# Supplemental Appendix
# From Population Growth to TFP Growth

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## A Spillovers calibration

In this section, we estimate the value of $\gamma$ for equation (19). To obtain a measure of $g_S$, we use BDS data and the following procedure. Use the data to construct the share of old establishments ($N_{old}/N$) and the share of workers in old establishments ($L_{old}/L$), where we consider an establishment as old if it is 16 years old or older. Then, note that from the equation for aggregate labor in production $L$, we can construct data on $\log([X_{old}/X]_t)$ as it is equal to $\log([L_{old}/L]_t) - \log([N_{old}/N]_t)$. Finally, we obtain $g_S$ as $g_S = \Delta\log([X_{old}/X]_t) + \Delta\log(X_t) = \Delta\log([X_{old}/X]_t) + g_X$.

The OLS estimation of equation (19) is in the first column of Table A1. That is the coefficient we use in our benchmark model. The second column is the same regression but adds a linear trend. It yields similar results.

Table A1: Estimates for calibration of spillovers

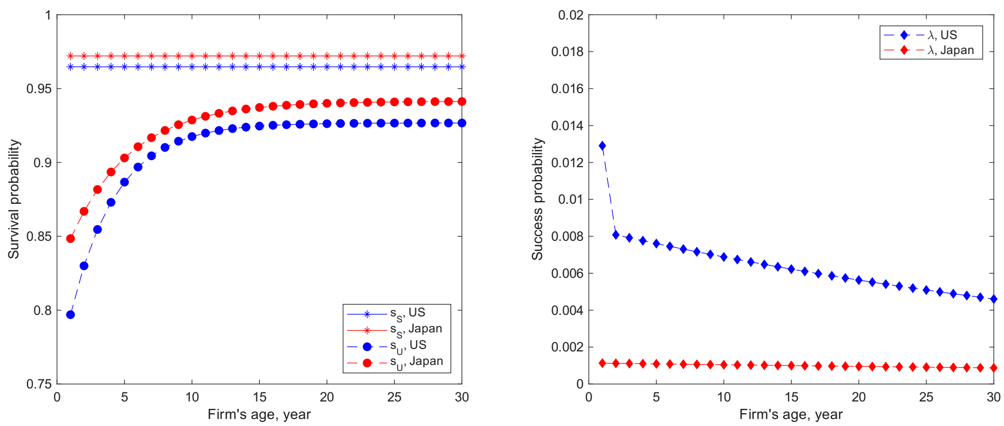
| Regression for $g_s$ | OLS | | Instrumental Variables | | | |
|---|---|---|---|---|---|---|
| $g_{x,t-1}$ | 0.342 | 0.304 | 0.385 | 0.389 | 0.460 | 0.468 |
| | (0.186) | (0.199) | (0.207) | (0.200) | (0.205) | (0.196) |
| Trend | no | yes | no | yes | no | yes |
| R squared | 0.124 | 0.137 | 0.108 | 0.091 | 0.115 | 0.091 |
| First stage statistic F | - | - | 14.549 | 12.659 | 25.452 | 22.187 |
| Hansen's $\chi^2$, p value | - | - | 0.126 | 0.142 | 0.166 | 0.195 |
| Instruments | - | - | VC | VC | VC & entry | VC & entry |
| Observations | 26 | 26 | 23 | 23 | 23 | 23 |

Note: "VC" stands for lag growth rate of (i) VC total investment, (ii) early stage investment, (iii) seed investment, and (iv) expansion stage investment. Similarly, "Entry" stands for lag growth rate in the entry rate.

One may be concerned that overall productivity growth in $t-1$ may be affected by the productivity of the old businesses in period $t$. Ideally, we want the variation in $g_X$ that is independent of the productivity growth of already leading businesses. We chose it as an instrument of $g_X$ venture capital (VC) investment because it is expected to affect $g_X$ through innovation by new firms. And VC investment should not directly affect the productivity growth of already leading businesses. Using this IV approach, we find slightly larger estimates.

# B Profiles of survival and success probabilities

Figure B1: Probability of survival and success over the life-cycle




# C Effect of population growth on the share of capital $\tilde{\alpha}$

Starting since the definition of $\tilde{\alpha}$, note that since $wL/Y = 1 - \alpha - \zeta$ from the definition of technology, we can obtain

$$\tilde{\alpha} = 1 - (1 - \alpha - \zeta)\frac{M}{L},$$

On the business side, the following equation holds:

$$Y = rK + wL + S, \tag{C1}$$

Note that $rK = \alpha Y, wL = (1 - \alpha - \zeta)Y$, and $S = \zeta Y$.

Conversely, on the household side, we have

$$Y = rK + wM + S - E. \tag{C2}$$

Note that $w(M - L) = E$ because entry, research, and development require only labor.

Combining equations (C1) and (C2), we derive

$$\frac{M}{L} = 1 + \frac{\zeta}{1 - \alpha - \zeta}\frac{E}{S},$$

so the ratio of $E$ to $S$ affects the share of workers $\tilde{\alpha}$ through the ratio of $M$ to $L$.

Additionally, in the context of the free entry condition, the initial cost of starting a business $E$ equals the expected future profit that entrants will earn until they exit. Therefore, the ratio $E/S$ represents the ratio of expected future profit to current profit. Along a BGP, this ratio transforms into:

$$\frac{E}{S} = \frac{\sum_{t=1}^{\infty} \left( \frac{g_w}{(1+r-\delta) g_X} \right)^t (\Lambda_{S,t}(g_S) + \Lambda_{U,t})}{\sum_{t=1}^{\infty} \left( \frac{1}{g_M g_X} \right)^t (\Lambda_{S,t}(g_S) + \Lambda_{U,t})} = \frac{\sum_{t=1}^{\infty} \left( \frac{\beta}{g_X^{\frac{\zeta(\epsilon-1)}{1-\alpha}+1}} \right)^t (\Lambda_{S,t}(g_S) + \Lambda_{U,t})}{\sum_{t=1}^{\infty} \left( \frac{1}{g_M g_X} \right)^t (\Lambda_{S,t}(g_S) + \Lambda_{U,t})}.$$

Given that $dg_X/dg_M$ is substantially smaller than one both from model and empirical results, an increase in $g_M$ decreases the denominator more than the numerator, as long as $\frac{\zeta(\epsilon-1)}{1-\alpha}$ is not large. Consequently, $d\tilde{\alpha}/dg_M$ is likely negative, amplifying the effect of population growth on TFP growth, although extreme parameter values can alter this relationship.

# D Cross-sectional IV regressions

Although the local projection analysis of the data and model reveals similar correlations between labor force growth and labor productivity growth, this analysis does not enable us to determine the *causal* impact of labor force growth on labor productivity growth. For example, one possible explanation for our result is that workers relocate to states with greater expected labor productivity growth. Nonetheless, our mechanism for the effect of labor force growth on labor productivity growth is quite specific, as it is based on a decrease in the number of new businesses. Karahan, Pugsley and Şahin (2024) validates this mechanism by demonstrating a causal relationship between labor force growth and the number of startups or the startup rate. In particular, they identify that a 1-percentage-point decrease in the working-age population growth rate roughly translates to a nearly 1-percentage-point decrease in the startup rate. Because our model was constructed using a firm-dynamics model similar to Karahan, Pugsley and Şahin (2024)'s framework, it is not surprising that we attain the same kind of relationship, as shown in panels for population growth and growth in average productivity in Figure 6.

Karahan, Pugsley and Şahin (2024), inspired by Shimer (2001), used a past birth rate as an instrumental variable for labor force growth. This variable is a powerful instrument because, as previous research has shown, there is a close connection between current labor force growth and the birth rate some 20 years ago. Furthermore, in our scenario, the birth rate many years ago is unlikely to have a direct impact on current labor productivity growth. Unfortunately, we find that this instrument is too weak to be used for the yearly dynamics across the states examined above using local projections. Lagged state-level birth rate, on the other hand, is a significant predictor of differences in labor force growth after averaging state data over 20 years. This fact enables us to use cross-sectional regressions in an attempt to identify the causal effect of labor force growth on labor productivity growth.

For the 20 years from 2004 to 2024, we average labor productivity growth, $g_{prod}$, and labor force growth, $g_M$. We use the birth rate pushed back 20 years as an instrument for $g_M$, so the average is from 2000 to 2004. We control by two potentially important variables. First, we control by the state's initial GDP per capita (average from 2004 to 2024) because state convergence would suggest a negative link between the initial level of development and future growth. Second, we include the state's population (average from 1990 to 2019), as many growth theories indicate that scale effects may exist.

The findings of four specifications are presented in Table D1. The first two columns show the results of OLS regressions, while the following two columns show the results of Instrumental Variable (IV) regressions. For each method, we perform the analysis using the average of labor force growth for the same period, as well as for 1999 and 2019, to avoid period overlap in the averages. The results are pretty similar.

Table D1: Impact of labor force growth on labor productivity growth, cross-sectional regressions for US states

| Dependent variable<br>Average $g_{prod}$ | OLS<br>year-lag on $g_M$<br>no lag | <br><br>5 years lag | IV, lagged birth rate<br>year-lag on $g_M$<br>no lag | <br><br>5 years lag |
|---|---|---|---|---|
| Average $g_M$ | 0.195 | 0.183 | 0.106 | 0.157 |
| | (0.070) | (0.100) | (0.042) | (0.059) |
| log(Initial gdp pc) | -0.004 | -0.005 | -0.004 | -0.005 |
| | (0.001) | (0.001) | (0.001) | (0.001) |
| log(Population) | 0.000 | 0.000 | 0.000 | 0.000 |
| | (0.001) | (0.001) | (0.001) | (0.001) |
| R-squared | 0.122 | 0.110 | 0.107 | 0.108 |
| First-stage reg F stat | – | – | 59.263 | 46.034 |
| Observations | 50 | 50 | 50 | 50 |

Note: There is also a constant in each regression, and the values in parentheses are robust standard errors. The states include all US states except the District of Columbia.

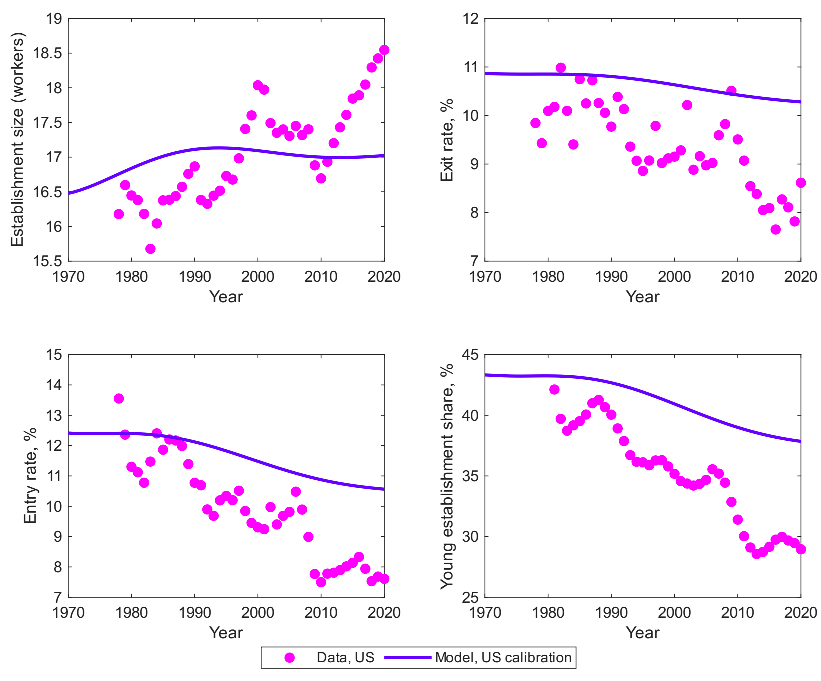
Table D1, regardless of specification, demonstrates that labor force growth has an effect on labor productivity of around a 0.2 percentage point change in labor productivity growth for every 1-percentage-point change in labor force growth. It should be noted that these estimates are comparable to the effect estimated in the local projections for the years following the shock. Note also that the magnitudes are comparable, although slightly smaller, than those estimated by Peters (2022) using forced population expulsions in post-war Germany.

# E The decline in US business dynamism

In this section, we examine how this model predicts the decline in business dynamism in the United States. Since we do not use trend data for the variables we compare, it may be challenging for the model to reproduce the US decline in business dynamism once it is fed the labor force trend. In that sense, the exercise in this section serves as a model validation exercise.

We use the calibration in the previous section to compute the model's transitional dynamics. To begin, we compute a new BGP based on the 1970 population growth. Then, we compute a transition in which the agents are aware of the change in $g_M$ exactly 100 years in advance. Thus, agents can predict the change—this appears appealing because low-frequency population size changes are predictable. Since we have $g_M$ data up to 2020, we assume that $g_M$ remains constant at the 2020 level thereafter.

Figure E1: Decline of business dynamism in the US




Because the time series for these variables in the US begins around 1980, the three plots in Figure E1 begin with the model's prediction in 1970. Overall, Figure E1 indicates that the model, to some extent, captures the decline in business dynamism in the US. The model predicts the decline in the exit rate, the entry rate, and the proportion of young establishments.[1] It also predicts a slight increase in the average business size. It is reassuring that the model performs reasonably well in this dimension. The mechanism is straightforward and similar to that described in Karahan, Pugsley and Şahin (2024) and Hopenhayn, Neira and Singhania (2022). As fewer establishments enter the market (bottom left panel) due to slowing population growth, the share of young establishments (bottom right panel) decreases. Then, since older establishments are less likely to exit and larger, the exit rate falls (top right panel), and the average business size increases (top left panel). Previous research has shown a link between population growth and business dynamism; therefore, our interest in business dynamism is primarily a validation exercise.

# F Dicussion: Our mechanism vis-à-vis scale effects

We have abstracted away from scale effects on growth up to this point by extending the firm-dynamics model of Hopenhayn (1992). However, those models have a long history, as elegantly explained by Jones (2022). In this section, we briefly discuss a particular type of model with scale effects, specifically a model in which each business produces a distinct type of good or variety.

Assume the technology for a project of productivity $x_i$, capital $k_i$ and labor $l_i$ is $y_i = x_i k_i^{\alpha} l_i^{1-\alpha}$, and the final consumption good is a CES combination of goods or varieties according to

$$Y = \left[ \sum_{i=1}^{N} y_i^{\frac{\tilde{\sigma}-1}{\tilde{\sigma}}} \right]^{\frac{\tilde{\sigma}}{\tilde{\sigma}-1}},$$

where $N$ is the number of firms, each producing a different variety as in Peters and Walsh (2022). These expressions give the following formula for total output,

$$Y = N^{\frac{1}{(\tilde{\sigma}-1)}} \tilde{X} K^{\alpha} M^{1-\alpha},$$

where $\tilde{X} \equiv \left( \sum_i \frac{1}{N} x_i^{\tilde{\sigma}-1} \right)^{\frac{1}{\tilde{\sigma}-1}}$ is the CES aggregation of productivity and $N$ is the number of goods or varieties ($K$ and $M$ are total capital and labor force, as before). Therefore, the growth in TFP is

$$g_{TFP} = g_{\tilde{X}} + \frac{1}{\tilde{\sigma}-1} g_N.$$

This equation implies that TFP growth is proportional to the growth of average productivity across firms and the growth in the number of businesses/varieties $g_N$. Recall that for comparisons of balanced-growth paths in our benchmark model, $g_{TFP}$ is proportional to $g_X$; therefore, the last term is the key difference when considering new varieties.

[1] We define young establishments as age 5 or younger.

In a BGP, the growth rate in the number of varieties, $g_N$, must equal the growth rate of the population, $g_M$. Thus, this equation implies that, unlike in our model, there is a direct effect of population growth on TFP growth across BGPs. The magnitude of this effect is given by the parameter $\tilde{\sigma}$. Calibrating $\tilde{\sigma} = 4$ such that it is consistent with the "degree of diminishing returns" calibrated in Jones (2022), this equation says that for each 1-percentage-point decline in population growth, there would be a 0.33 percentage point decline in productivity growth. This number is larger but comparable with the numbers we found for the change in $g_X$: 0.13 for the US's calibration and 0.22 for Japan's calibration. Thus, the new mechanism introduced in this paper would increase the impact of population growth on productivity growth between 30-60% (from 0.33 to 0.46-0.55 percentage points). Similarly, Peters and Walsh (2022) finds that in the long run, for each point of decline in population growth, productivity growth declines by about 0.23 percentage points. They also report that almost all of this is due to the decline in the number of varieties, which is the term highlighted in this section. Thus, the impact we find for our new mechanism would increase its effect on productivity growth in the US by 56% (from 0.23 to 0.36 percentage points).

# G Data sources

## G1 US data

**Civilian labor force.** Civilian labor force data come from the Bureau of Labor Statistics (BLS) Current Population Survey from 1949 to 2019 and from Lebergott (1966) from 1900 to 1948. The civilian labor force definition in BLS includes the population 16 years of age and over, while the definition in Lebergott (1966) includes the population 10 years of age and over. The labor force growth projections are based on the Bureau of Labor Statistics' (BLS) "A look at the future of the U.S. labor force to 2060," published in September 2016.

**Establishment data.** Establishment data come from the U.S. Census Bureau's Business Dynamics Statistics (BDS). It provides annual measures of establishment openings and closings, and job creation and destruction by age group, which allows computing life-cycle patterns of establishment that we use as targets in Figure 3 and other dynamism in Figure E1. The data is available since 1978. An establishment is a fixed physical location where economic activity occurs. A firm may have one establishment or many establishments.

**Total factor productivity.** Total factor productivity (TFP) data are calculated using Penn World Table (PWT) 10.0. While the value of TFP is directly available in PWT, we calculate TFP from real GDP, the number of persons engaged, and capital stock by assuming a

Cobb-Douglas production function. It ignores the effect of the change in human capital, which makes the data more consistent with our model. The data have been available since 1950.

**Venture capital investment.** Venture capital investment data are from the PwC/CB Insights MoneyTree™ Report.

## G2 Japan data

**Labor force.** Labor force data are sourced from the Statistics Bureau of Japan's (SBJ) Labor Force Survey from 1953 to 2019 and the National Institute of Population and Social Security Research (IPSS) from 1920 to 1952. The definition includes the population aged 15 years and over. The labor force growth projections are based on the Ministry of Internal Affairs and Communications (MIC) report, "Information and Communications in Japan 2022," published in July 2022. It offers only a projection of the working-age population, so we assume the ratio of the labor force to the working-age population has remained constant since 2020.

**Establishment data.** Establishment data come from SBJ's Establishment and Enterprise Census from 1981 to 2006 and from the Economics Census from 2009 to 2021. They provide the number of establishments and employment by age group, but these data are not annual (available only for the years 1981, 1986, 1991, 1996, 2001, 2004, 2006, 2009, 2012, 2014, 2016, and 2021). The number of establishments and employment by both age groups and three kinds of status (existing, newly organized, and closed) are only available in 2004, so the exit rate and growth of surviving establishments by age group, which are the targets, are calculated based on the data in 2004 by comparing with the data in 2001. Accurately, the calculation will not provide the annual exit rate and growth, but rather a three-year average. Therefore, the fittings for these targets in Figure 3 are based on this three-year average at an annual rate.

We have extracted the birth-year fixed effect for employment size by age, another target value. The life-cycle profile for Japan is primarily influenced by the birth year, while it remains very stable over time in the United States. In particular, we first assume that all ages in the same age group grow their establishment size at the same rate over the year. (For example, the establishment size for the age group between 3 and 7 grew by 1.2% annually from 2001 to 2004; we assume age 3, 4, 5, 6, and 7 establishments in 2001 increased their size by 1.2% every year between 2001 and 2004.) Then, we regress establishment size growth on fixed effects of age $a$ and year $t$: Establishment size growth$_{a,t} = u_a + v_t + \epsilon_{a,t}$, and extract $u_a$ to obtain the average growth in establishment size by age. Please note that with age $a$ and year $t$, the born year is identically defined by $t - a + 1$.

**Total factor productivity.** TFP data are calculated using PWT 10.0, as in the US case.

### G3 US data (state-level)

**Civilian labor force.** Civilian labor force data come from the BLS Current Population Survey for the years 1976 to 2019 and from *Historical Statistics of the United States, Colonial Times to 1970* issued by the Census Bureau for the years 1900 to 1950. We interpolate the data between 1950 and 1976. The projections for 2030 are based on the Projections Managing Partnership (PMP), which is grounded in employment data. We use only data between 1976 and 2019 to estimate empirical data, where yearly data are available. For the simulated data, we utilize the whole dataset to reproduce TFP growth.

**Real GDP and total nonfarm employees.** Real GDP data come from the Bureau of Economic Analysis (BEA) Gross Domestic Product by State and Personal Income by State. Total nonfarm employees for the corresponding years are from the BLS. We use these statistics to derive labor productivity.

**Population.** Population data are from the Census Bureau. This is one of the control variables in the cross-sectional regressions for US states.

## H More on BGP for the benchmark case

Section II illustrates the balanced growth path without two extensions and subsequently introduces two additional features (congestion and spillover). However, these two features somewhat change the property of the balanced growth path.

First, since the productivity growth of leading businesses, $g_S$, depends on $g_\chi$, which is equal to $g_X$ along a balanced growth path due to spillovers, the expected productivity for leading projects, $\Lambda_{S,a}(g_S)$, will also depend on $g_X$. Let it denote as $\Lambda_{S,a}(g_S(g_X))$. As such, the RHS of the equation for the step size $g^*$ (17) is written in this form:

$$g^* = \frac{\sum_{a=1}^{\infty} \left(\frac{1}{g_M}\right)^a \Lambda_{S,a}(1)}{\sum_{a=1}^{\infty} \left(\frac{1}{g_X g_M}\right)^a \Lambda_{S,a}(g_S(g_X))}. \tag{H1}$$

Next, because of congestion, the LHS of equation (17) (or equation (14)) is altered to

$$g^* = \left(\frac{2c_E(\tilde{n}/\tilde{M})^\phi z_R}{\iota - 2}\right)^{\frac{1}{\iota}}, \tag{H2}$$

so we need to specify $n/M$. Using the equation for labor demand, we can derive

$$\frac{N}{L} = \frac{w^{\frac{1-\alpha}{\zeta}}}{X} \left(\frac{\alpha}{r}\right)^{-\frac{\alpha}{\zeta}} (1 - \alpha - \zeta)^{-\frac{1-\alpha}{\zeta}}. \tag{H3}$$

Also, from equation (A2),

$$w^{\frac{1-\alpha}{\zeta}} = \chi (g^*)^{-\frac{\iota-2}{2}} \left(\frac{z_R z_D}{\iota}\right)^{\frac{1}{2}} \zeta \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{\zeta}} (1-\alpha-\zeta)^{\frac{1-\alpha-\zeta}{\zeta}} \times \sum_{t=1}^{\infty} \hat{\beta}_t (\Lambda_{S,t}(g_S) + \Lambda_{U,t}) \left(\frac{1}{g_w}\right)^{\frac{1-\alpha-\zeta}{\zeta}(t-1)}. \tag{H4}$$

Note that

$$\Gamma(w,r) = \zeta \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{\zeta}} \left(\frac{1-\alpha-\zeta}{w}\right)^{\frac{1-\alpha-\zeta}{\zeta}} \sum_{t=1}^{\infty} \hat{\beta}_t (\Lambda_{S,t}(g_S) + \Lambda_{U,t}) \left(\frac{1}{g_w}\right)^{\frac{1-\alpha-\zeta}{\zeta} t}$$

along a balanced growth path. Therefore, from equations (H3) and (H4),

$$\frac{N}{L} = \frac{\chi}{X} (g^*)^{-\frac{\iota-2}{2}} \left(\frac{z_R z_D}{\iota}\right)^{\frac{1}{2}} \frac{\zeta}{1-\alpha-\zeta} \sum_{t=1}^{\infty} \hat{\beta}_t (\Lambda_{S,t}(g_S) + \Lambda_{U,t}) \left(\frac{1}{g_w}\right)^{\frac{1-\alpha-\zeta}{\zeta} t}. \tag{H5}$$

Since

$$N = n \left( \sum_{a=1}^{\infty} \left(\frac{1}{g_M}\right)^{a} (\Lambda_{S,a}(1) + \Lambda_{U,a}/\theta) \right)$$

from equations (8) and (9), and

$$\frac{\chi}{X} = \frac{\left(\sum_{a=1}^{\infty} \left(\frac{1}{g_M}\right)^{a-1} (\Lambda_{S,a}(1) + \Lambda_{U,a}/\theta)\right) \left(\sum_{a=1}^{\infty} \left(\frac{1}{g_X g_M}\right)^{a-1} \Lambda_{S,a}(g_S(g_X))\right)}{\left(\sum_{a=1}^{\infty} \left(\frac{1}{g_M}\right)^{a-1} \Lambda_{S,a}(1)\right) \left(\sum_{a=1}^{\infty} \left(\frac{1}{g_X g_M}\right)^{a-1} (\Lambda_{S,a}(g_S(g_X)) + \Lambda_{U,a})\right)}$$

from equations (15) and (16), equation (H5) gives this equation:

$$\frac{n}{L} = (g^*)^{-\frac{\iota-2}{2}} \frac{\left(\sum_{a=1}^{\infty} \left(\frac{1}{g_X g_M}\right)^{a} \Lambda_{S,a}(g_S)\right)}{\left(\sum_{a=1}^{\infty} \left(\frac{1}{g_M}\right)^{a} \Lambda_{S,a}(1)\right) \left(\sum_{a=1}^{\infty} \left(\frac{1}{g_X g_M}\right)^{a} (\Lambda_{S,a}(g_S) + \Lambda_{U,a})\right)} \times \left[ \left(\frac{z_R z_D}{\iota}\right)^{\frac{1}{2}} \left(\frac{\zeta}{1-\alpha-\zeta}\right) \left( \sum_{t=1}^{\infty} \left( \frac{\beta}{g_w^{\frac{1-\alpha-\zeta}{\zeta}}} \right)^{t} (\Lambda_{S,t}(g_S) + \Lambda_{U,t}) \right) \right].$$

Substituting $g^*$ for the RHS of equation (17) and $g_w$ for $(g_X)^{\frac{\zeta}{1-\alpha}}$, we can derive the following expression for $n/L$:

$$\frac{n}{L} = \left(\frac{z_R z_D}{\iota}\right)^{\frac{1}{2}} \left(\frac{\zeta}{1-\alpha-\zeta}\right) \times \left(\frac{\sum_{a=1}^{\infty}\left(\frac{1}{g_X g_M}\right)^a \Lambda_{S,a}(g_S(g_X))}{\sum_{a=1}^{\infty}\left(\frac{1}{g_M}\right)^a \Lambda_{S,a}(1)}\right)^{\frac{\iota}{2}} \times \tag{H6}$$

$$\left(\frac{\sum_{t=1}^{\infty}\left(\frac{\beta}{g_X^{1+\frac{\zeta(\epsilon-1)}{1-\alpha}}}\right)^t (\Lambda_{S,t}(g_S(g_X)) + \Lambda_{U,t})}{\sum_{a=1}^{\infty}\left(\frac{1}{g_X g_M}\right)^a (\Lambda_{S,a}(g_S(g_X)) + \Lambda_{U,a})}\right). \tag{H7}$$

Also, from equations (4), (11), (A2), (A3), and (A4),

$$\frac{M}{L} = 1 + \frac{\zeta}{1-\alpha-\zeta}\left(\frac{\sum_{t=1}^{\infty}\left(\frac{\beta}{g_X^{1+\frac{\zeta(\epsilon-1)}{1-\alpha}}}\right)^t (\Lambda_{S,t}(g_S(g_X)) + \Lambda_{U,t})}{\sum_{a=1}^{\infty}\left(\frac{1}{g_X g_M}\right)^a (\Lambda_{S,a}(g_S(g_X)) + \Lambda_{U,a})}\right). \tag{H8}$$

Therefore, from equations (H7) and (H8), the step size $g^*$ is influenced by $g_M$ and $g_X$ through the number of entrants per capita. These equations and the equation for the step size

$$\left(\frac{2c_E(n/M)^{\phi} z_R}{\iota - 2}\right)^{\frac{1}{\iota}} = \frac{\sum_{a=2}^{\infty}\left(\frac{1}{g_M}\right)^{a-1} \Lambda_{S,a}(1)}{\sum_{a=2}^{\infty}\left(\frac{1}{g_X g_M}\right)^{a-1} \Lambda_{S,a}(g_S(g_X))}$$

define the equilibrium $g_X$ for each $g_M$.

# I Sensitivity analysis

We simulate a shock of $g_M$ from 1.02 to 1.01 to $g_{TFP}$ using the US calibration as a benchmark case. The impact size is the change in $g_{TFP}$ that persists in the long run as a result of that change. The elasticity of "Impact Size" represents the elasticity of the impact of $g_M$ on $g_{TFP}$ to a change in a parameter value. Specifically, an "Impact Size" elasticity equal to $X$ means that a 1% change in the parameter results in an X% increase in the response of $g_M$ to $g_{TFP}$.

"Convergence Speed" is the share of the change in $g_{TFP}$ that occurred 20 periods after the $g_M$ shock (relative to the long-run impact). The “Convergence Speed” elasticity equal to Z means that a 1% change in the parameter results in a Z% increase in the explained share of $g_{TFP}$ change 20 periods after the shock.

Finally, note that for the parameters that vary over age, the exact change to $\lambda$ and $s_U$ is applied to all ages.

Table I1: Role of parameters for the impact size and the convergence speed

| | Elasticity | |
|---|---|---|
| | Impact | Convergence |
| Parameter | Size | Speed |
| Survival rate of leading businesses, $s_S$ | 8.184 | -9.827 |
| Survival rate of laggard businesses, $s_U$ | 1.704 | 0.267 |
| Decreasing returns, $\zeta$ | 1.049 | 0.092 |
| Capital share, $\alpha$ | 0.017 | 0.187 |
| Spillover elasticity, $\gamma$ | 0.236 | 0.031 |
| Entry cost exponent, $\phi$ | 0.169 | -0.092 |
| Success probabilities, $\lambda$ | 0.028 | 0.071 |
| Depreciation rate, $\delta$ | -0.002 | -0.119 |
| Risk aversion, $\epsilon$ | -0.190 | 0.256 |
| Inverse of Jump in prod at success, $\theta$ | -0.049 | -0.079 |
| Research cost exponent, $\iota$ | -0.059 | -0.179 |
| Discount factor, $\beta$ | 2.412 | -4.212 |
| Productivity growth old businesses, $g_S$ | -28.165 | 6.856 |

# J Computational Details

In this section, we describe the method used to compute the model.

## J1 Balanced Growth Path

We first solve for a BGP using the 1980-1999 population and productivity growth averages as targets. Substituting these target values for $g_M$ and $g_X$ in equation (H1) gives the step size $g^*$ for the reference periods. We then derive the initial productivity growth to define the BGP state before the population growth shock. Given the step size and the discussion in Supplemental Appendix H, we can find an equation for $g_X$ as a function of the exogenous value $g_M$. Since an analytical solution does not exist, we use Newton's method. We can also derive the other variables using the properties of the BGP as described in Lemma 2.

## J2 Transitional Periods

After computing the BGP, we compute the transitional dynamics in two steps. First, we guess the growth rate of the number of entrants $\{g_{n_t}\}$ and solve for the equilibrium set of prices $\{g_{w_t}, r_t\}$ using the capital, goods, and labor market clearing conditions. We then derive $\{g_{n_t}\}$ using the free entry condition. Each step requires finding the roots numerically, so our code has a nested structure of two Newton's method computations. Importantly, we solve all periods simultaneously, not sequentially; innovators stand on the shoulders of previous innovators, which requires solving the model from the past to the future. However, they must also choose the step size of productivity, considering expected profits, which requires solving the model from the future to the past.

In the first step, we derive the number of businesses by age given the conjectured $\{g_n\}$ and the values in the initial BGP. We can also determine productivity by age, as the step size satisfies equation (H2) even during transitional periods. Next, we guess the interest rate $\{r_t\}$. As the number of businesses $N_t$ and average productivity $X_t$ are already known, $\{g_{w_t}\}$ can be derived easily by the labor market clearing condition (11) given $\{r_t\}$. With this set of prices $\{g_{w_t}, r_t\}$, we solve the household and business optimization problems. Lastly, we fix $\{r_t\}$ using Newton's method, incorporating capital and goods market clearing conditions.

In the second step, we compute the expected profits for entrants and, therefore, the value of businesses. Since it should equal the entry cost, we adjust $\{g_{n_t}\}$ using Newton's method. Since changes in $\{g_{n_t}\}$ affect $\{r_t\}$, we return to the first step for every repetition. When the value of businesses minus the entry cost converges to zero sufficiently ($< 10^{-6}$), the computation is finished.

# K Full transitions

This section shows the full transitions computed for the US and Japan. As mentioned in the main text, the transition's input is the labor force growth trend, which is shown with a black line in Figure K1.

# L Calibration of alternative productivity processes

Figure K1: Full Transitions

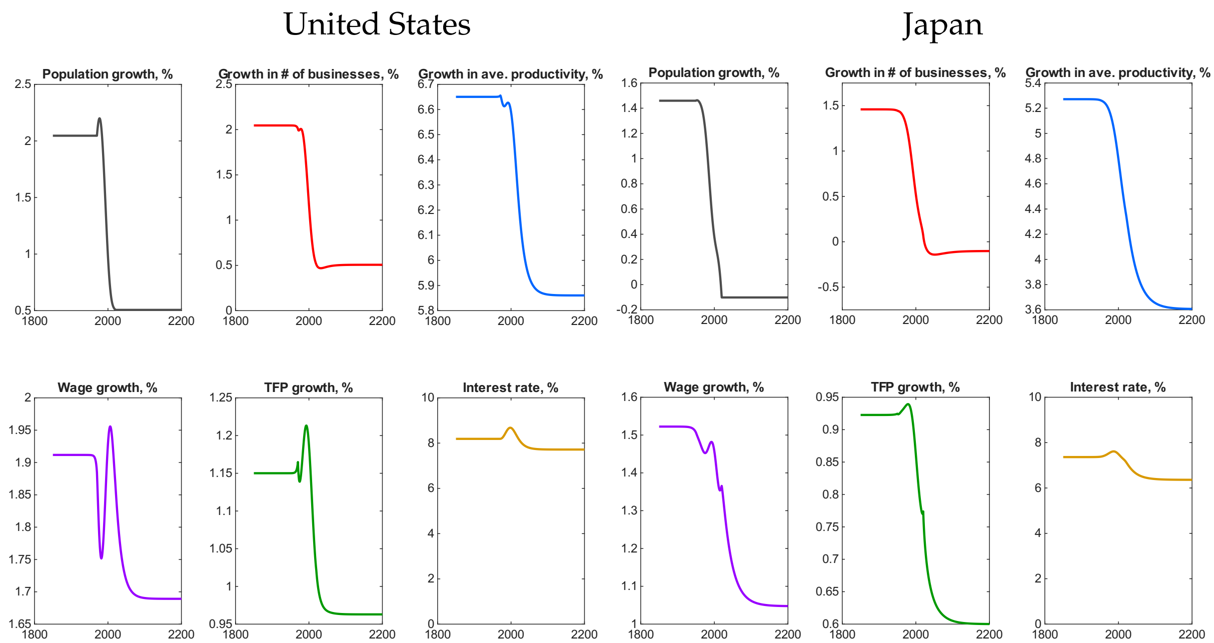


Figure L1: Fit of targets of alternative specifications

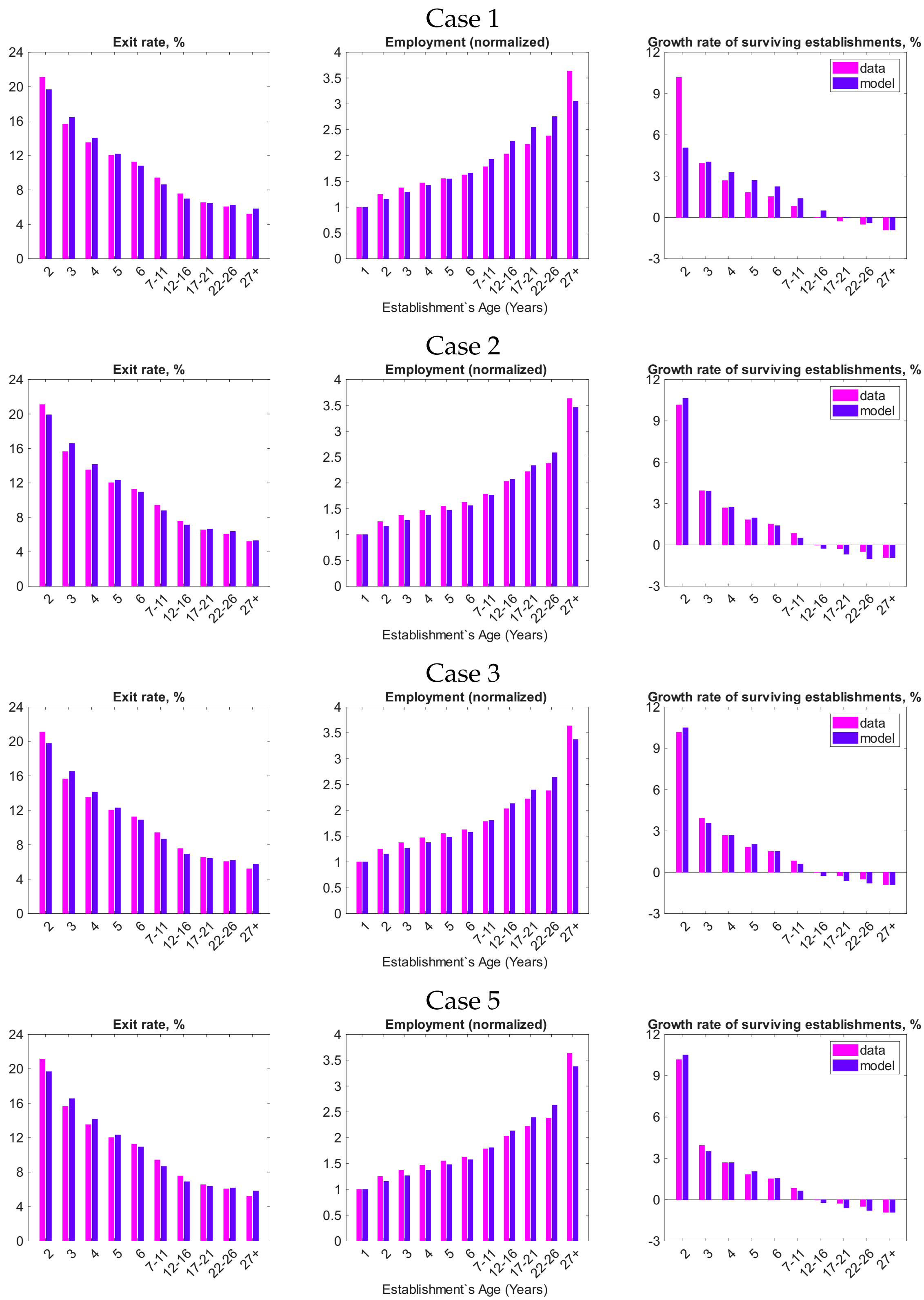
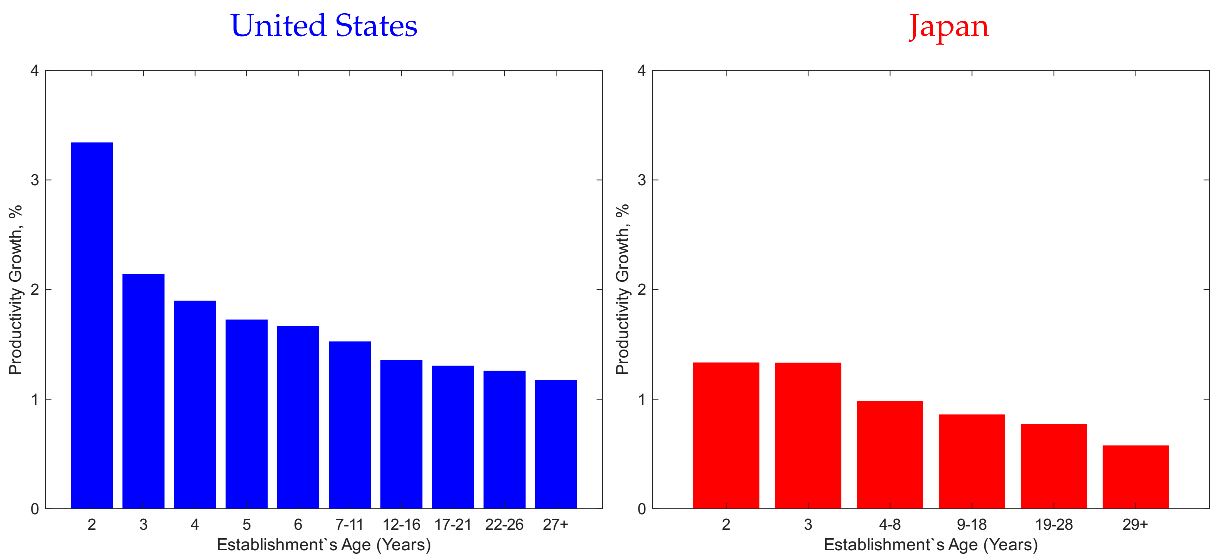


Figure L2: Productivity growth by establishment age



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