

Supplemental Appendix

Corporate Debt Maturity and Business Cycle Fluctuations

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A Equilibrium equations

This section reports the equilibrium equations of our baseline model.

- Workers

$$1 = \mathbb{E}_t \Lambda_{t+1} \frac{R_t^d}{\pi_{t+1}} \quad (1)$$

$$1 = \mathbb{E}_t \Lambda_{t+1} \frac{R_t}{\pi_{t+1}} \quad (2)$$

where $\Lambda_{t+1} = \beta \frac{U_c(t+1)}{U_c(t)}$ is the stochastic discount factor with the marginal utility of consumption given by

$$U_c(t) = \frac{\xi_t^c}{C_t - hC_{t-1}} - \beta h \mathbb{E}_t \frac{\xi_{t+1}^c}{C_{t+1} - hC_t}.$$

- Entrepreneurs

$$1 = \Psi'(\omega_t) \gamma_t \quad (3)$$

$$\begin{aligned} & \left(Q_t^l + \frac{dQ_t^l}{d\eta_t} \eta_t \right) \gamma_t - \frac{(1 - \lambda_l) \eta_{t-1}}{\pi_t} \frac{dQ_t^l}{d\eta_t} \frac{Q_t^k - Q_t^l \eta_t}{\Psi(\omega_t) \mu_t} \varphi_t \\ &= \mathbb{E}_t \Lambda_{t,t+1} \frac{c_l + (1 - \lambda_l) Q_{t+1}^l}{\pi_{t+1}} \varphi_{t+1} (1 - F_t(\bar{\xi}_{t+1}, \sigma_t)) \end{aligned} \quad (4)$$

$$X_t = \Psi(\omega_t) \left(\tilde{\xi}_t - \bar{\xi}_t \right) R_t^k Q_{t-1}^k K_{t-1} (1 - F(\bar{\xi}_t^l)) + T_t^e + \xi_t^n, \quad (5)$$

$$K_t = \frac{X_t}{Q_t^k - \eta_t Q_t^l}. \quad (6)$$

where

$$R_t^k = \frac{r_t^k + (1 - \delta)Q_t^k}{Q_{t-1}^k}, \quad (7)$$

$$R_t^l = \frac{1}{\pi_t} \cdot \frac{c_l + (1 - \lambda_l) Q_t^l}{Q_{t-1}^l}. \quad (8)$$

$$\Psi(\omega_t) = (1 - \bar{\psi})\omega_t - \frac{\psi}{2}(\omega_t - \bar{\omega})^2. \quad (9)$$

$$\gamma_t = \frac{1}{Q_t^k - Q_t^l \eta_t} \mathbb{E}_t \Lambda_{t,t+1} \varphi_{t+1} \int_{\bar{\xi}_{t+1}} (\xi_{t+1} - \bar{\xi}_{t+1}) R_{t+1}^k Q_t^k dF_t(\xi_{t+1}, \sigma_t) \quad (10)$$

$$\varphi_t = \{(1 - \omega_t) + \Psi(\omega_t) \gamma_t\} \quad (11)$$

$$\bar{\xi}_{t+1} = \frac{R_{t+1}^l Q_t^l}{R_{t+1}^k Q_t^k} \eta_t \quad (12)$$

$$\mu_t = \tilde{\xi}_t R_t^k Q_{t-1}^k - \frac{c_l + (1 - \lambda_l) Q_t^l}{\pi_t} \eta_{t-1}, \quad (13)$$

$$\tilde{\xi}_t = \frac{1}{1 - F_{t-1}(\bar{\xi}_t)} \int_{\bar{\xi}_t} \xi_t dF_{t-1}(\xi_t) \quad (14)$$

$$F_{t-1}(\bar{\xi}_t) = \Phi\left(\frac{\log(\bar{\xi}_t) + .5(\sigma_{t-1})^2}{\sigma_{t-1}^e}\right) \quad (15)$$

$$\int_{\bar{\xi}_t} \xi_t dF_{t-1}(\xi_t) = 1 - \Phi\left(\frac{\log(\bar{\xi}_t) - .5(\sigma_{t-1})^2}{\sigma_{t-1}}\right) \quad (16)$$

and $\Phi(\cdot)$ is the cdf of a standard Normal distribution.

- Financial intermediary

$$Q_t^l = \mathbb{E}_t \Lambda_{t+1} \kappa_t \left\{ [1 - F_t(\bar{\xi}_{t+1}, \sigma_t)] \frac{c_l + (1 - \lambda_l) Q_{t+1}^l}{\pi_{t+1}} + \gamma_l \frac{Q_t^k R_{t+1}^k}{\eta_t} \int_0^{\bar{\xi}_{t+1}} \xi_{t+1} dF_t(\xi_{t+1}, \sigma_t) \right\} \quad (17)$$

$$\begin{aligned} \frac{\partial Q_t^l}{\partial \eta_t} &= -\mathbb{E}_t \Lambda_{t+1} \kappa_t \left\{ \frac{\partial \bar{\xi}_{t+1}}{\partial \eta_t} f_t(\bar{\xi}_{t+1}) \left[Q_t^l R_{t+1}^l - \gamma_l \frac{Q_t^k R_{t+1}^k}{\eta_t} \bar{\xi}_{t+1} \right] + \gamma_l \frac{Q_t^k R_{t+1}^k}{(\eta_t)^2} \int_0^{\bar{\xi}_{t+1}} \xi_{t+1} dF_t(\xi_{t+1}) \right. \\ &\quad \left. + \mathbb{E}_t \Lambda_{t+1} \kappa_t [1 - F_t(\bar{\xi}_{t+1})] (1 - \lambda_l) \frac{\partial Q_{t+1}^l}{\partial \eta_{t+1}} \eta_{t,t+1}(\eta_t) \right\} \end{aligned} \quad (18)$$

- Intermediate goods producers

$$Y_t^m = A_t (u_t K_{t-1})^\chi (H_t)^{1-\chi}, \quad (19)$$

$$w_t = p_t^m (1 - \chi) \frac{Y_t}{H_t} \quad (20)$$

$$r_t^k = p_t^m \chi \frac{Y_t}{K_{t-1}} \quad (21)$$

$$\frac{r_t^k}{u_t} = a'(u_t). \quad (22)$$

$$a(u_t) = \bar{\gamma}_u (e^{\gamma_u (u_t - 1)} - 1) \quad (23)$$

- Retailers

$$\left(\frac{\pi_t}{\tilde{\pi}_t} - 1 \right) \frac{\pi_t}{\tilde{\pi}_t} = \left[P_t^m \frac{\varepsilon_t}{(\varepsilon_t - 1)} - 1 \right] \gamma_p + \beta \mathbb{E}_t \Lambda_{t,t+1} \left(\frac{\pi_{t+1}}{\tilde{\pi}_{t+1}} - 1 \right) \frac{\pi_{t+1}}{\tilde{\pi}_{t+1}} \frac{Y_{t+1}}{Y_t} \quad (24)$$

- Labor unions

$$\frac{\pi_t^w}{\tilde{\pi}_t^w} \left(\frac{\pi_t^w}{\tilde{\pi}_t^w} - 1 \right) = \left(\frac{\bar{\varphi} H_t^\varphi}{U_{ct}} \frac{\varepsilon_t^w}{\varepsilon_t^w - 1} - W_t \right) \gamma_w + \beta \mathbb{E}_t \Lambda_{t,t+1} \frac{\pi_{t+1}^w}{\tilde{\pi}_{t+1}^w} \left(\frac{\pi_{t+1}^w}{\tilde{\pi}_{t+1}^w} - 1 \right) \frac{H_{t+1}}{H_t}, \quad (25)$$

- Capital goods producers

$$Q_t^k = \mu_t^k \left(1 + \frac{\gamma_k}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 + \gamma_k \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right) - \beta \mathbb{E}_t \Lambda_{t+1} \mu_{t+1}^k \gamma_k \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \quad (26)$$

$$K_t = (1 - \delta) K_{t-1} + I_t. \quad (27)$$

- Monetary policy

$$\log(R_t) = (1 - \rho_r) \log(R_{SS}) + \rho_r \log(R_{i,t-1}) + (1 - \rho_r) \left(\kappa_\pi \log\left(\frac{\pi_t}{\bar{\pi}_t}\right) + \kappa_y \log\left(\frac{Y_t}{Y_{t-1}}\right) \right) + \xi_t^m. \quad (28)$$

- Goods market clearing

$$Y_t - \nu \left[K_{t-1} (1 - \gamma_l) \int_0^{\bar{\xi}_t} \xi_t dF_t(\xi_t) \right] = C_t + I_t + G_t + a(u_t) K_{t-1}, \quad (29)$$

B Model without equity adjustment costs

In this section, we show that in a model with defaultable long-term debt and costless equity issuance the equilibrium is not well-defined. By setting $\psi = 0$, we can rewrite the value function of the entrepreneur as

$$\begin{aligned} v_t(\eta_{t-1}, \xi_t) &= \max_{\eta_t, \omega_t} \left\{ \mu_t(\eta_t, \eta_{t-1}, \xi_t) (1 - \omega_t) + \mu_t(\eta_t, \eta_{t-1}, \xi_t) \frac{\omega_t (1 - \bar{\psi})}{Q_t^k - Q_t^l \eta_t} \mathbb{E}_t \Lambda_{t,t+1} \max\{0, v_t(\eta_t, \xi_{t+1})\} \right\} \\ &= \mu_t(\eta_t, \eta_{t-1}, \xi_t) + \omega_t g_t(\eta_t) \end{aligned}$$

where

$$\mu_t(\eta_t, \eta_{t-1}, \xi_t) = \frac{n_t}{k_{t-1}} = \left[\xi_t R_t^k q_{t-1}^k - \frac{(c_l + (1 - \lambda_l) Q_t^l)}{\pi_t} \eta_{t-1} \right] \quad (30)$$

$$g_t(\eta_t) = \mu_t(\eta_t, \eta_{t-1}, \xi_t) \left(-1 + \frac{(1 - \bar{\psi}) \mathbb{E}_t \Lambda_{t,t+1} \max\{0, v_t(\eta_t, \xi_{t+1})\}}{Q_t^k - Q_t^l \eta_t} \right) \quad (31)$$

where $g_t(\eta_t)$ represents the net return on an extra unity of equity inside the firm.

The first order conditions for equity and leverage, ω_t^* and η_t^* , are

$$g_t(\eta_t^*) = 0 \quad (32)$$

$$\frac{d\mu_t}{d\eta_t^*} + \omega_t^* \frac{dg_t(\eta_t^*)}{d\eta_t^*} = 0 \quad (33)$$

where

$$\frac{d\mu_t}{d\eta_t^*} = -(1 - \lambda_l) \frac{dQ_t^l}{d\eta_t} \frac{\eta_{t-1}}{\pi_t} > 0 \quad (34)$$

because $\frac{dQ_t^l}{d\eta_t} < 0$. However, these first order conditions necessarily select a saddle point. In fact, equations (33) and (34) imply that $\frac{dg_t(\eta_t^*)}{d\eta_t^*} < 0$. If this is the case, then we can find another ε such that $\hat{\eta}_t = \eta_t^* - \varepsilon$ and $g_t(\hat{\eta}_t) > 0$, and set ω_t to infinity to yield an infinite value for the entrepreneur. The entrepreneur could reduce leverage to increase the excess return on equity and then freely adjust equity to infinity to obtain an infinite return. Hence, without some increasing marginal costs of adjusting equity, the entrepreneur's problem does not have an interior optimum and consequently the macroeconomic model does not have an equilibrium. Quadratic costs of equity adjustment alter the first order conditions by making the return on equity depend on ω_t , that is $g_t(\eta_t, \omega_t)$, preventing the entrepreneur from pursuing the strategy described above.

On the other hand, if debt is short term, that is if $\lambda = 1$, an interior optimum would be possible even with $\psi = 0$. In fact, using $\frac{d\mu_t}{d\eta_t^*} = 0$, the first order conditions would become

$$g_t(\eta_t^{*,ST}) = 0 \quad (35)$$

$$\frac{dg_t(\eta_t^{*,ST})}{d\eta_t^{*,ST}} = 0. \quad (36)$$

In this case, equation (36) implies that the optimal leverage is chosen in order to maximize the expected return on equity, preventing any profitable deviation from the interior optimum.

This analysis suggests that in a model with defaultable long-term debt some form of equity adjustment cost is a necessary condition for the existence of an equilibrium. We can think of the model with $\psi = 0$ from Section 3 as a limit case with arbitrarily small equity adjustment costs.

An additional result for the model with $\psi = 0$ is that the derivative of the entrepreneurial leverage policy function is zero, that is $\eta_\eta = 0$. This can be seen by rewriting equation (32) as

$$\frac{1}{(1 - \bar{\psi})} = \frac{\mathbb{E}_t \Lambda_{t,t+1} \max \{0, v_t(\eta_t, \xi_{t+1})\}}{Q_t^k - Q_t^l \eta_t} \quad (37)$$

which is equivalent to equation (36) in the paper. In this case, equation (37) pins down η_t as a function of only the aggregate price of capital and the household stochastic discount factor. Leverage η_t will not depend on past leverage η_{t-1} or ω_t and hence $\eta_\eta = 0$. Equation (33) will instead determine ω_t .

C Algorithm to compute derivative of leverage

In our baseline model, the entrepreneur's individual choice of equity (ω_t^i) and leverage (η_t^i) is characterized by the following first order conditions

$$1 = \Psi'(\omega_t^i) \gamma_t^i \quad (38)$$

$$\begin{aligned}
& \left(Q_t^{l,i} + \frac{dQ_t^{l,i}}{d\eta_t^i} \eta_t^i \right) \gamma_t^i - \frac{(1-\lambda_l)\eta_{t-1}^i}{\pi_t} \frac{dQ_t^{l,i}}{d\eta_t^i} \frac{Q_t^k - Q_t^{l,i} \eta_t^i}{\Psi(\omega_t^i) \mu_t^i} \varphi_t^i \\
& = \\
& \mathbb{E}_t \Lambda_{t,t+1} \frac{c_l + (1-\lambda_l) Q_{t+1}^{l,i}}{\pi_{t+1}} \varphi_{t+1}^i (1 - F_t(\bar{\xi}_{t+1}^i, \sigma_t))
\end{aligned} \tag{39}$$

where

$$Q_t^{l,i} = \mathbb{E}_t \Lambda_{t+1} \kappa_t \left\{ [1 - F_t(\bar{\xi}_{t+1}^i, \sigma_t)] \frac{c_l + (1-\lambda_l) Q_{t+1}^{l,i}}{\pi_{t+1}} + \gamma_l \frac{Q_t^k R_{t+1}^k}{\eta_t^i} \int_0^{\bar{\xi}_{t+1}^i} \xi_{t+1} dF_t(\xi_{t+1}, \sigma_t) \right\} \tag{40}$$

$$\begin{aligned}
\frac{\partial Q_t^{l,i}}{\partial \eta_t^i} &= -\mathbb{E}_t \Lambda_{t+1} \kappa_t \left\{ \frac{\partial \bar{\xi}_{t+1}^i}{\partial \eta_t^i} f_t(\bar{\xi}_{t+1}^i) \left[Q_t^{l,i} R_{t+1}^{l,i} - \gamma_l \frac{Q_t^k R_{t+1}^k}{\eta_t^i} \bar{\xi}_{t+1}^i \right] + \gamma_l \frac{Q_t^k R_{t+1}^k}{(\eta_t^i)^2} \int_0^{\bar{\xi}_{t+1}^i} \xi_{t+1} dF_t(\xi_{t+1}) \right\} \\
&+ \mathbb{E}_t \Lambda_{t+1} \kappa_t [1 - F_t(\bar{\xi}_{t+1}^i)] (1 - \lambda_l) \frac{\partial Q_{t+1}^{l,i}}{\partial \eta_{t+1}^i} \eta_{t+1}^i (\eta_t^i)
\end{aligned} \tag{41}$$

$$\gamma_t^i = \frac{1}{Q_t^k - Q_t^{l,i} \eta_t^i} \mathbb{E}_t \Lambda_{t,t+1} \varphi_{t+1}^i \int_{\bar{\xi}_{t+1}^i} (\xi_{t+1} - \bar{\xi}_{t+1}^i) R_{t+1}^k Q_t^k dF_t(\xi_{t+1}, \sigma_t) \tag{42}$$

$$\varphi_t^i = \{ (1 - \omega_t^i) + \Psi(\omega_t^i) \gamma_t^i \} \tag{43}$$

$$\bar{\xi}_{t+1}^i = \frac{(c_l + (1-\lambda_l) Q_{t+1}^{l,i})}{\pi_{t+1}} \frac{\eta_t^i}{R_{t+1}^k Q_t^k} \tag{44}$$

$$\mu_t^i = \tilde{\xi}_t R_t^k Q_{t-1}^k - \frac{c_l + (1-\lambda_l) Q_t^{l,i}}{\pi_t} \eta_{t-1}^i, \tag{45}$$

These equations determine a policy function for *individual* leverage which we denote as $\tilde{\eta}^i(\eta_{t-1}^i, S_t, \sigma^\epsilon)$, where η_{t-1}^i is the *individual* leverage at time $t-1$, S_t is the aggregate state of the economy, which also includes the *aggregate* level of leverage at time $t-1$, and σ^ϵ is a vector collecting the standard deviations of all exogenous shocks. Note that this policy function is different from the policy function for *aggregate* leverage.

Our algorithm to approximate $\frac{\partial \tilde{\eta}^i(\eta_{t-1}^i, S_t, \sigma^\epsilon)}{\partial \eta_{t-1}^i}$ works in two steps. In the

first step, we compute $\tilde{\eta}^i(\eta_{t-1}^i, S^{ss}, 0)$ and its derivative $\frac{\partial \tilde{\eta}^i(\eta_{t-1}^i, S^{ss}, 0)}{\partial \eta_{t-1}^i}$ in the steady state. This is done with the following iterative procedure:

- Given an initial guess for S^{ss} and for associated prices and quantities in steady state, and a guess for $\eta_{\eta,ss}$, we use global methods to solve for $\tilde{\eta}^i(\eta_{t-1}^i, S^{ss}, 0)$ over a grid on entrepreneurial leverage η_{t-1}^i .
- Given the policy function $\tilde{\eta}^i(\eta_{t-1}^i, S_{ss}, 0)$, we can compute $\eta_{\eta,ss} = \frac{\partial \tilde{\eta}^i(\eta_{t-1}^i, S_{ss}, 0)}{\partial \eta^i}$ and update $\eta_{\eta,ss}$ and S_{ss} .
- These steps are repeated until convergence.

In the second step of our algorithm we approximate the derivative function denoted by

$$\tilde{g}(\eta_{t-1}^i, S_t, \sigma^\epsilon) = \frac{\partial \tilde{\eta}^i(\eta_{t-1}^i, S_t, \sigma^\epsilon)}{\partial \eta_{t-1}^i}$$

linearly around the steady state. To do so we write the policy function for leverage in an economy without aggregate risk, as an explicit function of previous leverage, aggregate prices at time t , expected prices at $t+1$, and all other aggregate quantities that the entrepreneur takes as given in the system (38)–(45). That is, letting $\Gamma_t = \{\eta_{t-1}^i, \tilde{\xi}_t, \sigma_t, \kappa_t, \mathbb{E}_t R_{t+1}^k, R_t^k, Q_t^k, Q_{t-1}^k, \mathbb{E}_t \Lambda_{t+1}, \mathbb{E}_t \pi_{t+1}, \pi_t\}$ we denote by $\eta(\Gamma_t)$ the policy function that solves the optimality conditions above without aggregate uncertainty and for a level of previous leverage, prices at time t , expected prices at $t+1$, and all other aggregate quantities given by Γ_t . We let $g(\Gamma_t) = \frac{\partial \eta}{\partial \eta_{t-1}^i}$ and our approximation be given by

$$\hat{g}(\Gamma_t) = g(\Gamma^{ss}) + \nabla g d\Gamma_t$$

where the gradient of g is computed numerically by solving the policy functions for η globally for small deviations of each argument in Γ_t from steady state.

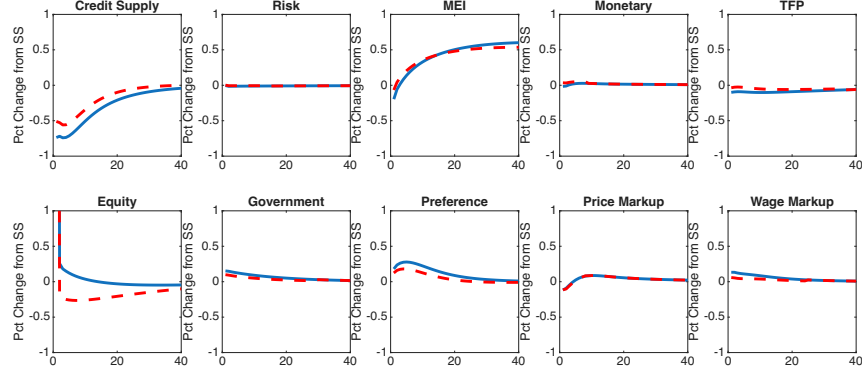


Figure C.1: Accuracy of derivative algorithm

Note: Figure reports the impulse responses of the derivative of the entrepreneurial leverage policy function computed either with our linear approximation method, that is $\hat{g}(\Gamma_t)$ (blue solid line), or computed non-linearly, that is $g(\Gamma_t)$ (red dashed line). In each panel we consider a one standard deviation of the shock, as obtained in our estimated baseline model.

C.1 Accuracy of the algorithm

For sufficiently small shocks, the linear approximation provides an accurate characterization of the behavior of the function $g(\cdot)$. As an additional validation of our computational approach, Figure (C.1) compares the impulse response implied by the linearized policy function, $\hat{g}(\Gamma_t)$ (solid blue line), with the response obtained from the derivative of the policy function computed using global solution methods, $g(\Gamma_t)$ (red dashed line). In both cases, the responses are evaluated along the same linear path for Γ_t following an unanticipated shock. The path of $g(\Gamma_t)$ captures the nonlinear relationship between entrepreneurial leverage and aggregate variables. As shown in Figure (C.1), the two solution methods deliver very similar paths for $g(\cdot)$, providing support for our linear algorithm.

D Estimation of equity adjustment costs

This appendix provides more detail on the data treatment, estimation, and robustness of the results presented in Section IV.B.

We perform the following cleaning steps on our quarterly Compustat sample (S&P Global Market Intelligence, 2025). We remove firms in the finance, government, and utility sectors. We remove non-U.S. firms. Observations with negative total assets are dropped. We transform cumulative variables `sstky|`, `dvy`, and `prstkcy` into quarterly variables.

Our dependent variable is $R_{i,t+1}^x = \frac{V_{i,t+1}}{X_{i,t}}$, where $V_{i,t+1}$ is the $t + 1$ cum-dividend market value of equity. It is constructed using Compustat variables `cshoq` × `prccq` + `netpayouts`, where `netpayouts` = `dvyq` - (`sstkyq` - `prstkcyq`). We remove negative observations of $V_{i,t+1}$. The denominator $X_{i,t}$ is the cum-dividend book value of equity, defined as `ceqq` + `netpayouts`. Again, we remove negative observations.

Our key independent variable is $\omega_{it} = 1 - \frac{\text{netpayouts}}{X_{i,t}}$, i.e., one minus the fraction of net payouts to shareholders over the cum-dividend book value of equity. We remove observations of ω_{it} that include negative values of $X_{i,t}$.

Finally, we trim the independent variable at the 5th and 95th percentile. The dependent variable is truncated at zero from below, we therefore only trim it at the 95th percentile.¹

The regression we run is

$$\frac{R_{i,t+1}^x}{1 + r_{t+1}} = \psi\omega_{i,t} + \delta_i + \delta_t + \nu_{it+1}, \quad (46)$$

where r_{t+1} is the quarterly realized real rate and δ_i and δ_t are firm- and time fixed effects.

Our results are shown in Table D.1. The first column shows the regression coefficient of the model estimated in (46). Columns (2) and (3) show the effect of removing the firm or time fixed effects.

¹Winsorizing instead of trimming the data leads to similar results.

Table D.1: Estimating the equity adjustment cost parameter

	(1)	(2)	(3)	(4)	(5)	(6)
ω	10.85 (0.13)	15.43 (0.15)	11.77 (0.13)	10.74 (0.13)	15.26 (0.15)	11.65 (0.13)
Firm fixed effects	yes	no	yes	yes	no	yes
Time fixed effects	yes	yes	no	yes	yes	no
Additional cleaning	no	no	no	yes	yes	yes
N	513,257	513,785	513,257	495,070	495,518	495,070

Robustness As a robustness exercise, we perform additional cleaning steps. We only keep firms with at least five quarters of information in the sample. Further, we remove firm-quarters that violate the accounting identity by more than 10 per cent of the book value of assets (as in [Covas and Den Haan \(2012\)](#)). Lastly, we drop firms involved in major mergers (Compustat footnote `saleq_fn1` code “AB”). As indicated in columns (4)–(6) of Table [D.1](#), these additional cleaning steps hardly affect our results.

E Additional model experiments

This appendix provides additional results for our model. Figures [E.1](#) and [E.2](#) show the impulse responses to a debt shock in the baseline model without equity adjustment costs and with infinite equity adjustment costs, respectively, and with no real default costs, i.e. $\nu = 0$. Figure [E.3](#) presents the impulse response to a credit supply shock in a model with a different calibration of the Taylor rule and of the slope of the Phillips curve. Figure [E.4](#) shows the impulse responses to a net worth shock in the baseline model. Figure [E.5](#) compares the impulse responses in the baseline model with those in a model with $\psi = \infty$. Table [E.1](#) shows the variance decomposition for that case. Finally, Figure [E.6](#) compares the impulse responses when debt is short- or long-term in the case of $\psi = \infty$.

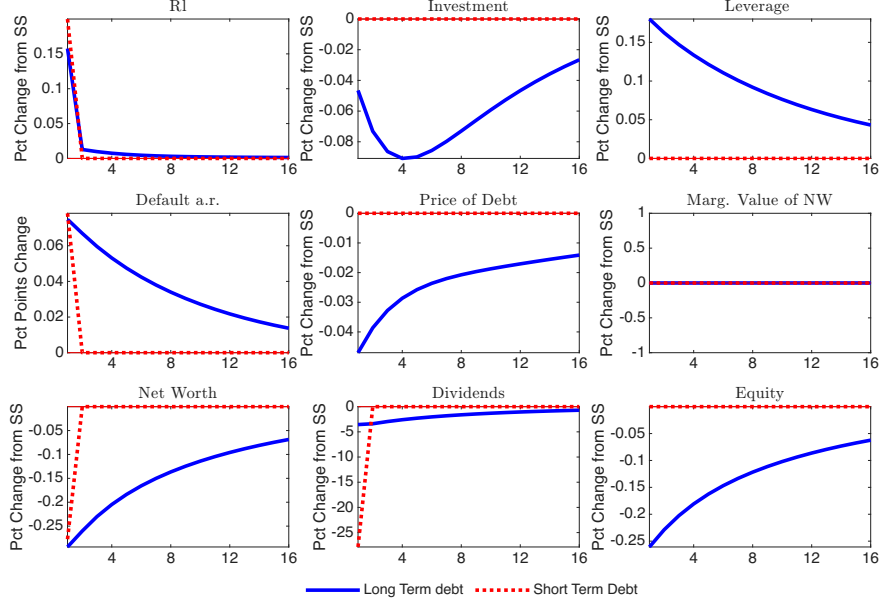


Figure E.1: Debt shock in model with sticky prices: no equity adjustment costs

Note: Figure shows the impulse response to a one-time shock to the value of outstanding debt in a version of the baseline model with no real default costs, i.e. $\nu = 0$.

Table E.1: Variance Decomposition: Model with $\psi = \infty$

Model with long-term debt and $\psi = \infty$								
	Credit Supply	Risk	M.E.I.	NW	Markup	Gov	M.P.	TFP
GDP	47	8	5	6	16	7	3	3
Investment	54	9	13	7	11	0	2	1
Consumption	8	7	24	2	2	1	0	4
Spread	9	68	13	8	0	0	0	0
Equity	18	2	12	64	1	0	1	0
Credit	17	10	37	31	1	1	1	0
Inflation	37	13	8	2	18	1	4	5
FFR	45	19	21	4	1	1	1	1
Hours	28	11	33	2	14	1	1	1
Wages	2	2	4	1	10	0	0	3

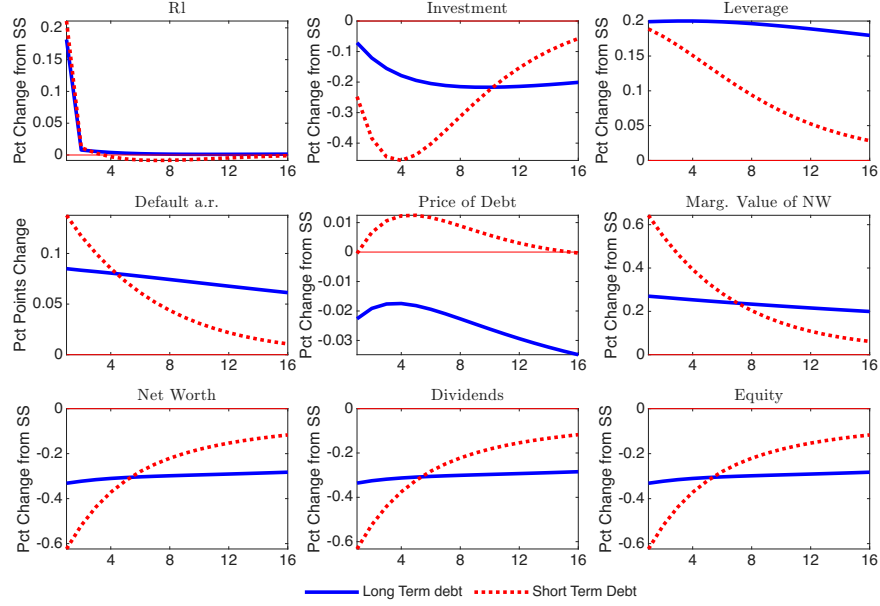


Figure E.2: Debt shock in model with sticky prices: infinite equity adjustment costs

Note: Figure shows the impulse response to a one-time shock to the value of outstanding debt in a version of the baseline model with infinite equity adjustment costs no real default costs, i.e. $\nu = 0$.

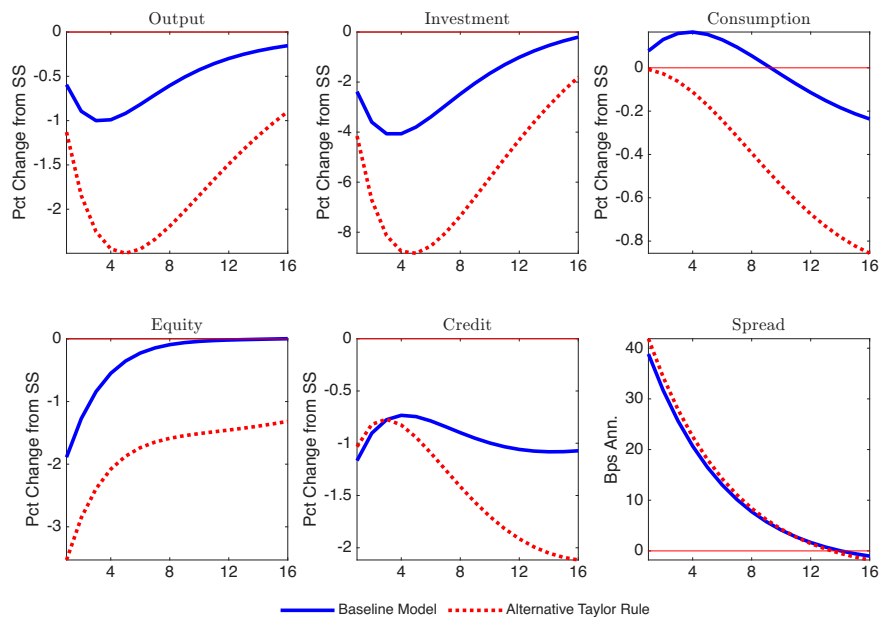


Figure E.3: Credit supply shock in model with alternative calibration

Note: Figure shows the impulse response to a credit supply shock in the baseline model (blue line) and in a version of the model with a lower Phillips curve slope of 2.5 percent and a less aggressive Taylor rule, with a coefficient of 1.5 on inflation and zero on output (red dotted line). The shock is scaled to deliver a 1 percent decline in output in the baseline model.

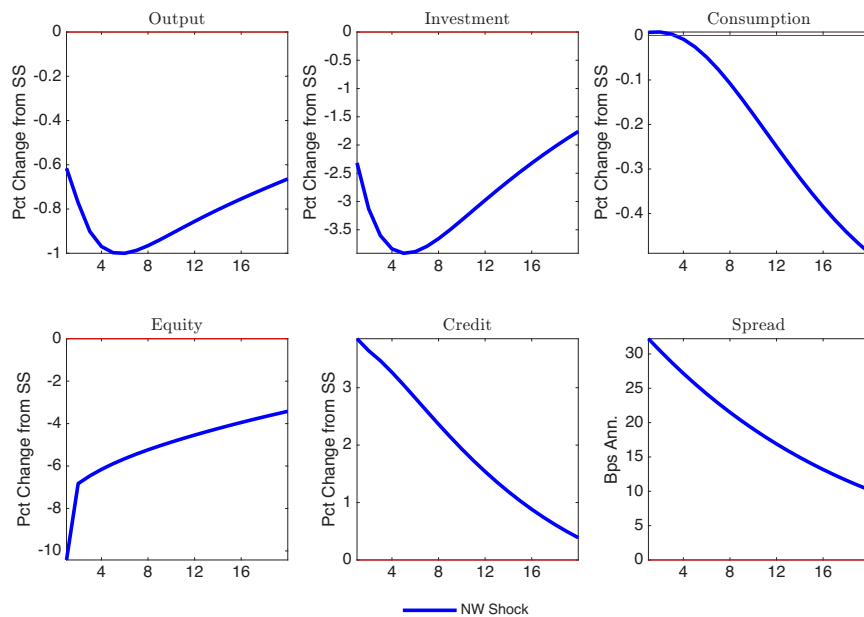


Figure E.4: Net worth shock in baseline model

Note: Figure shows the impulse response to shock to the entrepreneurial net worth in our baseline model

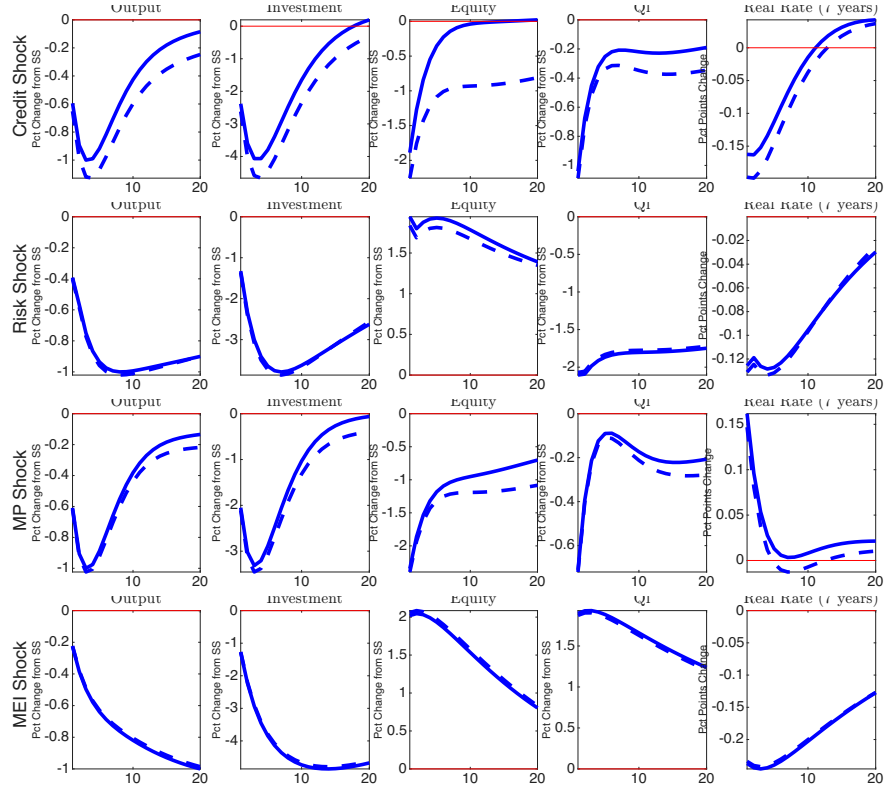


Figure E.5: Impulse responses in baseline model vs model with $\psi = \infty$

Note: Figure compares the impulse responses in our baseline model (blue solid line) to a model with the same parameters but with $\psi = \infty$ (dark blue dashed line).

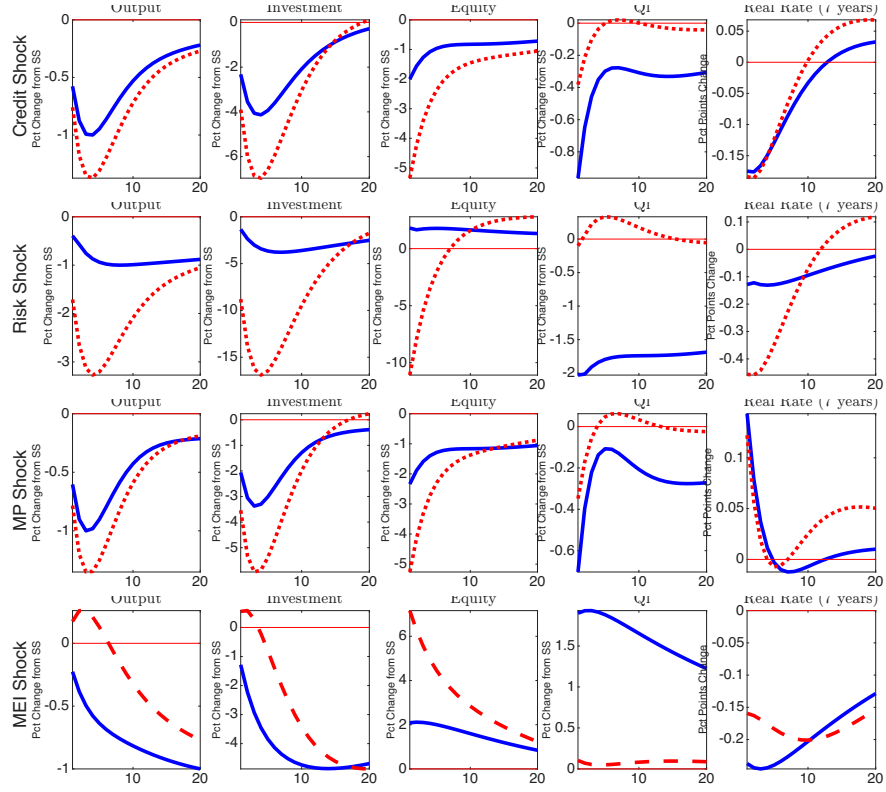


Figure E.6: Long-term debt vs short-term debt in baseline model with $\psi = \infty$
Note: Figure compares the impulse responses in our baseline model (blue solid line) and in a model with short-term debt (red dashed line), in both cases assuming that $\psi = \infty$.

F Financially constrained intermediaries

This section presents additional details on our extension of the baseline model discussed in Section V. We follow Gertler and Karadi (2011) and assume that, at each point in time, a fraction b of household members are bankers managing a financial intermediary. At the beginning of each period, a fraction $(1 - \sigma_b)$ of bankers returns to the family and is replaced by an equal number of new bankers, so that the relative proportion of bankers and non-bankers within the family remains constant. New bankers enter with some startup funds from the family, as we discuss below.

A financial intermediary uses its own net worth, N_t^b , and deposits to invest in loans:

$$Q_t^l L_t = N_t^b + D_t \quad (47)$$

A banker's net worth is equal to the difference between the return on assets and the return on liabilities:

$$N_t^b = R_{t+1}^b Q_t^l L_t - \frac{R_t^d}{\pi_{t+1}} D_t \quad (48)$$

The banker's objective is to maximize the payouts to the representative family upon exit, which can be written recursively as

$$V_t^b = E_t \Lambda_{t+1} [(1 - \sigma_b) N_{t+1}^b + \sigma_b V_{t+1}^b] \quad (49)$$

where Λ_{t+1} is the household stochastic discount factor between time t and $t+1$.

We assume that bankers can divert a portion θ^l of their loans and stop operating their financial subsidiary. As a result, depositors will limit the amount they lend to bankers to ensure that bankers do not have an incentive to divert funds:

$$V_t^b \geq \theta^l Q_t^l L_t. \quad (50)$$

The problem of the banker will be to choose L_t, D_t and N_t^b in order to optimize (49) subject to (47), (48) and (50). Given the linearity of the problem it can be shown that the value function satisfies $V_t^b = \psi_t N_t^b$ where ψ_t represents the bank's franchise value, that is the ratio between the marginal value of wealth inside the bank and the marginal value of wealth to the household. We will show that this variable does not depend on bank specific characteristic.

If we define bank leverage as $\phi_t^b = Q_t^l L_t / N_t^b$ we can then rewrite the bank problem as

$$\max_{\phi_t^b} E_t \Lambda_{t+1} [(1 - \sigma_b) + \sigma_b \psi_{t+1}] N_t^b \left[\phi_t^b \left(R_{t+1}^b - \frac{R_t^d}{\pi_{t+1}} \right) + \frac{R_t^d}{\pi_{t+1}} \right] \quad (51)$$

subject to

$$E_t \Lambda_{t+1} [(1 - \sigma_b) + \sigma_b \psi_{t+1}] \left[\phi_t^b \left(R_{t+1}^b - \frac{R_t^d}{\pi_{t+1}} \right) + \frac{R_t^d}{\pi_{t+1}} \right] \geq \theta^l \phi_t^b. \quad (52)$$

In our calibration the incentive constraint is binding and the optimality condition for leverage is given by the incentive constraint at equality:

$$E_t \Omega_{t+1} \left[\phi_t^b \left(R_{t+1}^b - \frac{R_t^d}{\pi_{t+1}} \right) + \frac{R_t^d}{\pi_{t+1}} \right] = \theta^l \phi_t^b, \quad (53)$$

where $\Omega_t = \Lambda_t [(1 - \sigma_b) + \sigma_b \psi_t]$. We can rewrite the optimality condition of leverage in (53) as:

$$E_t \Lambda_{t+1}^b R_{t+1}^b = 1, \quad (54)$$

where the banks stochastic discount factor, Λ_{t+1}^b , is:

$$\Lambda_{t+1}^b = \frac{\Omega_{t+1}}{\theta^l + E_t \Omega_{t+1} \frac{R_t^d}{\pi_{t+1}} \left(\frac{\phi_t^b - 1}{\phi_t^b} \right)} \quad (55)$$

Hence, we can replace the loan pricing equations in the model with

$$Q_t^l = \mathbb{E}_t \Lambda_{t+1}^b \left\{ [1 - F_t(\bar{\xi}_{t+1}, \sigma_t)] \frac{c_l + (1 - \lambda_l) Q_{t+1}^l}{\pi_{t+1}} + \gamma_l \frac{Q_t^k R_{t+1}^k}{\eta_t} \int_0^{\bar{\xi}_{t+1}} \xi_{t+1} dF_t(\xi_{t+1}, \sigma_t) \right\} \quad (56)$$

and

$$\begin{aligned} \frac{\partial Q_t^l}{\partial \eta_t} = & -\mathbb{E}_t \Lambda_{t+1}^b \left\{ \frac{\partial \bar{\xi}_{t+1}}{\partial \eta_t} f_t(\bar{\xi}_{t+1}) \left[Q_t^l R_{t+1}^l - \gamma_l \frac{Q_t^k R_{t+1}^k}{\eta_t} \bar{\xi}_{t+1} \right] + \gamma_l \frac{Q_t^k R_{t+1}^k}{(\eta_t)^2} \int_0^{\bar{\xi}_{t+1}} \xi_{t+1} dF_t(\xi_{t+1}) \right\} \\ & + \mathbb{E}_t \Lambda_{t+1}^b [1 - F_t(\bar{\xi}_{t+1})] (1 - \lambda_l) \frac{\partial Q_{t+1}^l}{\partial \eta_{t+1}} \eta_{t+1}(\eta_t) \end{aligned} \quad (57)$$

Finally, to close the model, we need to specify aggregate credit supply and the evolution of bankers net worth as:

$$\phi_t^b = \frac{Q_t^l L_t}{N_t^b}, \quad (58)$$

$$N_t^b = \sigma_b \left\{ R_t^{l,b} Q_{t-1}^l L_{t-1} - \frac{R_{t-1}^d}{\pi_t} D_t \right\} + T^b N_{t-1}^b \quad (59)$$

where T^b controls the transfer to new bankers.

The results in Section [V](#) are obtained by calibrating the extended model as follows. We set $\sigma_b = 0.97$ as in [Gertler and Karadi \(2011\)](#), and we calibrate the parameters $T^b = .0004$ and $\theta^l = .52$ to obtain a steady state bank leverage of 10 and a liquidity premium of 1 percent. These are standard values in the literature, see, for example [Gertler, Kiyotaki and Prestipino \(2020\)](#). All other parameters are the same as in the baseline model.

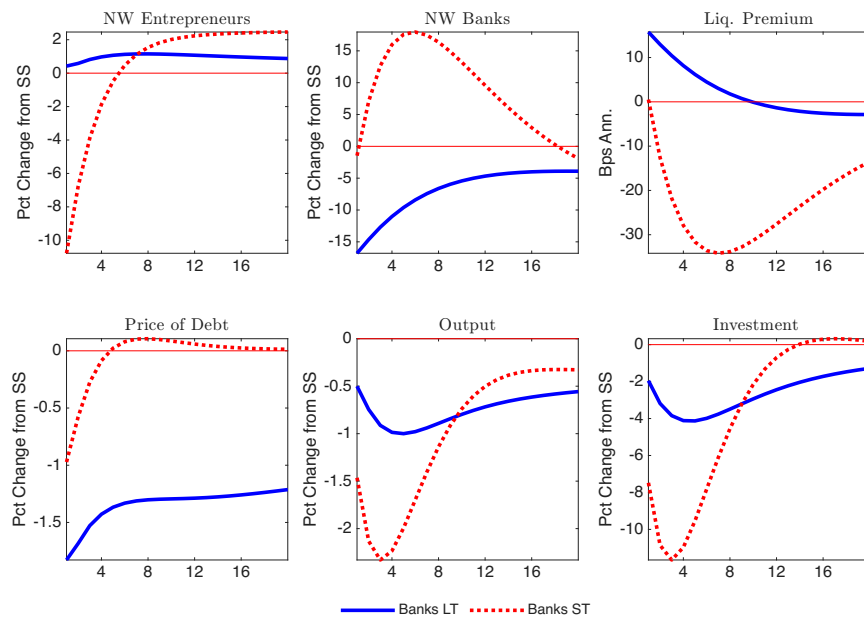


Figure F.7: Risk shock in model with constrained intermediaries

Note: Figure reports the impulse responses to a risk shock in a model extension with constrained financial intermediaries. The blue line represents a model with long term debt and the red dotted line represents a model with short term debt. The shock is calibrated to deliver a 1 percent decline in GDP in the model with long term debt.

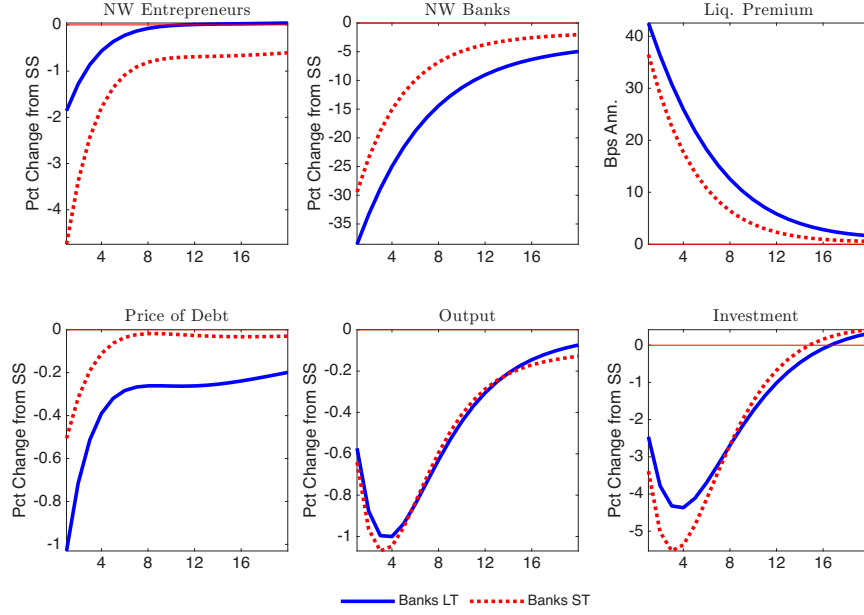


Figure F.8: Bank net worth shock in model with constrained intermediaries

Note: Figure reports the impulse responses to a bank net worth shock in a model extension with constrained financial intermediaries. The blue line represents a model with long term debt and the red dotted line represents a model with short term debt. The shock is calibrated to deliver a 1 percent decline in GDP in the model with long term debt.

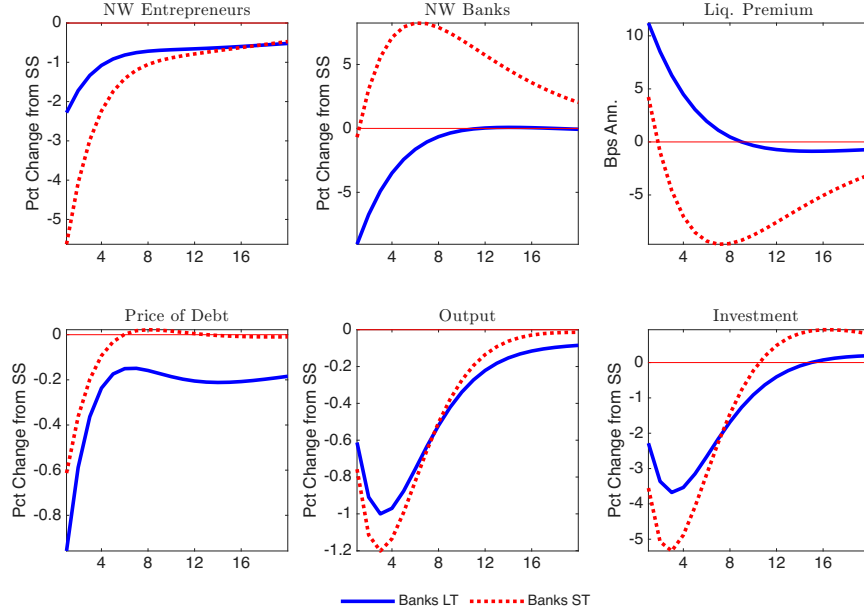


Figure F.9: Monetary shock in model with constrained intermediaries

Note: Figure reports the impulse responses to a monetary policy shock in a model extension with constrained financial intermediaries. The blue line represents a model with long term debt and the red dotted line represents a model with short term debt. The shock is calibrated to deliver a 1 percent decline in GDP in the model with long term debt.

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