Divisive by Design: Supplementary Appendix

Anja Prummer* Francesco Nava†

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*Freie Universität Berlin and Berlin School of Economics, anja.prummer@fu-berlin.de.
†London School of Economics, f.nava@lse.ac.uk.

S.A. Applications

Consider a celebrity auction. In these auctions, celebrities offer fans the opportunity to spend time with them for a specific activity.[1] The activity, which serves as a feature of the good, can either increase, decrease, or have no effect on the value of certain agents. To make this example more concrete, imagine a famous tennis player participating in such an auction. The tennis player could offer a casual tennis match, an ambitious training session, or, alternatively, a dinner with the auction winner. The type of good—whether a celebrity plus casual tennis, a celebrity plus intense training session, or a celebrity plus dinner—can significantly influence valuations and has the potential to help screen bidders' preferences. Some bidders may prefer an intense training session, others may simply want to meet their favorite tennis player and play a casual round, while some might favor the dinner to have the opportunity for more personal interaction. Our results suggest hat the activity chosen should be divisive – catering strongly to the preferences of specific bidders, thereby increasing surplus and facilitating better screening.

Similarly, our model applies to the art market. An artist creating an object with the goal of selling it should aim to incorporate features that increase surplus while minimizing information rents. The artist can select the art form (e.g., sculpture or painting), the materials used, and the size of the object. Each of these decisions has the potential to influence bidders' preferences and, in turn, their bids (Bocart, Gertsberg, and Pownall, 2022). For example, choosing the color blue in painting has been associated with higher revenues (Ma, Noussair, and Renneboog, 2022). Similarly, larger paintings generate more income, even after accounting for the increased material costs.[2] The artist's goal is then to create a piece that increases the value for buyers and, additionally, better screens their valuations. This helps explain why art often polarizes, with some customers valuing it substantially while others may feel alienated.

[1] For example, this can be booked through `https://www.charitybuzz.com/`.

[2] See, for instance, `https://www.gallerytoday.com/blog/122_what-determines-the-price-of-a-painting-does-size-matter.html`, which argues that this is due to larger paintings being seen as more prestigious.

S.B. Designing Mechanisms with General Distributions

Our analysis in the main paper relies on optimal value designs that frequently feature atomic distributions with mass points at specific values. In this section, we demonstrate how to apply classical revenue equivalence insights when agents' value distributions, F_A and F_B, may contain atoms or singular continuous parts. A more general treatment of similar topics appears in Monteiro and Svaiter (2010). In this section we focus on how to derive the specific revenue expressions appearing in the proofs of Propositions 1, 2 and 4.

We use Riemann-Stieltjes integrals to keep the notation compact. This means that the differential accounts for jumps at points of discontinuity of F and continuity points at which the density may not exist. To gain intuition, note that when the measure has no singular continuous part, for a cumulative distribution F with support V and a function $u : V \to \mathbb{R}$, we have

$$\int_V u(\mathbf{v})\, dF(\mathbf{v}) = \int_S u(\mathbf{v}) \frac{\partial F(\mathbf{v})}{\partial \mathbf{v}}\, d\mathbf{v} + \sum_{\mathbf{v} \in D} u(\mathbf{v})[F_+(\mathbf{v}) - F_-(\mathbf{v})],$$

where $D \subseteq V$ denotes the set of discontinuity points of F, $S \subseteq V$ denotes the set of points where F is differentiable, and $F_+(\mathbf{v})$ and $F_-(\mathbf{v})$ denote the right and left limits of F at $\mathbf{v}$, respectively.

To prove the desired result, it is convenient to proceed in the quantile space. For any $q \in [0, 1]$, let $v_i(q) = \inf\{v_i \in [\underline{v}, \overline{v}] | F_i(v_i) \geq q\}$ denote the value associated to the q-th quantile in agent i's distribution. Observe that by construction $v_i(q)$ is left-continuous since cumulative distributions are right-continuous.

Since the revelation principle holds irrespective of distributional assumptions, we begin by considering in the quantile space a direct mechanism $(y, p) : [0, 1]^2 \to \Delta(A, B, 0) \times \mathbb{R}^2$. Define the interim allocation probability and the expected transfer of agent i reporting quantile $q \in [0, 1]$, when others are sincere, as follows

$$y_i(q) = \int_0^1 y_i(q, q_{-i})\, dq_{-i},$$

$$p_i(q) = \int_0^1 p_i(q, q_{-i})\, dq_{-i}.$$

With this notation, the expected payoff of type q_i claiming to be q is given by

$$v_i(q_i)y_i(q) - p_i(q).$$

As usual, a mechanism (y, p) is (Bayesian) incentive compatible when truth-telling is optimal for all agents i all quantiles q_i, meaning that

$$v_i(q_i)y_i(q_i) - p_i(q_i) = \max_{q\in[0,1]} [v_i(q_i)y_i(q) - p_i(q)] \equiv u_i(q_i).$$

Even when value distributions are not well-behaved, incentive compatibility can be restated as requiring that the interim probability of winning the object increases in the agent's type, as the following lemma shows.

IC Lemma: A direct mechanism is incentive compatible for agent i if and only if $y_i(q)$ is non-decreasing and for all $q_i, q_i' \in [0, 1]$

$$u_i(q_i) - u_i(q_i') = \int_{q_i'}^{q_i} y_i(q)dv_i(q).$$

Proof (necessity): If incentive compatibility holds for agent i, for any two values $q_i, q_i' \in [0, 1]$, we have that

$$u_i(q_i') \geq v_i(q_i')y_i(q_i) - p_i(q_i) = u_i(q_i) + (v_i(q_i') - v_i(q_i))y_i(q_i).$$

The same inequality holds when switching q_i and q_i', so we obtain

$$(v_i(q_i') - v_i(q_i))y_i(q_i') \geq u_i(q_i') - u_i(q_i) \geq (v_i(q_i') - v_i(q_i))y_i(q_i). \quad (1)$$

Condition (1) implies that

$$(v_i(q_i') - v_i(q_i))(y_i(q_i') - y_i(q_i)) \geq 0.$$

When $v_i(q_i') > v_i(q_i)$ this implies $y_i(q_i') \geq y_i(q_i)$, so y_i is non-decreasing on every interval on which v_i is strictly increasing. On any interval on which v_i is flat, the Riemann-Stieltjes integral $\int y_i\, dv_i$ is unaffected by the values of y_i, so without loss we may take y_i to be constant on each such interval, and thus y_i is non-decreasing everywhere.

To establish the integral representation, fix any $q_i < q_i'$ and consider any interval partition $\{(q_{k-1}, q_k)\}_{k=1}^{n}$ of the interval $[q_i, q_i']$ satisfying $q_i = q_0 < q_1 < \cdots < q_n = q_i'$. Summing (1) for consecutive pairs (q_{k-1}, q_k) yields

$$\sum_{k=1}^{n} y_i(q_k)(v_i(q_k) - v_i(q_{k-1})) \geq u_i(q_i') - u_i(q_i) \geq \sum_{k=1}^{n} y_i(q_{k-1})(v_i(q_k) - v_i(q_{k-1})).$$

Since y_i is non-decreasing and v_i is non-decreasing, both sums are upper and lower Riemann–Stieltjes sums for $\int_{q_i}^{q_i'} y_i(q)\, dv_i(q)$. As the mesh of the partition tends to zero, both bounds converge to this integral, so

$$u_i(q_i') - u_i(q_i) = \int_{q_i'}^{q_i} y_i(q)\, dv_i(q).$$

Proof (sufficiency): If y_i is non-decreasing and $u_i(q_i') - u_i(q_i) = \int_{q_i}^{q_i'} y_i(q)\, dv_i(q)$, then for any $q_i, q_i' \in [0, 1]$

$$u_i(q_i') - u_i(q_i) = \int_{q_i}^{q_i'} y_i(q)\, dv_i(q) \geq (v_i(q_i') - v_i(q_i))\, y_i(q_i),$$

where the inequality holds because y_i is non-decreasing which amounts to incentive compatibility. ■

We can use this restatement of incentive compatibility to derive the classical revenue equivalence result for this setting.

Revenue Equivalence Theorem: In any incentive compatible, direct mechanism (y, p), the interim transfer of agent i with quantile q_i is given by:

$$p_i(q_i) = p_i(0) - v_i(0)y_i(0) + v_i(q_i)y_i(q_i) - \int_0^{q_i} y_i(q)\, dv_i(q).$$

Proof: Since $u_i(q_i) = v_i(q_i)y_i(q_i) - p_i(q_i)$ and $u_i(0) = v_i(0)y_i(0) - p_i(0)$, incentive compatibility implies that

$$v_i(q_i)y_i(q_i) - p_i(q_i) = v_i(0)y_i(0) - p_i(0) + \int_0^{q_i} y_i(q)\, dv_i(q).$$

Thus, interim transfers in any two incentive compatible mechanisms with the same allocation rule must coincide up to a constant. ■

Revenue equivalence implies that even when value distributions are discontinuous, the designer's problem coincides with the maximization of virtual surplus. To see this, recall that a mechanism is individually rational if

$$u_i(q_i) \geq 0 \quad \text{for all } q_i.$$

When incentive compatibility holds, individual rationality (IR) simplifies to

$$u_i(0) \geq 0 \quad \text{or} \quad p_i(0) \leq v_i(0)y_i(0).$$

By revenue equivalence, we also know that in an incentive-compatible and direct mechanism, the expected payment of agent i is given by

$$\mathbb{E}[p_i(q_i)] = -u_i(0) + \int_0^1 v_i(q_i)y_i(q_i)\,dq_i - \int_0^1 \int_0^{q_i} y_i(q)\,dv_i(q)\,dq_i. \tag{2}$$

Exploiting Riemann-Stieltjes integrals, we can change the order of integration in the last term. The outer integral of that term is a Lebesgue integral in dq_i and the inner integral is a Riemann-Stieltjes integral with respect to the measure induced by v_i. By the Fubini-Tonelli theorem for this product of measures,

$$\int_0^1 \int_0^{q_i} y_i(q)\,dv_i(q)\,dq_i = \int_0^1 \int_q^1 dq_i\, y_i(q_i)\,dv_i(q) = \int_0^1 (1-q_i)y_i(q_i)\,dv_i(q_i).$$

By replacing this expression in condition (2) and collecting terms, we obtain that

$$\mathbb{E}[p_i(q_i)] = -u_i(0) + \int_0^1 y_i(q)\,d[(q_i-1)v_i(q_i)].$$

With this notation, the principal's revenue from (y, p) can as usual be written as

$$\hat{R}(y, p|F_A, F_B) = \sum_{i\in N} \left[-u_i(0) + \int_0^1 y_i(q_i)\,d[(q_i-1)v_i(q_i)]\right]. \tag{3}$$

Next consider $(q_i-1)v_i(q_i)$ and $y_i(q_i)$ as the two parts of the integral. Observe that the two parts have bounded variation and no common jumps, since $(q_i-1)v_i(q_i)$ is left-continuous by construction, while $y_i(q_i)$ can be selected to be right-continuous without loss.[3] Thus,

[3]Allocation probabilities on gaps are irrelevant for revenue as long as incentive compatibility holds.

even if F has atoms or singular continuous parts, we can apply standard integration by parts to obtain that

$$\hat{R}(y,p|F_A,F_B) = \sum_{i\in\{A,B\}} \left[-u_i(0) + v_i(0)y_i(0) + \int_0^1 v_i(q_i)(1-q_i)dy_i(q_i) \right].$$

As with non-atomic distributions, the optimal mechanism can be found by maximizing this expression for revenue subject to incentive compatibility and individual rationality, which require $y_i(q_i)$ to be non-decreasing and $u_i(0) \geq 0$ for all agents i. Formally, a feasible interim allocation rule is optimal for suitable transfers if and only if it solves

$$\max_y \hat{R}(y,p|F_A,F_B) \ \text{ subject to } \ (v_i(q_i') - v_i(q_i))(y_i(q_i') - y_i(q_i)) \geq 0 \quad \text{for all } q_i', q_i \in [0,1],$$

and $u_i(0) = 0$ since individual rationality binds at the optimum.

Implication Proposition 4: To map back to the value space, define the corresponding direct mechanism $(x(\mathbf{v}), t(\mathbf{v})) = (y(F_A(v_A), F_B(v_B)), p(F_A(v_A), F_B(v_B)))$. This is well-defined because, by the normalization established in the IC Lemma, y_i is constant on every flat segment of v_i. Since flat segments of $v_i(q) = F_i^{-1}(q)$ correspond exactly to atoms of F_i, for any atom v_0, evaluating y_i at the single quantile $F_i(v_0)$ correctly identifies the allocation rule for the entire interval $(F_i^-(v_0), F_i(v_0)]$, where $F_i^-(v_0) \equiv \lim_{v\uparrow v_0} F_i(v)$ denotes the left-hand limit of F_i at v_0. Observe that because F_i is increasing and y_i is right-continuous, x_i must be right-continuous. With this notation and requiring individual rationality to bind at the lowest value $u_i(0) = 0$, equation (3) can be rewritten in the value space as

$$R(x,t|F_A,F_B) = \sum_{i\in\{A,B\}} \left[\underline{v}_i x_i(\underline{v}_i) + \int_{V_i} v(1-F_i(v))dx_i(v) \right]. \tag{4}$$

This is the revenue expression found in expression (4) of the proof of Lemma 3.

Implication Proposition 1: When F_i is a binary distribution with only values $\underline{v}$ and h in the support and $F_i(\underline{v}) = \underline{q}$, we can further simplify expression (3) since

$$\int_0^1 y_i(q_i)\, d[(q_i-1)v_i(q_i)] = \int_0^{\underline{q}} \underline{v} y_i(q_i)dq_i + \int_{\underline{q}}^1 h y_i(q_i)dq_i + y_i(\underline{q})(\underline{q}-1)(h-\underline{v}).$$

When $\underline{v} = 0$, this further reduces to

$$\int_0^1 y_i(q_i)\, d[(q_i - 1)v_i(q_i)] = \int_{\underline{q}}^1 h y_i(q_i) dq_i + y_i(\underline{q})(\underline{q} - 1)h$$

Define $\hat{\psi}_i(v_i) = h\mathbb{1}(v_i = h)$. Because in any optimal mechanism $y_i(\underline{q}) = 0$ (the good is never allocated when the value equals zero), the previous expression can be rewritten in the value space as

$$\int_0^1 y_i(q_i)\, d[(q_i - 1)v_i(q_i)] = \int_{V_i} \hat{\psi}_i(v_i) x_i(v_i) dF_i(v_i),$$

which amounts to expression (11) in the proof of Proposition 1. Here $\hat{\psi}_i$ represents the generalized virtual value of agent i.

Implication Proposition 2: Since the good is only ever allocated to an agent with the highest non-negative generalized virtual value (see Section 4.1 in Monteiro and Svaiter (2010)), we further obtain expression (13) in the proof of Proposition 2, namely

$$R(F_i, F_j) = \underline{q} \int_{V_j} \max\{\hat{\psi}_j(v_j), 0\} dF_j(v_j) + (1 - \underline{q}) \int_{V_j} \max\{\hat{\psi}_j(v_j), h\} dF_j(v_j).$$

S.C. Fixed Surplus Constraint: Additional Results

Binding Constraint We begin by showing that in any optimal design the surplus constraint must bind for all non-excluded values. A value $v \in [\underline{v}, \overline{v}]$ is non-excluded by the optimal mechanism if $x_A(v|F^*) + x_B(v|F^*) > 0$.

Proposition S.1. *Value design (F_A^*, F_B^*) is optimal among all designs satisfying (5) only if $F_A^*(v) F_B^*(v) = G(v)$ for all non-excluded values $v \in [\underline{v}, \overline{v}]$.*

The result follows because if the constraint was slack at some non-excluded value v in the support of a design F, one could construct a design F' that first-order stochastically dominates F while still satisfying the constraint. Revenue would increase strictly whenever the constraint is slack for some non-excluded values, precluding the optimality of F.

Binary and Ternary Distributions While we have demonstrated that asymmetric designs when accompanied by an appropriately asymmetric mechanism can improve upon symmetric designs, establishing optimality is challenging. The optimal design depends on whether the minimal value $\underline{v}$ equals or exceeds zero as well as the specific surplus distribution G. If the minimal value exceeds zero, the maximally divisive design, defined as $(F_i^+(v), F_j^+(v)) = (G(v), 1)$ for all $v \in [0, \overline{v}]$ and $j \neq i$, can be optimal. However, this is not the case if the minimum value equals zero. The optimal design in this case depends on the surplus distribution function, G. For ternary distributions, assigning mass to exactly three values, with minimal value $\underline{v} = 0$, w-threshold designs are always optimal. We summarize these results in Proposition S.2. For convenience, let V_G be the support of G.

Proposition S.2. *The unique revenue maximizing design is:*

- *the maximally divisive design F^+, when $\underline{v} > 0$ and $V_G = \{\underline{v}, \overline{v}\}$;*
- *the w-threshold design $F^{[w]}$, when $\underline{v} = 0$ and $V_G = \{\underline{v}, w, \overline{v}\}$.*

If $\underline{v} > 0$ and the surplus distribution G equals a binary distribution with mass at $\underline{v}$ and $\overline{v}$, the maximally divisive design is optimal. The principal can extract the full surplus by offering the good to the agent with distribution G at price $\overline{v}$. If the agent does not purchase the good, then the principal awards the good to the other agent at price $\underline{v}$. In contrast, any other design requires that both agents can have value $\overline{v}$. But then full surplus extraction is not possible, as the principal now faces agents with two different, strictly positive values.[4] This contrasts sharply, with the sub-optimality of maximally divisive design established in the main text for the case in which $\underline{v} = 0$.

If $\underline{v} = 0$ and the surplus distribution G is ternary, then threshold designs are optimal. In this case, each agent has exactly one strictly positive valuation by setting $F_A^{[w]}(0) = p/q$, $F_A^{[w]}(w) = 1$, while $F_B^{[w]}(0) = F_B^{[w]}(w) = q$, $F_B^{[w]}(\overline{v}) = 1$. Given these distributions, the principal first offers the good to agent B at price $\overline{v}$. If B declines, the good is then offered to agent A at price w. Consequently, the optimal mechanism allows the principal to fully extract surplus.

Threshold Design Uniform Example To illustrate the profitability of threshold designs, we work through a concrete example where values are uniformly distributed, so $G(v) = v$. This setting offers clean analytical solutions while capturing the essential economic forces at play.

[4] Either the principal has to provide information rents to at least one agent, or they can choose to not allocate the good to agents with low value. Alternatively, they could pool them and set a price equal to $\underline{v}$. Regardless, they cannot extract the entire surplus.

Consider first the symmetric case where both agents have identical value distributions: $F_A^-(v) = F_B^-(v) = \sqrt{v}$. The virtual values in this design are:

$$\psi_i^-(v) = 3v - 2\sqrt{v}.$$

These virtual values turn positive when $v > 4/9$, establishing the optimal reserve price. Since virtual values are strictly increasing for $v \geq 4/9$ (with derivative $3 - 1/\sqrt{v} \geq 3/2 > 0$), the optimal mechanism allocates the object to the highest-value bidder above this threshold. This yields revenue of:

$$R(F^-) = \int_{4/9}^{1} (3v - 2\sqrt{v})\, dv = 43/162 \approx 0.265.$$

At the other extreme, consider a maximally divisive design where only agent A values the good: $F_A^+(v) = v$ and $F_B^+(v) = 1$. This reduces the problem to a simple monopoly pricing problem with a single buyer. The optimal mechanism becomes a posted price of $1/2$, generating revenue of exactly $R(F^+) = 1/4 = 0.25$.

Notably, maximal divisiveness performs *worse* than the symmetric design. This may appear to be a striking reversal of the typical advantage of asymmetric designs we see in other settings. But the optimal design is not a symmetric one; instead it is a threshold design aimed at extracting surplus from types that would otherwise be excluded.

Now consider a w-threshold design for $w \in [0, 1]$. This design assigns to agent A a uniform distribution on $[0, w]$: $F_A^{[w]}(v) = \min\{v/w, 1\}$, and to agent B value 0 with probability $G(w)$ and uniform distribution on $[w, 1]$: $F_B^{[w]}(v) = \max\{w, v\}$. This creates a distinct segmentation: agent A is assigned to low values while agent B ends up with the high values and an atom at 0. The virtual values become

$$\psi_A^w(v) = 2v - w \text{ for } v \leq w \quad \text{and} \quad \psi_B^w(v) = 2v - 1 \text{ for } v \geq w.$$

Since these are increasing, the optimal mechanism allocates to the agent with the highest non-negative virtual value. This generates revenue:

$$R(F^{[w]}) = w \int_0^w \max\{2v_A - w, 0\}\, d\frac{v_A}{w} + \int_0^w \int_w^1 \max\{2v_A - w, 2v_B - 1, 0\}\, dv_B\, d\frac{v_A}{w},$$

where the two terms amount respectively to the case in which B has value 0 and the case in which they value above w.

The analysis splits into two cases based on whether w exceeds $1/2$, as depicted in Figure S.1.

Case $w > 1/2$: In this scenario, there is never exclusion. After working through the integration regions where different agents have higher virtual values, revenue becomes

$$R(F^{[w]}) = \frac{w^2}{4} + \frac{(1-w)(25w^2 - 2w + 1)}{24w}.$$

Case $w \leq 1/2$: In this scenario, it is possible that the good is not allocated to either agent. Revenue calculations are further complicated by exclusion, but yield

$$R(F^{[w]}) = \frac{w^2}{4} + \frac{6 + 3w - 5w^2}{24}.$$

Figure S.2 shows that revenue peaks at $w^* \approx 0.648$, achieving approximately 0.336—a

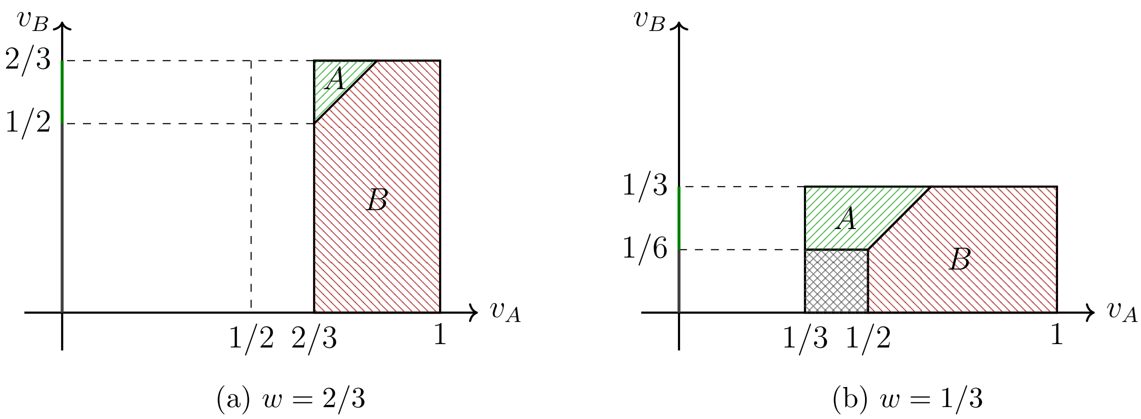

Figure S.1: Integration regions for w-threshold designs when $G(v) = v$.
Panels (a)–(b) depict integration regions for different values of w: in green north-east lines, value profiles for which A wins the good; in red north-west lines, value profiles for which B wins the good; and in gray crosshatch, value profiles for which the good is not allocated.

substantial improvement over both benchmarks. Revenue in this w^*-threshold design is 27% higher than in the symmetric design and 34% higher than in the maximally divisive design.

The threshold design maintains competitive pressure between agents (unlike the posted price solution) while creating specialized market segments that reduce information rents. Agent A faces competition only from low-value realizations of agent B, while agent B competes primarily in the high-value range where the surplus gains justify the information rents.

This example demonstrates that when surplus is constrained, optimal designs remain divisive but require nuanced market segmentation rather than extreme differentiation or symmetry.

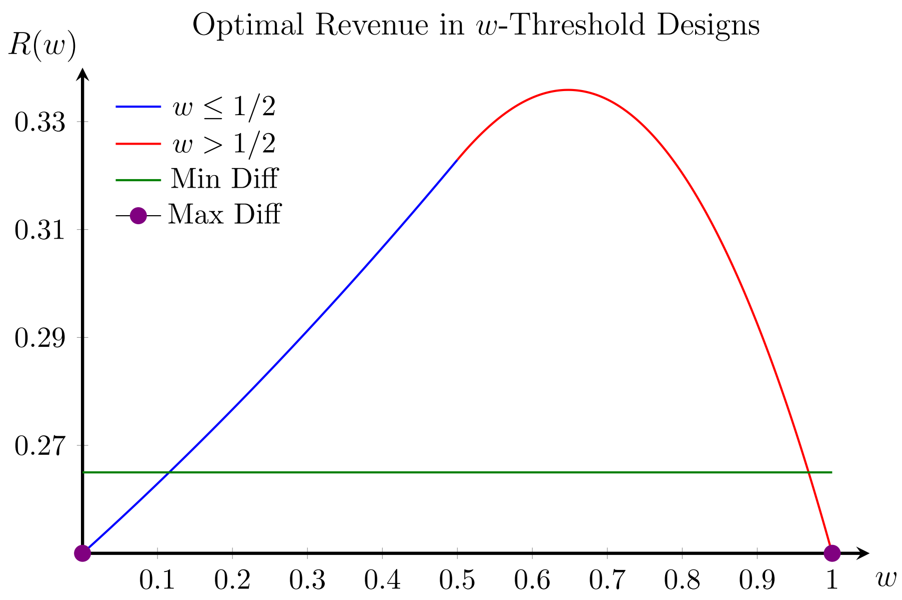

Figure S.2: The plot depicts optimal revenue as a function of w, as well as optimal revenue in both the maximally and minimally divisive designs.

S.D. Correlated Private Values

Maintaining the independence of value distributions across design problems is a natural assumption if one believes that agents should not be able to infer anything about their competitors' realized values based on their own. This assumption is not only plausible but also makes the problem more challenging, as it requires accounting for agents' information rents, rather than focusing solely on surplus design. However, many of our results extend to settings where signals are not independent, and bidders have some information about the realized values of their competitors.

Consider a correlated information setting, as discussed by Crémer and McLean (1985), where revenue generically coincides with surplus, $\mathbb{E}[\max\{v_A, v_B\}]$. To provide some insight while staying close to the core of our analysis, we allow the principal to design marginal value distributions for both bidders, but not the correlation structure between their values, which is fixed. Additionally, we assume that bidders know their own value and have some information about their competitors' values. This approach mirrors our baseline analysis, where the principal designs the marginal value distributions for both agents but cannot influence the independence of their signals.

To fix the correlation structure, consider any copula $W : [0,1]^2 \to [0,1]$, where $W(F_A(v_A), F_B(v_B)) = \Pr(V_A \leq v_A, V_B \leq v_B)$ identifies the joint probability of an event given the marginal distributions F_A and F_B. By Sklar's Theorem, copulas can be used to capture arbitrary correlation structures—such as independence, perfect positive correlation (concordance), and perfect negative correlation (discordance). Assume that W determines the underlying correlation of tastes across agents and cannot be influenced by the principal, as was the case in the original setup where $W(F_A(v_A), F_B(v_B)) = F_A(v_A)F_B(v_B)$. Instead, let the principal design the two marginal value distributions, $F_A(v_A)$ and $F_B(v_B)$. This approach is suitable for settings in which the features of the goods can be determined, but the correlation in tastes across agents must be taken as given due to factors beyond the principal's control.

No Spillovers When designing marginal distributions subject to the mean-bound constraint $\mathbb{E}_{F_i}[v_i] \leq k$ for all i, extreme bimodal designs remain optimal. This follows from the fact that variance increases the expected surplus, $\mathbb{E}[\max\{v_A, v_B\}]$, due to the convexity of the maximum operator. As an example, consider the case where values are discordant, such that $W(F_A, F_B) = \min\{F_A, F_B\}$ for any pair of probabilities (F_A, F_B). Additionally, assume $\overline{v} = 2$ and $k = 1$. In this case, surplus, or equivalently revenue, is maximized by a value design (F_A, F_B) in which both agents value the good at $v_i = 2$ or $v_i = 0$ with equal probability. This design corresponds to the maximally divisive design described in Proposition 2. Under such a maximally spread two-atom distribution, surplus is exactly equal to 2, since at least one agent must value the good at 2 due to the discordance. Thus, revenue also equals 2, and the principal secures this surplus by awarding the good with certainty to the agent with the realized value equal to 2. No other value design can lead to higher surplus, as values never exceed 2. Therefore, designs in which both distributions are maximally spread are optimal when values are negatively correlated.

With concordance $W(F_A, F_B) = \max\{0, F_A + F_B - 1\}$, the design in which one agent always values the good at the mean, $k = 1$, while the other agent's value is maximally spread, as described in Proposition 2, yields a surplus of 1.5. This exceeds the surplus from the design where both agents are maximally spread, which results in a surplus of 1. Similarly, if both agents have all mass at the mean, the surplus equals 1. Consequently, with positively correlated values, the principal is better off when one agent has all mass at the mean, while the other agent's value is maximally spread, in line with our finding when values are independent.

Spillovers When designing marginal distributions subject to the linear constraint, $F_A + F_B = H$, maximally divisive designs remain optimal. This follows because such designs increase surplus, $\mathbb{E}[\max\{v_A, v_B\}]$, by minimizing the chance of having two agents with high values.

As an example, consider the concordant copula discussed above, and assume that $H(v) = v$ for all $v \in [0, 2]$. In this case, surplus in the maximally divisive design, $(F_A, F_B) = (H - 1, H)$, coincides with the expected value, $\mathbb{E}_{H-1}[v] = 1.5$, of the high-value agent A. Thus, revenue also amounts to 1.5. This revenue can be achieved by never allocating the good to agent B, but always awarding it to agent A. To extract full surplus, the transfer of the high-value agent A is set to the value associated with the quantile reported by agent B, i.e., $t_A(\mathbf{v}) = \inf\{v \mid H(v) - 1 \geq H(v_B)\}$, while B never pays anything. In such a setting, revenue under the minimally divisive design, where both agents draw values from $H/2$, simply amounts to the expected value of one of the two agents, $\mathbb{E}_{H/2}[v] = 1$. This highlights that the optimality of maximally divisive value designs, as discussed in Proposition 4, does not depend on the independence of value distributions, but holds in more general settings. However, other designs may also be optimal for specific correlation structures.

In the same example, when the copula is discordant, the minimally divisive design yields the same surplus as the maximally divisive one. Further, discordance generally increases surplus since differentiation in values increases gains from trade.

S.E. Cost of Value Designs

It is natural in many contexts to assume that attributes can be added for free, as it is far from obvious whether and how much the addition of features costs. For completeness, we explore how our results change if we allow for costs, with $C(F)$ representing the cost of a particular value design F.

A natural starting point is to consider costs that increase with the "quality" of the design, measured by first-order stochastic dominance. Under such cost structures, the principal would first select a distribution according to costs, which subsequently serves as the boundary distribution that cannot be exceeded. This transforms our constrained optimization problems into cost-benefit trade-offs, where the principal balances the revenue gains from better designs against their implementation costs.

We characterize the types of cost functions that yield the same designs as those derived for our various constraints, demonstrating the robustness of our main insights.

No Spillovers If an attribute affects each agent individually, but there are no spillovers, then this corresponds to a setting with separable cost functions, i.e., $C(F) = c(F_A) + c(F_B)$. Designs that are optimal for $\mathbb{E}_{F_i}[v] \leq k$ would naturally arise if the separable cost functions depend solely on the mean of the value distribution chosen for each agent. Formally, this would be represented as $C(F) = c(\mathbb{E}_{F_A}[v]) + c(\mathbb{E}_{F_B}[v])$ for some increasing function $c : [\underline{v}, \overline{v}] \to \mathbb{R}_+$. In such environments, the designer would first select the mean based on cost considerations, and then choose the shape of the optimal design by maximizing revenue subject to the constraint $\mathbb{E}_{F_i}[v] \leq k$. Thus, even in costly design settings, the principal would still choose features that result in bimodal value designs.

Spillovers With spillovers, a cost that depends on the sum of average values for the good will still yield the result that it is optimal to assign all value to one agent, in the manner described in Proposition 3.

For the additive constraint, $H(v) = F_A(v) + F_B(v)$, cost functions must satisfy a weak linearity property to ensure the optimality of the maximal split distribution. This requires that any two value designs with the same sum incur the same cost. Formally, this would be represented as $C(F) = C(F')$ if $F_A + F_B = F'_A + F'_B$. This assumption is satisfied by common design cost functions, such as entropy, and would hold for integrable cost functions of the form:

$$\int_0^{\overline{v}} c(v)\, d[F_A(v) + F_B(v)]$$

for some function $c : [\underline{v}, \overline{v}] \to \mathbb{R}_+$.

Finally, designs that are optimal for the multiplicative constraint, $G(v) = F_A(v)F_B(v)$, would also be optimal in costly design settings where design costs depend solely on the distribution of surplus and increase according to first-order stochastic dominance.

S.F. Proofs: Corollaries and Appendix Results

Proof of Corollary 1: Asymmetric Mean Bound If agents faced different mean bounds, $k_A \neq k_B$, the logic of Proposition 2 continues to apply. To see this, suppose that both agents possess the maximal spread binary distribution subject to their respective mean constraint. With probability $\frac{(\overline{v}-k_A)(\overline{v}-k_B)}{(\overline{v}-\underline{v})^2}$, both agents would have the minimal value, and with the remaining probability, at least one agent would have the maximal value. Surplus would therefore amount to

$$\left(\frac{(\overline{v}-k_A)(\overline{v}-k_B)}{(\overline{v}-\underline{v})^2}\right)\underline{v}+\left(1-\frac{(\overline{v}-k_A)(\overline{v}-k_B)}{(\overline{v}-\underline{v})^2}\right)\overline{v}=\overline{v}-\frac{(\overline{v}-k_A)(\overline{v}-k_B)}{(\overline{v}-\underline{v})}.$$

Consider instead a scenario in which an agent i's distribution was maximally spread, while the other agent j had all the mass at the mean. If so, then agent i would receive the good if their valuation is maximal, and otherwise, agent j would obtain the good. This would result in the same surplus as if both agents had the maximal spread,

$$\left(\frac{\overline{v}-k_i}{\overline{v}-\underline{v}}\right)k_j+\left(1-\left(\frac{\overline{v}-k_i}{\overline{v}-\underline{v}}\right)\right)\overline{v}=\overline{v}-\frac{(\overline{v}-k_A)(\overline{v}-k_B)}{(\overline{v}-\underline{v})}.$$

Yet if $\underline{v}>0$, surplus cannot be fully extracted in the first scenario, since trading with some agent at a price smaller or equal to $\underline{v}$ precludes full surplus extraction from the same agent when their type is $\overline{v}$.■

Proof of Corollary 2: Second-Order Stochastic Dominance We show that F^* is second-order stochastically dominated by G. The proof then follows because any distribution that is second-order stochastically dominated by G must have a mean no higher than k, and because F^* was optimal among all distributions with mean no higher than k.

If $G=F^*$, then F^* is second-order stochastically dominated by G trivially. So, suppose that $G \neq F^*$. If so, for some $\hat{v}<\overline{v}$ in the support of G and all $v \in [0,\hat{v})$,

$$\Delta(v)=G(v)-F^*(v)<0,$$

because F^* maximizes the probability that the value is equal to 0 among all distributions with mean equal to k. Moreover $\Delta(v)$ is non-decreasing for any $v<\overline{v}$, because G is non-decreasing and F^* is constant for $v<\overline{v}$. Finally observe that at $\overline{v}$, Riemann-Stieltjes integration by parts yields

$$\int_0^{\overline{v}} \Delta(s)ds = [s\,(G(s) - F^*(s))]_0^{\overline{v}} - \int_0^{\overline{v}} sd\,(G(s) - F^*(s)) = \mathbb{E}_{F^*}[v] - \mathbb{E}_G[v] = 0,$$

where the final equality holds because both distributions have mean k. But if so, F^* is second-order stochastically dominated by G, since $\int_0^v \Delta(s)ds = \int_0^v G(s) - F^*(s)ds < 0$ for all $v < \overline{v}$. ■

Proof of Corollary 3: Profitable Surplus Reduction The observation follows directly from the proof of Proposition 1, but noting that equation (12) must now hold by the assumptions invoked on F_A and m.■

Proof of Lemma 3: Convex Supports We restrict attention to value designs F in which the designed distributions have supports $\hat{V}_B = [\underline{v}, w_1] \cup [w_2, w_3]$ and $\hat{V}_A \subseteq [w_1, w_2] \cup [w_3, \overline{v}]$, for $\underline{v} < w_1 < w_2 < w_3 \leq \overline{v}$, and establish that neither yields higher revenue than the maximally divisive design. Since further splitting of supports cannot improve on designs already shown to be suboptimal, this establishes convexity. Note that by the definition of the median value, $w_1 < v^M < w_3$.

As shown in Section S.B of the Supplementary Appendix, optimal revenue can always be written as

$$R(F) = \sum_{i \in \{A,B\}} \left[\underline{v}_i x_i(\underline{v}_i) + \int_{V_i} v(1 - F_i(v))dx_i(v) \right]. \tag{4}$$

As the supports of the two distributions are disjoint by the previous argument, we can redefine variables so that

$$(\hat{F}_A(v), d\hat{x}_A(v)) = \begin{cases} (F_A(v), dx_A(v)) & \text{if} \quad v \geq v^M \text{ and } v \in V_A \\ (F_B(v), dx_B(v)) & \text{if} \quad v \geq v^M \text{ and } v \in V_B \\ (0,0) & \text{if} \quad v < v^M \end{cases},$$

$$(\hat{F}_B(v), d\hat{x}_B(v)) = \begin{cases} (F_A(v), dx_A(v)) & \text{if} \quad v < v^M \text{ and } v \in V_A \\ (F_B(v), dx_B(v)) & \text{if} \quad v < v^M \text{ and } v \in V_B \\ (0,0) & \text{if} \quad v \geq v^M \end{cases}.$$

With this labeling, revenue can be rewritten as

$$R(F) = \underline{v}x_B(\underline{v}) + w_1 x_A(w_1) + \int_{\underline{v}}^{v^M} v(1-\hat{F}_B(v))d\hat{x}_B(v) + \int_{v^M}^{\overline{v}} v(1-\hat{F}_A(v))d\hat{x}_A(v).$$

For some non-negative number M, consider the alternative interim allocation rule

$$\hat{x}_B(v) = x_B(\underline{v}) + \int_{\underline{v}}^{v} d\hat{x}_B(v) \text{ for } v \in [\underline{v}, v^M],$$
$$\hat{x}_A(v) = M + \int_{v^M}^{v} d\hat{x}_A(v) \text{ for } v \in [v^M, \overline{v}].$$

This allocation rule is incentive compatible because all differentials are positive by incentive compatibility of the original allocation rule. Further it is interim feasible for the maximally divisive design $F^* = (F_A^*, F_B^*)$ as we establish in a separate part of the proof below. Thus to prove that the maximally divisive design raises more revenue than $R(F)$, it suffices to show that

$$\begin{aligned} &\underline{v}\hat{x}_B(\underline{v}) + v^M\hat{x}_A(v^M) + \int_{\underline{v}}^{v^M} v(1-F_B^*(v))d\hat{x}_B(v) + \int_{v^M}^{\overline{v}} v(1-F_A^*(v))d\hat{x}_A(v) \\ &> \underline{v}x_B(\underline{v}) + w_1 x_A(w_1) + \int_{\underline{v}}^{v^M} v(1-\hat{F}_B(v))d\hat{x}_B(v) + \int_{v^M}^{\overline{v}} v(1-\hat{F}_A(v))d\hat{x}_A(v). \end{aligned} \tag{5}$$

Showing that (5) holds would establish the result by the usual logic because $R(F^*)$, the revenue accruing to the principal when the design is F^* and the optimal mechanism is selected, must exceed the left-hand side of (5) which amounts to revenue under some other incentive compatible mechanism.

We consider three distinct cases: (i) $w_2 \leq v^M$, $w_1 > \underline{v}$, $w_3 < \overline{v}$; (ii) $w_2 > v^M$, $w_1 > \underline{v}$, $w_3 < \overline{v}$; (iii) $w_3 = \overline{v}$.

Case (i): In this scenario, inequality (5) can be amended to

$$\begin{aligned} &v^M M + \int_{w_1}^{v^M} v(1-F_B^*(v))d\hat{x}_B(v) + \int_{v^M}^{w_3} v(1-F_A^*(v))d\hat{x}_A(v) \\ &> w_1 x_A(w_1) + \int_{w_1}^{v^M} v(1-\hat{F}_B(v))d\hat{x}_B(v) + \int_{v^M}^{w_3} v(1-\hat{F}_A(v))d\hat{x}_A(v). \end{aligned} \tag{6}$$

This follows because the distributions are identical below w_1 and above w_3 and $v^M \in V_B$. Further, we have that $\hat{F}_A(v) = F_B(v) = H(v) - H(w_2) + H(w_1)$ and $F_A^*(v) = H(v) - 1$

for all $v \in [v^M, w_3]$, and thus

$$\int_{v^M}^{w_3} v(1-F_A^*(v))d\hat{x}_A(v) - \int_{v^M}^{w_3} v(1-\hat{F}_A(v))d\hat{x}_A(v) = \int_{v^M}^{w_3} v(\hat{F}_A(v)-F_A^*(v))d\hat{x}_A(v)$$
$$= (1-H(w_2)+H(w_1))\int_{v^M}^{w_3} v d\hat{x}_A(v).$$

We also have that $\hat{F}_B(v) = F_B(v) = H(v) - H(w_2) + H(w_1)$ for $v \in [w_2, v^M]$, $\hat{F}_B(v) = F_A(v) = H(v) - H(w_1)$ for $v \in [w_1, w_2]$, while $F_B^*(v) = H(v)$ for $v \in [w_1, v^M]$, and thus

$$\int_{w_1}^{v^M} v(1-F_B^*(v))d\hat{x}_B(v) - \int_{w_1}^{v^M} v(1-\hat{F}_B(v))d\hat{x}_B(v) = \int_{w_1}^{v^M} v(\hat{F}_B(v)-F_B^*(v))d\hat{x}_B(v)$$
$$= \int_{w_2}^{v^M} v(F_B(v)-H(v))d\hat{x}_B(v) + \int_{w_1}^{w_2} v(F_A(v)-H(v))d\hat{x}_B(v)$$
$$= -(H(w_2)-H(w_1))\int_{w_2}^{v^M} v d\hat{x}_B(v) - H(w_1)\int_{w_1}^{w_2} v d\hat{x}_B(v).$$

In light of the last two identities, condition (6) can be rewritten as

$$v^M M - w_1 x_A(w_1) + (1-H(w_2)+H(w_1))\int_{v^M}^{w_3} v d\hat{x}_A(v)$$
$$-(H(w_2)-H(w_1))\int_{w_2}^{v^M} v d\hat{x}_B(v) - H(w_1)\int_{w_1}^{w_2} v d\hat{x}_B(v) > 0.$$

As $H(w_1), H(w_2) - H(w_1) < 1$, it suffices to show that

$$v^M M - w_1 x_A(w_1) - \int_{w_1}^{v^M} v d\hat{x}_B(v) > 0.$$

Integrating by parts, this is equivalent to

$$\begin{aligned} v^M M - w_1 x_A(w_1) + \int_{w_1}^{v^M} \hat{x}_B(v)dv &> [\hat{x}_B(v)v]_{w_1}^{v^M} \\ &= v^M[x_B(v^M) - x_B(w_2) + x_A(w_2) - x_A(w_1) + x_B(w_1)] - w_1 x_B(w_1). \end{aligned} \tag{7}$$

Further because allocation rules are increasing, we have that $\int_{w_1}^{v^M} \hat{x}_B(v)dv > (v^M - w_1)x_B(w_1)$. Consequently equation (7) holds as long as

$$v^M(M - x_B(v^M) - x_B(w_2) + x_A(w_2)) + (v^M - w_1)x_A(w_1) > 0,$$

which holds for $M = x_B(v^M) - x_B(w_2) + x_A(w_2)$—thus establishing the result.

Case (ii): As in case (i), we need to establish inequality (5). As in case (i), we can establish that

$$\int_{w_2}^{w_3} v(1 - F_A^*(v))d\hat{x}_A(v) - \int_{w_2}^{w_3} v(1 - \hat{F}_A(v))d\hat{x}_A(v) = (1 - H(w_2) + H(w_1)) \int_{w_2}^{w_3} vd\hat{x}_A(v),$$

because $\hat{F}_A(v) = F_B(v) = H(v) - H(w_2) + H(w_1)$ and $F_A^*(v) = H(v) - 1$ for $v \in [w_2, w_3]$. Moreover, we have that

$$\int_{v^M}^{w_2} v(1 - F_A^*(v))d\hat{x}_A(v) - \int_{v^M}^{w_2} v(1 - \hat{F}_A(v))d\hat{x}_A(v) = (1 - H(w_1)) \int_{v^M}^{w_2} vd\hat{x}_A(v),$$

because $\hat{F}_A(v) = F_A(v) = H(v) - H(w_1)$ and $F_A^*(v) = H(v) - 1$ for $v \in [v^M, w_2]$. Finally, we have that

$$\int_{w_1}^{v^M} v(1 - F_B^*(v))d\hat{x}_B(v) - \int_{w_1}^{v^M} v(1 - \hat{F}_B(v))d\hat{x}_B(v) = -H(w_1) \int_{w_1}^{w_2} vd\hat{x}_B(v),$$

because $\hat{F}_B(v) = F_A(v) = H(v) - H(w_1)$ and $F_B^*(v) = H(v)$ for $v \in [w_1, v^M]$. In light of the last identities, condition (5) can be rewritten as

$$\begin{aligned} &v^M M - w_1 x_A(w_1) + (1 - H(w_2) + H(w_1)) \int_{w_2}^{w_3} vd\hat{x}_A(v) \\ &+(1 - H(w_1)) \int_{v^M}^{w_2} vd\hat{x}_A(v) - H(w_1) \int_{w_1}^{v^M} vd\hat{x}_B(v) > 0. \end{aligned}$$

As $H(w_1), H(w_2) - H(w_1) < 1$, this again reduces to

$$v^M M - w_1 x_A(w_1) - \int_{w_1}^{v^M} vd\hat{x}_B(v) > 0,$$

which from integration by parts, again reduces to

$$v^M M - w_1 x_A(w_1) + \int_{w_1}^{v^M} \hat{x}_B(v)dv > v^M(x_A(v^M) - x_A(w_1) + x_B(w_1)) - w_1 x_B(w_1). \quad (8)$$

Because $\int_{w_1}^{v^M} \hat{x}_B(v)dv > (v^M - w_1)x_B(w_1)$, equation (8) holds as long as

$$v^M(M - x_A(v^M)) + (v^M - w_1)x_A(w_1) > 0,$$

which holds for $M = x_A(v^M)$—thus establishing the result.

Case (iii): In this scenario, inequality (5) can be amended to

$$\begin{aligned} v^M M + \int_{w_1}^{v^M} v(1 - F_B^*(v))d\hat{x}_B(v) + \int_{v^M}^{w_2} v(1 - F_A^*(v))d\hat{x}_A(v) \\ > w_1 x_A(w_1) + \int_{w_1}^{v^M} v(1 - \hat{F}_B(v))d\hat{x}_B(v) + \int_{v^M}^{w_2} v(1 - \hat{F}_A(v))d\hat{x}_A(v). \end{aligned} \tag{9}$$

As before, we have that

$$\int_{v^M}^{w_2} v(1 - F_A^*(v))d\hat{x}_A(v) - \int_{v^M}^{w_2} v(1 - \hat{F}_A(v))d\hat{x}_A(v) = (1 - H(w_1)) \int_{v^M}^{w_2} v d\hat{x}_A(v),$$

because $\hat{F}_A(v) = F_A(v) = H(v) - H(w_1)$ and $F_A^*(v) = H(v) - 1$ for $v \in [v^M, w_2]$. Finally, we have that

$$\int_{w_1}^{v^M} v(1 - F_B^*(v))d\hat{x}_B(v) - \int_{w_1}^{v^M} v(1 - \hat{F}_B(v))d\hat{x}_B(v) = -H(w_1) \int_{w_1}^{w_2} v d\hat{x}_B(v),$$

because $\hat{F}_B(v) = F_A(v) = H(v) - H(w_1)$ and $F_B^*(v) = H(v)$ for $v \in [w_1, v^M]$. But then, condition (9) can be rewritten as

$$v^M M - w_1 x_A(w_1) + (1 - H(w_1)) \int_{v^M}^{w_2} v d\hat{x}_A(v) - H(w_1) \int_{w_1}^{v^M} v d\hat{x}_B(v) > 0. \tag{10}$$

which can be shown to hold following the same steps as in case (ii).

Interim Feasibility: To conclude, we establish that the proposed allocations are interim feasible. Define V_i^* as the support of distribution F_i^*. Theorem 1 in Border (1991) implies that our earlier interim allocation rules $(\hat{x}_A, \hat{x}_B)$ are feasible for the maximally divisive design F^* if and only if for any $(v_A, v_B) \in V_A^* \times V_B^*$

$$\begin{aligned} \int_{\underline{v}}^{v_B} \hat{x}_B(v)dH(v) + \int_{v^M}^{v_A} \hat{x}_A(v)dH(v) &\leq 1 - (1 - F_B^*(v_B))(1 - F_A^*(v_A)) \\ &= 1 - (1 - H(v_B))(2 - H(v_A)). \end{aligned} \tag{11}$$

To show that this condition holds, denote the difference between the left- and right-hand sides of (11) by

$$\Gamma(v_A, v_B) = \int_{\underline{v}}^{v_B} \hat{x}_B(v)dH(v) + \int_{v^M}^{v_A} \hat{x}_A(v)dH(v) - 1 + (1 - H(v_B))(2 - H(v_A)).$$

We want to show that $\Gamma(v_A, v_B) \leq 0$ for all $(v_A, v_B) \in V_A^* \times V_B^*$. Observe that $\Gamma(\overline{v}, v^M) \leq 0$ must hold, since

$$\Gamma(\overline{v}, v^M) = \int_{\underline{v}}^{v^M} \hat{x}_B(v)dH(v) + \int_{v^M}^{\overline{v}} \hat{x}_A(v)dH(v) - 1 = \int_{\underline{v}}^{\overline{v}} x_A(v) + x_B(v)dH(v) - 1 \leq 0,$$

where the second equality holds by definition of the new allocation rule and the disjoint support assumption, and where the inequality holds because the aggregate probability of trade in the original mechanism had to be smaller than 1. Because $\Gamma(\overline{v}, v^M) \leq 0$, we also have that $\Gamma(\overline{v}, v_B) \leq \Gamma(\overline{v}, v^M) \leq 0$ for all $v_B \leq v^M$. Further, we have that for $v_A = v^M$ and all $v_B \leq v^M$,

$$\Gamma(v^M, v_B) = \int_{\underline{v}}^{v_B} \hat{x}_B(v)dH(v) - H(v_B) \leq 0,$$

where the inequality holds as B can receive the good with probability no higher than 1. To conclude, observe that Γ is single-dip in v_A. To see this, note that

$$\frac{d\Gamma(v_A, v_B)}{dv_A} = dH(v_A)(\hat{x}_A(v_A) - 1 + H(v_B)).$$

But $d\Gamma(v_A, v_B)/dv_A$ can be negative only when $\hat{x}_A(v_A) < 1 - H(v_B)$ and must remain positive once it becomes larger than 0, given that $\hat{x}_A$ is increasing by incentive compatibility. Because of this, (11) must hold, since for all $(v_A, v_B) \in V_A^* \times V_B^*$,

$$\Gamma(v_A, v_B) \leq \max\{\Gamma(v^M, v_B), \Gamma(\overline{v}, v_B)\} \leq 0,$$

where the first inequality holds since $\Gamma(v_A, v_B)$ is single-dip in v_A by the previous argument, and the second one holds since $\Gamma(v^M, v_B) \leq 0$ and $\Gamma(\overline{v}, v_B) \leq 0$.■

Proof of Proposition S.1: No Value Destruction with Surplus Bound We establish the result by contradiction. Suppose by contradiction that there exists an optimal design (F_A^*, F_B^*), in which $F_A^*(w)F_B^*(w) > G(w)$ for some non-excluded value w. If so, an alternative value design (F_A', F_B') exists that first-order stochastically dominates the original design, $F' \succsim_1 F^*$, and that still satisfies the constraint

$$F_A^*(w)F_B^*(w) > F_A'(w)F_B'(w) \geq G(w).$$

Because revenue is increasing in first-order stochastic dominance, the optimal revenue under design F^* is no larger than the optimal revenue under design F'. Further, revenue strictly increases when value w is not excluded and the mechanism is fixed to $(x(F^*), t(F^*))$, because higher transfers will be paid more often in the F' design.■

Proof of Proposition S.2: Surplus Bounds and Optimality When the surplus distribution is binary ($G(v) = p$ for $v \in [\underline{v}, \overline{v})$ and $G(\overline{v}) = 1$), the allocation rule and the transfers in the optimal mechanism for the maximally divisive, $(F_i^+, F_j^+) = (G, 1)$, amount to

$$x_i(v_i, v_j|F^+) = \mathbb{1}(v_i = \overline{v}) \text{ and } x_j(v_j, v_i|F^+) = \mathbb{1}(v_i = \underline{v}),$$
$$t_i(v_i, v_j|F^+) = x_i(v_i, v_j|F^+)\overline{v} \text{ and } t_j(v_j, v_i|F^+) = x_j(v_j, v_i|F^+)\underline{v}.$$

These rules award the good to i in case of high-value at price $\overline{v}$, and sell the good to j at price $\underline{v}$ otherwise. This mechanism is incentive compatible since interim allocation rules are increasing. It is individually rational, as ex-post payoffs are non-negative. Further, it is optimal because revenue coincides with surplus $R(F^+) = S(F^+)$. When $\underline{v} > 0$, this design is uniquely optimal. To see this, consider any other value design $\hat{F}$ in which $\hat{F}_A\hat{F}_B = G$. If so, $\hat{V}_i = \hat{V}_j = \{\underline{v}, \overline{v}\}$. By construction, the designs $\hat{F}$ and F^+ generate the same surplus, $S(\hat{F}) = S(F^+)$. To show that $R(\hat{F}) < R(F^+)$, we argue that some agents must obtain a positive information rent in the optimal mechanism associated with value design $\hat{F}$. By incentive compatibility, if an agent reports the high value truthfully, it must be that

$$U_i(\overline{v}|\hat{F}) = x_i(\overline{v}|\hat{F})\overline{v} - t_i(\overline{v}|\hat{F}) \geq x_i(\underline{v}|\hat{F})\overline{v} - t_i(\underline{v}|\hat{F}).$$

If $x_A(\underline{v}) = x_B(\underline{v}) = 0$, the result follows because the mechanism does not generate full surplus, and thus $R(\hat{F}) < S(\hat{F})$. If instead $x_i(\underline{v}) > 0$ for at least some player i, by individual rationality, we further have that

$$U_i(\overline{v}|\hat{F}) \geq x_i(\underline{v})\overline{v} - t_i(\underline{v}) > x_i(\underline{v})\underline{v} - t_i(\underline{v}) = U_i(\underline{v}|\hat{F}) \geq 0.$$

If so, agent i's ex-ante utility is strictly positive, $U_i(\hat{F}) = \hat{F}_i(\overline{v})U_i(\overline{v}|\hat{F}) + \hat{F}_i(\underline{v})U_i(\underline{v}|\hat{F}) > 0$. Thus, we have that

$$R(\hat{F}) \leq S(\hat{F}) - U_A(\hat{F}) - U_B(\hat{F}) < S(F^+) = R(F^+).$$

This establishes that divisive designs maximize revenue.

To prove the second part of the result, consider any ternary distribution, $G(v) = p$ for $v \in [0, w)$, $G(v) = q > p$ for $v \in [w, \overline{v})$, and $G(\overline{v}) = 1$, and value design $F^{[w]}$. In such a value design, $F_i^{[w]}(0) = F_i^{[w]}(w) = q$, while $F_j^{[w]}(0) = p/q$, $F_j^{[w]}(w) = 1$. The optimal mechanism associated with this design amounts to

$$x_i(v_i, v_j|F^{[w]}) = \mathbb{1}(v_i = \overline{v}) \text{ and } x_j(v_j, v_i|F^{[w]}) = \mathbb{1}(v_j = w \text{ and } v_i = 0),$$
$$t_i(v_i, v_j|F^{[w]}) = x_i(v_i, v_j|F^{[w]})\overline{v} \text{ and } t_j(v_j, v_i|F^{[w]}) = x_j(v_j, v_i|F^{[w]})w.$$

These rules award the good to i in case of high-value at price $\overline{v}$, and otherwise offer to sell the good to j at price w. This mechanism is incentive compatible since interim allocation rules are increasing. It is individually rational, as ex-post payoffs are non-negative. Further, it is optimal because revenue coincides with surplus $R(F^{[w]}) = S(F^{[w]})$. It is uniquely optimal because any other design for which the constraint binds would lead to an agent having two positive values and accruing some information rent.■

References

Bocart, F. Y., M. Gertsberg, and R. A. Pownall (2022). An empirical analysis of price differences for male and female artists in the global art market. *Journal of Cultural Economics 46*(3), 543–565.

Border, K. C. (1991). Implementation of reduced form auctions: A geometric approach. *Econometrica: Journal of the Econometric Society*, 1175–1187.

Crémer, J. and R. P. McLean (1985). Optimal selling strategies under uncertainty for a discriminating monopolist when demands are interdependent. *Econometrica 53*, 345–361.

Ma, M. X., C. N. Noussair, and L. Renneboog (2022). Colors, emotions, and the auction value of paintings. *European Economic Review 142*, 104004.

Monteiro, P. K. and B. F. Svaiter (2010). Optimal auction with general distributions. *Journal of Mathematical Economics 46*, 21–31.