy adjusts from a regime of easy credit to one of tight credit? Suppose it is relatively easy for consumers and firms to borrow and the economy is in some stationary state with a given distribution of creditor and debtor positions. An unexpected shock hits—say a shock to the financial system—and borrowing gets harder, either in terms of higher credit spreads or of tighter borrowing limits. Now the consumers and firms with the largest debtor positions need to readjust towards lower levels of debt. Since the debtor position of one agent is the creditor position of another, this also means that lenders will have to reduce their holdings of financial claims. How are the spending and production decisions of borrowers and lenders affected by this economy-wide financial adjustment? What happens to aggregate activity? How long does the adjustment last?

In this paper, we address these questions in the context of a workhorse Bewley model, thus focusing on the household sector. Households borrow and lend to smooth transitory income fluctuations. The model captures two channels in the agents' response: a direct channel, by which constrained agents are forced to reduce their indebtness, and a precautionary channel, by which unconstrained agents increase their savings as a buffer against future shocks, once they perceive a reduction in future borrowing capacity. Both channels increase the net supply of lending in the economy, so the equilibrium interest rate has to fall in equilibrium.

Our analysis leads to two sets of results. First, we look at the short run dynamics of the interest rate and show that they are characterized by a sharp initial fall followed by a gradual adjustment to a new, lower steady state. The reasons for the interest rate overshooting is that, at the initial asset distribution, the agents at the lower end of the distribution try to adjust faster towards their higher target level of net savings. So the initial increase in net lending is stronger. To maintain the asset market in equilibrium, interest rates have to fall sharply. As the asset distribution converges to the new steady state the net lending pressure subsides and the interest rate moves gradually up.

Second, we look at the response of aggregate activity. Here our crucial observation is that overly indebted agents can deleverage in two ways: by spending less or by earning more. In the context of our model, this means that the shock leads both to a reduction in consumer spending and to an increase in labor supply. Whether a recession follows depend on the

relative strength of these two forces. In particular, if the consumer's precautionary motive is strong enough, the reduction in consumer spending dominates, and output declines. As for the case of interest rates, the contraction is stronger in the short run, when the distribution of asset holdings is far from its new steady state and some agents are far below their new savings target.

Our results on interest rate dynamics link our analysis to the idea of the liquidity trap. A liquidity trap is a situation where the economy is in a recession and the nominal interest rate is zero. In this situation, the central bank cannot lower the nominal interest rate to boost private spending as it would in normal times. The monetary policy literature—recently Krugman (1998) and Woodford and Eggertsson (2001)—has pointed out that the basic problem in a liquidity trap is that the real interest rate required to achieve full employment, the "natural" interest rate, is unusually low and possibly negative. If inflation is low, in line with the central bank target, or, even worse, if deflation has taken hold, the real interest rate corresponding to a zero nominal rate is higher than the natural rate and private spending is stuck at an inefficiently low level. In the context of a simple representative agent models it is not easy to identify shocks that push the economy in a liquidity trap and the literature has mostly resorted to introducing ad-hoc shocks to intertemporal preferences, which mechanically increase the consumer's willingness to save. Our analysis shows that shocks to the agents' borrowing capacity are precisely the type of shocks that can push down the "natural" rate by increasing the net demand for savings in the short run, and thus trigger a liquidity trap. Historically, liquidity traps have typically arisen following disruptions in the banking system, the most notable examples being the Great Depression, Japan in the 90s, and the current crisis. Our paper shows a natural connection between credit market shocks and the emergence of a liquidity trap.

Our paper is related to different strands of literature. First, there is the vast literature on savings in incomplete-markets economies, following the seminal work of Bewley (1977), Huggett (1993), and Aiyagari (1994).[1] Our paper is particularly related to a few recent contributions that focus on transitional dynamics after different types of shocks. For example, Mendoza, Rios Rull and Quadrini (2010) look at the response of an economy opening up to international asset trade.

Two papers that explore the effects of precautionary behavior on business cycle fluc-

[1] Heathcote, Storesletten, and Violante (2009) provide an exhaustive review.

tuations are Guerrieri and Lorenzoni (2009) and Ragot and Challe (2010). Both papers, derive analytical results under simplifying assumptions that essentially eliminate the wealth distribution from the state variables of the problem. The core of this paper is to show how the slow adjustment of the wealth distribution affects transitional dynamics. Another related paper is Chamley (2010), a theoretical paper which explores the role of precautionary motive in a monetary environment and focuses on the possibility of multiple equilibria.

The paper is also related to the growing literature that analyzes the real effects of a credit crunch in dynamic general equilibrium models, including Curdia and Woodford (2009), Hall (2009), Jermann and Quadrini (2009), Brunnermeier and Sannikov (2010), Gertler and Karadi (2010), Gertler and Kiyotaki (2010), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2010). Mostly these papers focus on the effects of a credit tightening on firms' investment, rather than on household consumption as we do here, with the notable exception of Hall (2009) that looks at both consumption and investment. A second distinctive feature of our paper is the focus on the dynamics of the distribution of creditor and debtor positions. Most of the literature, for reasons of tractability, departs minimally from a representative agent environment, assuming that there are two agents, a borrower and a lender, and focusing on the inefficiencies in channeling resources from one agent to the other. The relevant state variable then is the fraction of wealth held by the borrower. Our state variable instead is the whole distribution of net lending positions. The slow adjustment of this state variable is behind the long-lasting effects of a credit shock in our model. Finally, our focus on the household's problem brings to our attention the role of labor supply in financial adjustment.

Finally, there is a growing number of papers that focus on the dynamics of entrepreneurial wealth (Cagetti and De Nardi, 2006, Buera and Shin, 2007). Two recent papers that look at the response of the entrepreneurial sector to a credit shock are Goldberg (2010) and Khan and Thomas (2010). In particular, Goldberg (2010) shares with our paper the emphasis on precautionary behavior and on the scarcity of liquid assets, but focusing on its effects on entrepreneurs' decisions.

The rest of the paper is organized as follows. In Section 2, we introduce the environment and define an equilibrium. We also describe our main calibration exercise and characterize the steady state. In Section 3, we perform our main exercise, that is, we analyze the equilibrium transitional dynamics after an unexpected permanent tightening of the borrowing limit, or credit crunch. Section 4 studies the effects of simple fiscal policies. Section 5

explores a variant of the model with nominal rigidities where the Central Bank sets the interest rate path and studies the effects of a credit crunch under alternative monetary policies. Section 6 concludes. The appendix explains the computational strategy.

2 Model

Consider an economy populated by households hit by idiosyncratic income shocks, that smooth income risk by borrowing and lending. The model is a version of a standard Bewley model with endogenous labor supply and no capital. The only asset traded is a one-period risk-free bond. Households are allowed to hold negative amounts of bonds—i.e., to borrow—up to an exogenous limit. We will first analyze the steady state equilibrium of this economy for a given borrowing limit. Then we will study the economy transitional dynamics following an unexpected shock that permanently reduces this limit.

There is a continuum of infinitely lived households with preferences represented by the utility function

$$\mathrm{E}\left[\sum_{t=0}^{\infty}\beta^t U(c_{j,t},n_{j,t})\right],$$

where $c_{j,t}$ is consumption and $n_{j,t}$ is labor effort. Each household produces consumption goods using the linear technology

$$y_{j,t}=\theta_{j,t}n_{j,t},$$

where $\theta_{j,t}$ is an idiosyncratic productivity level which follows a Markov chain on the space $\left\{\theta^1,\dots\theta^S\right\}$. Let $\theta^1=0$, so that agents hit by the worst shock are unemployed. For the moment, there are no aggregate shocks.

The household budget constraint is

$$q_t b_{j,t+1}+c_{j,t}+\tau_{j,t}=b_{j,t}+y_{j,t},$$

where $b_{j,t}$ are the household bond holdings, q_t is the bond price and $\tau_{j,t}$ is the tax paid by household j. We assume that $\tau_{j,t}=\tau_t-z_t$ if $\theta_{j,t}=0$ and $\tau_{j,t}=\tau_t$, otherwise. That is, all the households pay the lump-sum tax τ_t and the unemployed receive the unemployment benefit z.[2] The budget constraint requires that the households's current resources, bonds plus current income, have to cover consumption, the tax payment, and the purchase of new

[2]The presence of the unemployment benefit ensures that the natural borrowing limit is strictly positive.

bonds. The household's debt position is bounded below by the exogenous limit $\phi \geq 0$, that is, bond holdings have to satisfy

$$b_{j,t+1} \geq -\phi. \tag{1}$$

The interest rate implicit in the bond price is $r_t = 1/q_t - 1$.

The government chooses the supply of real bonds B_t and the unemployment benefit z_t for all t and sets the lump sum tax τ_t to satisfy the budget constraint:

$$B_t + uz_t = q_t B_{t+1} + \tau_t,$$

where $u = \Pr(\theta_{j,t} = 0)$ is the fraction of unemployed agents. For the moment we assume that the supply of government bonds and the unemployment benefit are constant at some levels $\bar{B}$ and $\bar{z}$, while the tax τ_t adjusts to ensure government budget balance.

The main deviation from the approach taken in Aiyagari (1994) and in much of the following literature is that our model does not feature capital accumulation. The standard assumption in models with capital is that claims to physical capital are a perfect substitute for government bonds and other highly safe and liquid stores of value. Clearly, this would not be a satisfactory assumption here, given that we are trying to capture episodes of financial turmoil and of flights to liquidity. To introduce capital in our model requires introducing imperfect substitutability between different assets, possibly due to different risk profiles and/or different resaleability. We will extend the model in this direction in Section **??**.

A second simplification is to assume that the only outside source of safe/liquid assets are government bonds. A central role of the banking sector is to transform relatively risky and illiquid assets into safer and more liquid liabilities, like deposits. In the model, we abstract from intermediation. However, when we calibrate our model we include in $\bar{B}$ all the liquid assets held by the household sector, including bank deposits. Therefore, our simulations try to capture a situation in which the household sector perceives a tightening of credit and in which the total supply of liquid assets is fixed.

Finally, a stark simplification is that there is a single interest rate on bonds r_t, which applies both to positive and negative bond holdings. In other words, agents' liabilities and government bonds are perfect substitutes.

2.1 Equilibrium

Given a sequence of interest rates $\{r_t\}$, let $C_t(b,\theta)$ and $N_t(b,\theta)$ denote the optimal decisions for consumption and labor supply at time t for a household with bond holdings $b_t = b$ and current productivity $\theta_t = \theta$. Notice that, given consumption and labor supply, next period bond holdings are given by the budget constraint. Therefore, the transition for bond holdings is fully determined by the functions $C_t(b,\theta)$ and $N_t(b,\theta)$.

Let $\Psi_t(b,\theta)$ denote the joint distribution of bond holdings and current productivity levels in the population. The household's optimal transition for bond holdings together with the Markov process for productivity yield a transition probability for the individual states (b,θ). This transition probability can be used to compute the distribution $\Psi_{t+1}(b,\theta)$ given the distribution $\Psi_t(b,\theta)$. We are now ready to define an equilibrium.

Definition 1 *An equilibrium is a sequence of interest rates* $\{r_t\}$*, a sequence of decision rules for consumption and labor supply* $\{C_t(b,\theta), N_t(b,\theta)\}$*, a sequence of tax rates* $\{\tau_t\}$*, and a sequence of joint distributions for bond holdings and productivity* $\{\Psi_t(b,\theta)\}$ *such that, given the initial distribution* $\Psi_0(b,\theta)$*:*

(i) $C_t(b,\theta)$ *and* $N_t(b,\theta)$ *are optimal given* $\{r_t\}$ *for all* t*;*

(ii) $\Psi_t(b,\theta)$ *is consistent with consumption and labor supply policies for all* t*;*

(iii) *the tax* τ_t *satisfies*

$$\tau_t = u\bar{z} + \frac{r_t}{1+r_t}\bar{B};$$

(iv) *the bonds market clears:*

$$\int\int b d\Psi_t(b,\theta) = \bar{B}.$$

The optimal policies for consumption and labor supply are characterized by two optimality conditions. First, the Euler equation

$$U_c(c_t,n_t) \geq \beta(1+r_t)E_t\left[U_c(c_{t+1},n_{t+1})\right], \tag{2}$$

which must hold with equality if the borrowing constraint $b_{t+1} \geq -\phi$ is not binding. Second, the optimality condition for labor supply

$$\theta_t U_c(c_t,n_t) + U_n(c_t,n_t) = 0, \tag{3}$$

for all households with $\theta_t > 0$.

A key observation is that, when we lower the borrowing limit, agents face more uncertainty in future consumption, as consumption becomes more responsive to income shocks. With prudence in preferences, this implies that for a given average level of consumption tomorrow, the expected marginal utility on the right hand side of the Euler equation will be higher, by Jensen's inequality. This means that for a given level of interest rates, consumption today will fall, as if there was a negative preference shock reducing the marginal utility of consumption today. This will be the core mechanism reducing consumption demand after a credit shock. In this sense, our model with precautionary savings provides a microfoundation for models with preference shocks.

2.2 Calibration

We analyze the model by numerical simulations, therefore we need to specify preferences and calibrate the model parameters. We calibrate the model to capture households' accumulation and decumulation of liquid assets in response to income and employment shocks. Notice that we are abstracting from life-cycle considerations and from many important drivers of individual wealth dynamics, like purchases of durable goods and housing, health expenses, educational expenses and so on. However, our computational exercise will allow us to identify some general qualitative features of the response of an economy in which borrowers and lenders gradually adjust to a shock to credit access. We believe these qualitative implications survive in environments with richer motives for borrowing and lending.

The utility function is:

$$U(c,n) = \frac{c^{1-\gamma}}{1-\gamma} + \psi \frac{(1-n)^{1-\eta}}{1-\eta}.$$

We will discuss shortly the advantages of a non isoelastic specification for the disutility of labor. The preference parameters are reported in Table 1. The discount factor β corresponds to a discount factor of 0.98 at a yearly frequency. This value, higher than in most calibrations, is chosen to replicate the low-interest rate environment of the mid 2000. In our baseline, we choose a relatively high coefficient of risk aversion $\gamma = 4$. Clearly, this coefficient is crucial in determining the consumers' precautionary behavior, so we will experiment with different values as well. The parameters η and ψ are chosen so that the hours worked of employed workers are on average about 40% of their time endowment (normalized to

1) and so that the average Frisch elasticity of labor supply is 2. This high elasticity will be crucial in generating sizeable output responses in the baseline model without nominal rigidities.

β	0.995	ρ	0.974	$\bar{B}$	1.4
γ	4	σ_ε^2	0.025	ϕ	5.1
η	0.75	z	0.39		
ψ	1.26				

Table 1. Baseline calibration

For the wage process we assume that the log of θ follows a AR1 process with autocorrelation ρ and variance σ_ε^2. We choose the parameters ρ and σ_ε^2 in line with the evidence in Floden and Lindé (2001), who use yearly panel data from the PSID to estimate the stochastic process for individual wages. In particular, we choose parameters that yield a coefficient of autocorrelation for average yearly wages of 0.9 and an unconditional variance, also for average yearly wages, equal of 0.244. The corresponding parameters are reported in Table 1. The wage process is approximated by a 5-state Markov chain, following the approach in Tauchen (1991).

For the transitions between employment and unemployment we follow Shimer (2005), who estimates the finding rate and the separation rate from CPS data. At a quarterly frequency, we set the transition from employment to unemployment at 0.1, corresponding to jobs that lasts on average 2.5 years, and the transition from employment to unemployment at 0.833. We assume that when re-employed workers start at the lowest positive value of θ. For the unemployment benefit z, we also follow Shimer (2005) and calibrate it so that it corresponds to 40% of average labor income.

Finally, we need to choose values for the supply of government bonds $\bar{B}$ and for the borrowing limit ϕ. We choose these values to capture US households' balance sheets in 2006, just prior to the recent financial crisis, looking at the Federal Reserve Board flow of funds data. First, we look at the households' holdings of liquid assets, namely the sum of their holdings of all deposits plus treasury securities (line 9 plus line 16), which was 7.1 trillion dollars, or 53% of GDP. We choose $\bar{B}$ to match this ratio. Second, we interpret debt in our model as consumer credit (line 34 which corresponds, essentially, to total household liabilities minus mortgage debt), which was 2.4 trillion dollars, or 18% of GDP.

2.3 Steady State

We first compute the initial steady state decision rules and the initial bond distribution. Figure 1 shows the optimal steady state values of consumption and labor supply for each level of bond holdings. For ease of reading, we plot the policies for only two values of θ, θ^2 and θ^6.

Different responses at different levels of bond holdings are apparent. At high levels of bond holdings, consumers behavior is close to the permanent-income hypothesis and the consumption function is increasing almost linearly in b. For lower levels of bond holdings, the consumption function becomes concave. Carroll and Kimball (1996) show that this is a typical feature of the consumption function in this class of models. The optimality condition for labor supply implies that the behavior of labor supply mirrors that of consumption, capturing an income effect. In particular, a steeply increasing consumption function at low levels of b translates into a steeply decreasing labor supply function. As a consequence, the labor supply function is convex in b. Notice that for high levels of b, the substitution effect plays a larger role and labor supply is very responsive to the wage rate (the ratio θ^6/θ^2 is 1.3). For lower levels, labor supply is much less responsive to the wage rate.[3]

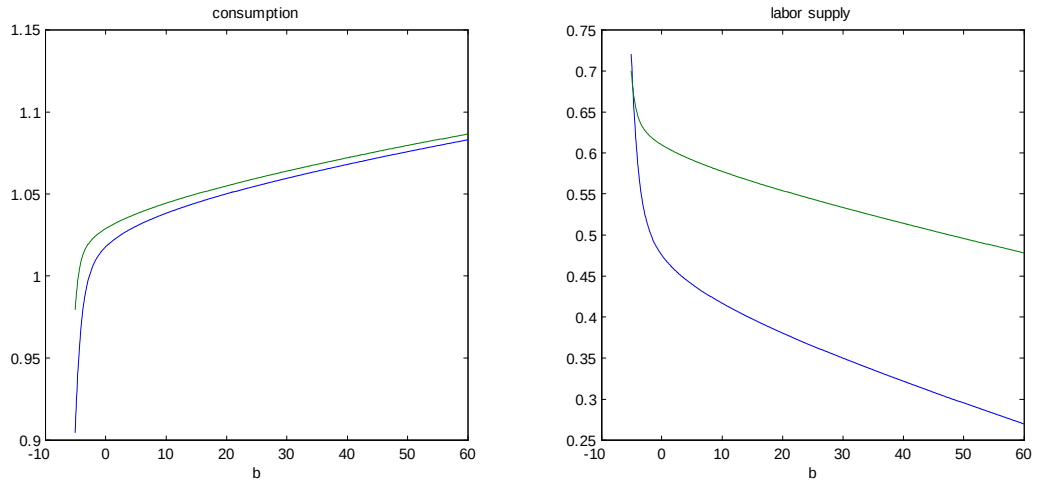


Figure 1: Optimal consumption and labor supply at the initial steady state (for $\theta = \theta^2$ and $\theta = \theta^6$).

[3]For very low levels of b, labor supply is decreasing in the wage rate. This reflects a strong income effect, associated with a highly persistent of wage shocks.

3 Credit Crunch

We now explore the response of our economy to a credit crunch. We consider an economy that at time -1 is in steady state, with a borrowing limit equal to ϕ' and a stationary wealth distribution Ψ'. At time 0, the economy is hit by an unexpected shock leading to a decrease in the borrowing limit to ϕ''. In particular, we look at the effects of a shock that halves the debt limit from $\phi' = 5.1$ to $\phi'' = 2.55$. As the initial wealth distribution is $\Psi_0 = \Psi'$, which is different from the new steady state distribution Ψ'', the economy goes through a gradual transition towards the new steady state.

Before looking at the transitional dynamics, let us briefly compare the two steady states. Figure 2 shows how the interest rate is determined in the two steady states. The solid line shows the average demand for bond holdings in the initial steady state, which is an increasing function of the interest rate, as it is common in Beweley models. The dashed line shows the average demand for bond holdings in the new steady state. The new demand curve is to the right of the old one due to two effects. First, there is a mechanical effect, all households with bond holdings below $-\phi'$ now need to hold at least $-\phi'' > -\phi'$. Second, there is a precautionary effect: for a given interest rate, households accumulate more wealth to stay away from the borrowing limit. Given that the supply of bonds is fixed at $\bar{B}$, it follows that the new interest rate r'' is lower than r' to convince the households to demand the same quantity of bonds.

3.1 Interest rate dynamics

To study the transitional dynamics, we assume that the borrowing limit ϕ_t adjusts gradually towards its new level along the linear adjustment path

$$\phi_t = \max\left\{\phi'', \phi' - \Delta t\right\}.$$

The reason for this assumption is to ensure that agents at all initial levels of debt can adjust without being forced into default. Since all debt in the model has a one-quarter maturity, a sudden adjustment in the debt limit would make it impossible for many borrowers to roll over their debt. An assumption of gradual adjustment of the debt limit is a simple way of capturing the fact that with longer debt maturities agents have some time to adjust to the new regime. In particular, we choose Δ so that the unemployment benefit is sufficient

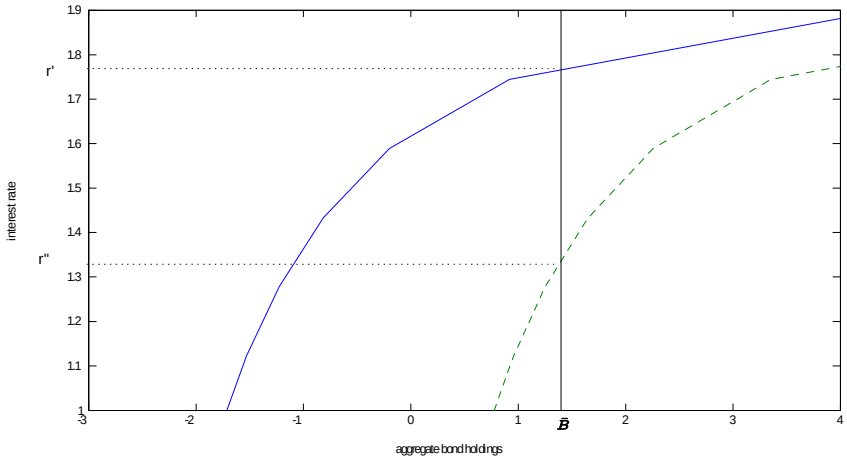


Figure 2: Steady state interest rate determination

to cover the minimum debt repayment $-b_t - q_t\phi_{t+1}$ for an agent starting at $b_t = -\phi_t$. Given the model parameters and the size of the shock this gives us an adjustment lasting 8 quarters. Default and bankruptcy are clearly an important element of the adjustment to a tighter credit regime, but for now we abstract from them.

In the top panel of Figure 3, we plot the exogenous adjustment path for ϕ_t. In the bottom panel we plot the interest rate path. The interest rate drops dramatically after the shock, going negative for more than a year. This is our first main result and we now investigate the mechanism behind it to argue that it is fairly general result and not just the outcome of our choice of parameters.[4]

The first observation to explain the interest rate overshooting is that the bond distribution converges gradually to its new steady state and that the new steady state distribution is more concentrated than the initial one. Let $F_t(b)$ denote the CDF of the marginal bond distribution, that is, $F_t(b) = \int \Psi_t(b,\theta)\, d\theta$. Let $F'(b)$ and $F''(b)$ denote the distributions, respectively, at the initial and at the final steady state. The middle panel of Figure 4 reports the densities f' (solid line) and f'' (dashed line), associated, respectively, to F' and F''. The panel suggests that indeed the distribution in the new steady state is more concentrated. Since the bond supply is fixed at $\bar{B}$ we know that the two distributions

[4]We have also explored the robustness of this result numerically and it holds for all the parameter configurations we have tried.

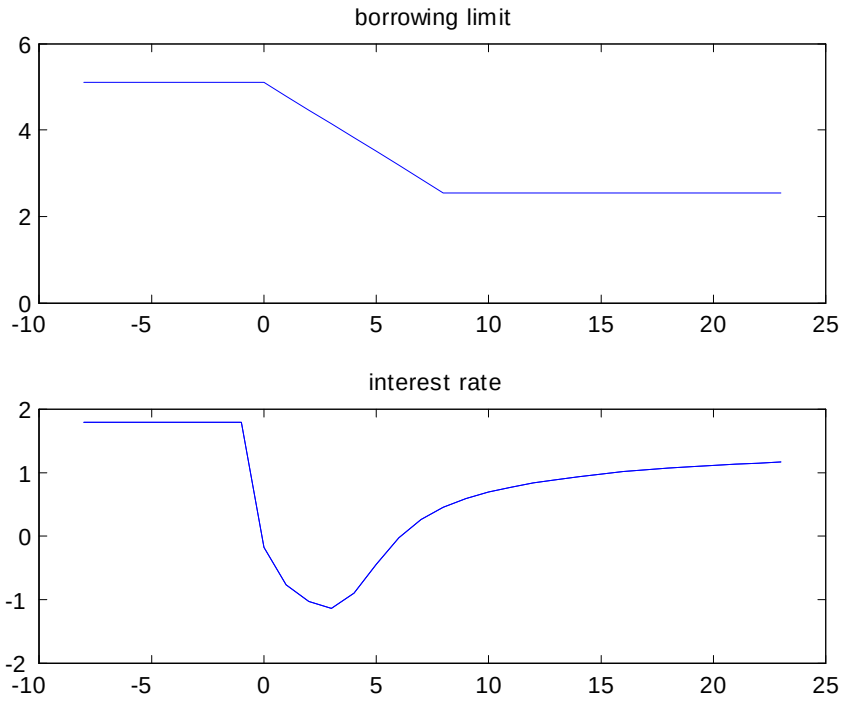


Figure 3: Interest rate dynamics

have the same mean. To check formally that F' is a mean-preserving spread of F'', in the bottom panel of Figure 4 we plot the integral $\int_{-\infty}^{b}(1-F(\tilde{b}))d\tilde{b}$ for the two distributions. The fact that integral for F'' is always above the one for F' confirms the visual impression from the middle panel. Why is the distribution in the new steady state more concentrated? Two forces are at work here. At low levels of bond holdings, the precautionary behavior induces agents in the new steady state to accumulate bonds faster. At high levels of bond holdings, the low equilibrium interest rate induces agents to decumulate bonds faster. This makes bond holdings to mean revert faster and makes the stationary distribution more concentrated.

Consider now the top panel of Figure 4. This panel plots the average bond accumulation $b_{j,t+1}-b_{j,t}$ (averaged over θ) as a function of the initial bond holdings $b_{j,t}$. The decision rules used for this plot are those that would arise at date 0 *if* the interest rate were to adjust immediately to its new steady state level and stay there at all following dates $t=0,1,2,....$ Note that this function is not exactly convex, but almost so. The reason for this convexity is the same reason behind the concavity of the consumption function and the convexity of the labor supply functions in Figure 1.[5]

We are now ready to put the pieces together. Let us make a mental experiment and suppose the interest rate jumps immediately to its new steady state at date 0. If the wealth distribution was already at its new steady state level, the average bond accumulation would average to zero, as we would just be in the new steady state. In other words, if we integrate the function in the top panel weighted by the density f'' in the second panel we get zero. If instead we integrate the same function weighted by the density f' we get a positive number, because the function in the top panel is (approximately) convex and f' is a mean-preserving spread of f''. This means that at the conjectured interest rate path households want to accumulate bonds on average. Since the bond supply is fixed this means that at the conjectured interest rate path there is excess demand of bonds. To equilibrate the bonds market we need a lower interest rate in the initial periods.

[5]The non-convexity at very low levels of b is due to the fact that at the new steady state, the labor supply for very low levels of b is very high for the low shocks and in that region it is less elastic (given our preferences).

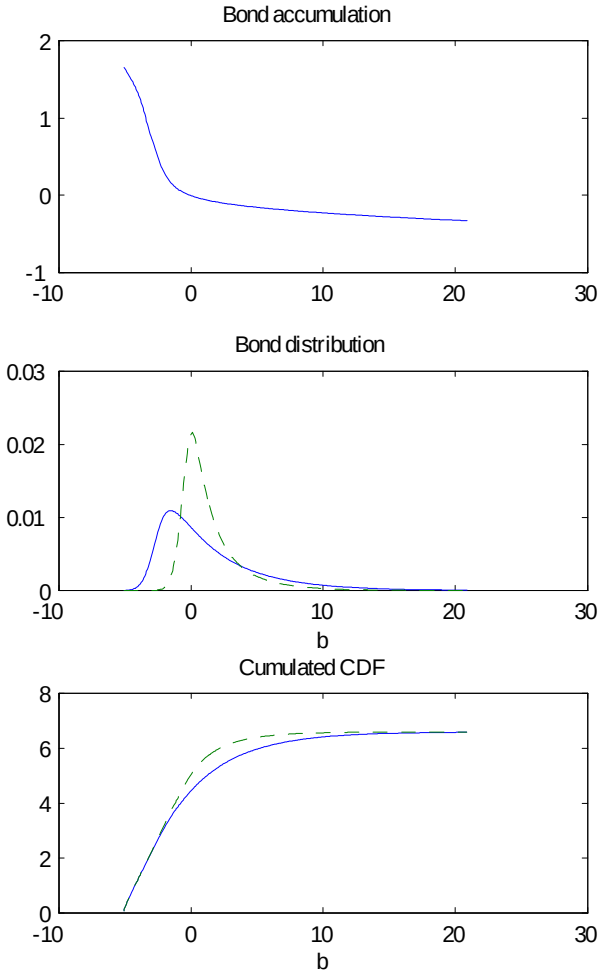


Figure 4: Explaining the overshooting: bond accumulation and bond distribution at the two steady states

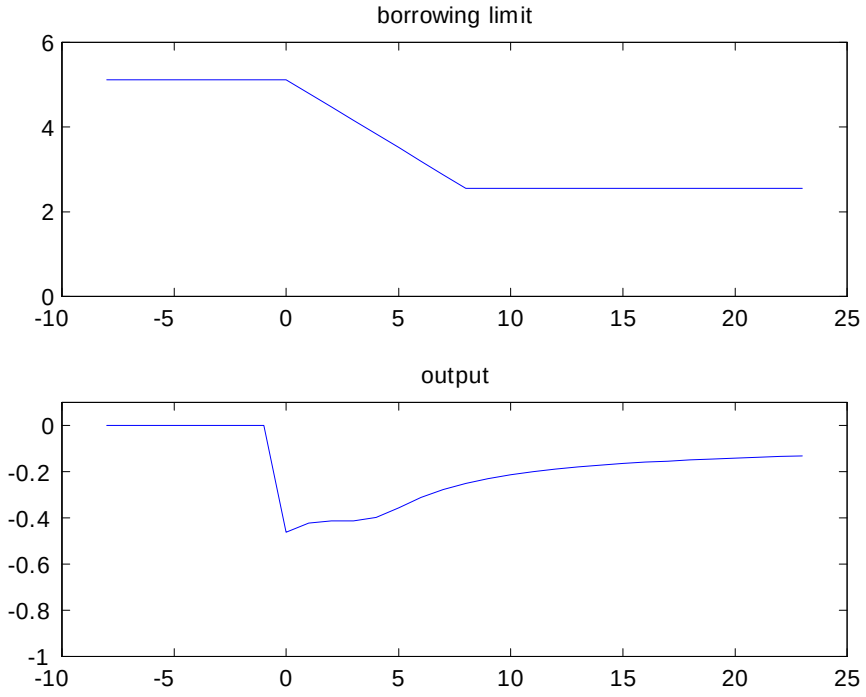


Figure 5: Output dynamics

3.2 Output response

Next, we want to understand what happens to output. Figure 5 shows that output converges to a lower level in the new steady state and overshoots in the short run. The economy goes through a recession and then converges to a permanently lower level of output. The scale for output is percentage deviations from the initial steady state, so, in terms of magnitude, the recession generated in our baseline model is small, with less than a 1/2 of a percent reduction in output. We will consider below mechanisms that can magnify this response. But first let us understand the mechanism behind the recession.

The output response depends both on consumption and on labor supply decisions. Let us focus on the transitional dynamics and try to understand the overshooting in Figure 5. Again, let us make a mental experiment and suppose the interest rate jumped directly to r''. As argued in the discussion of Figure 1, the consumption and the labor supply policies are, respectively, concave and convex functions of the household's bond holdings. Then, given

that the initial distribution is more dispersed than the the new steady state distribution (in the sense of second-order stochastic dominance), average consumption demand is lower than at the new steady state and average output supply is higher. Therefore, at the price r'' there is excess supply in the goods market, which corresponds to the excess demand in the bonds market discussed above.

To clear the goods markets (and the bonds market) the interest rate must be lower on the transition path. As we lower the interest rate towards its equilibrium value, the goods market adjusts on both sides: consumption increases and labor supply falls, due to intertemporal substitution. Therefore, the market clearing output level can, in general, be either above or below its steady state level. Two sets of considerations determine which side of the market dominates the adjustment path: (i) how large are the negative shift in consumption demand and the positive shift in labor supply due to the larger dispersion of bond holdings at the beginning of the transition; and (ii) how elastic are consumption demand and labor supply to a reduction of the interest rate from r'' to the equilibrium level r_0? Our parameters imply that the fall in consumption demand is the dominating factor, and output falls below its new steady state value. Building on this discussion, we can now better understand the role of our parameters.

On the demand side, the effect of a decrease in consumption demand is higher when γ is higher. Notice that γ is both the coefficient of relative risk aversion and the inverse elasticity of intertemporal substitution. On the one hand, when households are more risk averse, the precautionary motive is stronger, making their consumption policy more concave. Therefore, the initial shift in consumption demand is stronger. On the other hand, when the elasticity of intertemporal substitution is lower, consumption responds less elastically to the interest rate. Both effects tend to make the recession larger. Figure 6 shows the behavior of interest rate and output for the same economy with $\gamma = 2$ (red lines) instead of $\gamma = 4$ (blue lines). According to the intuition, the precautionary motive is less strong, making the interest rate decrease less in the short run and the recession milder. However, the recession is longer because the agents are less prepared to a credit crunch and hence take longer to adjust their wealth accumulation.

On the supply side, instead, the elasticity of labor supply to the interest rate and its reaction to a shock in ϕ are determined by the parameter η, but in different directions. When η is lower, the labor supply is more elastic to the interest rate, weakening the

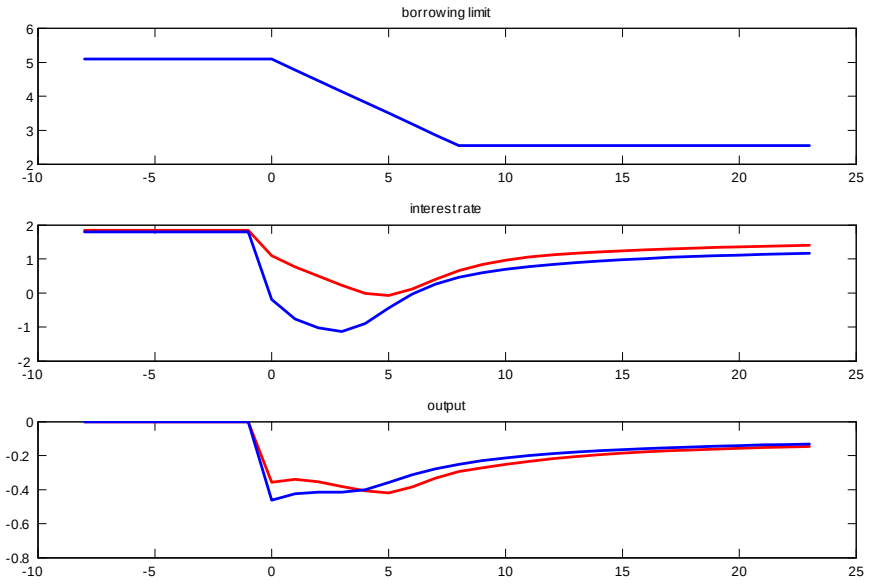


Figure 6: Changing the coefficient of relative risk aversion γ

increase in labor supply. However, at the same time, when η is lower the labor supply function becomes more convex, which implies that the reaction of average labor supply to a shock in ϕ is stronger. However, the choice of a not isoelastic preferences ensure that poor households are less sensitive to a decrease in wealth, making the labor supply policy less convex for any value of η. This ensures that on net, with our parameterization, output tend to overshoots in the short run. Figure 7 shows that when η increases (red lines) so that the Frish elasticity goes from 2 to 1, the short run decline in both the interest rate and output are smaller. This confirms that the effect of the elasticity of labor supply to the interest rate dominates the decrease in convexity.

4 Fiscal policy

We now explore the role of different government policies in mitigating the recession. In particular, let us consider changes in the supply of government bonds. Increasing the supply of bonds can be beneficial for two reasons. First, there is a direct increase in the supply of liquid assets that reduces the downward pressure on the real interest rate. Second, as the

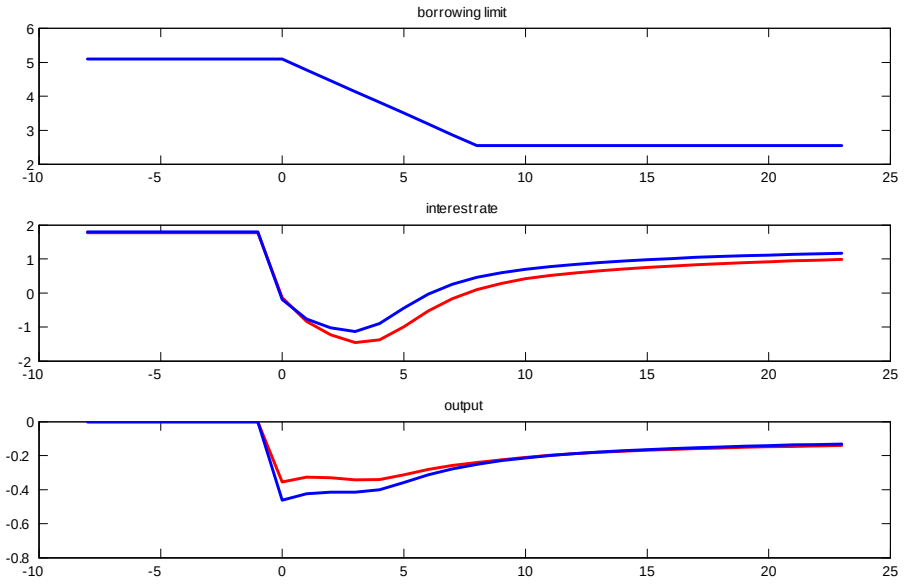


Figure 7: Changing the labor supply elasticity (η)

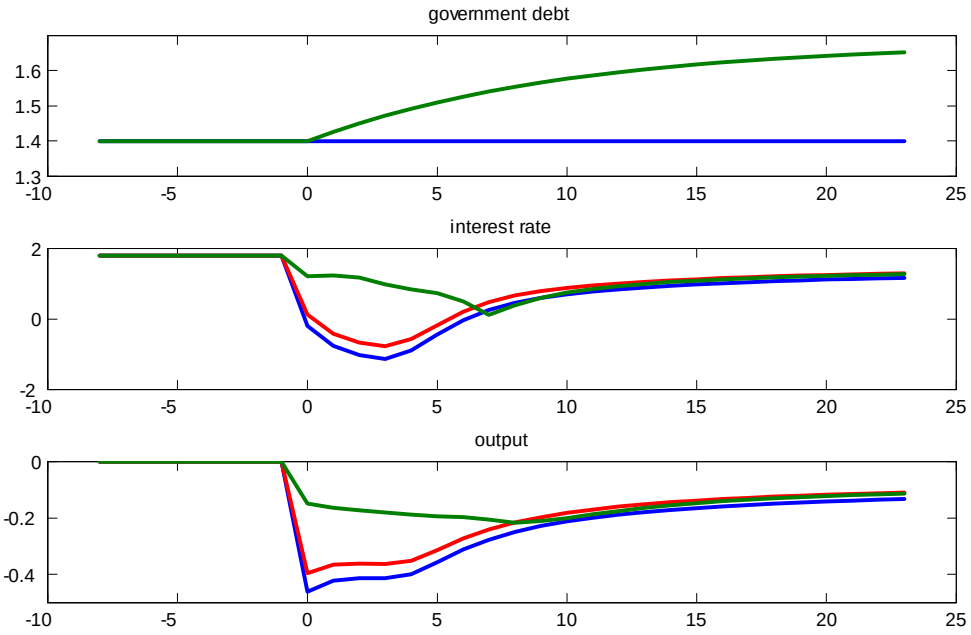
government increases bond supply, the associated deficit can be used to reduce taxes in the short run. Since Ricardian equivalence fails in our economy, this has a positive effect on spending.

Our model has the feature, common to many models with government supplied liquidity and lump sum taxation, that an increase in the supply of government bonds B can exactly offset a change in the borrowing limit ϕ. In particular, the only thing that matters for the equilibrium is the sum $B + \phi$. Here, however, we analyze the effects of policies that only partially offset the long run change in ϕ, possibly because of concerns with the distortionary effects of higher taxation in the long run.

Consider, in particular, a policy of increasing gradually the supply of real bonds to a level that is 20% higher in the new steady state. Namely, assume that B_t follows the path

$$B_t = \rho_b^t B' + \left(1 - \rho_b^t\right) B'',$$

for some $\rho_b^t \in (0, 1)$. We then consider two different ways of spending the deficit associated to this increase in bond supply. First, we look at a policy where taxes adjust to balance the government budget in every period. Second, we look at a policy where the government



deficit is used to finance a temporary increase in the unemployment benefit. In particular, we let the unemployment benefit to be 50% higher for the first two years after the shock. Figure **??** shows what happens to interest rate and output under these two policies. The red lines represent the policy where the increase in B finances a temporary reduction in the tax τ_t; the green lines represent the policy where the deficit goes partly to finance an increase in unemployment insurance.

The figure shows that increasing the supply of government bonds help the economy to reduce the overshooting both in interest rate and in output. Moreover, what is particular effective in this economy is to combine this deficit increase with an increase in unemployment insurance. Increasing the unemployment benefit in the short run is more beneficial than reducing the lump-sum tax because it is targeted to the fraction of the population who is more likely to be credit constrained.

5 Nominal Rigidities

The analysis so far was conducted under the assumption that prices are fully flexible and that the real interest rate adjusts to its equilibrium path to equilibrate the demand and supply of bonds. In this section we explore what happens in a variant of the model with nominal rigidities. Adding nominal rigidities is a simple way to focus on the response of consumer spending, assuming that output in the short run is demand-determined. Once we move in this direction, we can then let the central bank choose the interest rate path and see how different choices lead to different adjustment paths for the distribution of bond holdings and for real output.

The households' side of the model is as before, but output is now produced by a continuum of monopolistically competitive firms. Each firm produces a good $j \in [0,1]$ and consumption is a Dixit-Stiglitz aggregate of these goods. Namely, consumption of household i is given by

$$c_{i,t} = \left(\int_0^1 c_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $c_{i,t}(j)$ is consumption of good j. Each firm produces with a linear technology which produces one unit of good with one efficiency unit of labor. We interpret the shock $\theta_{i,t}$ as a shock to the efficiency of household j labor, so we have a common wage rate w_t per efficiency unit, while the wage rate per hour is given by $w_t \theta_{j,t}$.

The firms are owned by the consumers, so letting Π_t denote total profits, the budget constraint is now

$$q_t b_{j,t+1} + P_t c_{j,t} = b_{j,t} + W_t \theta_{j,t} n_{j,t} - \tau_{j,t} + \Pi_t.$$

Monopolist j faces the demand

$$y_{j,t} = \left(\frac{p_{j.t}}{P_t} \right)^{-\varepsilon} C_t,$$

where C_t is aggregate consumption in the economy.

If prices are flexible, the equilibrium is very similar to that of the perfectly competitive economy of the previous section. The only difference is that the real wage is

$$w_t = \frac{W_t}{P_t} = \frac{\varepsilon - 1}{\varepsilon}, \tag{4}$$

and that households receive profit income on top of labor income. Therefore, the response of the economy to the credit tightening are similar to the ones of the baseline model.

To analyze the case of nominal rigidities, we first consider an extreme form of rigidity, in which prices are fully rigid, that is, $P_t = P_{t-1} = 1$. In combination with this extreme assumption, we assume that the central bank chooses a path for the nominal (and real) interest rate r_t which converges to the new steady state level r''. This ensures that the private benefit from adjusting prices goes to zero in the long run.

To find an equilibrium we choose a path $\{r_t\}$ and look for a sequence of real wage rates $\{w_t\}$ and profits $\{\Pi_t\}$ such that given the optimal consumption and the labor supply decision rules, the bond market clears in each period.

Assume now that the central bank tries to replicate the flexible price path for the interest rate, with the only added constraint that the interest rate cannot go negative. The last panel of Figure 8 shows the output response in this case. The dashed line corresponds to the response in the flexible price case. With nominal rigidities the economy is in a liquidity trap and the output response is larger. As long as the economy is in the liquidity trap, output dynamics are fully dominated by the demand side.

6 Concluding Remarks

We have proposed a model with uninsurable idiosyncratic risk to show how a credit crunch can generate a recession due to precautionary motive. This helps to explain why recessions driven by financial market trouble are more likely to drive the economy into a liquidity trap.

In the current version of the paper, we interpret a credit crunch as a tightening of the borrowing limit. More generally, it would be interesting to explore versions of the model with a simple intermediation sector, introducing a spread between the interest faced by borrowers and by savers, as in Curdia and Woodford (2009) and Hall (2009).

Another simplifying assumption in our model is that the unemployment risk is exogenous and not affected by the credit crunch. It would be interesting to develop a version of the model with endogenous job creation. If firms need liquid assets also to invests in new job openings, a credit crunch can generate a drop in vacancy creation and hence an increase in the unemployment risk. This would generate a potentially interesting feedback effect between unemployment risk and the precautionary motive.

Finally, we believe it would be interesting to explore variants of the model in which the

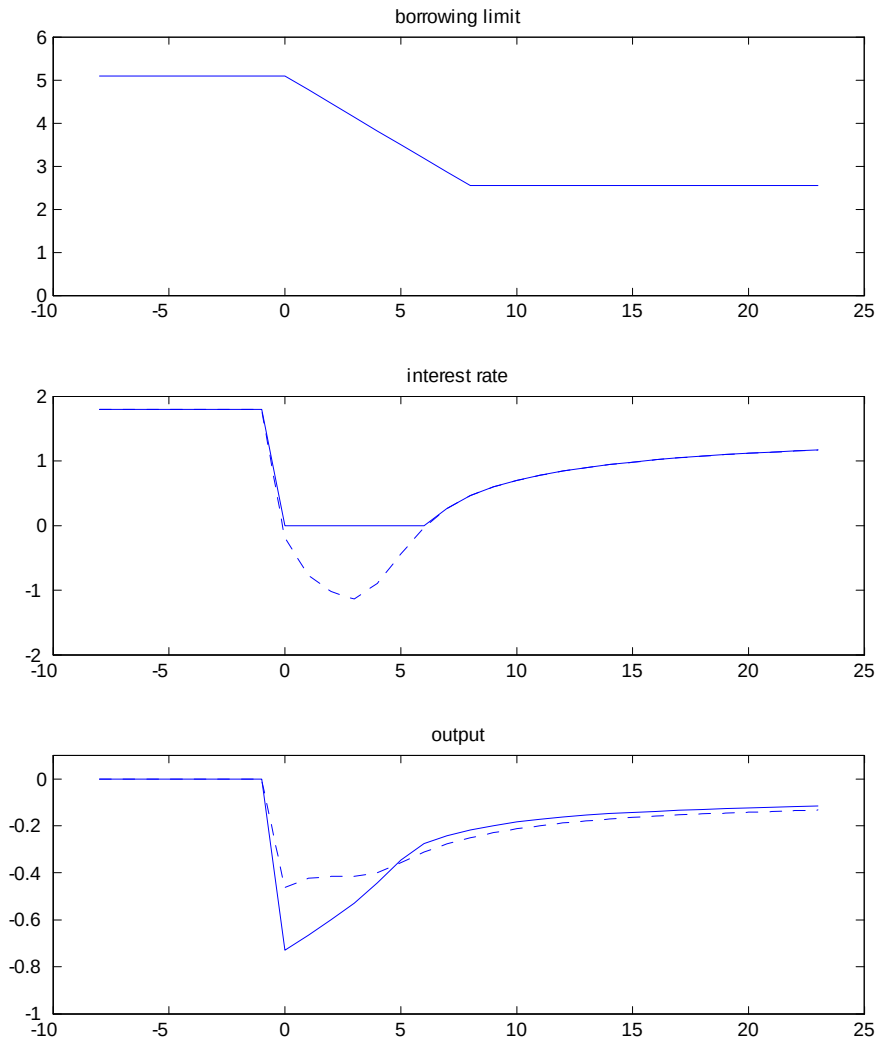


Figure 8: Interest rate and output responses with a lower bound on the interest rate (dashed lines: flexible price benchmark).

household holds a richer portfolio of assets. A natural extension would be to add durable consumption, which has been the most responsive component of consumption in the recent crisis. Durable consumption is a relatively illiquid form of saving. After a credit crunch, the precautionary motive can induce households to allocate their increased savings away from durable consumption goods and towards more liquid assets. This may lead to a larger drop in consumer spending. Another extension would be to introduce risky assets, representing claims to capital, in the household portfolio. It would be interesting to see if portfolio reallocation in favor of safe assets can help to explain the observed drop in asset prices and, through that channel, in investment.

Appendix

Here we describe the algorithm used to compute steady states and transitional dynamics.

To compute the steady state, given a candidate interest rate r, we iterate on the Euler equation and the optimality condition for labor supply to compute the policy functions $C(b,\theta)$ and $N(b,\theta)$ on a discrete grid for the state variable b. In particular, to iterate on the policy functions, we use the endogenous gridpoints approach of Carroll (2006). To compute the invariant distribution $\Psi(b,\theta)$ we derive the inverse of the bond accumulation policy, denoted by $g(b,\theta)$, from the policy functions, and update the conditional bond distribution using the formula $\Psi_{(k)}(b|\theta) = \sum_{\tilde{\theta}} \Psi_{(k-1)}\left(g\left(b,\tilde{\theta}\right)|\tilde{\theta}\right) T\left(\tilde{\theta}|\theta\right)$ for all $b \geq -\phi$, where k is stands for the k-th iteration and $T\left(\tilde{\theta}|\theta\right)$ is the probability of $\theta_{t-1} = \tilde{\theta}$ conditional on $\theta_t = \theta$. Due to the borrowing constraint, the inverse $g(b,\theta)$ is not well defined for $b = -\phi$, but the formula above is still correct if we define $g(-\phi,\theta)$ as the largest value of b such that $b' = -\phi$ is optimal. Finally, we search for the interest rate r that clears the bond market.

To compute transitional dynamics, we get the initial bond distribution $\Psi_0(b,\theta)$ from the initial steady state. We then compute the final steady at $\phi = \phi''$. We choose T large enough that the economy is approximately at the new steady state at $t = T$ (we use $T = 200$ in the simulations reported). Next, we guess a path of interest rates $\{r_t\}$ with $r_T = r''$. We take the consumption policy to be at the final steady state level at $t = T$, setting $C_T(b,\theta) = C''(b,\theta)$, and we compute the sequence of policies $\{C_t(b,\theta), N_t(b,\theta)\}$ using the Euler equation and the optimality condition for labor supply, going backward from $t = T-1$ to $t = 0$ (also using endogenous gridpoints). Next, we compute the sequence of

distributions $\Psi_t(b,\theta)$ going forward from $t=0$ to $t=T$, starting at $\Psi_0(b,\theta)$, using the optimal policies $\{C_t(b,\theta), N_t(b,\theta)\}$ to derive the bond accumulation policy (using the same updating formula as in the steady state). We then compute the aggregate bond demand B_t for $t=0,...T$ and update the interest rate path using the simple linear updating rule $r_t^{(k)} = r_t^{(k-1)} - \epsilon(B_t^{(k)} - \bar{B})$. Choosing the parameter $\epsilon > 0$ small enough the algorithm converges to bond market clearing for all $t=0,...T$. To check that T is large enough, we compare check that $\Psi_T(b,\theta)$ is close enough to $\Psi''(b,\theta)$.

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