Inequality and Risk-Taking Behaviour

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Abstract

This paper investigates social influences on attitudes to risk and offers an evolutionary explanation of risk-taking by young low-ranked males. Becker, Murphy and Werning (2005) found that individuals about to participate in a status tournament may take fair gambles even though they are risk averse in both wealth and status. Here their model is generalised by use of the insight of Hopkins and Kornienko (2010) that in a tournament or status competition one can consider equality in terms of the status or rewards available as well as in initial endowments. While Becker et al. found that risk-taking is increasing in the equality of initial endowments, it is found here that it is increasing in the *inequality* of rewards to status. Further, it is shown that the poorest will be risk loving if the lowest level of status awarded is sufficiently low. Thus, the disadvantaged in society rationally engage in risky behavior when social rewards are sufficiently unequal. Finally, as greater inequality in terms of social status induces gambling, it can cause greater inequality of wealth.

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1 Introduction

Many people undertake highly risky activities. They engage in crime, go to war, participate in extreme sports, take drugs or start fights. To an evolutionary psychologist, the fact that the vast majority of people engaging in such activities are young men is not a coincidence. Young men take risks in order to achieve social reputation or status which may improve mating success (Wilson and Daly, 1985). Such competition is more intense amongst men than women, as men face a greater variance in reproductive outcomes. To an economist, such behaviour is difficult to integrate into a tradition where decision makers are typically taken to be risk averse. Further, to my knowledge, economics has nothing to say about why risk attitudes should depend on either on gender or age. If anything, the lower wealth of the young compared to older adults should make them relatively risk averse.

A second question is the relation between risk-taking behaviour and inequality. Clearly, the traditional economic view of risk preferences being subjective and idiosyncratic says little of how such behaviour should vary with the wealth of others. Yet, there is evidence that such risk-taking behaviour is more common is more unequal societies. Crime has found to be increasing in inequality by Kelly (2000) and Fajnzylber et al. (2002), among others. Wilkinson and Pickett (2009) find a positive relationship between inequality and a wider range of risky behaviours. To explain their empirical findings, they argue that violence and other forms of risk taking are provoked by unfavourable social comparisons, and inequality increases such "evaluation anxiety".

The problem is that existing formal models of social rivalry find results that seem to run in the opposite direction. Hopkins and Kornienko (2004) find that, in a model of status, competition decreases not increases with inequality. Becker, Murphy and Werning (2005) analyse a similar model but concentrate on the implications for risk taking. They find that such behaviour will only take place when the initial distribution of wealth is sufficiently equal. That is, there is more risk-taking behaviour in more equal societies. Further, gambling will be done at intermediate levels of wealth, so that the middle class should be the most risk-taking.

In this paper, I show how a simple change of perspective can provide very different results. A large population of individuals choose how much of their initial endowments to allocate to competition in a tournament. Performance in the tournament determines how status or rewards are allocated. The great insight of Becker et al. (2005) is that the anticipation of taking part in such a competition for status may induce individuals to take fair gambles, even though their underlying preferences over consumption and status are concave. This is integrated into the framework of Hopkins and Kornienko (2010) which permits consideration of equality in terms of the status or rewards available as well as in terms of initial endowments. For example, the difference in return to occupying high versus low social position can and does vary across societies. Here, I find that risk-taking behaviour is increasing in inequality of final *rewards*, even if it is decreasing in the inequality of initial *endowments*.

Specifically, I find that if the minimum status awarded approaches zero, the lowest-ranked in society will be risk-loving. This result holds even if the lowest rank have substantial wealth. Thus, in this model, it is low status, independent of the affluence of society, that determines risk taking. So, it is possible that the low-ranked will be risk loving, and the high-ranked, risk averse. Many middle-ranked individuals will be risk loving with respect to losses, and risk averse with respect to gains, which is reminiscent of prospect theory. Further, an increase in inequality of status will make low-ranked agents more risk loving. All agents after the tournament, in "old age", will be risk averse. Thus, this model implies, when combined with the observation from evolutionary psychology that men face more dispersed rewards than women, that risk-taking behaviour will be most common amongst young, low-ranked men.

The basic intuition for these results is that social exclusion leads to desperation. More specifically, an individual who has an endowment that is low relatively to his rivals can expect only a low reward from participating in the tournament, even if his wealth is high in absolute terms. Further, if this reward is sufficiently low, the marginal value of doing better in the tournament can be arbitrarily high. For example, if low status means that marriage and children are unlikely to be attainable, then evolutionary considerations suggest that an individual in that situation should be desperate to change this outcome. This gives an incentive to gamble.

Robson (1992) was the first to integrate status concerns into risk preferences (see also Robson, 1996; Ray and Robson, 2010). The important difference in Robson (1992) is that there individual utility is directly assumed to be convex in relative wealth. This could be plausible in that it means that the difference between being first and second is more important than the difference between tenth and eleventh. However, here I explore an alternative idea. It is not the underlying preferences for high position that cause fierce competition for status. Rather it is the large objective difference in rewards to high and low position that is what is important. For example, a top tennis tournament typically has prize money that is highly convex in the ranking achieved, which will induce highly competitive behaviour by tennis professionals even though they may have utility that is concave in wealth. Second, the results in Robson (1992) and Ray and Robson (2010) are qualitatively similar to those of Becker et al. (1992). More gambling happens in more equal societies and those who gamble have intermediate levels of wealth and not the poor.

Becker et al. (2005) and Ray and Robson (2010) draw an important further conclusion from their analysis: there is an upper bound on the level of equality that can be supported in society. If the level of equality exceeds it, then some agents would have an incentive to gamble leading to a wider distribution of wealth. In this paper, I find that the maximum level of equality in wealth is increasing in the equality of status. In fact, an arbitrarily equal distribution of wealth can be supported without gambling, if status is sufficiently equally distributed. Equally, if society operates at the maximum level of wealth equality, an increase in the inequality of status will lead to greater inequality in wealth: status inequality can create inequality in wealth.

Thus, this model provides an explicit theoretical mechanism which would support the apparent empirical relationship between inequality and risk-taking behaviour. But the causation flows in a different way than is normally assumed. If a society is relatively egalitarian in its treatment of its citizens, so that advantages of being rich beyond higher consumption are relatively minor then individuals have little incentive to gamble and the wealth distribution will remain undispersed. However, where social advantage leads to highly differential mating opportunities, such as in polygamous societies, or where there is significant differences in treatment between the social classes, then this provides a strong incentive for the low ranked to take on risks. This in turn leads to a relatively dispersed distribution of wealth.

There are, of course, other explanations of the link between risky behaviour and inequality. Indeed, there are many more factors affecting crime than just one's attitude to risk. For example, İmrohoroğlu et al. (2000) find that crime increases with the level of inequality in a general equilibrium model. Such models may explain why inequality is associated with economic crimes like theft. However, the documented link (Fajnzylber et al., 2002) between inequality and violent crime is more difficult to explain using purely economic motives.

2 A Status Tournament

The model is similar to that found in Frank (1985), Hopkins and Kornienko (2004, 2010) and Becker, Murphy and Werning (BMW) (2005). A large population of agents compete in a tournament with a range of ranked rewards that could represent either material outcomes, such as cash prizes, or non-material awards of status. Agents make a strategic decision over how to allocate their endowment between performance in the tournament and private consumption. As BMW first discovered, this situation can lead to individuals being willing to take fair gambles if they are offered before the tournament. This is because the utility function implied by equilibrium behaviour in the tournament can be convex in initial endowments, even though an individual has preferences that are concave in both consumption and rewards. The model is solved backwards. This section analyses the tournament stage of the game. The next section looks at the implied incentives to take gambles prior to the tournament.

I assume a continuum of agents. Each has a different endowment of wealth z_0 with endowments being allocated according to the publicly known distribution $G_0(z_0)$ on $[\underline{z}_0, \bar{z}_0]$ with $\underline{z}_0 > 0$. The distribution $G_0(z_0)$ is twice differentiable with strictly positive density $g_0(z_0)$.

Next, and before the tournament, individuals may gamble with their wealth. Specifically, it is assumed that a fair gamble in the form of a continuous density over a bounded interval may be available. Given the assumption that the available gambles are continuous densities, the resulting distribution of wealth $G(z)$ is also continuous with strictly positive density $g(z)$. Further, I assume that bankruptcy is not allowed so that the support of the resulting distribution is $[\underline{z}, \bar{z}]$ with $\underline{z} > 0$.

Then, in the tournament, agents must divide their wealth between performance x and consumption c. Performance has no intrinsic utility, but rewards/status s are awarded on the basis of performance, with the best performer receiving the highest reward, and in general, one's rank in performance determining the rank of one's reward. A specific interpretation in BMW and Hopkins and Kornienko (2004) is that x represents expenditure on conspicuous consumption, and s is the resulting status. An alternative, first due to Cole, Mailath and Postlewaite (1992), is that s represents the quality of a marriage partner achieved. Relating this to evolutionary considerations, the range of rewards in a society which permits a high degree of polygyny would be wider than in a society in which strict monogamy is enforced. What is important here is that there is a schedule of rewards or status positions available, which are assigned by performance, but are otherwise exogenous with respect to wealth.

In any case, it is assumed that all individuals have the same preferences over consumption c and status or rewards s,

$$U(c, s) \tag{1}$$

where U is a strictly increasing, strictly concave, three times differentiable function with $U_c, U_s > 0$, and $U_{cc}, U_{ss} < 0$. So, agents are risk averse with respect to both consumption and status. I also assume that $U_{cs} \geq 0$, so that the case of additive separability $U_{cs} = 0$ and status and consumption being positive complements $U_{cs} > 0$ are both included.

The order of moves is, therefore, the following:

1. Agents receive their endowments z_0.
2. Agents are offered fair gambles which they are free to accept or to reject. Denote the resulting wealth z.
3. Agents commit a part x of their wealth z to performance in the tournament.
4. Each agent receives a reward s, the value of which is determined by performance in the tournament.
5. Agents consume their remaining endowment $c = z - x$ and their reward s, receiving utility $U(c, s)$.

To this point, the model is identical to that of BMW (and very similar to that of Hopkins and Kornienko, 2004). However, here I follow Hopkins and Kornienko (2010) in assuming that the rewards or status positions of value s whose publicly known distribution has an arbitrary twice differentiable distribution function $H(s)$ on $[\underline{s}, \bar{s}]$, with $\underline{s} > 0$, and strictly positive density $h(s)$. BMW assume that $H(s)$ is fixed as a uniform distribution on [0,1]. As they point out, for the existence of equilibrium, this represents a harmless normalisation. However, this clearly prevents the major exercise here: identifying the change of behaviour arising from changes in the distribution of rewards.

Rewards or status are assigned assortatively according to rank in performance. Let $F(x)$ be the distribution of choices of performance. One's position in this distribution will determine the award achieved. Precisely, an individual who chooses a performance level x will receive a reward

$$S(x, F(\cdot)) = H^{-1}\left(\theta F(x) + (1-\theta)F^{-}(x)\right) \tag{2}$$

where and $F^{-}(x) = \lim_{\xi \uparrow x} F(\xi)$ and for some $\theta \in (0,1)$. This is a way of breaking potential ties.[1] However, if all contestants choose according to a continuous strictly increasing strategy $x(z)$, then, first, $F(x) = F^{-}(x)$ for all x, and, second, $F(x(z)) = G(z)$.[2] Together, this implies, $H(s) = F(x) = G(z)$, one holds the same rank in wealth, performance and in reward achieved, or

$$S(x, F(x)) = H^{-1}(F(x)) = H^{-1}(G(z)) = S(z). \tag{3}$$

We can call $S(z)$ the reward or status function, as in a monotone equilibrium, it represents the relationship between initial endowment and the reward or status achieved.

Importantly, the reduced form equilibrium utility given a monotone equilibrium performance function $x(z)$ will then be

$$U(z) = U(z - x(z), S(z)). \tag{4}$$

We will see that this function $U(z)$ can be convex, even given our concavity assumptions on $U(c, s)$. Therefore, agents would accept a fair gamble over their endowment, if such a gamble was offered before the tournament.

If all agents follow a monotone strategy $x(z)$, then an individual with endowment z should choose $x(z)$. If she considers deviating to a different level of performance $x(\hat{z})$, she will have no incentive to do so if

$$-x'(\hat{z})U_c(z - x(\hat{z}), S(\hat{z})) + S'(\hat{z})U_s(z - x(\hat{z}), S(\hat{z})) = 0. \tag{5}$$

Setting $x(\hat{z}) = x(z)$ and rearranging, we have

$$x'(z) = \frac{U_s(z - x(z), S(z))S'(z)}{U_c(z - x(z), S(z))}. \tag{6}$$

The solution to the above differential equation with boundary condition,

$$x(\underline{z}) = 0 \tag{7}$$

will be our equilibrium strategy. This is shown in the next result, which is a slight generalisation of similar results in Hopkins and Kornienko (2004) and BMW (2005).

[1] Note that $F(x)$ and $F^{-}(x)$ are only distinct when a positive mass of agents choose the same performance x. For a full discussion, see Hopkins and Kornienko (2004).

[2] The probability that an individual i has higher status than an another individual j is therefore $F(x_i(z_i)) = \Pr[x_i(z_i) > x_j(z_j)] = \Pr[x_j^{-1}(x_i(z_i)) > z_j] = G(x_j^{-1}(x_i(z_i))) = G(z_i)$.

Proposition 1. *There exists a unique solution $x(z)$ to differential equation (6) with boundary condition (7). This is the unique symmetric equilibrium to the tournament.*

Having established the framework of the tournament, the next step is to proceed in solving backwards. The next section considers the risk attitudes of agents who are about to participate in the tournament.

3 Implied Risk Attitudes

An individual with wealth z participating in the tournament described in the previous section will anticipate equilibrium utility $U(z) = U(z - x(z), S(z))$. If this function is convex for some range of wealth, then individuals with wealth on that range would take fair bets if such bets were offered to them prior to the tournament. The analysis in this section focuses on the question as to when in fact this function will be convex.

We have by the envelope theorem $U'(z) = U_c(z - x(z), S(z))$ and

$$U''(z) = U_{cc}(z - x(z), S(z))(1 - x'(z)) + U_{cs}(z - x(z), S(z))S'(z). \tag{8}$$

By inspection one can immediately see that $U''(z)$ will be positive, even though $U_{cc} < 0$, if either $x'(z)$ or $S'(z)$ is sufficiently large. Note that $S'(z) = g(z)/h(S(z))$. Thus, BMW's result that equality in endowments would lead agents to be willing to accept lotteries follows quite directly. If the distribution of endowments $G(z)$ is strongly unimodal, then its density $g(z)$ will have a very high value at and around its mode.

One could also decompose the expression (8) into (suppressing arguments)

$$U''(z) = U_{cc} + (\ U_{cs}S'(z) - x'(z)U_{cc}\ )$$

which separates the negative and positive elements but also the traditional and non-traditional parts. The first part U_{cc} is negative and reflects risk aversion towards regular consumption. The second, in brackets, gives the competitive part which is positive. The problem in obtaining an unambiguous result is that both factors can become larger in absolute terms when wealth or status is low. With low wealth (and hence low c) traditionally one would be risk averse. But in the presence of status competition, low status leads to desperation and love of risk.

Nonetheless, one can find a sufficient condition for low status individuals to be risk loving. It is a condition on the marginal value of status.

Definition 1. Devil Take the Hindmost[3] (DTTH) condition: $\lim_{\underline{s}\to 0} U_s(c, \underline{s}) = \infty$ for any $c > 0$ and $U_c(c, s)$ is bounded above for $c > 0$.

[3] On the origin of this phrase: "It is said when a class of students have made a certain progress in their mystic studies, they are obliged to run through a subterranean hall, and the last man is seized by the devil" (Brewer (2001)).

For example, suppose $U = \log c + \log s$, then $U_s = 1/s$ so that as $\underline{s}$ tends to 0 then U_s tends to infinity. In general, since $U_{ss} < 0$ by assumption, as the lowest reward or level of status $\underline{s}$ decreases, it pushes its marginal value U_s higher. The DTTH assumption is simply that U_s is not bounded above. Thus, when the consequences of being last are sufficiently unattractive (for example, being seized by the devil), the value to the last-placed individual of moving up the field is arbitrarily high.

I now show that given the DTTH condition, the poorest individuals in any society must be risk loving if their status is sufficiently low. This is independent of the minimum level of wealth $\underline{z}$. That is, even in rich societies, the lowest ranked people can be risk loving. In the developed world, the poor may have consumption levels that are high by historic standards, but what this result shows is that if relative status is low, they still may be risk-taking.

The result is stated for the individual with the lowest possible status $\underline{s}$, but by continuity of the utility function, if the lowest ranked individual is risk loving, so will be an interval of others with higher wealth (see also Example 1 below).

Proposition 2. *Assume the DTTH condition, fix the distribution of wealth with $\underline{z} > 0$, and consider a distribution of rewards such that $S'(\underline{s}) > 0$ and $\underline{s} > 0$. Then, there is an $s^* > 0$ such that if the minimum reward level $\underline{s}$ is less than s^* then the poorest individual will be risk loving.*

Note that the effect of taking the minimum level of wealth to zero will have the opposite effect. For simplicity, suppose there is additive separability so that $U_{cs} = 0$. Then, if U_c becomes large as $\underline{z}$ goes to zero, x' will go to zero, and U'' will be negative. So low wealth leads to risk aversion. It is low status that leads to risk taking.

3.1 Effects of an Increase in the Dispersion of Rewards

Further, it is possible to show that making rewards more unequal leads to more risk taking behaviour. To do this, I will compare two distributions of rewards $H_a(s)$ and $H_p(s)$, a for ex ante and p for ex post. I suppose that the distribution of rewards changes from $H_a(s)$ to $H_p(s)$ for exogenous reasons. We then see how this affects risk attitudes.

To carry out this analysis, some notion of a distribution being more dispersed than another is needed. I use a strong version of the dispersive order. Specifically, I say that a distribution H_p is strictly larger in the dispersive order than a distribution H_a, or $H_p >_d H_a$ if

$$h_p(H_p^{-1}(r)) < H_a(H_a^{-1}(r)) \text{ for all } r \in [0,1]. \tag{9}$$

The original definition of this stochastic order (Shaked and Shanthikumar, 2007, pp148-9) has the same condition but with a weak inequality, and on (0,1). A simple example of distributions satisfying this stronger condition would be any two uniform distributions where one distribution has support on a strictly longer interval than the other (see Hopkins and Kornienko (2010) for further examples and discussion).

Lemma 1. *Suppose that the distribution of rewards becomes strictly more dispersed in terms of the dispersive order $H_p >_d H_a$ and that the lowest reward $\underline{s}$ is unchanged then the poor become more risk loving. That is, there is a $\hat{z} \in (\underline{z}, \bar{z}]$ such that $U_p''(z) > U_a''(z)$ on $[\underline{z}, \hat{z})$.*

While one might think that a general increase in the dispersion of rewards would lead to a general increase in risk-taking, this may not be the case. This is because an increase in the dispersion of rewards makes the tournament more competitive, which will tend to raise performance and lower consumption. For example, Hopkins and Kornienko (2010) find simple sufficient conditions for all agents to increase performance in response to more dispersed rewards. This matters as, other factors being equal, lower consumption typically increases risk aversion.

A further result is that reducing the minimum reward level will increase risk-taking by the poorest in society.

Lemma 2. *Consider two distributions of rewards $H_a(s)$ and $H_p(s)$ which differ in terms of minimum status such that $\underline{s}_a > \underline{s}_p$, but $h_p(\underline{s}_p) = h_a(\underline{s}_a)$. Either (a) assume additive separability so that $U_{cs} = 0$; or (b) assume the DTTH condition and that $U_{css}, U_{ccs} \leq 0$ and that $U_a''(\underline{z}) \leq 0$. Then, $U_p''(\underline{z}) > U_a''(\underline{z})$.*

Putting these two results together, it is possible to obtain the following result: greater inequality causes the poor to be more risk-taking. This case would include a form of mean preserving spread on rewards. For example, two uniform distributions having the same mean but with one H_p having a wider support would be suitable.

Proposition 3. *Suppose that the distribution of rewards becomes strictly more dispersed in terms of the dispersive order $H_p >_d H_a$, and the minimum reward decreases $\underline{s}_p \leq \underline{s}_a$. Either (a) assume additive separability so that $U_{cs} = 0$; or (b) assume the DTTH condition and that $U_{css}, U_{ccs} \leq 0$ and that $U_a''(\underline{z}) \leq 0$. Then the poor become more risk loving. That is, $U_p''(\underline{z}) > U_a''(\underline{z})$.*

See Figure 1 for an illustration of this result. It also gives typical results on how performance and the level of utility responds to the greater level of competition implied by greater inequality of rewards. While there are no such results in this paper, Hopkins and Kornienko (2010) already have shown that performance rises and utility falls for most, and sometimes for all, individuals. See also Example 1 below.

3.2 Effects of an Increase in the Dispersion of Wealth

BMW argue that increase in the dispersion of wealth, such as produced by gambling over wealth, should reduce the desire to gamble. However, it is not straightforward to transfer the above results on greater inequality of rewards to greater inequality of

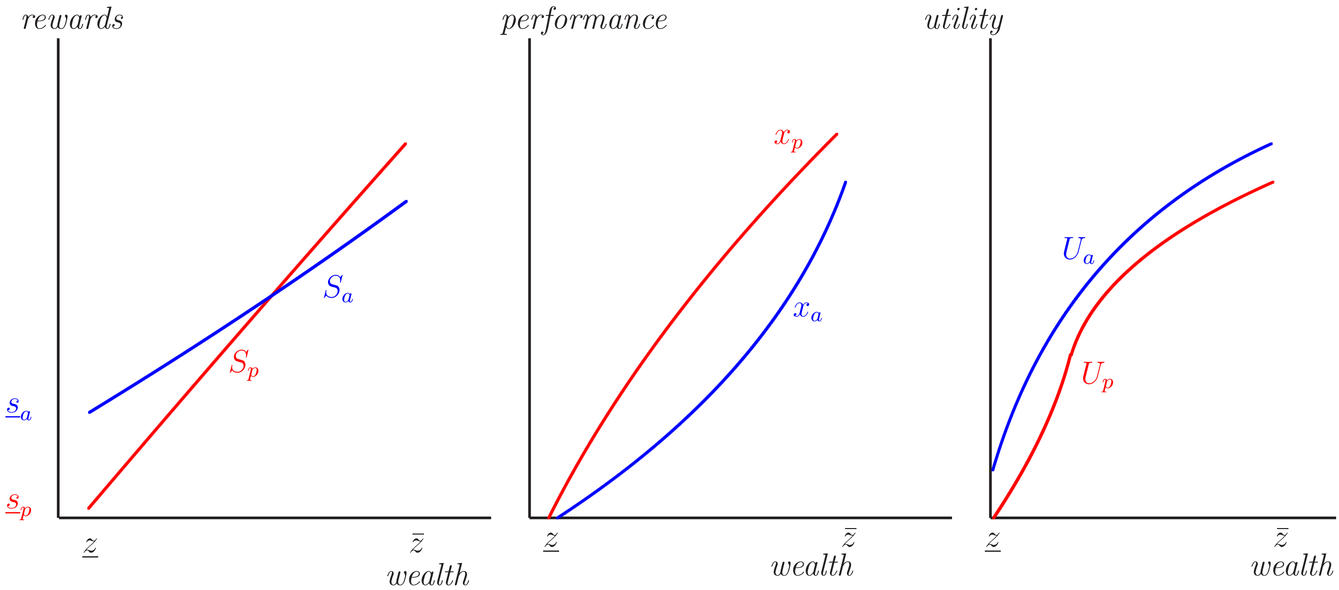

Figure 1: Illustration of Proposition 3: ex post status S_p is more dispersed than ex ante S_a leading to the slope of the equilibrium utility function becoming convex at low levels of wealth. Typically, ex post utility U_p is lower and ex post performance x_p is higher.

wealth. For example, it is possible to show that certain mean preserving spreads will make the poorest less, not more, risk averse.

For this, I use second order stochastic dominance. Specifically, let us say a distribution F is more dispersed than a distribution G in terms of second order stochastic dominance with single crossing, and we write $F >_{sc} G$ if they have the same mean and

$$\int_{\underline{z}}^{z} F(t) - G(t)\ dt > 0 \tag{10}$$

on $(\underline{z}, \bar{z})$ and are single crossing. That is, $F(z) > G(z)$ on $(\underline{z}, \hat{z})$ and $F(z) < G(z)$ on $(\hat{z}, \bar{z})$ for some $\hat{z} \in (\underline{z}, \bar{z})$. This represents a refinement of the standard definition of second order stochastic dominance (see, for example, Wolfstetter, 1999, pp. 140-4), in which the inequality (10) hold weakly and there is no single crossing condition.

I now consider changes in the distribution of wealth, assuming that the distribution changes from some distribution $G_a(z)$ ex ante, to another distribution $G_p(z)$ ex post. One can think about this change occurring for two different reasons. First, there could be some exogenous change. Second, the distribution of wealth could become more dispersed due to the gambling activity by individuals. But whatever the reason for the change, we will see how this affects individual risk attitudes.

Proposition 4. *Suppose that the distribution of wealth becomes more dispersed in terms of second order stochastic dominance with single crossing $G_p >_{sc} G_a$, and minimum wealth $\underline{z}$ is unchanged. Then the poor become less risk averse. That is, $U_p''(\underline{z}) > U_a''(\underline{z})$.*

The above result is based on the assumption that the dispersion of wealth rises without the support of the distribution widening. So, the density of people rises at the top and bottom ends of the distribution. In general, as found by Hopkins and Kornienko (2004), a higher density means greater competitiveness, and here the higher density of poor people leads to a higher willingness to undertake risky behaviour.

In contrast, to increase risk aversion at low incomes, it is necessary to disperse wealth over a greater range, and in particular to make the poorest poorer. Even then strong conditions are needed to ensure that risk-taking decreases. Nonetheless, the final result in this section establishes, as BMW supposed, that greater dispersion of wealth can lower individuals' willingness to gamble.

Proposition 5. *Suppose that the distribution of wealth becomes strictly more dispersed in terms of the dispersive order $G_p >_d G_a$, and minimum wealth decreases $\underline{z}_p \leq \underline{z}_a$. Assume that $U_{ccc} \geq 0$ and $U_a''(\underline{z}_a) \leq 0$ and assume either (a) that there is additive separability so that $U_{cs} = 0$; or (b) that $U_{ccs} \leq 0$ and U_{cc}/U_{cs} is increasing in c.*[4] *Then the poor become more risk averse. That is, $U_p''(\underline{z}_p) < U_a''(\underline{z}_a)$.*

3.3 Cobb-Douglas

In this section, for concreteness we look at Cobb-Douglas preferences, for which closed form solutions for equilibrium behaviour and preferences are possible. Suppose

$$U(c, S) = c^\alpha S^\beta = (z - x)^\alpha S^\beta \tag{11}$$

Let $\gamma = \beta/\alpha$. Then,

$$x'(z) = \gamma \frac{S'(z)}{S(z)}(z - x) \tag{12}$$

with again $x(\underline{z}) = 0$. This differential equation has the explicit solution

$$x = z - \frac{\underline{s}^\gamma \underline{z} + \int_{\underline{z}}^{z} S^\gamma(t)dt}{S^\gamma(z)}, \quad c = \frac{\underline{s}^\gamma \underline{z} + \int_{\underline{z}}^{z} S^\gamma(t)dt}{S^\gamma(z)}.$$

Thus

$$U(z) = \left(\underline{s}^\gamma \underline{z} + \int_{\underline{z}}^{z} S^\gamma(t)dt\right)^\alpha \tag{13}$$

and

$$U'(z) = \alpha S^\gamma(z)(\underline{s}^\gamma \underline{z} + \int_{\underline{z}}^{z} S^\gamma(t)dt)^{\alpha-1} = \alpha c^{\alpha-1} S^\beta(z)$$

and

$$U''(z) = \alpha(\alpha - 1)c^{\alpha-2} S^\beta(z)(1 - x') + \alpha\beta c^{\alpha-1} S^{\beta-1}(z) S'(z).$$

With Cobb-Douglas preferences, the expression for absolute risk aversion is particularly neat,

$$AR(z) = -\frac{U''(z)}{U'(z)} = -\frac{\gamma S'(z)}{S(z)} + \frac{1 - \alpha}{c(z)} \tag{14}$$

[4] This last condition certainly holds for Cobb-Douglas and CES utility functions.

where again $\gamma = \beta/\alpha$. That is, changes in the ratio $S'(z)/S(z)$ clearly change risk preferences (though $c(z)$ will also change). But we could also define a form of relative risk aversion as $AR(z)c(z)$ which would give us

$$RR(z) = -\frac{c(z)U''(z)}{U'(z)} = -\frac{\gamma c(z)S'(z)}{S(z)} + (1-\alpha) = -x'(z) + (1-\alpha) \tag{15}$$

Example 1. Suppose rewards are uniform on $[\varepsilon, 1-\varepsilon]$ and wealth is uniform on $[1,5]$ and $\alpha = \beta$ so that $\gamma = 1$. We have then

$$S(z) = \varepsilon + \frac{1-2\varepsilon}{4}(z-1)$$

and

$$U(z) = \left(\varepsilon + \int_1^z \varepsilon + \frac{1-2\varepsilon}{4}(t-1)dt\right)^\alpha = \left(\frac{(z-1)^2 + 2\varepsilon(-1+6z-z^2)}{8}\right)^\alpha$$

Take, for example, $\alpha = 0.4$. With a relatively equal distribution of rewards/status S_a, for example with $\varepsilon = 0.25$, all agents are risk averse. However, let us make rewards more unequal, $\varepsilon = .1$, label this new status function S_p. Then, $U_p(z)$ is convex on [1,2.44) and is concave on (2.44, 5]. See Figure 1. That is, take an individual with an endowment of about 2.5, then that individual will be risk loving with respect to losses and risk averse with respect to gains. Note that ex post equilibrium utility $U_p(z)$ is everywhere lower than ex ante $U_a(z)$ and ex post equilibrium performance $x_p(z)$ is everywhere higher ($x_a = (z-1)/2$ and $x_p = (z^2-1)/(2z-1)$). We can also verify that more dispersed wealth makes agents more risk averse. Keeping rewards dispersed with $\varepsilon = 0.1$ but making wealth also more dispersed, so for example wealth is now uniform on $[0.5, 5.5]$, utility will return to being concave at all wealth levels.

4 Interaction Between Inequality in Rewards and in Wealth

BMW, following Robson (1992), consider distributions of wealth that are *stable* in the sense that given such a distribution, no agent wishes to gamble and therefore the distribution of wealth does not change. Note that there will potentially be many wealth distributions that induce no gambling. Thus, BMW focus on the stable wealth distribution (which they call the $*$ allocation) that induces risk neutrality at all levels of wealth.[5] The idea is that distributions that are less dispersed than the stable distribution will induce gambling (indeed, see Proposition 5 in the previous section). Thus, this stable distribution represents an upper bound on sustainable equality of wealth. So, let us call it the *most equal stable distribution* or MESD.

[5]They show that marginal utility having a constant value λ, or $U'(z) = U_c(c(z), S(z)) = \lambda$ in current notation, is also a solution to the problem of a utilitarian social planner.

In this section, there are the following novel results. First, I prove the uniqueness of the MESD. Second, I show that greater inequality in rewards implies that the MESD becomes more unequal. Third, a result that follows directly from the second, the minimum level of inequality is as low as the degree of inequality in status. Societies that offer a high degree of equality of esteem can support very equal distributions of wealth.

Further, suppose we take the MESD as a prediction of the actual distribution of wealth in society. Then our second result implies that societies that are socially more unequal *result* in more unequal wealth outcomes. For example, consider two societies that have the same initial distribution of wealth that is quite equal. The first society is relatively socially egalitarian with the status assigned to the poorest individual relatively high so that there is no incentive to gamble and the initial distribution of wealth is unchanged. However, the second society treats its citizens more unequally with a low minimum status. This we have seen can induce gambling, which will result in a greater dispersion of wealth. Thus, more unequal social conditions can produce wealth inequality.

We now solve for the MESD, which is defined as the wealth distribution that induces risk neutrality at all levels of wealth. If we set the expression for $U''(z)$ in (8) to zero, we obtain the following differential equation (suppressing arguments)

$$S'(z) = \frac{U_c U_{cc}}{U_s U_{cc} - U_c U_{cs}} \tag{16}$$

with boundary condition

$$S(\underline{z}) = \underline{s}. \tag{17}$$

Using this differential equation (16) and the differential equation (6) for equilibrium performance, we can write a new differential equation for equilibrium consumption,

$$c'(z) = \frac{U_c U_{cs}}{U_c U_{cs} - U_s U_{cc}}. \tag{18}$$

Given the boundary condition (7) for equilibrium performance, the boundary condition for the above equation will be $c(\underline{z}) = \underline{z}$. A solution of the two equations simultaneously will provide the MESD. Specifically, the MESD $G^*(z)$ is defined as $G^*(z) = H(S^*(z))$, where $S^*(z)$ is the solution to the equation (16). Further, it is possible to prove the MESD is unique, for a given distribution of rewards and for a given mean wealth.

Proposition 6. *For a given distribution of rewards $H(s)$, there is a unique solution $(c^*(z), S^*(z))$ to the simultaneous differential equation system (16) and (18) with boundary conditions $c(\underline{z}) = \underline{z}$ and (17), such that $U(z) = U(c^*(z), S^*(z))$ is linear in z for all $z \in [\underline{z}, \bar{z}]$. Thus, $U''(z) = 0$ at all wealth levels. Further, assume that $U_{ccc} \geq 0, U_{ccs} \leq 0$ and U_{cc}/U_{cs} is increasing in c, then for fixed mean wealth μ there is a unique distribution of wealth $G^*(z)$ such that $H^{-1}(G^*(z)) = S^*(z)$.*

From this proposition, we can draw the following comparative statics result. The MESD moves with the distribution of rewards. If rewards become more (less) equal, the minimum level of wealth inequality falls (rises) in the sense of second order stochastic dominance with single crossing, a concept introduced in the previous section.

In what follows, it is assumed that there are ex ante and ex post distributions of rewards, $H_a(s)$ and $H_p(s)$ respectively. Under each distribution of rewards, we calculate $S_i^*(z)$ for $i = a, p$, the associated reward function that induces risk neutrality at all wealth levels. Further, by the previous result, Proposition 6, this will also define $G_a(z)$ and $G_p(z)$ the ex ante and ex post distributions of wealth. We find that a greater dispersion in rewards necessitates a greater dispersion in wealth in order to maintain risk neutrality. An example of this is illustrated in Figure 2.

Proposition 7. *Assume the ex post distribution of rewards H_p is more dispersed than the ex ante distribution H_a in terms of the dispersive order, $H_p >_d H_a$, that the minimum reward falls or $\underline{s}_p < \underline{s}_a$, that the maximum reward rises $\bar{s}_p > \bar{s}_a$ and the mean reward is unchanged. Assume further that $U_{css}, U_{ccs} \leq 0$, $U_{ccc} \geq 0$ and U_{cc}/U_{cs} is increasing in c. Then, the ex post MESD wealth distribution is more dispersed in terms of second order stochastic dominance with single crossing than ex ante. That is, $G_p >_{sc} G_a$.*

This has an important corollary. If we consider a sequence of distributions of rewards each progressively more equal than the previous, then the corresponding distributions of wealth would also become progressively more equal.

Corollary 1. *As the distribution of rewards approaches perfect equality, so does the Most Equal Stable Distribution of wealth.*

Despite the earlier results of BMW and Ray and Robson (2010), it is possible to sustain an equal society, even in the presence of status competition, provided there is an equality in terms of esteem.

4.1 Cobb-Douglas

Assume Cobb-Douglas preferences $U(c, s) = c^\alpha s^\beta$, then the differential equation (16) becomes

$$S'(z) = \frac{\alpha(1-\alpha)S(z)}{\beta c(z)}$$

and (18) becomes

$$x'(z) = 1 - \alpha.$$

This implies that performance and consumption are linear in wealth, specifically $x(z) = (1-\alpha)(z - \underline{z})$ and $c(z) = \alpha z + (1-\alpha)\underline{z}$. This in turn can be used to solve for $S^*(z)$:

$$S^*(z) = A[c(z)]^{(1-\alpha)/\beta} = A(\alpha z + (1-\alpha)\underline{z})^{(1-\alpha)/\beta},$$

where A is a constant of integration. One can check that this implies $U(z) = A^\beta c(z)$ which is linear as required. Of course, for strict concavity of the Cobb-Douglas utility

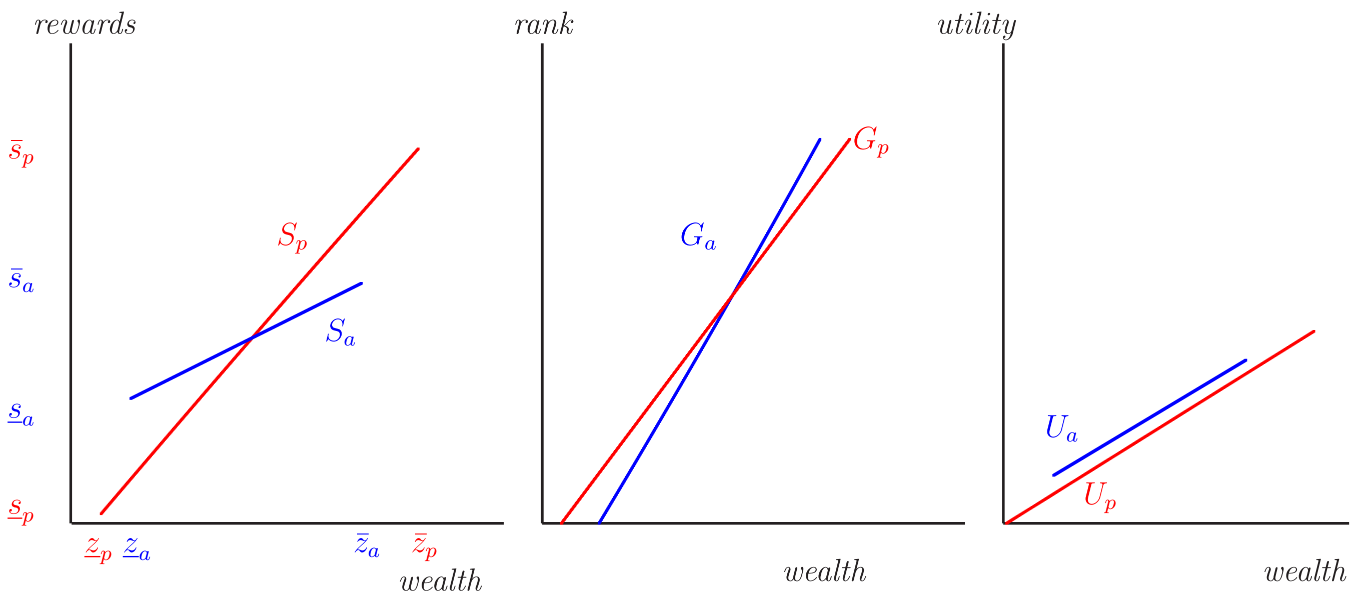

Figure 2: Illustration of Proposition 7: ex post rewards S_p are more dispersed than ex ante S_a. To maintain risk neutrality, ex post wealth G_p must be more dispersed than ex ante G_a, with ex post minimum wealth $\underline{z}_p$ being lower and maximum wealth $\bar{z}_p$ being higher than ex ante.

function, one needs $\alpha + \beta < 1$, so that $S^*(z)$ is therefore convex. Thus, as $G^*(z)$, the MESD, is equal to $H(S^*(z))$, this minimum inequality wealth distribution will be more convex than the distribution of rewards.

Example 2. Assume that rewards are distributed uniformly on $[\varepsilon, 1-\varepsilon]$. Assume further that $\alpha = \beta = 1/2$ (of course, this means that the utility function is not strictly concave, but as we will see it makes everything conveniently linear). Then, given mean wealth of $1/2$, the unique distribution $G^*(z)$ that solves for S^* is

$$G^*(z) = \frac{(1-\varepsilon)z - \varepsilon/2}{1-2\varepsilon}.$$

That is, it is uniform on $[\varepsilon/(2(1-\varepsilon)), (2-3\varepsilon)/(2(1-\varepsilon))]$. We have

$$S^*(z) = (1-\varepsilon)z + \varepsilon/2, \; U^*(z) = \frac{\varepsilon/2 + (1-\varepsilon)z}{\sqrt{2(1-\varepsilon)}}.$$

Clearly, a decrease in ε makes the distribution of rewards more dispersed. It will also make the equilibrium distribution of wealth $G^*(z)$ more dispersed. Equally, a more equal distribution of rewards, implies a more equal stable distribution of wealth. Indeed, as ε approaches $1/2$, then both the distribution of rewards and the distribution of wealth become entirely concentrated at $1/2$.

5 Discussion: Risk-Taking, Gender and Age

It is possible to link the formal results of this paper to some quite simple conclusions about risk taking by age and by gender. It follows quite directly that risk taking can be expected to be greater by young low-ranking males.

First, after the tournament, in "old age", all agents will be risk averse. In the final stage, after rewards have been assigned, an agent will have a reward s and will have her endowment less x, the amount spent on the tournament. She will have utility $U(z-x,s)$ if she goes ahead and consumes the remaining endowment and the reward. If offered a fair gamble over either, she will refuse as U is concave in both arguments by assumption. That is, gambling only occurs when young.

Second, from Proposition 3, one can see that a population facing a greater dispersion in rewards will have greater risk-taking by those with low endowments. So, if men as a population have more dispersed rewards than women, low-ranking men will be more risk-taking than low-ranking women.

It is well-recognised that, in an evolutionary sense, men's rewards are more variable than women's. As Wilson and Daly (1985, p60) write, "male fitness variance exceeds female fitness variance". This is because, while female fertility is limited by physiological constraints, male fertility can be much higher if access to multiple mates is possible. Wilson and Daly argue that therefore the effective degree of polygyny - the extent to which a single male can have multiple exclusive partners - determines the level of social competition amongst males and the "more intense this competition, the more we can expect males to be inclined to risky tactics" (p. 60). That is, the current model provides formal support for Wilson and Daly's argument.

The previous study closest to the current work is Robson (1996). He considers a model where men care about relative wealth because of the possibility of polygyny: high relative wealth means that a man can attract multiple partners. This gives men an incentive to gamble. He shows that in some cases the only stable wealth distribution is where one man has all the wealth. The effective difference here is that by varying the reward schedule, the rate at which relative wealth can be converted into marriage opportunities is altered. Thus, the incentive to gamble can itself be varied.

The idea that low ranked agents may have an incentive to gamble has an apparent similarity to the idea of "gambling for resurrection", in which agents who are near to bankruptcy have an incentive to gamble because any downside losses would be truncated. See, for example, Gollier et al. (1997). However, none of the results in this paper depend on any such mechanism. Here agents will take fair bets, even though they will have to suffer the downside in full. Clearly, if limited liability were a possibility, then the incentive to gamble would be increased.

An evolutionary approach also suggests how the findings in this paper - risk-taking by the poor is increasing in inequality of status, but decreasing in the inequality of wealth - might be distinguished empirically. The current model specifies a distribution of rewards or status outcomes that is exogenous and independent of the distribution of wealth. Marriage arrangements are one example of how rewards could vary in this way. Some societies explicitly allow polygamy, others condone polygyny while others are strictly monogamous. Thus, the evolutionary return to high status would be quite different across these different societies. Further, while the underlying causation for these differing customs may be economic, such institutions change slowly. Thus, most individuals would plausibly take them as fixed.

Thus, the apparent empirical relationship between economic inequality and risk-taking behaviour might be misleading. Rather, as in the model presented here, it is unequal social relationships that can cause risk-taking behaviour. Inequality in wealth then follows as a result, as social inequality provides individuals with an incentive to gamble over wealth. This is a fascinating possibility which merits further empirical investigation.

Appendix

Proof of Proposition 1: This proof follows that of Proposition 1 of Hopkins and Kornienko (2004). A sketch is as follows. Given $U_{cs} \geq 0$, best replies are (weakly) increasing in z. Given the tie breaking rule (2), a symmetric equilibrium strategy must in fact be strictly increasing. If the equilibrium strategy is strictly increasing then it can be shown to be continuous and, furthermore, differentiable. Thus, it satisfies the differential equation (6). This has a unique solution by the fundamental theorem of differential equations.

The first order condition (5) is a maximum, as if all others adopt the proposed equilibrium strategy utility is pseudoconcave in x for each individual. That is, $U(z - x, H^{-1}(F(x)))$ is increasing in x for x less than the equilibrium choice $x(z)$ and is decreasing in x for x greater than $x(z)$.[6] To show this, if all agents adopt a strictly increasing strategy $x(z)$ then an individual's utility can be written as $U(z - x, H^{-1}(F(x)))$ and $\partial U/\partial x = -U_c(z - x, H^{-1}(F(x))) + U_s(z - x, H^{-1}(F(x)))f(x)/h(\cdot)$. Then, one has $\partial^2 U/\partial x \partial z = -U_{cc} + U_{cs} f(x)/h(\cdot) > 0$. Take $\breve{x} < x(z)$ and let $\breve{z}$ be such that $x(\breve{z}) = \breve{x}$, so that $\breve{z} < z$. Hence, for any $\breve{x} < x(z)$, $dU(z - \breve{x}, H^{-1}(F(\breve{x})))/dx \geq dU(\breve{z} - \breve{x}, H^{-1}(F(\breve{x})))/dx = 0$. Thus, utility is increasing in x for x below the equilibrium choice $x(z)$. A similar argument can establish that it is decreasing in x for x above $x(z)$.

The boundary condition (7) must hold as the agent with lowest wealth $\underline{z}$ in a symmetric equilibrium has status $S(\underline{z}) = \underline{s}$ and thus chooses performance x to maximise

[6] That is, utility $U(z - x, H^{-1}(F(x)))$ is pseudoconcave in performance x, even though, as we will see, a major point of this paper is that the indirect equilibrium utility function $U(z) = U(z - x(z), S(z))$ can be convex in wealth z.

$U(\underline{z} - x, \underline{s})$. Clearly, the optimal choice of performance for the agent with wealth $\underline{z}$ is zero. $\square$

Proof of Proposition 2: One has from (7) that $c(\underline{z}) = \underline{z}$, so that from (8) it follows that

$$U''(\underline{z}) = U_{cc}(\underline{z}, \underline{s})(1 - x'(\underline{z})) + U_{cs}(\underline{z}, \underline{s})S'(\underline{z}) \tag{19}$$

and from (6) that

$$x'(\underline{z}) = \frac{U_s(\underline{z}, \underline{s})S'(\underline{z})}{U_c(\underline{z}, \underline{s})}. \tag{20}$$

It can be calculated that

$$\frac{\partial x'(\underline{z})}{\partial \underline{s}} = \frac{U_{ss}U_c - U_{cs}U_s}{U_c^2} S'(\underline{z}) < 0.$$

But this implies that $x'(\underline{z})$ is monotone in $\underline{s}$. Further, applying the DTTH condition, one obtains $\lim_{\underline{s} \to 0} x'(\underline{z}) = \infty$ (note that as $U_{cs} \geq 0$ then $U_c(\underline{z}, \underline{s})$ will not increase as $\underline{s}$ decreases). Putting these together, there is clearly an $s_0 > 0$ such that $x'(\underline{z}) = 1$ for $\underline{s} = s_0$. Therefore, given the continuity of $U''(\underline{z})$, there exists s^* such that $U''(\underline{z})$ is strictly positive for $\underline{s} < s^*$ where $s^* \geq s_0 > 0$ (with $s^* = s_0$ only if $U_{cs} = 0$). $\square$

Proof of Lemma 1: The second derivative of the utility function risk aversion for the poorest agent is $U_i''(\underline{z}) = U_{cc}(\underline{z}, \underline{s})(1 - x'(\underline{z})) + U_{cs}(\underline{z}, \underline{s})S_i'(\underline{z})$ for $i = a, p$. That is, as $c(\underline{z}) = \underline{z}$ and $S(\underline{z}) = \underline{s}$ under both distributions, the only way that $U''(\underline{z})$ can differ is in terms of S' and x'. The dispersive order by its definition (9) implies that $h_p(H_p^{-1}(r)) < h_a(H_a^{-1}(r))$ for $r \in [0, 1]$. Now, $S'(z) = g(z)/h(S(z)) = g(z)/h(H^{-1}(r))$. Thus, given $g(z)$ is unchanged, the dispersive order implies that $S_p'(z) > S_a'(z)$ for all $z \in [\underline{z}, \bar{z}]$. It is easy to verify that an increase in $S'(\underline{z})$ will also increase $x'(\underline{z})$ as given in (20). The result follows. $\square$

Proof of Lemma 2: (a) Under additive separability, the second derivative of the utility function for the poorest agent becomes $U''(\underline{z}) = (1 - x'(\underline{z}))U_{cc}(\underline{z})$. It is is easy to verify that a decrease in $\underline{s}$ will increase $x'(\underline{z})$ as given in (20), but given separability will not affect U_c or U_{cc}. The result follows.
(b) When there is not additive separability, one has

$$\frac{\partial U''(\underline{z})}{\partial \underline{s}} = U_{ccs}(1 - x'(\underline{z})) + U_{css}S'(\underline{z}) - \frac{\partial x'(\underline{z})}{\partial \underline{s}} U_{cc},$$

which, given our assumptions, is certainly negative where $x'(\underline{z}) < 1$. In the proof of Proposition 2 it was shown that $x'(\underline{z})$ is monotone in $\underline{s}$. Thus, as noted, there must be a value s_0 such that if $\underline{s} = s_0$ then $x'(\underline{z}) = 1$. If, as assumed, $\underline{s}_a$ is such that $U_a''(\underline{z}) \leq 0$, then $\underline{s}_a > s_0$. If also $\underline{s}_p > s_0$, then it follows that $U_p''(\underline{z}) > U_a''(\underline{z})$, as $U''(\underline{z})$ is monotone in $\underline{s}$ on $(s_0, \underline{s}_a)$. If $\underline{s}_p \leq s_0$, then $U_p''(\underline{z}) > 0 \geq U_a''(\underline{z})$ and the result follows. $\square$

Proof of Proposition 3: This follows directly from Lemma 1 and Lemma 2. $\square$

Proof of Proposition 4: As wealth and status of the poorest agent is unchanged, the only effect on $U''(\underline{z})$ as given in (19) is from a change in the density $S'(\underline{z}) = g(\underline{z})/h(\underline{s})$.

Now, as second order stochastic dominance by definition requires $\int_{\underline{z}}^{z} G_p(t) - G_a(t)dt > 0$ on $(\underline{z}, \bar{z})$, we have (generically) $g_p(\underline{z}) > g_a(\underline{z})$ and the result follows. $\square$

Proof of Proposition 5: We again consider $U''(\underline{z})$ as given in (19). From (20) it can be calculated that

$$\frac{\partial x'(\underline{z})}{\partial \underline{z}} = \frac{U_{sc}U_c - U_{cc}U_s}{U_c^2} S'(\underline{z}) > 0,$$

and

$$\frac{\partial U''(\underline{z})}{\partial \underline{z}} = U_{ccc}(1 - x'(\underline{z})) + U_{ccs}S'(\underline{z}) - \frac{\partial x'(\underline{z})}{\partial \underline{z}} U_{cc}. \tag{21}$$

(a) Given additive separability, if $U''(\underline{z}) \leq 0$ then $x' \leq 1$. It follows that the derivative (21) is strictly positive. Thus, the decrease in minimum wealth considered by itself leads to greater risk aversion for the poorest. Further, by the dispersive order we have $g_p(\underline{z}_p) = g_p(G_p^{-1}(0)) < g_a(G_a^{-1}(0))$ and so $S_p'(\underline{z}) = g_p(\underline{z}_p)/h(\underline{s}) < g_a(\underline{z}_p)/h(\underline{s}) = S_a'(\underline{z})$ and thus the greater dispersion also increases risk aversion, and the result follows.
(b) If $U''(\underline{z}) \leq 0$ then $x'(\underline{z}) < 1$ and $S'(\underline{z}) \leq -(1 - x'(\underline{z}))U_{cc}/U_{cs}$. Thus, the derivative (21) satisfies

$$\frac{\partial U''(\underline{z})}{\partial \underline{z}} \geq (1 - x'(\underline{z}))(U_{ccc} - U_{ccs}\frac{U_{cc}}{U_{cs}}) - \frac{\partial x'(\underline{z})}{\partial \underline{z}} U_{cc}.$$

If U_{cc}/U_{cs} is increasing, then $U_{ccc} - U_{ccs}U_{cc}/U_{cs} \geq 0$ and the derivative (21) is positive. The result then follows as in part (a). $\square$

Proof of Proposition 6: By the definition of the differential equation (16), the solution $(c^*(z), S^*(z))$ implies that $U''(c^*(z), S^*(z)) = 0$. Thus, $U(z)$ is linear as $U'(z) = U_c > 0$. Such a solution must exist by the fundamental theorem of differential equations because both (16) and (18) are continuously differentiable and bounded. The solution is unique for a given initial condition, that is, for a given minimum wealth level $\underline{z}$. That is, there is a family of distributions $\langle G_i \rangle$ that each satisfy $H^{-1}(G_i(z)) = S^*(z)$, each corresponding to a different level of minimum wealth $\underline{z}_n$. Finally, I prove that in this family, average wealth μ is strictly increasing in $\underline{z}$.

The equation system (c', S') as defined by (16) and (18) is autonomous, that is a function of c and S alone and only a function of z through c and S. It thus follows by fundamental theory of differential equations, that two solution curves $(c(z), S(z))$ cannot cross on the (c, S) plane. So, given two solutions with initial conditions $(\underline{z}_i, \underline{s})$ and $(\underline{z}_j, \underline{s})$ for some $\underline{z}_i < \underline{z}_j$, it follows that $c_j > c_i$ for any given value of S. Now consider the two associated solutions for rewards, $S_i(z)$ and $S_j(z)$ on the (z, S) plane. I claim there is no value of z such that $S_i(z) = S_j(z)$. Suppose not, then because $S_i(\underline{z}_i) = S_j(\underline{z}_j) = \underline{s}$ and $\underline{z}_j > \underline{z}_i$, at the first such crossing S_j must cross S_i from below. But as

$$\frac{\partial S'(c, s)}{\partial c} = \frac{-U_cU_{cs}U_{cc}^2 + U_{cc}^3U_s - U_c^2(U_{ccc}U_{cs} - U_{cc}U_{ccs})}{(U_sU_{cc} - U_cU_{cs})^2} < 0$$

(this follows from the assumptions on U_{ccc}, U_{ccs} and U_{cc}/U_{cs}) and as $c_j > c_i$, we have $S_i' > S_j'$ at such a point of crossing. Thus, such a crossing is not possible and so, given distinct initial values of endowments $\underline{z}_j > \underline{z}_i$, it must hold that $S_i(z) < S_j(z)$ for all

z. Hence, for a fixed $H(s)$, we have $G_j(z) = H^{-1}(S_j(z)) < H^{-1}(S_i(z)) = G_i(z)$. That is, $G_j(z)$ stochastically dominates $G_i(z)$ and $\mu_j > \mu_i$. This implies, that for any given level of average wealth μ, there exists a unique $\underline{z}$ such that the mean of $G^*(z)$ is μ. $\square$

Proof of Proposition 7: By the dispersive order we have $h_p(H_p^{-1}(r)) < h_a(H_a^{-1}(r))$. Together with our other assumptions on minimum and and maximum rewards, it implies that $H_p(s)$ and $H_p(s)$ are single crossing, with a unique reward $\hat{s}$ such that $H_p(\hat{s}) = H_a(\hat{s})$.

Let us assume that $\underline{z}_p < \underline{z}_a$, the minimum wealth level is lower under the new distribution (this later will be shown to hold). Given that solutions $(c(z), S(z))$ to the differential equation system cannot cross on the (c, S) plane, given our initial conditions $c_a(\underline{z}_a) = \underline{z}_a$, $S_a(\underline{z}_a) = \underline{s}_a$ and $c_p(\underline{z}_p) = \underline{z}_p$, $S_p(\underline{z}_p) = \underline{s}_p$ respectively, we have $c_p < c_a$ for a given level of S. Turning to solutions $S_a(z)$ and $S_p(z)$ graphed as a function of z alone, points of crossing of $S_a(z)$ and $S_p(z)$ are possible. However, as $c_a > c_p$ and we have $\partial S'(c, s)/\partial c < 0$ as shown in the proof to the previous proposition then $S_p'(z) > S_a'(z)$ at any such crossing. Thus, there is at most one crossing where $S_a(z) = S_p(z)$. There must be a crossing as otherwise clearly the mean reward could not be the same in both cases. Hence there is a unique crossing at endowment $\hat{z}$ where $S_a(\hat{z}) = S_p(\hat{z}) = \hat{s}$.

I now establish that $\underline{z}_p < \underline{z}_a$, the minimum wealth level is lower under the new distribution. Suppose not so that $\underline{z}_p \geq \underline{z}_a$. Then as solutions $(c(z), S(z))$ to the differential equation system cannot cross on the (c, S) plane, given our initial conditions that $c_p(\underline{z}_p) = \underline{z}_p \geq \underline{z}_a = c_a(\underline{z}_a)$ we have $c_p > c_a$ for a fixed level of S. Thus, given $\partial S'/\partial c < 0$ as established above, we would have $S_p'(z) < S_a'(z)$ at any potential point of crossing. Since we have $S_p(\underline{z}_p) = \underline{s}_p < \underline{s}_a = S_a(\underline{z}_a)$, S_p would never in fact cross S_a. Thus, the average reward must be higher ex post than ex ante, which is not possible. Thus, in summary, $S_p(z)$ has a strictly larger support than $S_a(z)$, and $S_p(z)$ and $S_a(z)$ are single crossing, with $S_p(z)$ crossing from below.

But this also implies that the inverses of $S_p(z)$ and $S_a(z)$ are also single crossing. That is, the two functions $G_p^{-1}(H_p(s)) = S_p^{-1}(s)$ and $G_a^{-1}(H_a(s)) = S_a^{-1}(s)$ are single crossing, with $G_p^{-1}(H_p(\hat{s})) = \hat{z} = G_a^{-1}(H_a(\hat{s}))$. But if the inverse of the distribution functions are single-crossing then so are distribution functions $G_p(z)$ and $G_a(z)$ with clearly $G_p(z) > G_a(z)$ on $(\underline{z}_p, \hat{z})$ and $G_p(z) < G_a(z)$ on $(\hat{z}, \bar{z}_p)$. Single crossing of this form with an equal mean implies second order stochastic dominance (Wolfstetter, 1999, Proposition 4.6). $\square$

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