Uncertainty Driven Business Cycle[1]

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ABSTRACT

This paper studies the relationship between micro-level uncertainty and macroeconomic activity. The study builds upon the irreversible investment model of Dixit(1989a,b) and extends it with a time varying uncertainty in a full-blown general equilibrium. In the presence of irreversible start up costs, a temporary increase in uncertainty directly affects entry/exit decisions by altering the value of marginal firms. The resulting change in the number of active firms strongly affects aggregate demand for capital and its equilibrium rental rate, leading to a long and peristent aggregate investment cycle. The time varying uncertainty, in conjunction with irreversibility, generates a realistic business cycle with a number of desirable properties such as strong internal propagation with a substantial degree of forecasted variance of output, countercyclicality of idiosyncratic uncertainty and procyclicality of net business formation. The issue is approached in two different ways: (i) individual value maximization under free entry condition and (ii) a constrained optimum(Spence(1976) and Mankiw and Whinston(1988)) where the total value of firms is maximized subject to the mark-up pricing rule implied by monopolistic competition. It is shown that the two allocations may not be equivalent depending on the returns to specialization and the free entry allocation can magnify the uncertainty driven business cycles.

[1] The vews expressed in the paper are those of the author and do not necessarily represent those of the Federal Reserve System.

1 Introduction

An important feature of U.S. business cycle is that the amount of idiosyncratic uncertainty is strongly countercyclical. Bloom, Floetotto and Jaimovich(2007) document that the correlation between cross-sectional spread of firm-level sales growth rates and GDP growth rates is -0.404(1967-2004) and the correlation between cross-sectional spread among firm-level stock market returns and GDP growth rates is -0.423(1968-2006). Eisfeldt and Rampini(2006) also show that dispersion in standard deviation of capacity utilization and total factor productivity growth rates(four digit SIC code level) are negatively correlated with GDP(-0.672 and -0.384, respectively, 1967-2000). Campbell et al(2001) reports that the dispersion of firm-level stock market returns is negatively correlated(-0.508) with NBER business cycle dates(1962-1998).

On the other hand, Chatterjee and Cooper(1993), Devereux et al(1996), Bergin and Corsetti (2005), Dos Santos and Dufourt(2006), and Jaimovich(2007) all document strongly procyclical net business formations. For instance, Bergin and Corsetti(2005) reports that the correlation between the net business formation and output is 0.73 for the U.S. economy(1959-1995), while Dos Santos and Dufourt(2006) reports a value of 0.60 for France(1993-2002). A logical conclusion from these two observations is that net business formation is negatively correlated with changes in idiosyncratic volatility.

This paper examines these phenomena in an equilibrium business cycle model with endogenous firm entry/exit, where the driving force of the business cycle comes from time-varying idiosyncratic uncertainty. In the presence of irreversible entry cost, a postive shock to the level of uncertainty, in the sense of mean-preserving spread, directly affects entry/exit decisions through changes in the value of marginal firm. At the time of heightened uncertainty, the equilibrium value of marginal firm declines because firm creation now faces a greater risk of inefficient shut-down tomorrow without compensating increase in profitability.[2] Since the value of the marginal firm serves as the benefit of creating a new firm as well as the opportunity cost of destroying an existing firm, the decrease in the marginal value can lead to a decline in the equilibrium number of firms.

The fluctuation in the equilibrium number of firms initiates a complete demand-driven aggregate cycle: because the aggregate demand for capital is partially determined by the total number of active firms and the supply of capital is fixed in the short run, the decrease in the number of firms directly lowers the equilibrium rental rate of capital. If the hysteresis effect of the model, measured by the mess of firms existing between the entry and exit boundaries, is strong enough, the effect on the total number of firms is maximized only slowly and the effect is likely to be accelerated in ensuing periods after the initial shock, generating hump-shaped cycle for firm turn-over and all related endogenous quantities such as aggregate investment. The value of marginal capital slowly deteriorates and recovers together with the value of marginal firm, and the temporary increase in uncertainty leads to a long and persistent aggregate investment cycle.

The essential mechanism behind this cycle is very similar to the one envisioned by Bernanke (1983)

[2] The expression, "inefficient " is in the sense that the initial start up cost has to be re-incurred when the production unit is reactivated.

despite the differences in the shock process and the equilibrium setting. In the framework of Bernanke(1983), the nature of uncertainty is different. Uncertainty stems from the fluctuations in the precision of agents' perception regarding the data generating process for the uncertainty. Agents have to adopt a dynamic inference in selecting the optimal stopping time. The current study is short of such an elegant structure of multi-layered uncertainty. However, one essential feature is common: uncertainty fluctuates, following a stationary Markov process known to all economic agents. Waiting can be a profitable and active form of policy only when economic agents understand that there will be a temporal resolution of uncertainty.

On the other hand, the current research is in line with a broader tradition of emphasizing the role of uncertainty in explaining aggregate investment found in Caballero and Bertola(1994), Caballero and Pindyck(1996) and Pindyck and Solimano(1993). A common feature of this continuous time literature is that the effect of uncertainty is analyzed only in the form of comparative statics about the consequences of different long run levels of uncertainty. However, to many decision makers of firms and governments, what matters the most is the short-run fluctuation in uncertainty and its implication for macroeconomy. In this sense, the current research makes an important contribution to that tradition.

A theoretical innovation in the current research is that the equilibrium concept of firm dynamics satisfies the consistency condition implied by a general equilibrium. In a fast growing body of literature on firm dynamics including Melitz(2003), Ghironi and Melitz(2005) and Alessandria and Choi(2007)) among others, a predominant equilibrium concept for firm dynamics is individual firm's value maximization starategy based on free entry condition. However, several non-neoclassical features adopted in this literature such as monopolistic competition, nonconvexity of adjustment costs and externality from returns to specialization imply that the resulting equilibrium allocation under free entry condition is not necessarily the best kind for the assumed owners of the firms in this literature, i.e., the households.

The current paper avoids this potential inconsistency between the maximization strategy of individual firm and the ownership of the firm by employing a different allocation device: a centralized venture capitalist fund. The venture capital fund, owned by household, owns all active firms and claims on future profits of potential entrants. It maximizes the total value of firms by making only firm creation/destruction decisions. In its optimization, the fund takes as given the mark-up pricing rules of monopolistically competitive firms in the sense of constrained optimum(Spence(1976) and Mankiw and Whinston(1986)). Although it is arguable that such a fund can overcome all conflicts within the corporate sector and between the corporate and household sectors in reality, it is certainly more consistent with general equilibrium requirements.

It turns out that neither the monopolistic competition nor the nonconvexity itself create any difference between the two allocations under the constrained optimum and the free entry allocation. However, the returns to specialization do make a difference. In the free entry allocation, firms follow a "myopic" decision rule in the sense that all firms take as given the total number of active firms in the market and do not internalize the consequences of their own entry and exit decisions on the market prices. In the constrained optimum, entry/exit decisions are regulated to maximize the total value of the firms, exploiting the structure of the returns to specialization. It is shown that with a nontrivial degree of returns to speacialization, the free entry allocation can either magnify or dampen the volatility of business cycle relative to the constrained optimum. To the limited knowlege of the author, there is only one analysis on the (sub)optimality of this myopic behavior in a dynamic setting with irreversible investment problem, Leahy(1993). In an industry equilibrium with no externality, Leahy(1993) shows the equivalency between the myopic competitive equilibria and social planner's allocation. The current research extends his anlayisis into a general equilibrium under monopolistic competition with returns to specialization.

The resulting equilibrium dynamics are promising. It is shown that a small sunk entry cost in conjunction with time varying uncertainty creates a powerful internal propagation mechanism regardless of its equilibrium setup(free entry vs. constrained optimum): in an environment without aggregate TFP shocks, the model delivers equilibrium dynamics where all equilibrium quantities and prices exhibit hump-shaped responses to an uncertainty shock. Furthermore, the forecastable variances(Rotemberg and Woodford(1996)) of the model's endogenous quantities are broadly consistent with those in the data. This shows the strong internal propagation mechanism generated by the model. The model is also very successful in explaining the joint behaviors of uncertainty, firm dynamics and aggregate output. In contrast to Jaimovich(2007), the current paper accomplishes this without the help of sunspot shocks.

The paper is organized as follows: section 2 describes the structure of the model and analyzes the equilibrium allocation of this paper. Section 3 compares the two allocations under the constrained optimum and the free entry equilibrium. Section 4 explains main findings. Section 5 concludes.

2 The Model

The model economy consists of a continuum of households, a continuum of intermediate goods producers, a competitive final goods industry and a competitive venture capital industry. Without loss of generality, I assume that the venture capital industry is centralized by one large fund, a fictitious capital planner.

The households consume final consumption goods, provide labors for competitive market wages and invest in productive capital and the shares of the centralized venture capital fund. The capital planner decides which projects should be initiated(entry) and terminated(exit). Conditioned upon the continuation(or the initiation decision in the case of new entry), managers operate firms(projects) to produce and sell output, and distribute dividends to the fund. Finally the venture capital fund redistributes dividends to households.

2.1 Production and Market Structure

Production uses two different types of capital: the first type is completely indivisible in the sense that "it must either be constructed in its entirety or not at all"(Moene(1985)). This capital outlay, denoted by γ_S in units of consumption goods, is completely irreversible in the sense that it must be re-incurred if a firm exits and re-enters the market later. The scrap value of this capacity is assumed to be zero. However, this assumption can be weakened in a straightforward way to allow for partial irreversibility. Although γ_S is time-invariant, its economic valuation is time-varying according to the value of consumption goods.

The second type is completely flexible. Firms rent the flexible capital from households by paying the rental cost and use it without any delivery lag(or time to build). Without the first type of capital, the second type of capital is not functional. For this reason, the total demand for flexible capital is also affected by the economy-wide investment in irreversible capital. In this sense, the production structure is almost similar to the one in Caballero and Pindyck(1996). The production technology is specified as a CRS Cobb-Douglas production function,

$$y(i) = m(i)z(i)^{\nu}k(i)^{\alpha}n(i)^{1-\alpha}, \;\; m(i) \in \{0, 1\}$$

where $m(i)$ is an indicator which takes 1 if the irreversible investment is made and the firm is still in operation and takes zero when the firm is idle. The number of firms in operation is then given by $M = \int_0^1 m(i)di$. The total number of active firms should be interpreted as the stock of irreversible capital or the stock of "beachheads"(Baldwin(1988)).

Production is subject to the idiosyncratic technology shock, $z(i)$ where $z(i)$ follows a lognormal distribution $\log z(i) \sim N(-0.5\sigma^2, \sigma^2)$. The first moment of the shock process does not vary over time. However, the second moment or the volatility of the shock process is a random variable and is assumed to follow a Markov process, $\log \sigma = (1 - \rho_\sigma) \log \bar{\sigma} + \rho_\sigma \log \sigma_{-1} + \varepsilon$, $\varepsilon \sim N(0, \Sigma_\varepsilon^2)$. Since the mean of the technology is $E(z|\sigma) = \exp(-0.5\sigma^2 + 0.5\sigma^2) = 1 = E(z)$, the first moment of the process is insulated from the changes in uncertainty level. There is no aggregate shock other than the shock to the second moment of the idiosyncratic technology as in Bloom(2007). The second moment shock can also be thought of as "dispersion shock" in Mortensen and Pissarides(1994)

Production is also subject to a fixed operation cost, γ_F in units of final goods. Owing to the fixed cost, a firm drawing a substantially bad idiosyncratic technology may optimally stop producing. Therefore, in contrast to Melitz(2003) and Bilbiie, Ghironi and Melitz(2006, 2007a and 2007b) and Bergin and Corsetti(2005), exit in this paper is completely endogenous. In this respect, the structure of firm dynamics is closer to Dixit(1989a,b) and Alessandria and Choi(2007a).

One important assumption in this paper is that the venture capital fund, which owns all operating projects(firms), does not have any decision rights over production, pricing and sales of firms. The venture capitalist only decides whether or not to fund a particular project. This seems to be in line with what the venture capitalists actually do in reality. In this sense, the fund behaves like a Ramsey planner in a constrained optimum where the planner maximizes the total profit of firms subject to the mark-up pricing rule implied by monopolistic competition.

Later, I show why the resulting allocation is more consistent with the notion of general equilibrium than that of a free-entry equilibrium. As pointed out by Spence(1976) and Mankiw and Whinston(1988), the allocation based on free entry condition does not necessarily coincide with the allocation of total profit maximization if there are "business stealing" or "business enhansing" effects owing to monopolistic competition, returns to specialization and non-convext adjustment costs such as sunk entry costs. Under the assumption of the representative household's ownership of firms, a centralized venture capitalist should maximize the total value of the fund, not the value of individual firm.[3] The venture capitalist fund in this paper is a general equilibrium device for consistency between the firm problem and its discounting factor.

The structure of the industrial organization in the economy is standard. The final goods are produced by a CES technology which combines intermediate goods. The set of intermediate goods used in final good production varies over time. The final goods firm does not have any control over the variety of intermediate goods. The final good firm maximizes $PY - \int_0^M p(i)y(i)di$ subject to a CES technology

$$Y = M^{1+v-1/\theta}\left[\int_0^M y(i)^\theta di\right]^{1/\theta}$$

where $0 < \theta < 1$. $M \in [0,1]$ is the total measure of active firms. The return to specialization(Benassy(1996), Devereux et al(1996)) is determined by $1+v$. If $v > 0(v < 0)$, there are increasing(decreasing) returns to scale due to specialization. To see this, consider the fact that if all active firms produce an identical quantity, say, y, the final good output becomes yM^{1+v} and $v > 0(v < 0)$ makes the output increasing(decreasing) in M more than proportionately. For convenience, I define $\xi \equiv 1 + v - 1/\theta$. The price index implied by a zero profit condition is given by

$$P = M^{-\xi}\left[\int_0^M p(i)^{\frac{\theta}{\theta-1}} di\right]^{\frac{\theta-1}{\theta}}$$

The demand for a specialized product i is derived as

$$y(i) = M^{-\xi\frac{\theta}{\theta-1}}\left[\frac{p(i)}{P}\right]^{\frac{1}{\theta-1}} Y$$

Upon entry decision(or upon continuation decision in the case of firms in place last period), the static maximization is independent of sunk start-up cost and fixed operation cost. The firm's manager follows a standard mark-up pricing rule implied by monopolistic competition. Using the definitions of production function and demand for intermediate goods, it is straightforward to show that the gross profit function of firm i(before netting out fixed cost) is given by

$$\pi(i) = m(i)\,(1-\theta)\,\theta^{\frac{\theta}{1-\theta}}\left[z(i)^\nu\left(\frac{\alpha}{r^K}\right)^\alpha\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\right]^{\frac{\theta}{1-\theta}} YM^{-\xi\frac{\theta}{\theta-1}} \tag{1}$$

where w and r^K are competitive real wage and the rental rate of capital, respectively. The profit function does not depend on installed capital owing to the rental market assumption. $m(i) = 0$ implies that the firm is inactive this time period either because the firm exits this period, or because the firm stays out of the market

[3] The problem also can be characterized as the optimization problem of a large multi-products firm. The firm must internalize the effects of introducing a new variety on its total profits.

in the case of inactive firms.

Two parameters deserve special attention. First, if the elasticity of the production function with respect to the idiosyncratic shock ν is equal to 1, the profit can be either a convex or a concave function of $z(i)$ depending on θ value. If $\nu = (1-\theta)/\theta$, the profit becomes a linear function of $z(i)$ and the expected profit is not affected by σ. To separate the uncertainty effect on firm dynamics from any effects directly stemming from the concavity/convexity of the profit function, I set $\nu = (1-\theta)/\theta$ throughout this paper.

Second, the elasticity of profit with respect to the total number of active firms is given by $-\xi\theta/(\theta-1) = [1-(1+v)\theta]/(\theta-1)$. If $-\xi\theta/(\theta-1) > 0$, an additional entry increases aggregate profit not only from the addition of the marginal unit but from the "business enhansing" effect. On the contrary, If $-\xi\theta/(\theta-1) < 0$, the positive effect from the addtion of the marginal unit will be diminished by increased competition and we have instead "business stealing" effect. If $v = 1/\theta - 1$, the profit does not depend on the total number of active firms, i.e., there is no externality stemming from the returns to specialization. Note that the zero exteranlity actually implies a small degree of increasing returns to specialization at the final goods production level since $v = 1/\theta - 1 > 0$. I consider this as a baseline case and relax this condition later.[4]

2.2 Constrained Optimum Allocation

The venture capital fund maximizes its value by selecting the most productive firms to operate. The expression, "the most productive" should be understood with care, as the irreversible investment complicates the problem. The presence of sunk cost implies that a positive current profit is not a sufficient condition to initiate a new firm or project. For the same reason, a negative net profit, which is possible due to the fixed cost, is not a sufficient condition to terminate an active firm. This implies that the selection decision must be conditioned upon the current status of heterogenous firms. This also creates the element of "hysteresis" or "tyranny of status quo"(Baldwin(1988), Baldwin and Krugman(1989) and Dixit(1989a,b)). The venture capital fund must compare the benefit and the cost of the entry/exit of the marginal firms in a forward looking manner.

To streamline notations, let $s \equiv [\sigma, K, M_{-1}]$ denote the set of aggregate state variables where K is aggregate flexible capital stock and M_{-1} is the number of active firms in the last period. The value of the fund $\bar{V}(s)$ can be defined in a recursive fashion,

$$\bar{V}(s) = \max_{m(i)\in\{0,1\}} \left\{ \int_0^1 m(i)\left\{\pi(z(i),s) - [(1-m_{-1}(i))\gamma_S + \gamma_F]\right\} di + \int \Lambda(s,s')\bar{V}(s')dQ(\sigma'|\sigma) \right\}$$

where $\Lambda(s,s')$ is the intertemporal substitution rate of the representative household's consumptions, $Q(\sigma'|\sigma)$ is the conditional distribution function of the volatility level.

The one period return function is the sum of individual profits minus the sum of all nonconvex costs. When $m(i) = 0$, the firm i does not generate any profits nor fixed costs regardless of its activity status yester-

[4]Under the baseline case of $\xi = 0$, the price index becomes a decreasing function of M. To a certain degree, this is a desirable property of the price index as a measure of living cost. See Broda and Weinstein(2007) for implication of upward bias in fixed basket CPI index. See also Bilbiie, Ghironi and Melitz(2007b) for its monetary policy implication.

day. However, when a firm is active today($m(i) = 1$), the firm's current cash flow depends on yesterday's activity status. If the firm was inactive yesterday, but has decided to enter the market today($[1 - m_{-1}(i)]\, m(i) = 1$), the sunk start-up cost has to be paid. If the firm was active yesterday, i.e., $m_{-1}(i) = 1$, today's decision to terminate this firm($m(i) = 0$) generates an economic cost because the scrap value of γ_S is nil. This is an economic cost rather than an accounting cost because the accounting cost was already incurred when the firm entered the market. The essence of the constrained optimum allocation is to measure this economic cost and to relate it with decision rules regarding entry and exit to maximize the total value of the firm.

Although the above formulation is the most straightforward, it is not the most efficient representation of the problem in the following sense: consider two firms under consideration for entry with $z(i) > z(j)$. Given the monotonicity of the profit function with respect to z, if $m(j) = 1$ is optimal, $m(i) = 1$ must be optimal as well. Also consider two firms under consideration for exit with $z(k) > z(l)$. If $m(k) = 0$ is optimal, $m(l) = 0$ must be optimal as well. This implies the existence of reservation property and suggests that the problem of specifying $m(i)$ for each firm can be replaced by a much more simple task of choosing a set of critical technology levels, one for entry and the other for exit.

Let $z_0(s)$ and $z_1(s)$ denote the entry and the exit cut-off level technology, respectively. According to the cut-off rules, active firms must exit if and only if today's productivity draw is lower than the cut-off $z_1(s)$. Similarly, inactive firms must enter the market if and only if today's productivity is higher than the cut-off $z_0(s)$. We can then transform the problem of deciding each individual firm's activity status into the one of choosing the set of threshold technology levels. However, in order to do that, we need to define the aggregates of the problem as functions of the threshold technology levels.

Let $F(z|\sigma)$ denote a cdf of z conditioned upon today's uncertainty realization σ. Since there is a continuum of firms, we have $E(z) = \int_0^1 z(i)di = \int zdF(z|\sigma)$. The definitions of the critical technologies imply the following law of motion for the total measure of active firms,

$$M(s) = [1 - F(z_1(s)|\sigma)]M(s_{-1}) + [1 - M(s_{-1})]\,[1 - F(z_0(s)|\sigma)]\,. \tag{2}$$

In words, among $M(s_{-1})$ firms who were active yesterday, only the fraction $1 - F(z_1(s)|\sigma)$ whose productivity is greater than $z_1(s)$ continues to produce. Similarly, out of $1 - M(s_{-1})$ firms, only the fraction $1 - F(z_0(s)|\sigma)$ whose productivity is greater than $z_0(s)$ starts producing by making an irreversible investment.[5]

Following Melitz(2003), I define a set of "average" productivity levels, one for entering firms whose productivity levels are greater than the entry threshold level $z_0(s)$ and the other for incumbent firms whose

[5] Later in this paper, I also use the following expression for the law of motion,

$$M(s) = M(s_{-1})[1 - \Phi(\mu_1(s))] + [1 - M(s_{-1})]\,[1 - \Phi(\mu_0(s))]$$

where $\mu_j(s) \equiv \sigma^{-1}\left[\log z_j(s) + 0.5\sigma^2\right]$, the standardization of the original threshold value $z_j(s)$. The two expression are equivalent.

productivity levels are greater than the exit threshold level $z_1(s)$,

$$\bar{z}_j(s) \equiv \left[\frac{1}{1 - F(z_j(s)|\sigma)} \int_{z \geq z_j(s)} z^{\frac{\nu\theta}{1-\theta}} dF(z|\sigma) \right]^{\frac{1-\theta}{\nu\theta}} \quad \text{for } j = 0, 1. \tag{3}$$

Melitz(2003) shows that Dixt-Stiglitz aggregator implies that the average profit conditioned upon a particular threshold level $z_j(s)$, i.e., $E[\pi(z,s)|z \geq z_j(s), \sigma]$, is equal to the individual profit of a firm whose productivity draw is equal to the average productivity level defined above, i.e.,

$$\frac{1}{1 - F(z_j(s)|\sigma)} \int_{z \geq z_j(s)} \pi(z,s) dF(z|\sigma) = \pi(\bar{z}_j(s), s) \quad \text{for } j = 0, 1.$$

This implies that the aggregate profit can be constructed as if there were only two firms, i.e., a representative entering firm and a representative incumbent firm. Since the total measure of the entering firms is $M_0(s) \equiv [1 - M(s_{-1})]\,[1 - F(z_0(s)|\sigma)]$ and the total measure of incumbent firms is $M_1(s) \equiv M(s_{-1})\,[1 - F(z_1(s)|\sigma)]$, the expression for aggregate profit can be greatly simplified as an weighted average,

$$\Pi(s) = \int_0^1 m(i)\pi(z(i), s) di = \sum_{j=0,1} M_j(s)\pi(\bar{z}_j(s), s). \tag{4}$$

Aggregate fixed cost and aggregate sunk costs, denoted by $\Gamma_F(s)$ and $\Gamma_S(s)$ respectively, can be constructed in a similar way, i.e.,

$$\Gamma_F(s) = \gamma_F \int_0^1 m(i) di = \gamma_F M(s) \tag{5}$$

and

$$\Gamma_S(s) = \gamma_S \int_0^1 m(i)(1 - m_{-1}(i)) di = \gamma_S M_0(s). \tag{6}$$

The venture capital fund problem can then be expressed in terms of aggregate quantities as follows:

$$\bar{V}(s) = \max_{z_0(s), z_1(s)} \left\{ \Pi(s) - \Gamma_F(s) - \Gamma_S(s) + \int \Lambda(s, s')\bar{V}(s') dQ(\sigma'|\sigma) \right\}$$
$$\text{s. t. (2), (4), (5) and (6).}$$

In the above formulation, the capital planner exploits the reservation property of the problem and just optimizes over the set of cut-off rules instead of a continuum of choice varibles, $m(i)$ for all $i \in [0,1]$. The first order conditions for the two threshold technology levels are given by

$$0 = \frac{\partial \Pi(s)}{\partial z_j(s)} - \frac{\partial \Gamma_F(s)}{\partial z_j(s)} - \frac{\partial \Gamma_S(s)}{\partial z_j(s)} + \frac{\partial M(s)}{\partial z_j(s)} \int \Lambda(s, s') \frac{\partial \bar{V}(s')}{\partial M(s)} dQ(\sigma'|\sigma) \quad \text{for } j = 0, 1. \tag{7}$$

Using an analogy from capital theory, let $q^M(s)$ denote the value of the marginal unit of irreversible capital stock $M(s)$, i.e.,

$$q^M(s) \equiv \int \Lambda(s,s') \frac{\partial \bar{V}(s')}{\partial M(s)} dQ(\sigma'|\sigma)$$

Dividing through (7) by $\partial M(s)/\partial z_j(s)$, we can rewrite it as

$$\frac{1}{\partial M(s)/\partial z_j(s)} \left[\frac{\partial \Gamma_S(s)}{\partial z_j(s)} + \frac{\partial \Gamma_F(s)}{\partial z_j(s)} - \frac{\partial \Pi(s)}{\partial z_j(s)} \right] = q^M(s) \quad \text{for } j = 0, 1. \tag{8}$$

Using the definitions of the aggregates constructed above, the FOCs can be further simplified as

$$\gamma_S + \gamma_F - \left[\pi(z_0(s), s) + \sum_{j=0,1} M_j(s) \frac{\partial \pi(\bar{z}_j(s), s)}{\partial M(s)} \right] = q^M(s) \tag{9}$$

and

$$\gamma_F - \left[\pi(z_1(s), s) + \sum_{j=0,1} M_j(s) \frac{\partial \pi(\bar{z}_j(s), s)}{\partial M(s)} \right] = q^M(s) \tag{10}$$

The left hand side of (9) measures the marginal cost of adjusting aggregate firm stock net of the effects on the aggregate profit, which are captured by the bracketted term on the left hand sides. $q^M(s)$ measures the expected value of the marginal firm. The FOC then says that the marginal costs and benefits of introducing one more firm must be equalized. For the entry, the marginal adjustment cost involves the sunk cost.[6]

In the case of exit, the roles of the left and right hand sides are reversed – the right hand side of (10) measures the marginal cost whereas the left hand side measures the marginal benefit of adjustment. By dropping the marginal unit, the centralized fund loses the expected present value of the net profit generated by the marginal unit. This marginal cost must be equalized with the saving of opertating loss(the left hand side) that would be generated if the marginal unit were kept producing.

Note that the effects on the current profits have two components: a direct effect from the marginal unit itself(the first term inside the bracket) and indirect effect of the entry(exit) decision on all incumbent firms(the second term inside the bracket). The direct effect of adding(dropping) the marginal unit is simply the profit generated by the marginal unit, which is captured by the term $\pi(z_0(s), s)(\pi(z_1(s), s))$. The indirect effect is owing to the returns to specialization – when a firm is added to or dropped from the inputs variety of the CES production, it generates an externality effect. In case of increasing returns to specialization, an entry(exit) has a positive(negative) effect on incumbents' profits. In case of decreasing returns to specialization, an entry(exit) has a negative(positive) effect on other firms' profits. Since the fictitious capital planner maximizes the total value of the firms, it internalizes the externality effects in comparing the marginal benefits and costs associated with changing the entry/exit margins.

[6] The venture capitalist fund has two margins, one for entry and the other for exit. However, in the above, there is only one expression for the value of marginal firm $q^M(s)$ owing to the normalization. The fact that the value of marginal firm at the entry margin and the counterpart at the exit margin deffer from each other only by a factor, $\partial M(s)/\partial z_j(s)$ has to do with the fact that the first conditional moment does not vary, i.e., the position at today's distribution does not have a prediction for tomorrow's position. Owing to the lack of persistency, once admitted, the firm at the entry margin and the one at the exit margin have an identical additional value for the fund.

The FOCs (9) and (10) cannot determine the threshold technology levels unless the value of marginal firm is simultaneously determined. The envelope condition for the Bellman equation provides a stochastic law of motion for the value of marginal firm. By directly differentiating the value function with respect to the total firm measure, we obtain

$$\frac{\partial \bar{V}(s)}{\partial M(s_{-1})} = \frac{\partial \Pi(s)}{\partial M(s_{-1})} + \frac{\partial M(s)}{\partial M(s_{-1})}\frac{\partial \Pi(s)}{\partial M(s)} - \frac{\partial \Gamma_F(s)}{\partial M(s_{-1})} - \frac{\partial \Gamma_S(s)}{\partial M(s_{-1})} + \frac{\partial M(s)}{\partial M(s_{-1})}\int \Lambda(s,s')\frac{\partial \bar{V}(s')}{\partial M(s)} dQ(\sigma'|\sigma).$$

Updating this expression one period, discounting it with $\Lambda(s, s')$ and integrating over future provides us with an Euler equation,

$$\underbrace{q^M(s)}_{\text{Marginal Q}} = \int \Lambda(s,s')\Bigg[\underbrace{\left(\frac{\partial \Pi(s')}{\partial M(s)} + \frac{\partial M(s')}{\partial M(s)}\frac{\partial \Pi(s')}{\partial M(s')} - \frac{\partial \Gamma_F(s')}{\partial M(s)}\right.}_{\text{Effects on Net Profits}} \underbrace{\left.-\frac{\partial \Gamma_S(s')}{\partial M(s)}\right)}_{\text{Adj. Costs}} + \underbrace{\frac{\partial M(s')}{\partial M(s)} q^M(s')}_{\text{Capital Gain}}\Bigg] dQ(\sigma'|\sigma). \quad (11)$$

There exists a remarkable analogy between $q^M(s)$ and traditional Tobin's marginal q^K in capital theory. The left hand side of the Euler equation is the shadow value of firm. The right hand side of the Euler equation shows that the shadow value of firm is equal to the sum of two effects: the effect on the future dividend(the parenthesized term) and the effect on the capital gain captured by the future shadow value.

The first effect can be further broken down into the effect on the profit and the effect on adjustment costs just as in the Euler equation arising from convex adjustment problem. Again, the effects on the net aggregate profit can be decomposed into two components, the direct one and the indirect one. The direct effect is from the future profit of the marginal unit itself and the indirect effect is from the changes made to existing firms' profits by the externality. The effect on adjustment cost tomorrow is essentially the saving of sunk costs brought by changing the entry margin today.

An important difference between Tobin's marginal q^K and $q^M(s)$ is that the effect on tomorrow's shadow value $q^M(s')$ is discounted by a time-varying factor $\partial M(s')/\partial M(s)$ rather than by $1 - \delta$ where δ is the depreciation rate of flexible capital. However, even this time-varying discounting factor has an exact analogy with $1 - \delta$ because $\partial M(s')/\partial M(s)$ measures the effect of changing today's irreversible capital on tomorrow's irreversible capital just like $\partial K'/\partial K$ measures the same effect in the case of conventional capital stock. Note that $0 < \partial M(s')/\partial M(s) < 1$ because $\partial M(s')/\partial M(s) = F(z_0(s')|\sigma') - F(z_1(s')|\sigma')$. In this sense, $\partial M(s')/\partial M(s)$ measures the fraction of undepreciated irreversible capital stock. The difference is that the survival rate of today's irreversible capital is stochastic and endogenous while the survival rate of conventional capital is deterministic and exogenous, i.e., $\partial K'/\partial K = 1 - \delta$.

It is noteworthy that by aggregating discrete choices of infinitesimal decision units, the capital planner transforms the optimal stopping problem into a barrier control problem with two linear costs γ_S(entry cost) and 0(exit cost).[7] In fact, the FOCs (9) and (10) are discrete time counterparts of smooth-pasting condition arsing from barrier control problem in continuous time. Furthermore, the Envelope condition provides a

[7] See Dixit(1993), Harrison and Taksar(1983) and Abel and Eberly(1996).

discrete time counterpart of differential equations for a value function arising from the same control problem. The difference is that the heterogeneity of each firm requires the barriers to apply to each unit of capital rather than to apply to the total capital stock. The situation can be thought of as the one in which the planner wants to control the total number of machines, but does so by selecting the best machines today given the consideration of costs associated with ins and outs of machines.

Whether or not increase in uncertainty increase or decrease the value of marginal firm is not obvious, either from the FOCs (9) and (10), or from the envelope condition (11). It is more likely that the increased uncertainty will lower the value of marginal firm because, with a greater amount of uncertainty, there is a temporary increase in the probability of inefficient shut-down of firms without compensating increase in overall profitability of firm. However, given the lack of analytical solution, a definitive answer to the question should wait until extensive numerical analysis. In fact, in section 4, I show that this prediction is not always validated depending the equilibriun set up and the degree of returns to specialization although the particular calibration that leads to this counterintuitive response is not highly realistic.

2.3 Capital Accumulation

To close the model, this section analyzes how the rental market for capital clears and how the firm creation/destruction decision of the venture capital fund affects household capital accumulation decision. The equilibrium number of active firms is an important determinant of aggregate capital demand. If the uncertainty shock affects the number of active firms by altering the marginal valuation of firms, it will also have an important impact on the capital accumulation of the economy.

It is straightforward to show that the demand for capital of an active firm with productivity draw z is equal to

$$k^D(z,s) = \theta^{\frac{1}{1-\theta}} z^{\frac{\nu\theta}{1-\theta}} \left[\frac{r^K(s)}{\alpha}\right]^{\frac{1}{\theta-1}-\varphi} \left[\frac{w(s)}{1-\alpha}\right]^{\varphi} Y(s) M(s)^{-\frac{\xi\theta}{\theta-1}} \tag{12}$$

where $\varphi \equiv (1-\alpha)\theta/(\theta-1)$. Since the capital demand is isomorphic to the profit function, the same aggreation framework works for aggregate capital demand. The aggregate capital demand can be written as a weighted average of group(entrants and incumbents) average capitals, i.e.,

$$K^D(s) = \sum_{j=0,1} M_j(s) k^D(\bar{z}_j(s), s) \tag{13}$$

where $\bar{z}_j(s)$ is $E[z|z \geq z_j(s), \sigma]$ for $j = 0, 1$, as defined in (3).

The equilibrium rental rate is then pinned down by the market clearing condition, $K^D(s) = K(s_{-1})$ where the supply of capital $K(s_{-1})$ is predetermined. The dynamic capital accumulation decision is made by households who maximize $E_0 \sum_0^\infty \beta^t u(C_t, H_t)$ subject to a period-by-period budget constraint,

$$C(s) + I(s) - \frac{\lambda}{2}\left[\frac{I(s)}{K(s_{-1})} - \delta\right]^2 K(s_{-1}) + W(s)S'(s) \leq w(s)H(s) + r^K(s)K(s_{-1}) + (D(s) + W(s))S(s_{-1})$$

where $D \equiv \int_0^1 d(i)di$ with $d(i) \equiv m(i)\left[\pi(i,s) - (1 - m_{-1}(i))\gamma_S - \gamma\right]$. W is the after-dividend price of centralized venture capital fund and S is the number of shares.[8] A convex adjustment cost with a parameter λ is specified to explore the implications of time-varying volatility on the asset value of flexible capital. The period utility function is specified as a CRRA form over consumption and hours, $u(C_t, H_t) = \frac{C^{1-\rho}}{1-\rho} - \chi\frac{H^{1+\psi}}{1+\psi}$ where $1/\rho$ and $1/\psi$ are the elasticity of intertemporal substitution rate and the Frisch elasticity of hours. The efficiency conditions for households are summarized by FOCs with respect to hours, shares and capital,

$$\begin{aligned}
\chi H(s)^{\psi} &= w(s)C(s)^{-\rho} \\
W(s) &= \int_S \Lambda(s,s')\left[D(s') + W(s')\right]Q(\sigma, d\sigma') \\
&\text{and} \\
q^K(s) &= \int_S \Lambda(s,s')\left\{r^K(s') + \frac{\lambda}{2}\left[\left(\frac{I(s')}{K(s)}\right)^2 - \delta^2\right] + (1-\delta)q^K(s')\right\}Q(\sigma, d\sigma') \qquad (14)
\end{aligned}$$

where $\Lambda(s,s') = \beta[C(s')/C(s)]^{-\rho}$ and $q^K(s) \equiv 1 + \lambda(I_K(s)/K(s_{-1}) - \delta)$.

2.4 Endogenous Capacity and Measured TFP

The aggregation formulas (4) and (13) can be used to derive an important implication of firm dynamics for endogenous capacity and measured TFP of the economy. To that end, it is useful to derive an expression for a productivity differential between different groups of firms. In Appendix 2, using the property of lognormal distribution, I show the relationship between a conditional mean of a group of firms whose productivity levels are greater than a particular threshold, $z_j(s)$ and the unconditional mean of the shock process can be expressed as

$$\frac{\bar{z}_j(s)}{\bar{z}} = \left[\frac{1 - \Phi\left(\mu_j(s) - \Sigma(\sigma)\right)}{1 - \Phi(\mu_j(s))}\right]^{\frac{1-\theta}{\nu\theta}} \quad \text{for } j = 0, 1 \qquad (15)$$

where Φ is the standard normal cdf, $\mu_j(s) \equiv \sigma^{-1}\left[\log z_j(s) + 0.5\sigma^2\right]$ and $\Sigma(\sigma) \equiv \sigma\nu\theta/(1-\theta)$. Note that $\Sigma(\sigma) = \sigma$ when $\nu = 1/\theta - 1$ as assumed throughout this paper. Since $\bar{z}$ is the unconditional mean of the shock, i.e., the aggregate productivity when all firms were active, (15) measures the productivity differential of entrants and incumbents against the maximum capacity level of productivity.

By a straightfoward algebra, the average capital demand and profit of each group can be written in terms of (15), i.e.,

$$\begin{aligned}
\pi(\bar{z}_j(s), s) &= \pi(\bar{z}, s)\left[\bar{z}_j(s)/\bar{z}\right]^{\frac{\nu\theta}{1-\theta}} \\
&\text{and} \\
k^D(\bar{z}_j(s), s) &= k^D(\bar{z}, s)\left[\bar{z}_j(s)/\bar{z}\right]^{\frac{\nu\theta}{1-\theta}}
\end{aligned}$$

[8] In this and the next section, I use the terminology of the constrained optimum, but the other choice would not make any difference in the analysis in the sections.

where $k^D(\bar{z}, s)$ and $\pi(\bar{z}, s)$ are the hypothetical maximum capacity levels of aggregate capital demand and profits. Using the last two expressions, the aggregate capital demand and aggregate profit can be shown to be equal to

$$\begin{aligned}\Pi(s) &= \pi(\bar{z}, s) \sum_{j=0,1} M_j(s) \left[\frac{1 - \Phi\left(\mu_j(s) - \sigma\right)}{1 - \Phi(\mu_j(s))} \right] \equiv \Xi\left(s\right) \pi(\bar{z}, s) \\ &\text{and} \\ K^D(s) &= k^D(\bar{z}, s) \sum_{j=0,1} M_j(s) \left[\frac{1 - \Phi\left(\mu_j(s) - \sigma\right)}{1 - \Phi(\mu_j(s))} \right] \equiv \Xi\left(s\right) k^D(\bar{z}, s).\end{aligned}$$

Note that $\Xi(s)$ can be thought of as measured TFP. To see this point, consider the fact that aggregate production is isomorphic to aggregate profit function so that $Y(s) = \Xi(s)\, y(\bar{z}, s)$, where $y(\bar{z}, s)$ is the capacity level aggregate output. Therefore, the ratio, $\Xi(s) = Y(s)/y(\bar{z}, s)$ can be considered as the ratio of actual output to maximum level of output, which is less than 1 by construction. Now imagine an econometrician who does not seperately observe $\Xi(s)$ and $y(\bar{z}, s)$, but instead observe only the aggregate output $Y(s)$. He then constructs residual time series that cannot be explained by movements in capital stock and labor hours and interepnets them as exogenous shocks to total factor productivity even though the true aggregate TFP is a constant, $\bar{z}$. The true reason for the productivity fluctuation is that as the uncertainty level changes, the capital planner(or the firms) reevaluates the value of marginal firm and modifies the optimal levels of entry and exit. The resulting fluctuation in the number of firms is the source of measured TFP in the model.

$\Xi(s)$ can also be thought of as endogenous capacity utilization. For this reason, it is important to distinguish it from the simple meausre of the active firms. To see the difference between them, we can expand the expression for $\Xi(s)$ as follows,

$$\Xi\left(s\right) = \frac{1 - \Phi\left(\mu_1(s) - \sigma\right)}{1 - \Phi(\mu_1(s))} \left[1 - \Phi(\mu_1(s))\right] M(s_{-1}) + \frac{1 - \Phi\left(\mu_0(s) - \sigma\right)}{1 - \Phi(\mu_0(s))} \left[1 - \Phi(\mu_0(s))\right] \left[1 - M(s_{-1})\right].$$

The above expression is very similar to the law of motion for the total number of active firms. An important difference is that the components of the original law of motion are weighted by the productivity differential. More productive firms contribute more to the capacity. As the uncertainty level changes, the value of marginal firm also changes. From the FOCs of the venture capitalist fund(or the value matching conditions), one can see that the threshold level shocks must repond to the changes in the value of marginal firm. A consequence of this process is that the average qualities of both entry and incumbent firms change as the cut-off technologies are modified as a result of the uncertainty shock. In section 4, we analyze in details how a shock to the uncertainty level affects the average quality of firms and how the latter modifies the course of the business cycle.

3 Relation with Free Entry Allocation

Several authors approached the issue using the notion of a free entry (or equivalently zero profit) condition in which each individual firm makes its own entry/exit decisions taking as given the other firms' strategies. Influential analysis can be found in Melitz(2003) in the context of an industry dynamics and Ghironi and Melitz(2005) and Alessandria and Choi(2007) in two country general equilibrium models.[9] A common feature of this literature is the implicit assumption of equivalency of competitive equilibria and planner's allocation.

However, there is an important exception, Leahy(1993) who analyzes the issue in the context of industry equilibrium. Leahy shows that a myopic equilibrium in which all firms make irreversible entry/exit decisions completely ignoring the effects that their own actions exert on the equilibrium price process can be coincident with the planner's allocation. In this subsection, I generalize the analysis to the case of monopolistic competition and general equilibrium and show under what conditions two allocations may or may not differ from each other in this environment.[10]

In this analysis, following the literature above, I assume that each firm is directly owned by the representative household. Accordingly, the firms discount future cash flows using intertemporal substitution rate of the representative household's consumption. The firm index is suppressed for simplified notation. The individual value maximization problem can be stated in a recursive way:

$$\begin{aligned} V(z,s,m_{-1}) \quad &= \quad \max_{m\in\{0,1\}} \left\{ m\left[\pi(z,s)-(1-m_{-1})\gamma_S-\gamma_F\right] + \int\int \Lambda(s,s')V(z',s',m)dF(z'|\sigma')dQ(\sigma'|\sigma) \right\} \\ & \quad \quad \text{s.t. (1) and (2).} \end{aligned}$$

The differences between this problem and the venture capital fund's problem earlier are obvious. An individual firm does not integrate the current dividend over heterogenous firms. On the other hand, the firm integrates its continuation value over the idiosyncratic uncertainty. The value function of an individual firm is a function of the same set of aggregate state variables, i.e., $s = [\sigma, K, M_{-1}]$, together with the firm's activity status last period. Furthermore, the maximization problem is subject to the same law of motion for the aggregate number of active firms, implying that all firms understand how the the total number of firms evolves over time. However, an important difference is that the threshold rules, which determine the law of motion, are all taken as given.

When $m = 0$, the firm becomes idle until the next period. In this case, the value of the firm is merely the continuation value, the value of an opportunity to enter the market. The cost of start-up investment, γ_S is incurred if and only if $m(1-m_{-1}) = 1$. Let $W(z,s,m_{-1},m)$ denote an auxiliary value function with a

[9] Bergin and Corsetti(2005) and Bilbiie, Ghironi and Melitz(2006) analyze the relationship between the free entry allocation and the social planner's allocation in a general equilibrium economy with homogeneous productivity and exogenous exit. The key difference between the approach in this paper and Bergin and Corsetti(2005) and Bilbiie et al(2006) is in the fact that the venture captial fund in this paper takes the aggregate demand shifter(Y) as given while a social planner would exploit the opportunity of affecting the aggregate demand itself by manipulating the number of firms.

[10] The stochastic environment in Leahy(1993) is very different from the one in this paper. Leahy assumes that the the shock follows a geometric Brownian motion while the current paper assumes that the first moment of the shock does not vary over time, but the second moment follwos a stationary Markov process.

particular policy $m \in \{0,1\}$ given the activity status m_{-1}, i.e.,

$$W(z,s,m_{-1},m) = m\left[\pi(z,s) - (1-m_{-1})\gamma_S - \gamma_F\right] + \int\int \Lambda(s,s')V(z',s',m)dF(z'|\sigma')dQ(\sigma'|\sigma). \qquad (16)$$

The value of this auxiliary function is potentially lower than the true value function because of the commitment to a particular policy m. Using the auxiliary value functions, the firm problem can be recast in the form of optimal stopping problem, i.e.,

$$V(z,s,m_{-1}) = \max\{W(z,s,m_{-1},1), W(z,s,m_{-1},0)\}. \qquad (17)$$

Since the profit function is monotonically increasing in idiosyncratic technology, the value maximization problem has a reservation property, i.e., the optimal stopping problem has its own critical technologies which are defined by a set of value matching conditions,

$$W(z^*_{m_{-1}}(s),s,m_{-1},0) = W(z^*_{m_{-1}}(s),s,m_{-1},1) \text{ for } m_{-1} \in \{0,1\}. \qquad (18)$$

The value matching conditions say that given yesterday's activity status m_{-1}, the firm should be indifferent between the two strategies $m \in \{0,1\}$ at the critical technology $z^*_{m_{-1}}(s)$. That is, $z^*_0(s)$ is the level of technology at which a firm is indifferent between entering and staying out of the market. Similarly $z^*_1(s)$ is the level of technology at which a firm is indifferent between exiting and staying in the market. An asterisk is used to distinguish the values from the counterparts of the venture capital fund allocation. The definitions of auxiliary value function (16) and value matching conditions imply that at the threshold shocks $z^*_{m_{-1}}(s)$, it must be the case that

$$\gamma_S + \gamma_F - \pi(z^*_0(s),s) = J(s) \qquad (19)$$

and

$$\gamma_F - \pi(z^*_1(s),s) = J(s) \qquad (20)$$

where the right hand sides of the above is defined as

$$J(s) \equiv \int\int \Lambda(s,s')\left[V(z',s',1) - V(z',s',0)\right]dF(z'|\sigma')dQ(\sigma'|\sigma). \qquad (21)$$

$J(s)$ measures the expected surplus value that an active firm has over the value of idle firm. Note that $V(z',s',1)$ is always no less than $V(z',s',0)$ because, absent any explicit costs associated with exit, active firms can always obtain the idle status for free if they find doing so profitable. [11]

[11] $J(s)$ is essentially the same as what Das, Roberts and Tybout(2007) define as the value of being able to continue operating next period without paying the start-up cost again. In Dixit(1989b), $J(s)$ can be negative owing to a strictly positive exit cost. If there is a persistency in the first conditional moment of the shock process, it can be shown that the surplus value function

The left hand side of (19) is the cost of entry net of the immediate profit. The right hand side is the benefit of entry which is the extra continuation value of being active over the continuation value of being idle next period. By paying the net cost, the firm obtains the surplus value. In case of exit, the left hand side of (20) measures the benefit of exit and the right hand side measures the opportunity cost of exit. By giving up the surplus value of being active, the firm saves the operating loss $\gamma_F - \pi(z_1^*(s), s)$ which is always positive at the exit threshold $z_1^*(s)$ becasue exit is never optimal when the net profit is positive.

The value matching conditions are necessary but not enough to characterize the free entry allocation because how the surplus value $J(s)$ evolves over time is yet to be determined. To analyze the dynamics of the surplus value, it is useful to rewrite the integrand of (21) as

$$V(z', s', 1) - V(z', s', 0) = \max\{W(z', s', 1, 1), W(z', s', 1, 0)\} - \max\{W(z', s', 0, 1), W(z', s', 0, 0)\}.$$

Consider three different cases: (i) $z' < z_1^*(s')$, (ii) $z' > z_0^*(s')$ and (iii) $z_1^*(s') \leq z' \leq z_0^*(s')$. In the first case, the technology level is not high enough to justify entry, but low enough to justify exit. Therefore $V(z', s', 1) - V(z', s', 0) = W(z', s', 1, 0) - W(z', s', 0, 0) = 0$. In the second case, the technology level is high enough to validate entry and of course, exit is never optimal in this case. Therefore $V(z', s', 1) - V(z', s', 0) = W(z', s', 1, 1) - W(z', s', 0, 1) = \gamma_S$. This is because the firm chooses to be in the market tomorrow regardless of today's activity status and the difference between the two value functions is simply the start-up investment cost γ_S. In the third case, the technology is neither high enough for entry nor low enough for exit and therefore status-quo is optimal for both idle and active firms. In other words, no firm with its technology level between the two threshold levels changes its current status. In this status-quo region, $V(z', s', 1) - V(z', s', 0) = W(z', s', 1, 1) - W(z', s', 0, 0)$, which, from (16), can be seen as

$$W(z', s', 1, 1) - W(z', s', 0, 0) = \pi(z', s') - \gamma_F + \int \int \Lambda(s', s'') \left[V(z'', s'', 1) - V(z'', s'', 0)\right] dF(z''|\sigma'') dQ(\sigma''|\sigma').$$

Note that the right hand side of the above is equal to $\pi(z', s') - \gamma_F + J(s')$. Therefore, by combining the three different cases, it can be readily verified that (21) is equivalent to

$$J(s) = \int \Lambda(s, s') \left[\int_{z' \geq z_0^*(s')} \gamma_S dF(z'|\sigma') + \int_{z_1^*(s')}^{z_0^*(s')} [\pi(z', s') - \gamma_F + J(s')] dF(z'|\sigma') \right] dQ(\sigma'|\sigma). \tag{22}$$

Clearly, the above is a Bellman equation of which the fixed point is the surplus value function $J(s)$.[12] Since the profit function is monotonically increasing in indiosyncratic technology level, the value matching conditions

depends on current idiosyncratic productivity level as well as macroeconomic variables, i.e., $J = J(z, s)$. However, in this case, a nonlinear solution method will be needed to solve to the model. Alessandria and Choi(2007b) introduces a persistent shock to firm dynamics, but assumes that exit is exogenous and starting value of the idiosyncratic shock is exogenously fixed. In the context of labor search, Elsby and Michaels(2008) analyze the consequences of persistency of idiosyncratic shock. However, they only compare invariant joint distributions of productivity and firm size(the number of employment) corresponding to different parameter sets and do not provide exact analysis of how the joint distribution evolves over time in the short run.

[12] This is essentially the same as the Bellman equation that was derived by Alessandria and Choi(1997).

can be inverted for the threshold shocks, i.e., $z_0^*(s') = \pi^{-1}(\gamma_S + \gamma_F - J(s'); s')$ and $z_1^*(s') = \pi^{-1}(\gamma_F - J(s'); s')$. Suibstituting these expressions in (22) shows that the Bellman equation is a functional equation for $J(\cdot)$, which only depends on the parameters of the model, γ_S, γ_F, the profit function $\pi(\cdot)$, and the shock processes.

Now we can compare the two allocations in the following way.

- FE: the value matching condition (19) and (20), the Bellman equation (22) for the surplus value function, and the law of motion (2) for the total number of active firms constitute a free entry competitive equilibrium for given wage and rental rate processes.

- CO: the two FOCs with respect to the entry and exit threshold shocks (9) and (10), the envelope condition of the centralized venture capital fund(11), and the law of motion (2) for the total number of active firms constitute the constrained optimum allocation for given wage and rental rate processes.

Compare the left hand sides of the value matching conditions, (19) and (20) with the FOCs of the constrained optimum with respect to the threshold shocks, (9) and (10). One can immediately see that the only difference is the existence of the indirect effect term $\sum_{j=0,1} M_j(s)\partial\pi(\bar{z}_j(s), s)/\partial M(s)$ for the constrained optimum. If the indirect effects are equal to zero, the left hand sides of the two sets of efficiency condtions are identical. Now compare the Bellman equation (22) and the envelope condition (11). To see better the difference between the two expressions, it is more useful to rewrite the envelope condition (11) in terms of the entry/exit threshold shocks. Appendix 1 shows that (11) is equivalent to

$$\begin{aligned} q^M(s) \quad &= \quad \int \Lambda(s, s') \left[\int_{z_1(s')}^{z_0(s')} \sum_{j=0,1} M_j(s') \frac{\partial \pi(\bar{z}_j(s'), s')}{\partial M(s')} dF(z'|\sigma') \right] dQ(\sigma'|\sigma) \\ &+ \int \Lambda(s, s') \left[\int_{z' \geq z_0(s')} \gamma_S dF(z'|\sigma') + \int_{z_1(s')}^{z_0(s')} \left[\pi(z', s') - \gamma_F + q^M(s') \right] dF(z'|\sigma') \right] dQ(\sigma'|\sigma). \end{aligned} \tag{23}$$

Again one can immediately recognize that the only difference is the presence of the truncated expectation of future indirect effects on the aggregate profit, $\sum_{j=0,1} M_j(s')\partial\pi(\bar{z}_j(s'), s')/\partial M(s')$. If the indirect effects do not exist, the two functional equations are identical.

From this analysis, we can conclude that when individual firm's profit does not directly depend on the total number of firms in the market, the value of marginal firm for the society in the costrained optimum allocation is equal to the surplus value for an entrant in the myopic equilibrium and the firm dynamics under the two allocations, summarized by their entry/exit cut-off rules, are identical. The two allocations, however, differ from each other in general if the assumption, $\partial\pi(\bar{z}_j(s), s)/\partial M(s) = 0$ does not hold. The difference stems from the fact that the capital planner in the constrained optimum, unlike individual firms in the myopic equilibria, internalizes the effects on inclumbent firms' profit of changing the entry/exit barriers. Thus, when there is a positive(negative) externality arising from returns to specialization, the planner wants to lower(raise) the entry and exit thresholds to the levels that would not be justified by the myopic entry and exit decision rules.

4 Results

In this section, I analyze the properties of the model using quantitative analysis. In doing so, I pay special attentions to whether or not the uncertainty shock in the model can replicate (i) the unconditional moments of major macroeconomic aggregates (ii) the comovements between uncertainty and output on one hand and between the firm dynamics and output on the other hand (iii) the forecasted variances and covariances of aggregates that are obvserved in the data. This section starts by discussing the calibration strategy.

4.1 Calibration

The model has 17 parameters to calibrate: β, θ, ν, δ, α, φ, ρ, ψ, λ, υ, γ_F, γ_S, σ_{ss}, ρ_σ, σ_ϵ, $z_1(\bar{s})$ and $z_0(\bar{s})$ where the last two are the steady state exit and entry cut-offs. Among these the last 8 parameters are non-standard ones. The choices for the standard parameters are fairly conventional. For the time discount rate β, I chose 0.980. Regarding the CES parameter θ which governs the degree of market power, I chose 3/4 following Ghironi and Melitz(2005). One might think that the implied mark-up might be too high. However, as pointed out by Ghironi and Melitz(2005), given the fixed operation cost, this choice does not imply an implausibly high net mark-up. The elasticity of the production function with respect to the technology shock ν is set equal to $(1-\theta)/\theta$ so that the profit function becomes linear in z.

The depreciation rate δ and the capital share α are set equal to 0.025 and 0.3, respectively. For the preference parameters, $\rho = 1$ and $\psi = \infty$ are chosen mainly for the ease of comparison with existing literature. This implies that the representative household has a log utility for consumption and indivisible labor(Rogerson(1988)). For the capital adjustment cost parameter λ, I chose 3.5. This choice helps the model replicate the basic business cycle moments of the data. For the returns to specialization, I take $\upsilon = 1/\theta - 1$ as the baseline case. However, I also try a range of different numbers to analyze the difference between the allocations of free entry and the centralized venture capital fund.

It is not easy to accurately pin down the long run level of idiosyncratic volatility. The calibration(or estimation) of this parameter in the literature ranges from a low of 15% in Veracierto(2002) to a high of 64% in Cooper and Haltiwanger(2006). I simply take the middle of this range, i.e., $\sigma_{ss} = 0.40$. For the persistency of the volatility process, following Bloom(2007), $\rho_\sigma = 0.850$ is chosen so that the annualized mean reversion of the volatility is approximately 0.48. Bloom(2007) shows that this choice is roughly consistent with micro level data of U.S. and U.K. manufacturing sectors. For the variance of the second moment shock, I take $\sigma_\epsilon = 0.20$ so that two standard deviation shock to the second moment increases the uncertainty level temporarily 100% from its long run level. This high level of increase in volatility is not unusual in the stock market data.

For the rest of parameterization, I follow the following strategy. I first calibrate the long run exit threshold shock, $z_1(\bar{s})$ from the long run entry and exit behaviors in U.S. economy. It is assumed that the total number of active firms in the long run is stationary. This implies that there must be a single number for both firm creation and destruction in the long run so that the net entry rate is zero. Following Dunne, Roberts and Samuelson(1988), Baldwin and Gorecki(1991), Hopenhayn(1992), and Hopenhayn and Rogerson(1993)[13], the long run exit rate is set equal to 2.5% per quarter.

Recently, D'erasmo(2006), in a model with endogenous entry and exit but without aggregate uncertainty, shows that 3% of sunk-cost-to-capital ratio($\gamma_S/k(\bar{s})$) is consistent with the invariant distribution of firm size and age in U.S. data. When $\gamma_S/Y(\bar{s}) = 0.15$ is chosen, the current model delivers 3.7% for the sunk-cost-to-capital ratio. I take this as a baseline case. I also provide a sensitivity analysis for the effect of change in this ratio. This choice allows us to pin down the long run entry threshold shock $z_0(\bar{s})$. The long run entry and exit threshold shocks jointly determine the long run measure of active firms $M(\bar{s})$ and capacity measure $\Xi(\bar{s})$. Combining these information and the steady state version of the Euler equation for $q^M(\bar{s})$, we can determine the fixed cost parameter γ_F, again as a ratio to output. The details of this procedure can be found in Appendix 3.

4.2 Impulse Responses

Figure 1 shows how the endogenous quantities of the model repond to an unexpected increase in uncertainty level. In the figure, it is assumed that the uncertainty level increases by 100% in the impact period, which is about a two standard deviation shock from the assumed Markov process. This amount of increase in volatility is not unusual in the data. The sunk-cost-to-capital ratio is fixed at 3.7%. The returns to specialization are specified as $v = 1/\theta - 1$ so that the allocations of free entry and constrained optimum coincide with each other.

The figure shows that the uncertainty shock leads to a strong recession in which all major aggregates such as consumption, investment, output and labor hours decline persistently. It takes about 14 quarters before the economic activity comes back to the normal level. The responses of all macroeconomic quantities exhibit strong hump-shaped cycles. Initial responses of endogenous quantities are small. But over time the initial effects are followed by much stronger deterioration in overall economic activities. This fact is more pronounced in the case of aggregate investment. The initial response is about 20% of the maximum response of aggregate investment. After the initial period, however, decline in aggregate investment is accelerated in ensuing 2 periods. The downward cycle is then followed by a long period of moderate recovery to the normal level.

Figure 2 and 3 show the driving force of the hump-shaped business cycle. Figure 2 shows that the uncertainty shock instantaneously increases the entry and exit thresholds by substantial margins. Because the variance level of the distribution itself is changing, the changes in the entry and exit thresholds do not necessarily imply that entry is less likely and exit is more likely. However, Figure 2 shows that the conditional probability of entry actually goes up substantially whereas the conditional probability of exit increases substabtially. This implies that time-varying uncertainty shocks induce a negative correlation between firm

[13] Hopenhayn(1992) and Hopenhayn and Rogerson(1993) report 5-year interval value as 39%, which implies a quarterly rate 2.5%. Campbell(1998) adopts a substantially lower value for the long run exit rate, 0.83%.

creation and firm destruction.

Figure 3 explains why both the entry and exit thresholds move in the same direction: in the presence of sunk entry cost, increase in short run uncertainty makes the benefit of making additional unit of firm decreasing. The figure shows that the marginal value of creating a new firm ($q^M(s) = J(s)$) drops immediately with the uncertainty shock. This is because the heightened uncertainty increases the probability of a future shut-down, without compensating increase in the mean profitability. The decline in the value of marginal firm(the right hand sides) implies that both the benefit of firm creation and the opportunity cost of firm destruction decrease. In order to recover the equilibrium, the cost of firm creation and the benefit of firm destruction must decline as well. Given the fixity of the non-convex costs, the only way to accomplish this is to increase both the entry and exit thresholds so that the profitabilities at both margin improve. Entry is delayed while exit is expedited.[14]

As a consequence, the equilibrium number of firms should decrease. Figure 3 shows that this is actually the case. The total number of firms drop 12% immediately. More importantly, the firm turnover cycle is accelerated from the second period, reaching its maximum response of 17% in the third period. After that, the number of firms slowly moves back to the normal level only after 30 quarters. An important mechanism behind this hump-shaped cycle is the hysteresis created by the sunk entry cost. To see this point, consider the law of motion for the number of firms

$$M(s) = [\Phi(\mu_0(s)) - \Phi(\mu_1(s))]\, M(s_{-1}) + 1 - \Phi(\mu_0(s))$$

Roughly speaking, the entry probability $1 - \Phi(\mu_0(s))$ works as a shock to the stock of firms while $\Phi(\mu_0(s)) - \Phi(\mu_1(s))$ works as an AR(1) parameter of the law of motion. In the impact period, the equilibrium firm number decreases by the amount of the decrease in entry probability, i.e., $d[1 - \Phi(\mu_0(s))]$. Although the degree of hysteresis, $[\Phi(\mu_0(s)) - \Phi(\mu_1(s))]$ changes as well, the movements in $\Phi(\mu_0(s))$ and $\Phi(\mu_1(s))$ tend to offset each other. Since $M(s_{-1})$ is predetermined, the initial response is almost entirely determined by the change in the entry probability. Beginning with the second period, however, the decrease in firm number is accelerated due to the lagged response of $M(s_{-1})$. Although the impact on the entry probability tends to be weakened at the second period, the lagged response of $M(s_{-1})$ outweighs this tendency, resulting in a capital-like accelerated swing in the stock of firms. A closer look reveals that the trough of the business cycle

[14] This is different from Dixit and Pindyck(1994) where an increased uncertainty (in the sense of comparative statics) tends to move the entry/exit barriers in opposite directions, inducing a positive correlation between firm creation and firm destruction. In their framework, an increase in uncertainty tends to increase the entry threshold value whereas it decreases the exit threshold value. The former is referred to as "bad news principle"(Bernanke(1983)) while the latter is called "good news principle". The upshot is a widened inaction region measured by $z_0(s) - z_1(s)$. The differences between their results and those herein are possibly related with the fact that the shock processes are very different. In their frame work, the idiosyncratic shock follows a geometric Brownian motion while the volatility of the process is fixed. In this paper, the assumption is quite opposite in that the first moment of the shock does not vary while the second moment follows a persistent Markov process. If the first moment follows a Markov process as well, then the marginal value of firm satisfying the entry condition does not need to coincide with the one for the exit condition, i.e., there will be two seperate marginal values of firm, one for satisfying the entry condition($q^M(z_0(s), s)$) and the other for exit condition($q^M(z_1(s), s)$). The two marginal values, in principle, can move in opposite directions in the short run, thereby allowing movements of threshold shocks in opposite directions.

is actually determined by the trough of firm turn-over cycle.

Figure 3 also shows that the response of the capacity utilization rate is following a similar dynamic pattern to that of the total firm measure. In fact, except the level differences, the two measures are showing exactly the same dynamic patterns. The reason for the smaller response of the capacity utilization can be found in the fact that the average quality of the firms are being improved during the downturn: the combination of higher entry and higher exit thresholds imply that the average qualities of both incumbents and new entries are improved. However, this "cleansing" effect of the recession is not strong enough to overcome the direct effect from the drop in the absolute number of firms. Since the measured TFP in the current environment is identical to the capacity utilization rate, the measured TFP also follows the same hump-shaped downward cycle.

The response of the capacity utilization rate is an important nexus which allows us to understand the connection between the movements of market prices and quantities. Given the capital supply fixed in the impact period, the drop in the capital utilization directly translates into a decreased demand for rental capital and leads to a substantial drop in the rental rate. As the utilization rate undergoes a persistent decline with the uncertainty shock, the rental rate follows a similar dynamic pattern over time. Because the marginal value of capital is determined by the expectation of the future path of the rental rate, Tobin's marginal $q^K(s)$ exhibits a hump-shaped downward cycle as well(see Figure 4), which in turn, magnifies the business cycle by decreasing the accumulation of flexible capital and output.

4.3 Moments

In this subsection, I analyze the performance of the model in matching business cycle moments of U.S. data. In doing so, I highlight the model's success in replicating conditional moments as well as unconditional moments of aggregate variables. The moments of U.S. aggregate data are based on time periods from 1955Q2 to 2008Q2. All data for U.S. and the model are filtered by Hodrick-Prescott filter. The moments of the model are the averages of 200 simulations with the same data length.

Table 1 shows the basic unconditional moments of U.S. business cycle and their model counterparts. In the table, we can see that the unconditional volatilities of the model line up very well with the stylized facts of U.S. business cycle. For instance, the model generates the same degree of volatility for output and three times greater volatility for investment than that of output as in the data. The model also generates a realistic degree of consumption volatility. However, the volatility of hours generated by the model only amounts to two thirds of that of hours in the data. This happens despite the choice of the preferences that are most favorable to match the volatility of hours (the linear disutility of labor or indivisible labor).

The third and the fourth columns of Table 1 compare the comovement properties of the model and the data. Although the overall degree of comovements measured by the correlation coefficients with output are slightly stronger than that of the data, especially for consumption (0.97 in the model and 0.80 in the data), the model can be considered very successful in matching the direction and the magnitude of comovements among conventional macroeconomic variables in the data. The model also performs relatively well in replicating a positive correlation between the firm measure and aggregate output. It may be argued that the degree of the comovement of the firm measure with output is too strong as compared to the counterpart of the data(0.60 in Dos Santos and Dufourt(2006) and 0.73 in Bergin and Corsetti(2005)). In the next section, I suggest a possible resolution for this problem.

The unconditional moments provide a useful summary of model's performance in explaining the basic properties of time series data. However they have clear limits in testing the ability of the model to replicate the propagation mechanisms in the data. For instance, Cogley and Nason(1995) and Rotemberg and Woodford(1996) showed that prototype real business cycle models, despite their success in matching the unconditional moments of the data, suffer from a lack of internal propagation mechanisms and fails to match the conditional moments of the data. As a result, the forecastable movements in output implied by these models only amount to 1% of the counterparts of the data over 12 months horizon. The strong internal propagation mechanism seen in the impulse response analysis suggests that the current model can substantially do better in matching forecastable movements of the data. This subsection reports the performances of the model in this respect.

To compute the forecasted changes in output implied by the model, I simulate the model for the same length of time periods that are available for U.S time series from 1995Q1 to 2008Q2. In order to construct conditional moments of the model that are comparable with those of the data, I estimate an identical times series model for the model and the data and construct conditional moments. More specifically, following Rotemberg and Woodford(1996), I estimate parsimonious VAR models of Δy, $c - y$, and h, where Δy is the log difference of output in subsequent time periods, $c - y$ is the log difference of consumption and output and h is the detrended hours. Let $E_t \Delta x_{t,t+k}$ denote a k - period ahead forecast of change in x based on time t information. The conditional moments can then be constructed as the moments of $E_t \Delta x_{t,t+k}$ (see Rotemberg and Woodford(1996) for the details of the procedure). I repeat this process 200 times and report the sample averages of the experiments.[15]

Table 2 shows the results. The first panel shows the standard deviations of cumulative changes in actual and forecasted output from U.S. data over various time horizons. The second panel exhibits the counterparts of simulated data. In the baseline parameterization with the low sunk-cost-to-capital ratio(3.7%), the model performs fairly well in matching the ratio of the forecasted movements to the actual movements of output observed in U.S. data. This implies that the model produces as much propagation as shown in the U.S. data. The baseline model tends to create greater forecastability than that of output in the data over short horizons(from one to two quarters) while it generates less forecastability over longer horizons(from four to

[15] Rotemberg and Woodford use "private output" of CITIBASE data for y_t. The straightforward counterpart of BLS database would be non-farm business output. However, the ratio of nondurable/service consumption (including housing services) to non-farm business output has been trending down over the entire sample period. For this reason, I use real GDP for y_t because the ratio of nondurable/service consumption to real GDP is found to be stationary in a whole battery of unit root tests. However, this difference does not appear to affect the results. Rotemberg and Woodford also use "private hours" of CITIBASE, which is "the private sector employee hours for wage and salary workers". I use aggregate labor hours of non-farm business (employed and self-employed) of BLS. However, this difference does not affect the results here either.

twelve quarters). However, on average, the model generates 65.0% of forecastable movements in outputs over the various time horizens while the data shows 56.7%.

A more important aspect of business cycle is that the forecatable changes in output are generally correlated with the forecastable counterparts of other aggregates such as consumption, investment and hours. In U.S. data, forecasted consumption, investment and hours are positively correlated with forecasted output. In the first panel of Table 3, we can see that the coefficients of correlations with forecasted output are 0.85, 0.98 and 0.92 for consumption, investment and hours, respectively, on average over different time horizons. A well-known problem with prototype RBC models is that they generally fail to replicate these positive comovements. Jaimovich(2007) shows that a RBC model with the same preferences utilized in this paper delivers a correlation of -1 between the predicted consumption and output for all forecasting horizons. On the other hand, Rotemberg and Woodford(1996) shows that RBC models with more standard preferences with a lower intertemporal substitution and a lower Frisch elasticity of labor supply generate a positive correlation between the forecasted consumption and output, but result in a negative correlation between the forecasted hours and output.

The third panel of Table 3 shows the performance of the model in this regard. The baseline model delivers reasonable degrees of comovements: the correlation coefficients of the forecasted changes in consumption and investment with that of output are 0.94 and 0.97 on average, remarkably similar to those observed in U.S. data. The model is less successful in replicating the high correlation (0.92) observed in the data between forecasted hours and output, although it still generates as high a correlation as 0.68 for the two forecasted series.

4.4 Flexibility and Volatility

In this subsection, I provide a comparative static analysis regarding how changes in inflexibility of the economy in terms of a higher start up sunk cost affect overall aggregate volatility. A simple and intuitive prediction can be made: a model with a higher sunk entry cost will tend to generate a stronger hysteresis effect, as the opportunity costs of entry and exit are simultaneously increased with this parameterization. Other things equal, the higher sunk entry cost makes both entry and exit harder to validate because of its implication for the costs of irreversible actions. On the other hand, a higher sunk entry cost means that there will be a larger fluctuation in the resources used for net business formation at a given degree of firm turn-over. This, together with the greater hysteresis effect, can undermine the ability of the economy to smooth its consumption and investment expenditures, resulting in greater volatilities in equilibrium quantities.

In order to analyze the effect of greater inflexibility, I reset the sunk-cost-to-capital ratio at 15%, approximately 4 times greater than the baseline parameterization. With this parameterization, the hysteresis effect in the steady state measured by the fraction of firms between the entry and exit cut-off values, i.e.,$[\Phi(\mu_0(\bar{s})) - \Phi(\mu_1(\bar{s}))]$, is increased to 0.94 from 0.56 of the baseline case. This implies that 94% of the firms in the steady state will not change the activity status from yesterday. Even with the higher sunk entry cost, the basic moments of the data can be matched by changing the other parameters of the model. However, in order to isolate the effects of higher sunk entry cost on the dynamics of the model, all other parameters are kept intact.

The third column of Table 1 reports the unconditional moments of the high sunk cost economy. As expected, the high sunk cost economy is characterized by greater volatilities for aggregates: output, consumption, investment and hours are 49%, 46%, 34% and 181% more volatile than those of the baseline economy. It is noteworthy that the increase in the volatility of investment is rather muted relative to that of hours. This can be understood in terms of relative size of costs in adjusting capital stock and labor. The current specification of capital adjustment cost penalizes fast adjustment of capital stock whereas the infinite Frisch elasticity of labor supply imposes almost no economic cost in terms of foregone utility associated with increase in labor supply. As a result, the economy is better off by using the labor adjustment margin more frequently and intensively than the capital adjustment margin.

The last column of Table 1 shows the comovement properties of the high sunk cost economy. It is interesting that the increase in inflexibility of firm entry and exit leads to a slightly greater comovement of investment with output while it leads to smaller degree of comovement with output for consumption and hours. The higher hysteresis also results in a lower comovement level with output for the total firm measure. The last aspect is somewhat expected given that the high entry cost generates too smooth time series for the firm measure, thereby dampening the correlation with output and other aggregates.

The high sunk entry cost has also important consequences for the conditional moments of the economy. The third panel of Table 3 exhibits the standard deviations of cumulative changes in forecasted output for the high sunk cost economy. In this panel, we can see that the higher sunk entry cost, not only increases unconditional moments of the model economy, but also increases the portion of forecastable changes in total ouput change. In the case of baseline model, the ratio of the standard deviations in forecastable and total output changes is 0.65 on average over various time horizens. As the sunk-cost-to-capital ratio increases to 15%, the ratio also increases to 0.77. The stronger hysteresis effect created by the higher entry cost is strengthening the internal propagation mechanism and the output is more forecastable under this environment.

The above findings has an imporant implication for the source of recent decline in the volatilities of macroeconomic data. As well documented by McConnell and Perez-Quiros(2000), Stock and Watson(2003), the output volatility has declined approximately 50% since the beginning of the Great Moderation (Bernanke(2004)). The result in this paper suggests that any technological innovations that lower the effective costs of entry or any institutional developments that can increase the scrap values of exiting firms' capital asset could have greatly dampened the volatility of business cycle. The development in financial market that facilitates firm entry and exit, such as growing importance of venture capital financing could also have had simlar effects on the economy.

However, if the underlying cause of the Great Moderation is in the structural changes of the economy, for instance, such as described in the above paragraph, we should be able to observe in the real data the decline in the forecastability of output as well as the decline of unconditional volatility of output. To check this possibility, in Table 4, I reestimate the VAR model for the real data for subsample periods before and after 1984. As expected, the standard deviations in cumulative output changes have substantially declined since the beginning of the Great Moderation.

More importantly, the share of the forecastable changes in total output has declined considerably as well. Since the great moderation, only 32% of output change over 24 quarters horizon is forecastable whereas 56% of output change over the same horizon was forecastable before the Great Moderation. This confirms that the Great Moderation is not simply the result of "good lucks" since the decomposition results suggest that the structure and the strength of internal propagation mechanism has been going through fundamental changes.[16] Although this finding also invites other explanations regarding the cause of the decline of forecastability of ouput and dampened propagation, for instance, such as optimal monetary policy and innovations in inventory managements, the analysis of this paper suggests that greater flexibility in firm creation/destruction margin could also have played an important role.

4.5 Constrained Optimum and Free Entry Allocations

So far we have assumed that the returns to specialization are equal to $v = (1-\theta)/\theta$, which makes individual profit function of intermediate good firms not directly dependent upon the equilibrium number of firms. Remind that

$$\pi(i) = m(i)\,(1-\theta)\,\theta^{\frac{\theta}{1-\theta}} \left[z(i)^{\nu} \left(\frac{\alpha}{r^K}\right)^{\alpha} \left(\frac{1-\alpha}{w}\right)^{1-\alpha} \right]^{\frac{\theta}{1-\theta}} Y M^{-\xi\frac{\theta}{\theta-1}}$$

where $\xi \equiv 1 + v - 1/\theta$. Also remind that $1+v$ measures the returns to specialization. The assumption $1 + v = 1/\theta$ makes the elasticity of individual profit with respect to the firm measure zero. In this special case, I showed that the two allocations under the constrained optimum(CO) and the free entry(FE) coincide with each other.

In this subsection, I show how the two allocations can be different when there is a meaningful degree of externality that makes individual profit directly depend on the total number of firms. To that end, I consider a range of returns to specializations so that the elasticity of profit with respect to the total firm measure is ranged from -0.7 to 0.7. The idea is to create a small degree of 'business stealing' effect (negative externality) associated with a higher degree of competition or to create a small degree of 'business enhancing' effect (positive externality) from a higher degree of competition. Again, to isolate the externality effect, I keep all parameters unchanged from the baseline simulation. It is important to note that the externality from returns to specialization exists for both equilibrium setups. The difference is whether or not the decision makers internalize this externality.

I first consider how the externality modifies the allocation of the constrained optimum. Figure 5 displays the impulse responses of aggregate investment for economies with different returns to specialization, ranging

[16] Recently a similar argument, though based upon a different statistical approach, is made by Gali and Gambatti (2008).

from -0.7 to 0.7 in the case of the constrained optimum. With a large amount of positive externality, the uncertainty shock leads to much stronger and longer periods of recession. Any given response of the total number of firms brings a greater impact on the economy because the positive externality created by returns to specialization is also destroyed by the same mechanism. On the contrary, with an extreme negative externality, the uncertainty shock, surprisingly, causes a brisk expansion cycle. The decrease in the number of active firms in this case works as a sufficiently good fundamental for existing firms' capital investments.

Therefore, both positive and negative externalities can increase the volatility of business cycle for different reasons, and strongly modify the basic properties of the business cycle. A positive externality strengthens the negative correlation between uncertainty and output that we observe in the data. A negative externality weakens the correlation, and if the externality is strong enough, it actually turns the negative correlation into a postive one. Uncertainty creates a boom in the extreme cases.

Table 5 summarizes the basic business cycle properties of the two allocations when there are small degrees of positive(0.3) and negative(-0.3) externalities from competition. More importantly, an interesting consequence of introducing externality from competition is that, regardless of its sign or the equilibrium settings under which its effects are considered, the externality tends to increase absolute volatilities of equilibrium quantities. The result shows that the free entry allocation *relatively* dampens the volatility of business cycle as compared to the constrained optimum allocation *in the presence of negative externality.* However, it amplifies the volatility as compared to the constrained optimum allocation *in case of positive externality.* The key to understanding why the free entry allocation stabilizes the business cycle in one case and amplifies it in the other case can be found in the behaviors of the value of marginal firm, and the resulting firm dynamics. To that end, I compare the impulse responses of these variables in Figure 6(negative externality case) and 7(positive externality case). By comparing two pictures, we can see that the responses of the value of marginal firm and the firm measure are greater for the constrained optimum allocation in case of negative externality, while the responses are greater for the free entry allocation in case of positive externality.

The reason behind this difference is not hard to grasp. When there is a negative externality from new entry, individual firms under free entry condition do not realize that they can collectively do better for the representative household by coordinating to reduce the number of firms further, improving the business environments of existing firms. However, the venture capitalist fund, knowing this externality, responds more aggressively in reducing the number of firms in recession. The opposite mechanism works for the positive externality case. In this case, individual firms under free entry condition do not exploit the fact that they can collectively do better by responding less to the shock. Again, the venture capitalist fund, knowing this positive externality, tends to be less aggressive in reducing the number of firms during the recession. The differences in the responses of the firm measure in two cases directly translate into the differences in the responses of the rental rate of capital and the marginal value of capital, and ultimately lead to the relative stabilization and amplification results of the business cycle by the free entry allocation.

This research is motivated by two key observations: a negative correlation between the idiosyncratic uncertainty and aggregate output and a positive correlation between the net business formation and aggregate output. Two correlations imply a negative correlation between the uncertainty and the net business formation. I conclude this section by summarizing the model's performance in these regards in Table 6 under a range of degrees of externality with the two equilibrium settings.

Table 6 reports the correlation coefficients of macroeconomic variables in interest with aggregate output. The list of variables includes (i) the uncertainty (ii) the total number of active firms (iii) the forecasted share of the total variance of output (iv) labor productivity (v) measure TFP. The choice of the first two variables is obvious given the motivation of the paper. The forecasted share of output variance is included to see how the strength of the model's internal propagation is being modified by different degrees of externality and by the two different equilibrium settings. The last two variables regarding productivity are included as the most important regularities of the business cycles in the data. All simulations are performed with the baseline parameterization for the sunk-cost-to-capital ratio.

The results in the table show that the model delivers a negative correlation between the uncertainty and aggregate ouput as long as the degree of externality is kept in a reasonable range. The more positive externality, in general, strengthens the countercyclical role of time varying uncertainty. However, whether it is positive or negative, an extreme degree of externality does not match with the data very well. While too high a positive externality results in too strong countercyclical role of the uncertainty, too low strong a negative externality induces a positive correlation of the uncertainty with the output. As shown earlier, with a negative externality greater than -0.5 in absolute term, an increased uncertainty brings in a boom period because of its positive implication for the business environment for incumbent firms. The last aspect is robust with respect to the equilibrium settings.

The flip side of the fact that the uncertainty shock can lead to a boom period with too strong a negative externality is that the uncertainty shock can induce a negative correlation between net business formation and aggregate output with a similar degree of negative externality. The table shows that with a negative externality greater than -0.5 in absoute term, the model generates a zero or negative correlation for the total number of active firms with aggregate output. This finding, together with the one in the previous paragraph, suggests that as much as the countercyclicality of the uncertainty and the procyclicality of net business formation can be regarded as reasonable facts, too strong a degree of externality may be considered unrealistic. On the contrary, with too high a degree of externality, the model generates unrealistically high degrees of countercyclicality for the uncertainty and procyclicality for the net busness formation.

As for the forecasted share of the total output variance, the experiment produces rather inconclusive results: the model performs well regardless of the absolute magnitude of externality or its sign. The forecasted share of the output variance ranges between 0.55 and 0.76 with its maximum value at the externality level equal to -0.4. In either direction from this level, the forecasted share of the output variance declines, but only gradually. This finding suggests that the externality is not the main source of the propagation mechanism. With or without the externality, the uncertainty shock and the model generate essentially similar amount of forecastability. On the other hand, the discussion in the previous section regarding the flexibility and the volatility suggests that the most important source of the strong internal propagation in the model should be found in the role sunk entry costs.

As the true source of measured TFP in this paper being the endogenous capacity utilization, which is nothing but the quality adjusted measure of active firms, it is not surprising that the correlation of measured TFP with ouput shows a similar pattern to the correlation of the total measure of firms with output as we change the level of externality: an extreme level of externality makes the measured TFP negatively correlated with output, adding one more reason to doubt the plausibility of that degree of negative externality in reality.[17] On the other hand, the table shows that even a moderate degree of positive externality generates a countercyclical labor productivity. From a perspective that emphasizes the positive comovement of output and productivity as the most prominent feature of U.S. business cycle (for instance, Hall(1987)), we can conclude that too strong a positive externality in the current economic environment is hard to reconcile with the established facts in the data. Overall, the results shown in Table 6 suggest that a zero to low degree of negative externality is the most realistic in the economic environment under analysis.

5 Conclusion

The current research project is motivated by the negative correlation between the uncertainty measure and aggregate output observed in U.S. data. The causation can run either way in principle: (i) uncertainty fluctuates randomly and exogenously and behaves as a driving force for the fluctuation of aggregate output and (ii) change in aggregate output can work its way to generate countercyclical fluctuations in the dispersion of individual firms' performances via endogenous fluctuation in agency problem over the business cycle. Rampini(2004) takes the second route, for instance. The current paper, like Bloom(2007), has taken the first route to explain the joint behaviors of uncertainty, firm dynamics and aggregate cycles. Perhaps, in reality, both causal directions play important roles. It would be definitely useful to explore a macroeconomic environment where financial frictions can interact with irreversible decisions of firm creation/destruction. I leave this as a future research project. It would be also interesting to see whether or not sunspot equilibria analyzed by Jaimovich(2007) in the context of firm dynamics with a *finite* number of players can survive in an economic environment where there is not only fixed cost but also irreversible start up cost such as the one in this paper. Introducing nominal frictions such as price rigidity and analyzing implications of temporal increase in uncertainty for monetary policy would be a straightforward extension of the current paper along the line of Bergin and Corsetti(2005) and Bilbiie, Ghironi and Melitz(2007b).

[17] The measured TFP shock in the table is constructed as $TFP_t = \hat{Y}_t - \alpha\hat{K}_t - (1-\alpha)\hat{H}_t$where $\hat{X}_t$ is a log deviation of a variable X_t from its trend. As emphasized by Basu(1995), the measured TFP is constructed with value added components of gross output, i.e., net of changes in capital adjustment cost and nonconvex costs associated with firm dynamics.

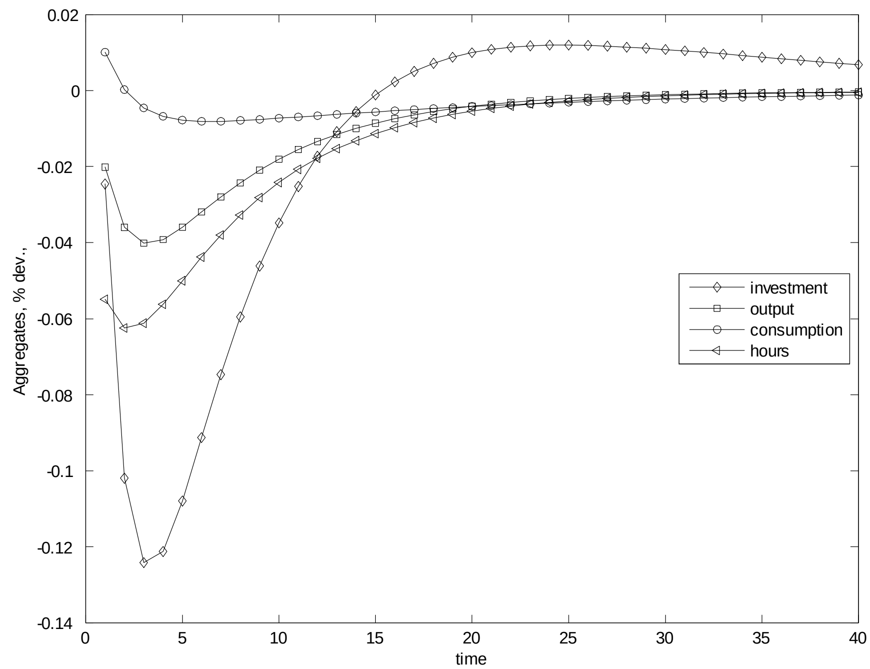

Figure 1. The Effects of Uncertainty Shock on Aggregate Quantities

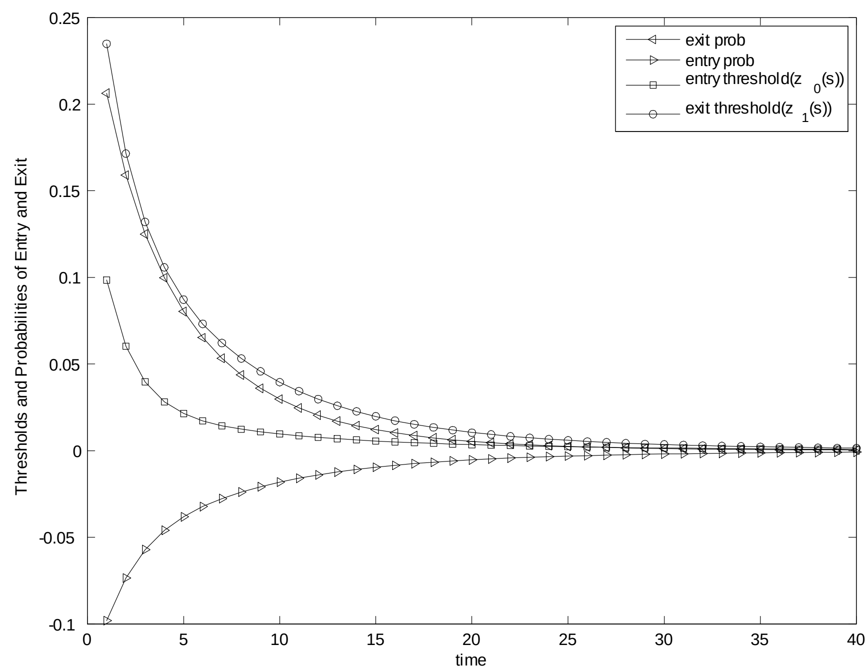

Figure 2. The Effects of Uncertainty Shock on Entry/Exit Thresholds and Probabilities

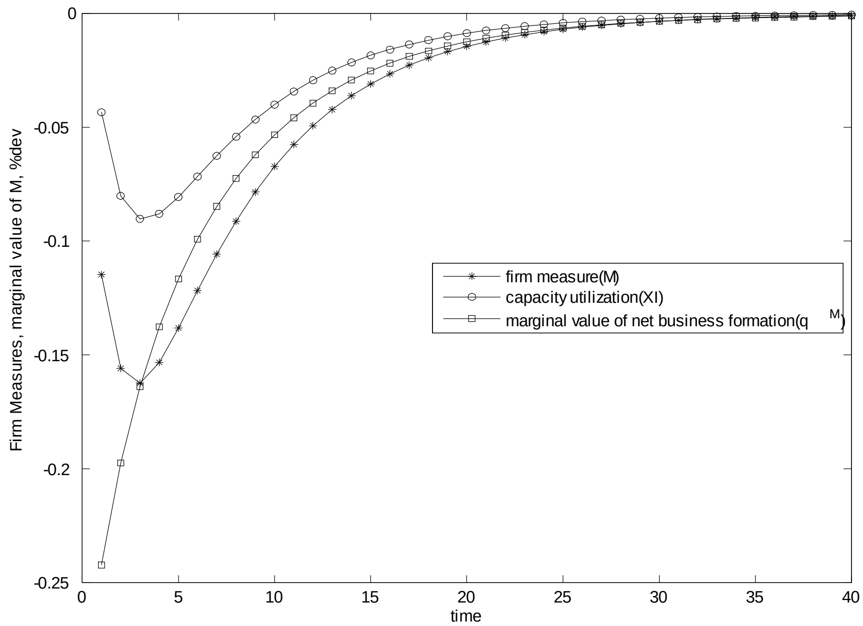

Figure 3. The Effects of Uncertainty Shock on the Value of Marginal Firm, Firm Measure and Capacity Utilization.

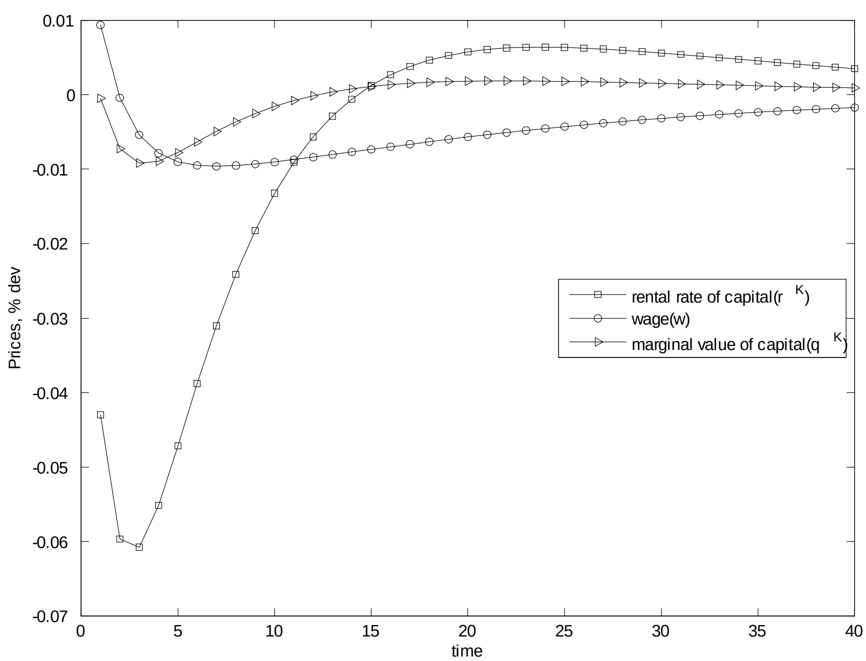

Figure 4. The Effect of Uncertainty Shock on Market Prices and Marginal Value of Capital

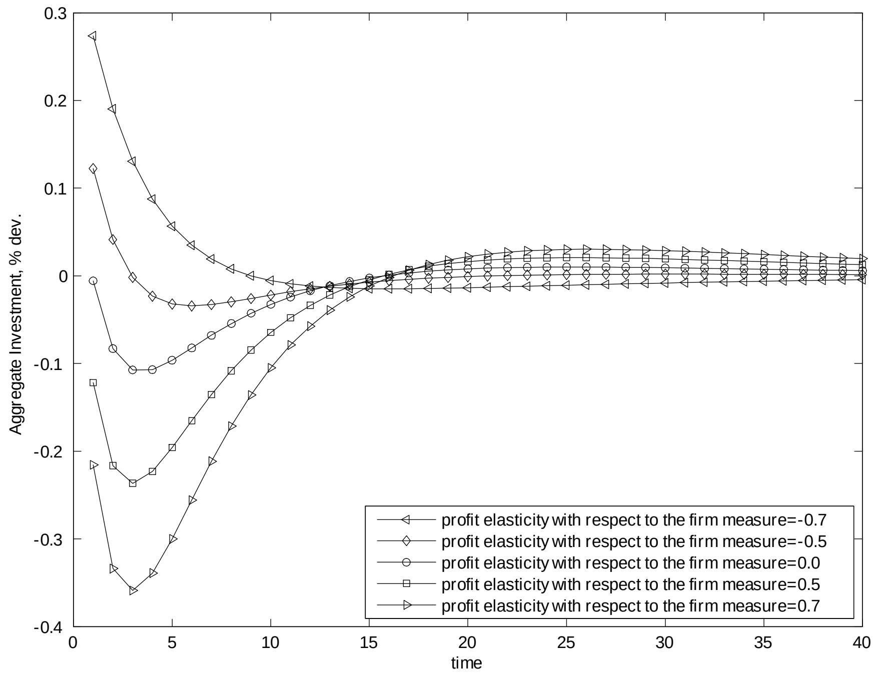

Figure 5. Externality and the Effect of Uncertainty Shock on Aggregate Investment.

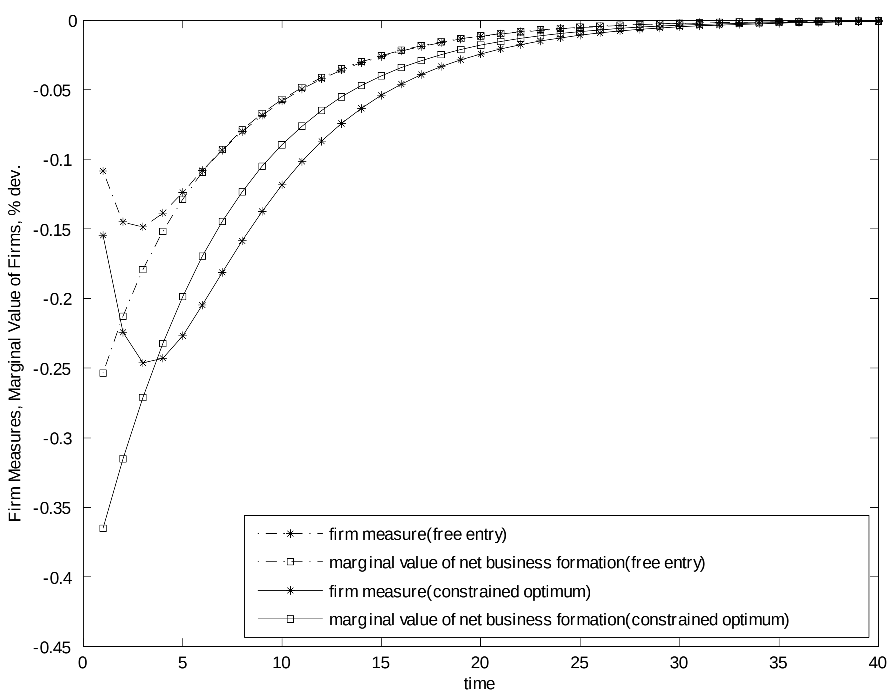

Figure 6. Uncertainty, Negative Externality and Firm Dynamics

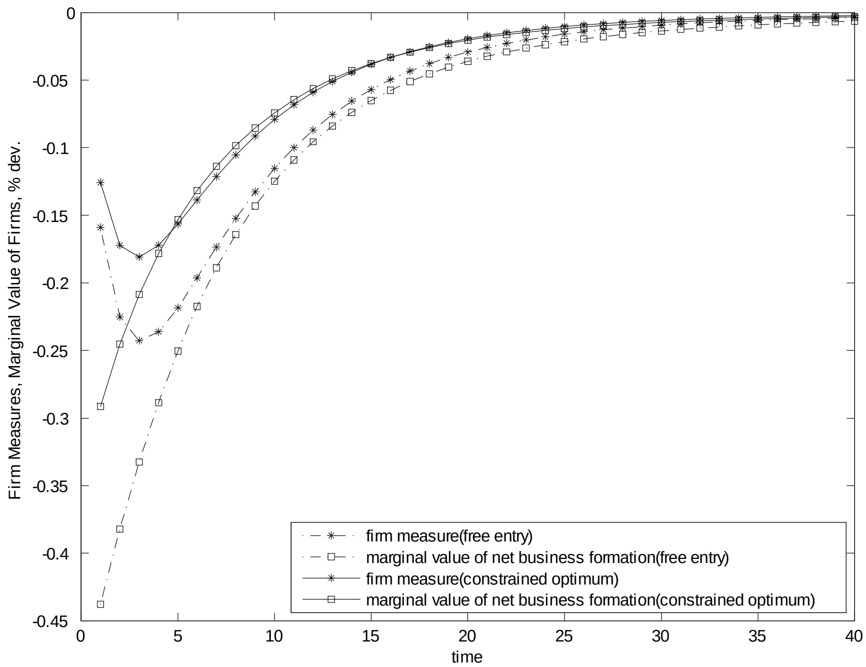

Figure 7. Uncertainty, Positive Externality and Firm Dynamics: Free Entry vs. Constrained Optimum

Table 1. Business Cycle Moments

	Standard Deviations				Comovement with Output		
	U.S.	M(3.7%)	M(15.0%)		U.S.	M(3.7%)	M(15.0%)
Output	1.57	1.67	2.48	Consumption	0.80	0.97	0.67
Consumption	1.18	1.00	1.46	Investment	0.91	0.95	0.98
Investment	7.02	7.29	9.74	Hours	0.88	0.86	0.77
Hours	1.52	1.09	3.07	No. of Firms	0.73	0.95	0.53

Hodrick-Prescott filtered. Average of 200 simulations of each data length of 216 quarters. 3.7% and 15.0% are two parameterizations of the sunk entry cost in terms of sunk-cost-to -capital ratio.

Table 2. Standard Deviation of Cumulative Changes in Output

Horizon	1	2	4	8	12	24	Ave.
U.S. data							
Forecastable Change in Output	0.005	0.008	0.013	0.020	0.024	0.026	0.016
Actual Change in Output	0.009	0.014	0.023	0.033	0.039	0.049	0.028
Ratio	0.539	0.546	0.551	0.608	0.626	0.529	0.566
The Model(Sunk Cost: 3.7% of capital)							
Forecastable Change in Output	0.007	0.010	0.014	0.020	0.024	0.029	0.017
Actual Change in Output	0.008	0.015	0.025	0.034	0.039	0.044	0.028
Ratio	0.850	0.664	0.542	0.572	0.616	0.656	0.650
The Model(Sunk Cost: 15.0% of capital)							
Forecastable Change in Output	0.016	0.025	0.032	0.034	0.036	0.040	0.030
Actual Change in Output	0.019	0.027	0.037	0.048	0.053	0.059	0.041
Ratio	0.817	0.909	0.855	0.709	0.667	0.665	0.770

Hodrick-Prescott filtered. Average of 200 simulations of each data length of 216 quarters. 3.7% and 15.0% are two parameterizations of the sunk entry cost in terms of sunk-cost-to -capital ratio. The periods for U.S. data are 1955Q2 to 2008Q2. For the methodology to construct the forecasted time series, and the data source, see the main text, footnote 15 and Rotemberg and Woodford(1996).

Table 3. Correlation and Regression Coefficients among Forecasted Changes

Horizon	1	2	4	8	12	24	Ave.
U.S data							
Correlation between forecasted output and:							
Consumption	0.80	0.83	0.86	0.88	0.88	0.83	0.85
Investment	0.98	0.98	0.99	0.99	0.99	0.97	0.98
Hours	0.87	0.89	0.90	0.94	0.96	0.96	0.92
Regression on forecasted output of:							
Consumption	0.29	0.30	0.28	0.28	0.30	0.36	0.30
Investment	3.02	3.00	3.04	3.05	3.00	2.81	2.99
Hours	1.10	1.18	1.12	1.08	1.12	1.19	1.13
The Model(Sunk Cost: 3.7% of capital)							
Correlation between forecasted output and:							
Consumption	0.98	0.96	0.91	0.90	0.92	0.95	0.94
Investment	0.99	0.98	0.97	0.97	0.97	0.95	0.97
Hours	0.51	0.47	0.61	0.80	0.85	0.86	0.68
Regression on forecasted output of:							
Consumption	0.65	0.64	0.58	0.52	0.53	0.61	0.59
Investment	6.32	6.07	5.40	4.76	4.45	3.81	5.14
Hours	0.19	0.23	0.42	0.58	0.59	0.51	0.42
The Model(Sunk Cost: 15.0% of capital)							
Correlation between forecasted output and:							
Consumption	0.94	0.92	0.86	0.68	0.54	0.49	0.74
Investment	1.00	1.00	1.00	0.99	0.98	0.97	0.99
Hours	0.95	0.93	0.86	0.77	0.79	0.84	0.86
Regression on forecasted output of:							
Consumption	0.45	0.46	0.48	0.41	0.31	0.26	0.40
Investment	4.12	4.08	3.99	3.80	3.64	3.27	3.82
Hours	0.83	0.81	0.79	0.93	1.11	1.23	0.95

Hodrick-Prescott filtered. Average of 200 simulations of each data length of 216 quarters. 3.7% and 15.0% are two parameterizations of the sunk entry cost in terms of sunk-cost-to -capital ratio.

Table 4. The Great Moderation and the Forecastability of Output

	1	2	4	8	12	24
Before 1985						
Forecastable Changes in Output(A)	0.007	0.011	0.020	0.031	0.034	0.031
Actual Output Changes(B)	0.011	0.018	0.028	0.040	0.045	0.056
C=A/B	0.602	0.622	0.691	0.794	0.748	0.557
Since 1985						
Forecastable Changes in Output(A')	0.002	0.004	0.006	0.008	0.009	0.010
Actual Output Changes(B')	0.005	0.008	0.013	0.020	0.025	0.030
D=A'/B'	0.429	0.481	0.460	0.393	0.348	0.322
D/C	0.712	0.773	0.666	0.495	0.465	0.579

Table 5. Externality and Business Cycle Moments

Negative Externality(Profit Elasticity in No. of Firms=-0.3)

Volatility	FE	CO	FE/CO	Comovement	FE	CO	FE/CO
Output	2.33	2.67	0.87	Consumption	1.00	0.99	1.00
Consumption	1.05	1.38	0.76	Investment	0.99	1.00	1.00
Investment	11.56	13.88	0.83	Hours	0.99	1.00	1.00
Hours	1.62	1.49	1.09	No. of Firms	-0.30	0.13	

Positive Externality(Profit Elasticity in No. of Firms=0.3)

Volatility	FE	CO	FE/CO	Comovement	FE	CO	FE/CO
Output	7.28	5.04	1.45	Consumption	0.26	0.21	1.21
Consumption	2.64	2.19	1.21	Investment	0.99	0.99	1.00
Investment	18.81	13.83	1.36	Hours	0.95	0.92	1.02
Hours	12.27	8.61	1.42	No. of Firms	0.96	0.92	1.04

Hodrick-Prescott filtered. Average of 200 simulations of each data length of 216 quarters.

Table 6. Externality and Its Effects on Comovements

	Firm Dyncs		Measured TFP		Uncertainty Measure		Forecasted Variance		Labor Prod.	
U.S.	0.73	0.73	0.81	0.81	-0.42	-0.42	0.57	0.57	0.56	0.56
Ext.	CO	FE	CO	FE	CO	FE	CO	FE	CO	FE
-0.7	-0.64	-0.73	-0.31	-0.26	0.99	0.94	0.55	0.60	-0.50	-0.51
-0.6	-0.30	-0.48	0.10	0.06	0.78	0.79	0.68	0.67	-0.06	-0.16
-0.5	0.21	0.01	0.62	0.53	0.32	0.38	0.75	0.74	0.53	0.41
-0.4	0.62	0.51	0.91	0.88	-0.17	-0.13	0.76	0.75	0.90	0.86
-0.3	0.81	0.77	0.99	0.98	-0.46	-0.46	0.75	0.74	1.00	1.00
-0.2	0.90	0.89	0.99	0.99	-0.62	-0.62	0.70	0.71	0.96	0.96
-0.1	0.94	0.94	0.98	0.97	-0.71	-0.71	0.67	0.67	0.82	0.81
0.0	0.95	0.95	0.96	0.96	-0.76	-0.76	0.65	0.65	0.58	0.58
0.1	0.98	0.98	0.94	0.94	-0.80	-0.79	0.63	0.63	0.27	0.27
0.2	0.98	0.98	0.92	0.93	-0.82	-0.81	0.61	0.61	-0.04	-0.03
0.3	0.99	0.99	0.91	0.92	-0.84	-0.82	0.59	0.60	-0.28	-0.28
0.4	0.99	0.99	0.90	0.91	-0.85	-0.83	0.58	0.58	-0.46	-0.46
0.5	0.99	1.00	0.89	0.90	-0.86	-0.83	0.58	0.58	-0.58	-0.59
0.6	1.00	1.00	0.88	0.89	-0.86	-0.82	0.57	0.57	-0.66	-0.67
0.7	1.00	1.00	0.87	0.88	-0.85	-0.81	0.55	0.55	-0.72	-0.74

Hodrick-Prescott filtered. Average of 200 simulations of each data length of 216 quarters. The counterparts of U.S. data are 0.81(Bils(1998)) for measured TFP, 0.56(Bils(1998)) for labor productivity, -0.42 for uncertainty measure(Bloom et al(2007)), 0.60 for firm dynamics(Bergin and Corsetti(2005)) and 0.57 for the forecasted variance of output, a computation of my own. The forecasted vari- -ance is the average of forecasted variances on the horizons of 1, 2, 4, 8, 12 and 24 quarters.

Appendix 1. The Equivalence of the Constrained Optimum and the Free Entry, Eq. (23)

The total derivative of tomorrow's aggregate profit with respect to today's number of firms is given by

$$\begin{aligned}\frac{d\Pi(s')}{dM(s)} &= \frac{\partial\Pi(s')}{\partial M(s)} + \frac{\partial M(s')}{\partial M(s)}\frac{\partial\Pi(s')}{\partial M(s')} \\ &= \sum_{j=0,1}\frac{\partial M_j(s')}{\partial M(s)}\pi(\bar{z}_j(s'),s') + \sum_{j=0,1} M_j(s')\frac{\partial M(s')}{\partial M(s)}\frac{\partial \pi(\bar{z}_j(s'),s')}{\partial M(s')}\end{aligned}$$

The partial effect of changes in $M(s)$, holding constant the direct effects on the conditional means of individual profits, is given by

$$\begin{aligned}\frac{\partial\Pi(s')}{\partial M(s)} &= \sum_{j=0,1}\frac{\partial M_j(s')}{\partial M(s)}\left[\frac{1}{1-F(z_j(s')|\sigma')}\int_{z'\geq z_j(s')}\pi(z',s')dF(z'|\sigma')\right] \\ &= -\int_{z'\geq z_0(s')}\pi(z',s')dF(z'|\sigma') + \int_{z'\geq z_1(s')}\pi(z',s')dF(z'|\sigma') \\ &= \int_{z_1(s')}^{z_1(s')}\pi(z',s')dF(z'|\sigma') \qquad \text{(A-1)}\end{aligned}$$

Using the definitions of aggregate fixed and sunk costs, we can see that the effects on the costs are given by

$$-\frac{\partial\Gamma_F(s')}{\partial M(s)} = -\int_{z_1(s')}^{z_0(s')}\gamma_F dF(z'|\sigma') \text{ and } -\frac{\partial\Gamma_S(s')}{\partial M(s)} = \int_{z'\geq z_0(s')}\gamma_S dF(z'|\sigma') \qquad \text{(A-2)}$$

From the law of motion for the total measure of firms, we can alse see that

$$\begin{aligned}\frac{\partial M(s')}{\partial M(s)}q^M(s') &= \left\{-[1-F(z_0(s')|\sigma')] + [1-F(z_1(s')|\sigma')]\right\}q^M(s') \\ &= \int_{z_1(s')}^{z_0(s')} q^M(s')dF(z'|\sigma') \qquad \text{(A-3)}\end{aligned}$$

The second line of (23) can be constructed by simply combing (A-1)∼(A-3). The first line of (23) captures the aggregation of the direct effect of changing today's number of firms on tomorrow's individual profits. Using the definition of aggregate profit (4), the aggregation of the direct effects can be seen as

$$\begin{aligned}\frac{\partial M(s')}{\partial M(s)}\frac{\partial\Pi(s')}{\partial M(s')} &= \frac{\partial M(s')}{\partial M(s)}\sum_{j=0,1}M_j(s')\frac{\partial\pi(\bar{z}_j(s'),s')}{\partial M(s')} \\ &= \int_{z_1(s')}^{z_1(s')}dF(z'|\sigma')\sum_{j=0,1}M_j(s')\frac{\partial\pi(\bar{z}_j(s'),s')}{\partial M(s')} \\ &= \int_{z_1(s')}^{z_1(s')}\sum_{j=0,1}M_j(s')\frac{\partial\pi(\bar{z}_j(s'),s')}{\partial M(s')}dF(z'|\sigma') \qquad \text{(A-4)}\end{aligned}$$

When $\partial\pi(\bar{z}_j(s'),s')/\partial M(s') = 0$ identically for all s', the Euler equation collapses into the following Bellman equation,

$$q^M(s) = \int \Lambda(s,s') \left[\int_{z' \geq z_0(s')} \gamma_S dF(z'|\sigma') + \int_{z_1(s')}^{z_0(s')} \left[\pi(z',s') - \gamma_F + q^M(s')\right] dF(z'|\sigma') \right] dQ(\sigma'|\sigma), \quad \text{(A-5)}$$

which is an identical functional equation with (22). Therefore, (A-5), together with two FOCs, (9) and (10) establishes the equivalence of the constrained optimum to the free entry allocaion under the special assumption, $\partial\pi(\bar{z}_j(s'), s')/\partial M(s') = 0$.

Appendix 2. The Productivity Differential, Eq. (15)

A lognormal distribution with parameter 0 and σ^2 is defined as $\log z \sim N(-0.5\sigma^2, \sigma^2)$. This implies

$$\log z^{\frac{\nu\theta}{1-\theta}} \sim N\left(-\frac{1}{2}\frac{\nu\theta}{1-\theta}\sigma^2, \left(\frac{\nu\theta}{1-\theta}\right)^2 \sigma^2\right)$$

Let $y = z^{\frac{\nu\theta}{1-\theta}}$ and $y_c(s) = z_c(s)^{\frac{\nu\theta}{1-\theta}}$. Also let H be the distribution function of y. Using the change of variable formula $dy = \frac{\nu\theta}{1-\theta} z^{\frac{(1+\nu)\theta-1}{1-\theta}} dz$, we can write

$$\begin{aligned} \int_{z_j(s)}^{\infty} z^{\frac{\nu\theta}{1-\theta}} f(z|\sigma) dz &= \int_{z_j(s)}^{\infty} z^{\frac{\nu\theta}{1-\theta}} \left[\frac{1-\theta}{\nu\theta} z^{1-\frac{\nu\theta}{1-\theta}} f(z|\sigma)\right] \left[\frac{\nu\theta}{1-\theta} z^{\frac{(1+\nu)\theta-1}{1-\theta}} dz\right] \\ &= \int_{y_j(s)}^{\infty} y h(y|\sigma) dy \end{aligned}$$

where $f(z|\sigma)$ is the log-normal pdf of z and $h(y|\sigma)$ is the log-normal pdf of y with parameter $-\frac{1}{2}\frac{\nu\theta}{1-\theta}\sigma^2$ and $\left(\frac{\nu\theta}{1-\theta}\right)^2 \sigma^2$. It is readily verifiable that $h(z^{\frac{\nu\theta}{1-\theta}}|\sigma) = \frac{1-\theta}{\nu\theta} z^{1-\frac{\nu\theta}{1-\theta}} f(z|\sigma)$. It then follows (See Johnson, Kotz and Balakrishnan(1994)) that

$$\begin{aligned} \int_{z_j(s)}^{\infty} z_j(s)^{\frac{\nu\theta}{1-\theta}} dF(z|\sigma) &= \int_{y_j(s)}^{\infty} y dH(y|\sigma) = \left[1 - \Phi(\mu_j(s) - \Sigma(\sigma))\right] \int_0^{\infty} y dH(y|\sigma) \\ &= \bar{z}(\sigma)^{\frac{\nu\theta}{1-\theta}} \left[1 - \Phi\left(\mu_j(s) - \Sigma(\sigma)\right)\right] \end{aligned}$$

where $\Sigma(\sigma) \equiv \frac{\nu\sigma\theta}{1-\theta}$, $\bar{z}(\sigma) \equiv \left[\int_0^{\infty} z^{\frac{\nu\theta}{1-\theta}} dF(z|\sigma)\right]^{\frac{1-\theta}{\nu\theta}} = \exp\left[\frac{1}{2}\left(\frac{\nu\theta}{1-\theta} - 1\right)\sigma^2\right]$ and

$$\mu_j(s) \equiv \Sigma(\sigma)^{-1}\left[\frac{\nu\theta}{1-\theta}\log z_j(s) + \frac{1}{2}\sigma\Sigma(\sigma)\right] = \frac{1}{\sigma}\left(\log z_j(s) + \frac{1}{2}\sigma^2\right)$$

This implies that the productivity differential between the unconstrained mean $\bar{z}(\sigma)$ and the truncated mean $\bar{z}_j(s)$ is given as

$$\left[\frac{\bar{z}_j(s)}{\bar{z}(\sigma)}\right]^{\frac{\nu\theta}{1-\theta}} = \frac{1 - \Phi\left(\mu_j(s) - \Sigma(\sigma)\right)}{1 - \Phi(\mu_j(s))}$$

In the special case of $\nu = (1-\theta)/\theta$ as assumed in this paper, $y = z$, $\Sigma(\sigma) = \sigma$ and $\bar{z}(\sigma) = \bar{z} = 1$.

Appendix 3. Determination of Steady State

It is assumed that the total number of active firms in the long run is stationary. This implies that there must be a single number for both business creation and destruction in the long run so that the net entry rate should be zero. Following Dunne, Roberts and Samuelson(1988), Baldwin and Gorecki(1991), Hopenhayn(1992), and Hopenhayn and Rogerson(1993), I set the long run exit rate as 2.5% per quarter,

$$\text{long run entry rate} = \text{long run exit rate} = \Phi\left(\mu_1(\bar{s})\right) = 0.025$$

This determines the actual threshold value $\log z_1(\bar{s}) = \bar{\sigma}\Phi^{-1}(0.025) - 0.5\bar{\sigma}^2$ given the long run volatility level $\bar{\sigma}$. One difficulty in calibration comes from the fact that the entry threshold value and therefore, the steady state measure of active firms,

$$M(\bar{s}) = \frac{1 - \Phi\left(\mu_0(\bar{s})\right)}{1 - \left[\Phi\left(\mu_0(\bar{s})\right) - \Phi\left(\mu_1(\bar{s})\right)\right]}$$

are neither restricted by the theory nor directly recorded by the data.

To detour this problem, I calibrate the ratio of the sunk entry cost relative to the average(also aggregate) level of output, i.e., $\gamma_S/Y(\bar{s})$. Recently, D'erasmo(2006), in a model with endogenous entry and exit but without aggregate uncertainty, shows that 3% of sunk-cost-to-capital ratio(γ_S/k_{ss}^D) is consistent with the invariant distribution of firm size and age in U.S. data. When $\gamma_S/Y(\bar{s}) = 0.15$ is chosen, the current model delivers 3.7% for the sunk-cost-to-capital ratio. I take this as a baseline case. I also provide a sensitivity analysis for the effect of change in this ratio.

To pin down the entry threshold value using this ratio, first note that by subtracting the first order condition for exit from the first order condition for entry, we have $\gamma_S = \pi(z_0(\bar{s}), \bar{s}) - \pi(z_0(\bar{s}), \bar{s})$ which implies

$$\frac{\gamma_S}{Y(\bar{s})} = (1-\theta)\,\theta^{\frac{\theta}{1-\theta}}\left[z_0(\bar{s})^{\frac{\nu\theta}{1-\theta}} - z_1(\bar{s})^{\frac{\nu\theta}{1-\theta}}\right]\left[\left(\frac{\alpha}{r^K(\bar{s})}\right)^{\alpha}\left(\frac{1-\alpha}{w(\bar{s})}\right)^{1-\alpha}\right]^{\frac{\theta}{1-\theta}} M(\bar{s})^{-\xi\frac{\theta}{\theta-1}} \tag{24}$$

For us to pin down the entry threshold($z_0(\bar{s})$ and its standardization $\mu_0(\bar{s})$), $r^K(\bar{s})$, $w(\bar{s})$ and $M(\bar{s})$ must be either constant or functions of the entry threshold. We know that in the steady state, $r^K(\bar{s}) = r(\bar{s}) + \delta = \beta^{-1} - 1 + \delta$, following the formula for Jorgensonian user cost. This can be verified from the first order condition for flexible capital. We also know that $M(\bar{s})$ is a function of the entry and exit threshold values. Therefore, I only need to show that the equilibrium wage is actually pinned down by the threshold values.

To show this, consider the fact that the final good production function implies that

$$\begin{aligned}\frac{Y(\bar{s})^{\theta}}{M(\bar{s})^{\xi\theta}} &= M(\bar{s})\left[1 - \Phi\left(\mu_1(\bar{s})\right)\right]\left[\frac{1}{1-\Phi\left(\mu_1(\bar{s})\right)}\int_{z\geq z_1(\bar{s})} y(z,\bar{s})^{\theta}dF(z|\bar{\sigma})\right] \\ &\quad + \left[1 - M(\bar{s})\right]\left[1 - \Phi\left(\mu_0(\bar{s})\right)\right]\left[\frac{1}{1-\Phi\left(\mu_0(\bar{s})\right)}\int_{z\geq z_0(\bar{s})} y(z,\bar{s})^{\theta}dF(z|\bar{\sigma})\right]\end{aligned}$$

Using the factor demand functions, one can derive a reduced form individual output function as following

$$y(z,\bar{s})^{\theta}=\theta^{\frac{\theta}{1-\theta}}z^{\frac{\nu\theta}{1-\theta}}\left[\left(\frac{\alpha}{r^K(\bar{s})}\right)^{\alpha}\left(\frac{1-\alpha}{w(\bar{s})}\right)^{(1-\alpha)}\right]^{\frac{\theta}{1-\theta}}Y(s)^{\theta}M(s)^{-\frac{\xi\theta^2}{\theta-1}}$$

By substituting this expression in the aggregate formula and evaluating the resulting expression, one can show that $Y(\bar{s})^{\theta}=\Xi(\bar{s})y(\bar{z},\bar{s})^{\theta}M(\bar{s})^{\xi\theta}$ which implies

$$1=\Xi(\bar{s})\theta^{\frac{\theta}{1-\theta}}\bar{z}(\sigma)^{\frac{\nu\theta}{1-\theta}}\left[\left(\frac{r^K(\bar{s})}{\alpha}\right)^{-\alpha}\left(\frac{w(\bar{s})}{1-\alpha}\right)^{-(1-\alpha)}\right]^{\frac{\theta}{1-\theta}}M(\bar{s})^{-\frac{\xi\theta}{\theta-1}}$$

This shows that the equilibrium wage is a function of $\Xi(\bar{s})$ and $M(\bar{s})$ which, in turn, are functions of the threshold values. After solving this expression for the equilibrium wage and substituting it in (24) provides us with a nonlinear equation for an unknown $\mu_0(\bar{s})$. A numerical equation solver can be used to determine this value.

To complete the parameterization of the model, I need to solve for the ratio of fixed cost relative to average output $\gamma_F/Y(\bar{s})$. Note that the euler equation for the capital planner can be transformed into in the steady state,

$$q^M(s)=\int\Lambda(s,s')\left[\begin{array}{c}\pi(s')[\Phi(\mu_0(s')-\sigma')-\Phi(\mu_1(s')-\sigma')]+[1-\Phi(\mu_0(s'))]\gamma_S\\+[\Phi(\mu_0(s'))-\Phi(\mu_1(s'))]\left[\Xi(s)\pi_M(s')-\gamma_F+q^M(s')\right]\end{array}\right]Q(\sigma,d\sigma').$$

where $\pi(s')=\pi(\bar{z},s')$. In the steady state, we can solve for the steady state value of marginal firm as follows

$$\begin{aligned}\frac{q^M(\bar{s})}{Y(\bar{s})} &= \beta\left[\frac{\pi(\bar{s})}{Y(\bar{s})}[\Phi(\mu_0(\bar{s})-\bar{\sigma})-\Phi(\mu_1(\bar{s})-\bar{\sigma})]\right. \\ &\quad\left.+[1-\Phi(\mu_0(\bar{s}))]\frac{\gamma_S}{Y(\bar{s})}-[\Phi(\mu_0(\bar{s}))-\Phi(\mu_1(\bar{s}))]\frac{\pi(z_1(\bar{s}),\bar{s})}{Y(\bar{s})}\right]\end{aligned} \tag{25}$$

where $q^M(\bar{s})$ is normalized by aggregate output because all terms on the right side are proportional to the average output. Substituting the calibrated sunk cost-to-output ratio $\gamma_S/Y(\bar{s})$, we can evaluate the ratio $q^M(\bar{s})/Y(\bar{s})$ as a function of $\mu_0(\bar{s})$. The first order condition for exit can be used to determine fixed cost-to output ratio,

$$\frac{\gamma_F}{Y(\bar{s})}=\pi(z_1(\bar{s}),\bar{s})-\frac{\xi\theta}{\theta-1}\frac{\Xi(\bar{s})}{M(\bar{s})}\frac{\pi(\bar{s})}{Y(\bar{s})}+\frac{q^M(\bar{s})}{Y(\bar{s})} \tag{26}$$

Because the set of threshold shocks completely pin down the values of $\Xi(\bar{s})$, $M(\bar{s})$, $\pi(\bar{s})/Y(\bar{s})$ and $\pi(z_1(\bar{s}),\bar{s})/Y(\bar{s})$, we can determine the value $\gamma_F/Y(\bar{s})$.

Since all factor prices are determined, we can solve for factor input/output ratios which are functions of equilibrium input prices. Combining (**??**) and (12), we can evaluate $\frac{K(\bar{s})}{Y(\bar{s})}$. Also using the cost minimization condition, $\frac{h(z,\bar{s})}{k^D(z,\bar{s})}=\left(\frac{1-\alpha}{\alpha}\right)\frac{r_K(\bar{s})}{w(\bar{s})}$, one can also that $H(\bar{s})=\left(\frac{1-\alpha}{\alpha}\right)\frac{r^K(\bar{s})}{w(\bar{s})}k^D(\bar{z},\bar{s})\Xi(\bar{s})$ which provides an expression for $\frac{H(\bar{s})}{Y(\bar{s})}$. Transforming the aggregate output relationship, we can solve for the dividend/output ratio as

$$\frac{D(\bar{s})}{Y(\bar{s})} = 1 - w(\bar{s})\frac{H(\bar{s})}{Y(\bar{s})} - r^K(\bar{s})\frac{K(\bar{s})}{Y(\bar{s})}$$

Finally, the consumption/output is determined as

$$\frac{C(\bar{s})}{Y(\bar{s})} = 1 - \frac{\Gamma_S(\bar{s})}{Y(\bar{s})} - \frac{\Gamma(\bar{s})}{Y(\bar{s})} - \delta\frac{K(\bar{s})}{Y(\bar{s})}$$

Finally, from the asset pricing equation for the venture capital fund, we get $\frac{D(\bar{s})}{W(\bar{s})} = \frac{1-\beta}{\beta}$. This completes the set of coefficients for linear equations. Therefore, we can see that the calibration of two parameter, the sunk cost-to-output ratio and the long run exit threshold value determines all the other equilibrium quantities and prices.

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