Supplemental Appendix of "Identifying preference for early resolution from asset prices"

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Proofs for Theorems 1 and 2

Proof for Theorem 1

Because the underlying probability space Ω is finite dimensional, for any random variable V defined on Ω, we can identify V as a finite dimensional vector $V = [V(1),\ V(2),\ \cdots,\ V(n)]$ and think of the certainty equivalent functional $\mathcal{I}$ as a function from $\boldsymbol{R}^n$ to $\boldsymbol{R}$. For $s = 1,\ 2,\ \cdots,\ n$, we denote $\frac{\partial}{\partial V(s)}\mathcal{I}[V]$ as the partial derivative of $\mathcal{I}$ with respect to the sth element of V. The stochastic discount factor can be computed from the marginal rate of substitution of the representative agent. Given the form of the utility function in (1), the SDF is given by:

$$SDF(s_0, s_1) = \beta \frac{1}{\mu(s_1)} \frac{\frac{\partial \mathcal{I}[V_1]}{\partial V_1(s_1)} u'(\bar{c}_1)}{u'(\bar{c}_0)} = \lambda \frac{\partial \mathcal{I}[V_1]}{\partial V_1(s_1)}, \tag{A1}$$

where $\lambda = \beta \frac{1}{\mu(s_1)} \frac{u'(\bar{c}_1)}{u'(\bar{c}_0)}$ is a constant that does not depend on s_1. Recall that $\mu(s_1) = \frac{1}{n}$ for all s_1 due to the assumption of equal probability.

To prove Theorem 1, we first set up some notation and introduce a useful lemma. Note that given the SDF, no arbitrage implies that the price of any period-1 payoff X denominated in period-0 consumption goods is given by $P_0(X) = E_0[SDF(s_0, s_1)X(s_1)]$. The one-period risk-free rate paid in period 1 is $R_f(0) = \frac{1}{E_0[SDF(s_0,s_1))]}$. The risk-premium for an asset with payoff X is therefore given by $E_0\left[\frac{X}{P_0(X)}\right] - R_f(0)$.

LEMMA 1: *Suppose that $\mathcal{I} : \mathcal{L}(\Omega, \mathcal{F}, P) \to \boldsymbol{R}$ is strictly increasing and continuously differentiable. The following conditions are equivalent:*

(i) The risk premium received in period 1 is non-negative for all payoffs that are comonotone with respective to V_1.

(ii) $\mathcal{I}$ is non-decreasing in second order stochastic dominance, that is, $\forall$ V and $\tilde{V} \in \mathcal{L}(\Omega, \mathcal{F}, P)$, if V second order stochastic dominates $\tilde{V}$ then $\mathcal{I}[V] \geq \mathcal{I}\left[\tilde{V}\right]$.

(iii) For any $V \in \mathcal{L}(\Omega, \mathcal{F}, P)$,

$$\left[\frac{\partial}{\partial V(s)}\mathcal{I}[V] - \frac{\partial}{\partial V(s')}\mathcal{I}[V]\right]\left[V(s) - V(s')\right] \leq 0. \tag{A2}$$

PROOF:

Here, we prove the equivalence between statements (i) and (iii). The equivalence between (ii) and (iii) is based on a characterization of Schur concavity that can be found in Marshall, Arnold and Olkin (2011) or Muller and Stoyan (2002).

First, we assume that statement (i) is true and prove (iii) by contradiction. Suppose there exists $V \in \mathcal{L}(\Omega, \mathcal{F}, P)$ and s, s' such that

$$V(s) > V(s'), \text{ and } \frac{\partial}{\partial V(s)}\mathcal{I}[V] > \frac{\partial}{\partial V(s')}\mathcal{I}[V]. \tag{A3}$$

Consider the following payoff:

$$X(i) = V(i) \quad for \quad i = s_1, s_1'; \quad X(i) = 0 \quad otherwise.$$

Given condition (A3), X is strictly positively correlated with $\frac{\partial \mathcal{I}[V_1]}{\partial V_1(s_1)}$ and, therefore, the SDF defined in (A1). As a result,

$$P_0(X) = E[SDF(s_0, s_1) X(s_1)] > E[SDF(s_0, s_1)] E[X(s_1)] = \frac{E[X(s_1)]}{R_f(0)},$$

That is, the risk premium for X is strictly negative. However, by the definition of comonotonicity in equation (12), X is comonotone with V_1, a contradiction.

Next, we assume that statement (iii) in the lemma is true and prove (i). Take any X that is comonotone with V_1. By condition (A2), X is negatively comonotone with respect to $\frac{\partial \mathcal{I}[V_1]}{\partial V_1(s_1)}$ and the SDF defined in (A1). As a result, X and SDF are negatively correlated and

$$P_0(X) = E[SDF(s_0, s_1) X(s_1)] \leq E[SDF(s_0, s_1)] E[X(s_1)] = \frac{E[X(s_1)]}{R_f(0)}$$

as needed.

It is straightforward to show that the strict inequality version of Lemma 1 also holds. That is, under the same assumptions in Lemma 1, the following statements are also equivalent:

(i') The risk premium received in period 1 is strictly positive for all payoffs that are strictly comonotone with respect to V_1.

(ii') $\mathcal{I}$ is strictly increasing in second order stochastic dominance, that is, $\forall$ V and $\tilde{V} \in \mathcal{L}(\Omega, \mathcal{F}, P)$, if V strictly second order stochastic dominates $\tilde{V}$ then $\mathcal{I}[V] > \mathcal{I}\left[\tilde{V}\right]$.

(iii') For any $V \in \mathcal{L}(\Omega, \mathcal{F}, P)$,

$$\left[\frac{\partial}{\partial V(s)}\mathcal{I}[V] - \frac{\partial}{\partial V(s')}\mathcal{I}[V]\right]\left[V(s) - V(s')\right] \leq 0. \tag{A4}$$

and the strict inequality holds as long as $V(s) \neq V(s')$.

To prove Theorem 1, we note that statement 2 in Theorem 1 is equivalent to statement (ii) in Lemma 1. In addition, statement 3 in Theorem 1 is equivalent to statement (iii) in Lemma 1. It is enough to show that statement 1 is equivalent to (i). Given that u is a strictly increasing function, the definition of V_1 in equation (11) implies that $c_2(s_1)$ is strictly comonotone with $V_1(s_1)$. This establishes the equivalence between statement 1 in Theorem 1 and statement (i) in Lemma 1. The strict inequality version of the theorem can be similarly proved by using the strict inequality version of Lemma 1.

Proof for Theorem 2

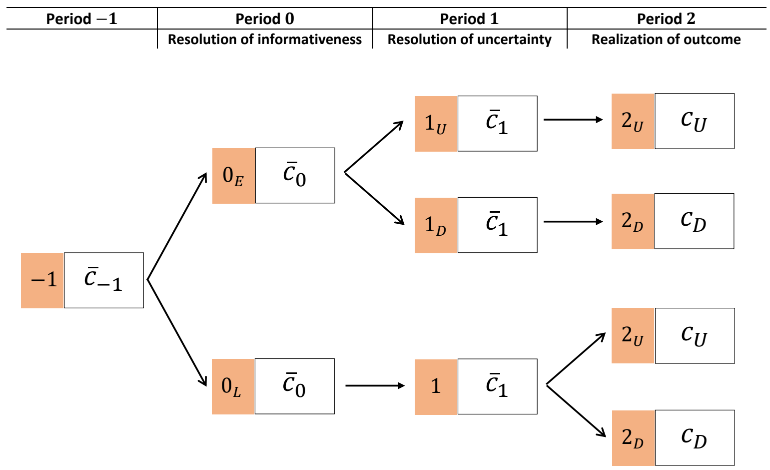
First, we assume condition 1 in Theorem 1 is true, that is, the risk premium for any asset with payoff comonotone with informativeness is non-negative. To prove condition 2, it is enough to show that $V_0(s_0)$ is comonotone with informativeness. We prove by contradiction. Assume $\exists s_0$ and s_0' such that $\iota(s_0) < \iota(s_0')$ and $V_0(s_0) < V_0(s_0')$. Consider the following payoff:

$$X(i) = \frac{1}{\iota(i)} \quad if \quad i = s_0, s_0';\ X(i) = 0 \ otherwise. \tag{A5}$$

Clearly, X is comonotone with informativeness. By condition 1, the risk premium of X must be non-negative. Note that X is also strictly negatively comonotone with $V_0(s_0)$. By Lemma 1, we know that under the assumption of strict GRS, the risk premium for X must be strictly negative, which is a contradiction.

Next, we assume that condition 2 in Theorem 1 holds and prove condition 1. Note that preference for early resolution of uncertainty is equivalent to $V_0(s_0)$ being comonotone with respect to informativeness. As a result, any payoff that is comonotone with respect to informativeness is also comonotone with $V_0(s_0)$. By the assumption of GRS, we know that the risk premium on this asset must be non-negative. The strict inequality version of this theorem can be proved similarly.

Examples 1 and 3



B1. Notations

We adopt the following general convention for notation. We use subscripts for the time period, and write in parenthesis the node at which the quantity or price is calculated. All nodes in period -1, 0, and 1 are illustrated in Figure 2. In addition, we maintain our assumption that 0_E and 0_L happen with probability 0.5, respectively. In addition, $\ln c_2$ follows a Gaussian distribution with mean μ and variance σ^2, denoted $N\left(\mu, \sigma^2\right)$. Below we first set up notations.

Utility

- Period -1: V_{-1} is the utility in period -1.
- Period 0: There are two nodes in period 0. The corresponding utilities are $V_0(0_E)$ and $V_0(0_L)$, or $V_0(s_0)$ in general.
- Period 1: There are three nodes in period 1. The corresponding utilities are $V_1(1_U)$, $V_1(1_D)$, and $V_1(1)$, or $V_1(s_1)$ in general.

SDF

We also compute the SDF that prices one-period cash flows. We use the following convention: $SDF(node_t, node_{t+1})$, which prices period $t+1$ cash flow into period t consumption units (note that we are looking at one-period SDFs). To be precise, the notation $(node_t, node_{t+1})$ emphasizes that the SDF computes the price of a cash flow delivered at $node_{t+1}$ in terms of $node_t$ consumption units. In this notation, $node_{t+1}$ must be a node that immediately follows $node_t$.

- Pricing period 0 cash flow into period -1 consumption units: $SDF(-1, s_0)$. There is only one node in period -1, and $SDF(-1, 0_E)$, $SDF(-1, 0_L)$ are the realizations of the random variable $SDF(-1, s_0)$.
- Pricing period 1 cash flow into period 0 consumption units: $SDF(0_L, s_1)$ and $SDF(0_E, s_1)$. There are two nodes in period 0. For 0_E, $SDF(0_E, 1_U)$ and $SDF(0_E, 1_D)$ are the two realizations of the random variable $SDF(0_E, s_1)$. At node 0_L, $SDF(0_L, 1)$ is the only possible value, as there is only one following node.
- Pricing period 2 cash flow into period 1 consumption units: $SDF(1_U, s_2)$, $SDF(1_D, s_2)$, and $SDF(1, s_2)$. There are three nodes in period 1. At node 1_U: $SDF(1_U, 2_U)$ is the only possible realization of the SDF, as there is only one following node. At node 1_D it is $SDF(1_D, 2_D)$, as again there is only one following node. At node 1: $SDF(1, 2_U)$ and $SDF(1, 2_D)$ are the two realizations of the random variable $SDF(1, s_2)$.

PRICE

Here, we calculate the price of a levered consumption claim paid in period 2: $C_2^{1+\zeta}$. The prices below refer to the price of this consumption claim evaluated at different nodes. We continue to adopt the same notation convention for evaluating utilities. That is, we use subscripts for the time period, and we write in parenthesis the node at which the prices are calculated.

- Period -1: P_{-1}.
- Period 0: $P_0(0_E)$, $P_0(0_L)$.
- Period 1: $P_1(1_U)$, $P_1(1_D)$, $P_1(1)$.

VARIANCE CLAIMS

We calculate the price of a claim to stock market returns variance. The stock market refers to the claim to the levered consumption as described above. We use subscripts for the horizon of the variance, and we write in parenthesis the node at which the expected variance is calculated. In general, $IV_{t\to t+1}(s_t)$ denotes the s_t expectation of the return variance from the end of period t to the end of $t+1$.

B2. Example 1: utility calculations

We focus on the multiplier robust control preference discussed in Example I.A. In period 1, let $V_1(1_U)$ and $V_1(1_D)$ denote the continuation utility at node 1_U and 1_D, respectively.

$$\begin{aligned} V_1(1_U) &= \ln \overline{c_1} + \beta \ln c_2, \\ V_1(1_D) &= \ln \overline{c_1} - \beta\theta \ln E[e^{-\frac{1}{\theta}\ln c_2}] = \ln \overline{c_1} + \beta(\mu - \frac{1}{2}\frac{1}{\theta}\sigma^2). \end{aligned}$$

In period 0, the continuation utility at node 0_E and 0_L can be computed as:

$$\begin{aligned}
V_0(0_E) &= \ln \overline{c_0} - \beta\theta \ln E[e^{-\frac{1}{\theta}V_1(s_1)}] \\
&= \ln \overline{c_0} - \beta\theta \ln E[e^{-\frac{1}{\theta}(\ln \overline{c_1} + \beta \ln c_2)}] \\
&= \ln \overline{c_0} + \beta \ln \overline{c_1} + \beta^2 \mu - \frac{1}{2}\frac{\beta^3}{\theta}\sigma^2 \\
V_0(0_L) &= \ln \overline{c_0} - \beta\theta \ln E[e^{-\frac{1}{\theta}\ln V_1(s_1)}] \\
&= \ln \overline{c_0} + \beta V_1(1) \\
&= \ln \overline{c_0} + \beta \ln \overline{c_1} + \beta^2 \mu - \frac{1}{2}\frac{\beta^2}{\theta}\sigma^2.
\end{aligned}$$

In period -1:

$$V_{-1} = \ln \bar{c}_{-1} - \beta\theta \ln E[e^{-\frac{1}{\theta}V_0(s_0)}].$$

B3. Example 3: SDF and asset price calculations

SDF calculations

The stochastic discount factors that convert period-2 cash flow into period-1 consumption units are:

$$\begin{aligned}
SDF(1_U, s_2) &= \beta \frac{e^{-\frac{1}{\theta}\ln c_2}}{E[e^{-\frac{1}{\theta}\ln c_2}]}(\frac{c_2}{\overline{c_1}})^{-1} = \beta(\frac{c_2}{\overline{c_1}})^{-1}, \\
SDF(1_D, s_2) &= \beta \frac{e^{-\frac{1}{\theta}\ln c_2}}{E[e^{-\frac{1}{\theta}\ln c_2}]}(\frac{c_2}{\overline{c_1}})^{-1} = \beta(\frac{c_2}{\overline{c_1}})^{-1}, \\
SDF(1, s_2) &= \beta \frac{e^{-\frac{1}{\theta}\ln c_2}}{E[e^{-\frac{1}{\theta}\ln c_2}]}(\frac{c_2}{\overline{c_1}})^{-1}.
\end{aligned}$$

Similarly, the SDF that prices period 1 cash flow is given by:

$$SDF(0_E, s_1) = \beta \frac{e^{-\frac{1}{\theta}V_1(s_1)}}{E[e^{-\frac{1}{\theta}V_1(s_1)}]}(\frac{\overline{c_1}}{\overline{c_0}})^{-1} = \beta \frac{e^{-\frac{1}{\theta}\beta \ln c_2}}{E[e^{-\frac{1}{\theta}\beta \ln c_2}]}(\frac{\overline{c_1}}{\overline{c_0}})^{-1}, \tag{B1}$$

$$SDF(0_L, 1) = \beta(\frac{\overline{c_1}}{\overline{c_0}})^{-1}. \tag{B2}$$

The above implies that the risk-free rates from period 0 to 1 are given by: $R_f(0_E) = \frac{1}{E[SDF(0_E,s_1)]} = \frac{1}{\beta}(\frac{\overline{c_1}}{\overline{c_0}})$, and $R_f(0_L) = \frac{1}{\beta}(\frac{\overline{c_1}}{\overline{c_0}})$. Finally, the SDF that prices period-0 cash flow into period -1 consumption is

$$SDF(-1, s_0) = \beta \frac{e^{-\frac{1}{\theta}V_0(s_0)}}{E[e^{-\frac{1}{\theta}V_0(s_0)}]}(\frac{\overline{c_0}}{\overline{c_{-1}}})^{-1}, \tag{B3}$$

and the risk-free rate from -1 to 0: $R_f(-1) = \frac{1}{E[SDF(-1,s_0)]} = \frac{1}{\beta}(\frac{\overline{c_0}}{\overline{c_{-1}}})$.

Asset prices

We consider the asset that pays $c_2^{1+\zeta}$ in period 2, where $\zeta > 0$, as described in Example 3. We compute its prices throughout the tree. Its prices in period 1 can be computed using the SDF listed in the last section. At nodes 1_U and 1_D,

$$P_1(s_1) = E[SDF(s_1, s_2)c_2^{1+\zeta}] = \beta(\frac{c_2}{c_1})^{-1}c_2^{1+\zeta} = \beta\overline{c_1}c_2^{\zeta}. \tag{B4}$$

At node 1,

$$P_1(1) = E[SDF(1, s_2)c_2^{1+\zeta}] = \beta\overline{c_1}e^{\zeta\mu+\frac{1}{2}(\zeta^2-2\frac{\zeta}{\theta})\sigma^2}. \tag{B5}$$

Similarly, we can compute the price at nodes 0_E and 0_L as

$$P_0(0_E) = E[SDF(0_E, s_1)P_1(s_1)] = \beta^2\overline{c_0}e^{\zeta\mu+\frac{1}{2}(\zeta^2-2\frac{\zeta\beta}{\theta})\sigma^2}, \tag{B6}$$

$$P_0(0_L) = SDF(0_L, 1)P_1(1) = \beta^2\overline{c_0}e^{\zeta\mu+\frac{1}{2}(\zeta^2-2\frac{\zeta}{\theta})\sigma^2}. \tag{B7}$$

Announcement premiums

Here, we compute the announcement returns, that is, returns from 0_E to 1_E in Example 3. The announcement return of the claim to $c_2^{1+\zeta}$ is

$$R_A = \frac{P_1(s_1)}{P_0(0_E)} = \frac{\beta\overline{c_1}c_2^{\zeta}}{\beta^2\overline{c_0}e^{\zeta\mu+\frac{1}{2}(\zeta^2-2\frac{\zeta\beta}{\theta})\sigma^2}} = \frac{1}{\beta}\frac{\overline{c_1}}{\overline{c_0}}\frac{c_2^{\zeta}}{e^{\zeta\mu+\frac{1}{2}(\zeta^2-2\frac{\zeta\beta}{\theta})\sigma^2}}.$$

As a result, the expected return is $E[R_A] = \frac{1}{\beta}\frac{\overline{c_1}}{\overline{c_0}}e^{\frac{\zeta\beta}{\theta}\sigma^2}$. Given that $R_f(0_E) = \frac{1}{\beta}\frac{\overline{c_1}}{\overline{c_0}}$, $\frac{E[R_A]}{R_f(0_E)} = e^{\frac{\beta\zeta}{\theta}\sigma^2} > 1$ as long as $\theta > 0$.

B4. PER premium and implied variance dynamics

Risk neutral variances

We first compute the conditional variance of announcement return, $\ln\frac{P_1}{P_0}$, in period 0 under the risk-neutral measure. Consider node 0_E first. At node 0_E, by Equation (B4), $\ln P_1 = \ln\beta + \ln\bar{c}_1 + \zeta\ln c_2$. Under the objective measure, $\ln c_2 \sim N\left(\mu, \sigma^2\right)$.

Let $f\left(x|\,\mu,\sigma^2\right)$ be the density of a Gaussian distribution with mean μ and variance σ^2. Because the risk-neutral density is proportional to $SDF\left(0_E,s_1\right)\times f\left(x|\,\mu,\sigma^2\right)$, given the form of $SDF\left(0_E,s_1\right)$ in Equation (B1), the risk-neutral distribution of $\ln c_2$ at node 0_E is $f\left(x|\,\mu-\frac{\beta}{\theta}\sigma^2,\sigma^2\right)$. It follows immediately that

$$Var^*\left[\ln\frac{P_1}{P_0}\bigg|\,0_E\right]=Var^*\left[\zeta\ln c_2|\,0_E\right]=\zeta^2\sigma^2. \tag{B8}$$

In addition, given $P_0\left(0_E\right)$ in Equation (B6),

$$\begin{aligned} E^*\left[\ln\frac{P_1}{P_0}\bigg|\,0_E\right] &= E^*\left[-\ln\beta+\left(\ln\bar{c}_1-\ln\bar{c}_0\right)+\zeta\left(\ln c_2-\left(\mu-\frac{\beta}{\theta}\sigma^2\right)\right)-\frac{1}{2}\zeta^2\sigma^2\bigg|\,0_E\right]\\ &= -\ln\beta+\left(\ln\bar{c}_1-\ln\bar{c}_0\right)-\frac{1}{2}\zeta^2\sigma^2. \end{aligned} \tag{B9}$$

At node 0_L, the distribution of P_1 is degenerate (Equation (B5)). As a result, $Var^*\left[\ln\frac{P_1}{P_0}\Big|\,0_L\right]=0$, and

$$E^*\left[\ln\frac{P_1}{P_0}\bigg|\,0_L\right]=-\ln\beta+\left(\ln\bar{c}_1-\ln\bar{c}_0\right).$$

PER Premium

The price of the variance claim, $Var^*\left(\ln\frac{P_1}{P_0}\Big|\,s_0\right)$ at time -1 is

$$E\left[SDF(-1,s_0)Var^*\left(\ln\frac{P_1}{P_0}\bigg|\,s_0\right)\right]=\beta(\frac{\overline{c_0}}{\overline{c_{-1}}})^{-1}\frac{1}{2}\frac{e^{-\frac{1}{\theta}V_0(0_E)}\zeta^2\sigma^2}{E[e^{-\frac{1}{\theta}V_0(s_0)}]}$$

The expected return of this asset can be calculated as

$$E[R_{Var}(s_{-1},s_0)]=\frac{\frac{1}{2}\zeta^2\sigma^2}{E\left[SDF(-1,s_0)Var^*\left(\ln\frac{P_1}{P_0}\Big|\,s_0\right)\right]}=\frac{1}{\beta}(\frac{\overline{c_0}}{\overline{c_{-1}}})\frac{E[e^{-\frac{1}{\theta}V_0(0_E)}]}{e^{-\frac{1}{\theta}V_0(0_E)}}$$

Given the risk-free rate, $R_f(-1)=\frac{1}{\beta}\left(\frac{\bar{c}_0}{\bar{c}_{-1}}\right)$, calculated in the last section,

$$\begin{aligned} \frac{E[R_{Var}(s_{-1},s_0)]}{R_f(-1)} &= \frac{E[e^{-\frac{1}{\theta}V_0(0_E)}]}{e^{-\frac{1}{\theta}V_0(0_E)}}\\ &= \frac{\frac{1}{2}e^{\frac{\beta^3}{2\theta^2}\sigma^2}+\frac{1}{2}e^{\frac{\beta^2}{2\theta^2}\sigma^2}}{e^{\frac{\beta^3}{2\theta^2}\sigma^2}}>1\text{ iff }\beta<1 \end{aligned}$$

Proof for Inequality (18)

We show that a sufficient condition for inequality (18) is:

$$\beta(1-\beta)\frac{1}{\theta} > \frac{\zeta}{\sqrt{2}}. \tag{B10}$$

Clearly, Equation (B8) and $Var^*\left[\ln\frac{P_1}{P_0}\middle| 0_L\right] = 0$ imply that the expected risk-neutral variance is

$$E\left[Var^*\left[\ln\frac{P_1}{P_0}\middle| s_0\right]|s_{-1}\right] = \frac{1}{2}\zeta^2\sigma^2. \tag{B11}$$

To compute the term $Var^*\left[\ln\frac{P_1}{P_0}\middle| s_{-1}\right]$ in (18), we note that at node -1, 0_E and 0_L each happens with probability $Prob(0_E|-1) = Prob(0_L|-1) = \frac{1}{2}$. Given the SDF in (B3), the risk neutral probability is $Prob^*(0_E|-1) = \frac{e^{\frac{\beta^3}{\theta^2}\sigma^2}}{e^{\frac{\beta^3}{\theta^2}\sigma^2}+e^{\frac{\beta^2}{\theta^2}\sigma^2}}$, and $Prob^*(0_L|-1) = \frac{e^{\frac{\beta^2}{\theta^2}\sigma^2}}{e^{\frac{\beta^3}{\theta^2}\sigma^2}+e^{\frac{\beta^2}{\theta^2}\sigma^2}}$. Here, $\ln\frac{P_1}{P_0}$ is a mixture Gaussian distribution: conditioning on reaching 0_E, $\ln\frac{P_1}{P_0}$ is a Gaussian distribution with mean $-\ln\beta+(\ln\bar{c}_1-\ln\bar{c}_0)-\frac{1}{2}\zeta^2\sigma^2$ and variance $\zeta^2\sigma^2$ as shown in (B9) and (B8), and conditioning on reaching 0_L, $\ln\frac{P_1}{P_0} = -\ln\beta+(\ln\bar{c}_1-\ln\bar{c}_0)$ is a constant. As a result,

$$Var^*\left[\ln\frac{P_1}{P_0}\middle| s_{-1}\right] = q[1-q]\left(\frac{1}{2}\zeta^2\sigma^2\right)^2 + q\zeta^2\sigma^2, \tag{B12}$$

where we denote $q = Prob^*(0_E|-1)$ to save notation. Comparing (B12) with (B11), $Var^*\left[\ln\frac{P_1}{P_0}\middle| s_{-1}\right] < E\left[Var^*\left[\ln\frac{P_1}{P_0}\middle| s_0\right] s_{-1}\right]$ is equivalent to

$$1-2q > q(1-q)\frac{1}{2}\zeta^2\sigma^2. \tag{B13}$$

Denote $a = e^{\frac{\beta^3}{\theta^2}\sigma^2}$ and $b = e^{\frac{\beta^2}{\theta^2}\sigma^2}$, then $q = \frac{a}{a+b}$, and inequality (B13) can be written as $\frac{b-a}{a+b} > \frac{ab}{(a+b)^2}\frac{1}{2}\zeta^2\sigma^2$, which is equivalent to:

$$b^2 - \frac{1}{2}\zeta^2\sigma^2 ab - a^2 > 0. \tag{B14}$$

The equation $b^2 - \frac{1}{2}\zeta^2\sigma^2 ab - a^2 = 0$ has two solutions. A sufficient condition for (B14) to hold is for b to be greater than the larger root of the quadratic equation

$b^2 - \frac{1}{2}\zeta^2\sigma^2 ab - a^2 = 0$. That is,

$$\frac{b}{a} > \frac{1}{2}\left[\frac{1}{2}\zeta^2\sigma^2 + \sqrt{4 + \left(\frac{1}{2}\zeta^2\sigma^2\right)^2}\right]. \tag{B15}$$

A sufficient condition for (B15) is

$$\ln\left(\frac{b}{a}\right) > \frac{1}{2}\zeta^2\sigma^2. \tag{B16}$$

To see this, we define $f(x) = e^x$ and $g(x) = \frac{1}{2}\left[x + \sqrt{4 + x^2}\right]$. Note that the left-hand side of (B15) is $f\left(\ln\left(\frac{b}{a}\right)\right)$ and the right-hand side of (B15) is $g\left(\frac{1}{2}\zeta^2\sigma^2\right)$. To establish that (B16) is a sufficient condition for (B15), it is enough to show $f\left(\ln\left(\frac{b}{a}\right)\right) > g\left(\ln\left(\frac{b}{a}\right)\right) > g\left(\frac{1}{2}\zeta^2\sigma^2\right)$. The second part of this inequality holds because $g(x)$ is a strictly increasing function. To see the first part of the inequality, we note that $f(0) = g(0) = 1$. In addition, $f'(x) = e^x$ and $g'(x) = \frac{1}{2}\left(1 + \frac{x}{\sqrt{4+x^2}}\right)$ imply that $f'(x) > g'(x) > 0$ for all $x > 0$. Given the definitions of a and b, (B16) can be written as $(1-\beta)\frac{\beta^2}{\theta^2}\sigma^2 > \frac{1}{2}\zeta^2\sigma^2$, which is equivalent to $\sqrt{1-\beta}\frac{\beta}{\theta} > \frac{\zeta}{\sqrt{2}}$. Because $\beta < 1$, inequality (B10) is a sufficient condition for $\sqrt{1-\beta}\frac{\beta}{\theta} > \frac{\zeta}{\sqrt{2}}$.

Details of results in Section IV

C1. Proof for Equation (27)

PROOF:

To compare the certainty equivalent of the utility associated with early resolution, Equation (25), and that associated with late resolution, Equation (26), it is convenient to define $e(s) = \left[\left(1 - \frac{1}{\psi}\right) V_2(s)\right]^{\frac{1-\gamma}{1-\frac{1}{\psi}}}$ and define

$$f(e|\,\bar{c}_1) = \left(\bar{c}_1^{1-\frac{1}{\psi}} + \beta e^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}}. \tag{C1}$$

Using the above notations, we can write $V_1\left(C^E\right)(s_1) = \frac{1}{1-\frac{1}{\psi}}\left[f\left(e\left(s_1\right)\right)\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}$, and $V_1\left(C^L\right)(s_1) = \frac{1}{1-\frac{1}{\psi}}\left\{f\left(E\left[e\left(s_2\right)\right]\right)\right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}}$.

The certainty equivalent of $V_1\left(C^E\right)(s_1)$, evaluated at time 1, before the revelation of s_1

can be computed as

$$\mathcal{I}\left[V_1\left(C^E\right)(s_1)\right] = h^{-1}\left(E\left[h \circ V_1\left(C^E\right)(s_1)\right]\right) = \frac{1}{1-\frac{1}{\psi}}\left(E\left[f\left(e\left(s_1\right)\right)\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}. \tag{C2}$$

Because $V_1\left(C^L\right)(s_1)$ is constant, the certainty equivalent is simply itself:

$$\mathcal{I}\left[V_1\left(C^L\right)(s_1)\right] = V_1\left(C^L\right)(s_1) = \frac{1}{1-\frac{1}{\psi}}\left\{f\left(E\left[e\left(s_2\right)\right]\right)\right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}}. \tag{C3}$$

Because s_1 and s_2 have the same distribution, the distinction between s_1 and s_2 in (C2) and (C3) is not necessary. In what follows, to simplify notation, we will suppress the state variable s and use e for both $e(s_1)$ in Equation (C2) and $e(s_2)$ in (C3). It is also clear that PER depends on the convexity of f defined in (C1).

The log difference between the two certainty equivalents can therefore be computed as:

$$\ln \mathcal{I}\left[V_1\left(C^E\right)\right] - \ln \mathcal{I}\left[V_1\left(C^L\right)\right] = \frac{1-\frac{1}{\psi}}{1-\gamma}\left\{\ln E\left[f(e)\right] - \ln f\left(E[e]\right)\right\}.$$

Denote $\bar{e} = E[e]$. Using Taylor expansion for $f(e)$ around $\bar{e}$, we have:

$$\begin{aligned} E\left[f(e)\right] - f(\bar{e}) &\approx E\left[f(\bar{e}) + f'(\bar{e})(e-\bar{e}) + \frac{1}{2}f''(\bar{e})(e-\bar{e})^2\right] - f(\bar{e}) \\ &= \frac{1}{2}f''(\bar{e})\,Var[e]. \end{aligned}$$

As a result, $\ln E[f(e)] - \ln f(\bar{e})$ can be approximated by: $\ln E[f(e)] - \ln f(\bar{e}) \approx \frac{1}{f(\bar{e})}\left[E[f(e)] - f(\bar{e})\right] = \frac{1}{2}\frac{f''(\bar{e})}{f(\bar{e})}Var[e]$. We can write

$$\ln \mathcal{I}\left[V_1\left(C^E\right)\right] - \ln \mathcal{I}\left[V_1\left(C^L\right)\right] \approx \frac{1}{2}\frac{1-\frac{1}{\psi}}{1-\gamma}\frac{f''(\bar{e})}{f(\bar{e})}Var[e]. \tag{C4}$$

We can represent the term $Var[e]$ as a linear function of the variance of $\ln V_1\left(C^E\right)$ by using a log linear approximation. Using the definition of $f(e)$, (C1), we can write $\ln V_1\left(C^E\right)$ as:

$$\ln V_1\left(C^E\right) = \ln\left[\frac{1}{1-\frac{1}{\psi}} f^{\frac{1-\frac{1}{\psi}}{1-\gamma}}(e)\right] = \ln\frac{1}{1-\frac{1}{\psi}} + \frac{1-\frac{1}{\psi}}{1-\gamma}\left[\ln f(\bar{e}) + \frac{1}{f(\bar{e})}f'(\bar{e})(e-\bar{e})\right].$$

Computing variance on both sides, we have:

$$Var\left[\ln V_1\left(C^E\right)\right] = \frac{\left(1-\frac{1}{\psi}\right)^2}{(1-\gamma)^2}\left(\frac{f'(\bar{e})}{f(\bar{e})}\right)^2 Var[e],$$

which allows us to write (C4) as

$$\begin{aligned}\ln\mathcal{I}\left[V_1\left(C^E\right)\right]-\ln\mathcal{I}\left[V_1\left(C^L\right)\right] &\approx \frac{1}{2}\frac{1-\gamma}{1-\frac{1}{\psi}}\frac{f''(\bar{e})f(\bar{e})}{(f'(\bar{e}))^2}Var\left[\ln V_1\left(C^E\right)\right]\\ &= \frac{1}{2}\frac{1-\gamma}{1-\frac{1}{\psi}}\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\left\{\frac{\bar{c}_1^{1-\frac{1}{\psi}}}{\beta\bar{e}^{\frac{1-\frac{1}{\psi}}{1-\gamma}}}\right\}Var\left[\ln V_1\left(C^E\right)\right]\\ &= \frac{1}{2}\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}\left\{\frac{\frac{1}{1-\frac{1}{\psi}}\bar{c}_1^{1-\frac{1}{\psi}}}{\beta\mathcal{I}\left[V_1\left(C^L\right)\right]}\right\}Var\left[\ln V_1\left(C^E\right)\right],\end{aligned} \tag{C5}$$

where the last line uses the definition of $\bar{e}$ to write $\bar{e}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} = \mathcal{I}\left[V_1\left(C^L\right)\right]$. We obtain Equation (27) by dividing both sides of (C5) by $1-\frac{1}{\psi}$.

LEMMA 2: *(Utility wealth relationship) Suppose the representative consumer has the Epstein-Zin preference of the form (1), where* $u(C) = \frac{1}{1-\frac{1}{\psi}}C^{1-\frac{1}{\psi}}$ *and* $\mathcal{I}[V] = h^{-1}(E[h(V)])$ *with* h *given in Equation (23). In any competitive equilibrium, the wealth-to-consumption ratio of the representative consumer satisfies*

$$\frac{W}{C} = \frac{V}{u(C)}. \tag{C6}$$

PROOF:

Let W denote aggregate wealth and C the representative consumer's consumption. The dynamic programming problem of the consumer's optimal portfolio choice problem can in general be written as

$$V\left(x,W\right)=\max_{C,\left\{\omega_j\right\}_{j=0}^{J}}\left\{\frac{1}{1-\frac{1}{\psi}}C^{1-\frac{1}{\psi}}+\beta\frac{1}{1-\frac{1}{\psi}}\left(E\left[\left(\left(1-\frac{1}{\psi}\right)V\left(x',W'\right)\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right\},$$

$$subject\ to: W'=\left(W-C\right)\sum_{j=0}^{J}\omega_jR_j$$

where $\left\{R_j\right\}_{j=0}^{J}$ is a vector of asset returns the conditional distribution of which depends on the Markov state variable x, and $\left\{\omega_j\right\}_{j=0}^{J}$ is a vector of portfolio weights that satisfies $\sum_{j=0}^{J}\omega_j=1$. We focus on the infinite horizon formulation to avoid time subscripts, but the horizon of the problem makes no difference to our argument.

The envelope condition of the above maximization problem implies:

$$\frac{\partial}{\partial W}V\left(x,W\right)=C\left(x,W\right)^{-\frac{1}{\psi}},$$

where $C\left(x,W\right)$ is the equilibrium optimal consumption policy. By homogeneity, $V\left(x,W\right)$ must be of the form $V\left(x,W\right)=\frac{1}{1-\frac{1}{\psi}}v\left(x\right)W^{1-\frac{1}{\psi}}$ for some function $v\left(x\right)$. The above envelope can therefore be written as $v\left(x\right)W^{-\frac{1}{\psi}}=C\left(x,W\right)^{-\frac{1}{\psi}}$,or, equivalently,

$$\frac{v\left(x\right)W^{-\frac{1}{\psi}}}{C\left(x,W\right)^{-\frac{1}{\psi}}}=1.$$

Multiply both sides of the above equation by $\frac{W}{C(x,W)}$, we obtain

$$\frac{W}{C\left(x,W\right)}=\frac{\frac{1}{1-\frac{1}{\psi}}v\left(x\right)W^{1-\frac{1}{\psi}}}{\frac{1}{1-\frac{1}{\psi}}C\left(x,W\right)^{1-\frac{1}{\psi}}},$$

which is (C6).

C2. Proof of Theorem 3

In this section, we first provide a proof for Theorem 3 that links the structural parameter $\bar{\eta}_{PER}$ to moments of the return on the variance claim, $Var^{*}\left[\ln W_1\right|s_0]$.

PROOF:

We first establish $Var\left[\ln W_1\right|s_0]=Var^{*}\left[\ln W_1\right|s_0]$. Similar to Equation (29), we can write the stochastic discount factor $SDF\left(s_0, s_1\right)$ as

$$\ln SDF\left(s_0, s_1\right) = cons - \frac{\gamma - \frac{1}{\psi}}{1 - \frac{1}{\psi}} \ln V_1\left(s_1\right).$$

Given the above form of the SDF, and using the fact that the conditional distribution of $\ln V_1\left(s_1\right)$ is Gaussian given s_0, the risk-neutral density is proportional to

$$e^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}x} \times f\left(x\middle|\,\mu\left(s_0\right), Z\left(s_0\right)\right),$$

where $f\left(x\middle|\,\mu, Z\right)$ is the Gaussian density with mean μ and variance Z. It is straightforward to show that the above density is also a Gaussian distribution with the same variance, $Z\left(s_0\right)$, but a different mean.

To save notation, we denote $Z = Var\left[\ln W_1\middle|\, s_0\right]$ and assume that Z follows a Gamma distribution with parameter (α, β). That is, the density of Z is given by, for $z > 0$,

$$f\left(z\middle|\,\alpha, \beta\right) = \frac{\beta^\alpha}{\Gamma\left(\alpha\right)} z^{\alpha-1} e^{-\beta z}, \tag{C7}$$

where $\Gamma\left(\alpha\right)$ is the Gamma function. Using Equation (29), we can write $SDF\left(s_{-1}, s_0\right) = e^{\hat{\eta}_0 - \eta Z}$, where $\eta = \frac{1}{2}(\gamma - \frac{1}{\psi})\bar{\eta}_{PER}$. This allows us to compute the expected payoff of the variance portfolio as:

$$E\left[Z\right] = \frac{\beta^\alpha}{\Gamma\left(\alpha\right)} \int_0^\infty z \times z^{\alpha-1} e^{-\beta z} dz = \frac{\alpha}{\beta},$$

the present value of the variance portfolio as

$$E\left[e^{\hat{\eta}_0 - \eta Z} Z\right] = \frac{\beta^\alpha}{\Gamma\left(\alpha\right)} \int_0^\infty e^{\hat{\eta}_0 - \eta z} z \times z^{\alpha-1} e^{-\beta z} dz = e^{\hat{\eta}_0} \frac{\alpha \beta^\alpha}{\left(\eta + \beta\right)^{\alpha+1}},$$

and the risk-free rate from period -1 to period 0 as

$$R_f\left(-1\right) = \frac{1}{E\left[ASDF_{-1,0}\right]} = \left[\frac{\beta^\alpha}{\Gamma\left(\alpha\right)} \int_0^\infty e^{\hat{\eta}_0 - \eta z} \times z^{\alpha-1} e^{-\beta z} dz\right]^{-1} = \left[e^{\hat{\eta}_0} \left(\frac{\beta}{\eta + \beta}\right)^\alpha\right]^{-1}.$$

The risk premium, $\frac{E[R_{-1,0}]}{R_f(-1)} - 1$ can therefore be computed as

$$\frac{E\left[R_{Var}(-1, 0)\right]}{R_f\left(-1\right)} - 1 = \frac{E\left[Z\right]}{E\left[e^{\hat{\eta}_0 - \hat{\eta} Z} Z\right]} \frac{1}{R_f\left(-1\right)} - 1 = \frac{\eta}{\beta}.$$

Given the definition of $\eta = \frac{1}{2}(\gamma - \frac{1}{\psi})\bar{\eta}_{PER}$, the above implies

$$\eta = \frac{1}{2}\left(\gamma - \frac{1}{\psi}\right)\bar{\eta}_{PER} = \beta \times \left\{\frac{E\left[R_{Var}(-1,0)\right]}{R_f\left(-1\right)} - 1\right\}.$$

To prove Equation (30), we note that $\beta = \frac{E[Z]}{Var[Z]}$ due to the property of the Gamma distribution.

Equation (30) relates the value of the structural parameter $\bar{\eta}_{PER}$ to moments of the variance claim, $Var^*\left[\ln W_1 | s_0\right]$. Empirically, we do not observe directly the price of the claim to the announcement-day variance. Instead, the variance portfolio we construct in Section III contains both announcement-day variance and non-announcement-day variances. The following corollary shows that, under the assumption that non-announcement day variance is a constant—as in Johannes, Kaeck and Seeger (2023)—we can replace the moments of the variance claim in Equation (30) by moments of the variance portfolio after adjusting for a leverage parameter, which we can estimate in the data.

We denote the payoff of the variance portfolio we construct in Section III as $X = Var^*\left[\ln W_1 | s_0\right] + Y$, where Y is total variance on non-announcement days and is assumed to be a constant. We continue to assume that the announcement-day variance, $Var^*\left[\ln W_1 | s_0\right]$ follows a Gamma distribution with parameters (α, β).

COROLLARY 1: *The structural parameter $\bar{\eta}_{PER}$ in Equation ((30)) can be related to the PER premium on the variance portfolio, X, by:*

$$\bar{\eta}_{PER} = \frac{2}{\gamma - \frac{1}{\psi}} \frac{\lambda_0}{\lambda_{-1}} \frac{E\left[X\right]}{Var\left[X\right]} \left\{\frac{E\left[R_X(-1,0)\right]}{E\left[R_f(-1)\right]} - 1\right\}, \tag{C8}$$

where $\lambda_0 = \frac{E[Var^*[\ln W_1|s_0]]}{E[X|s_0]}$ *and* $\lambda_{-1} = \frac{PV_{-1}[Var^*[\ln W_1|s_0]]}{PV_{-1}[X]}$.

PROOF:

We continue to assume that $Z = Var^*\left[\ln W_1 | s_0\right]$ follows the Gamma distribution as in Equation (C7). Given $ASDF_{-1,0} = e^{\hat{\eta}_0 - \eta Z}$, where $\eta = \bar{\eta}_{GRS}\bar{\eta}_{PER}$, we can compute the expected payoff of the variance portfolio as:

$$E\left[Z + Y\right] = \frac{\beta^\alpha}{\Gamma\left(\alpha\right)} \int_0^\infty \left(z + Y\right) \times z^{\alpha - 1} e^{-\beta z} dz = \frac{\alpha}{\beta} + Y,$$

the risk-free rate as:

$$R_f(-1,0) = \frac{1}{E\left[ASDF_{-1,0}\right]} = \left[\frac{\beta^\alpha}{\Gamma\left(\alpha\right)} \int_0^\infty e^{\hat{\eta}_0 - \eta z} \times z^{\alpha - 1} e^{-\beta z} dz\right]^{-1} = \left[e^{\hat{\eta}_0}\left(\frac{\beta}{\eta + \beta}\right)^\alpha\right]^{-1},$$

and the present value of the variance portfolio as:

$$PV(X) = E\left[e^{\hat{\eta}_0-\eta Z}(Z+Y)\right] = \frac{\beta^\alpha}{\Gamma(\alpha)}\int_0^\infty e^{\hat{\eta}_0-\eta z} z\times z^{\alpha-1}e^{-\beta z}dz = e^{\hat{\eta}_0}\frac{\alpha\beta^\alpha}{(\eta+\beta)^{\alpha+1}}+\frac{Y}{R_f(-1)} = \frac{\frac{\alpha}{\eta+\beta}+Y}{R_f(-1)}.$$

The risk premium, $\frac{E[R_X(-1,0)]}{R_f(-1)} - 1$ can therefore be computed as

$$\frac{E\left[R_X(-1,0)\right]}{R_f(-1)} - 1 = \frac{E\left[Z+Y\right]}{E\left[e^{\hat{\eta}_0-\eta Z}(Z+Y)\right]}\frac{1}{R_f(-1)} - 1 = \frac{\frac{\alpha}{\beta}+Y}{\frac{\alpha}{\eta+\beta}+Y} - 1 = \frac{\frac{\alpha}{\eta+\beta}}{\frac{\alpha}{\eta+\beta}+Y}\frac{\eta}{\beta}.$$

Here, $\frac{\frac{\alpha}{\eta+\beta}}{\frac{\alpha}{\eta+\beta}+Y} = \frac{PV(Z)}{PV(Z+Y)} = \lambda_{-1}$ is the market value share of the announcement variance before the revelation of τ (i.e., before the period of ROI). $\frac{\eta}{\beta} = \frac{E[R_{Var}(-1,0)]}{R_f(-1)} - 1$ is the PER premium on the variance claim Z. It follows that the risk premiums on the variance claim Z and that on the variance portfolio X are related by:

$$\frac{E\left[R_X(-1,0)\right]}{R_f(-1)} - 1 = \lambda_{-1}\left(\frac{E\left[R_{Var}(-1,0)\right]}{R_f(-1)} - 1\right). \tag{C9}$$

Furthermore, $E[X] = E[Z+Y] = \frac{\alpha}{\beta} + Y = \frac{\frac{\alpha}{\beta}+Y}{\frac{\alpha}{\beta}}\frac{\alpha}{\beta} = \frac{\frac{\alpha}{\beta}+Y}{\frac{\alpha}{\beta}}E[Z]$. Denote $\lambda_0 = \frac{\frac{\alpha}{\beta}}{\frac{\alpha}{\beta}+Y}$. It is the market value share of the announcement variance after the revelation of τ (i.e., after the period of ROI). $Var[X] = Var[Z+Y] = Var[Z]$ as Y is a constant. It then follows that:

$$\frac{E[X]}{Var[X]} = \frac{1}{\lambda_0}\frac{E[Z]}{Var[Z]}. \tag{C10}$$

Combining (C9) and (C10), we have:

$$\eta = \frac{1}{2}(\gamma-\frac{1}{\psi})\bar{\eta}_{PER} = \frac{E[Z]}{Var[Z]}\left\{\frac{E\left[R_{Var}(-1,0)\right]}{R_f(-1)} - 1\right\} = \frac{\lambda_0}{\lambda_{-1}}\frac{E[X]}{Var[X]}\left\{\frac{E\left[R_X(-1,0)\right]}{R_f(-1)} - 1\right\},$$

which is (C8).

In Appendix D.D2, we show how to use the estimation procedure in Johannes, Kaeck and Seeger (2023) to compute the values of λ_{-1}, λ_0, $E[X]$, and $Var[X]$.

C3. Repeated one-period early resolution of uncertainty

To compute the welfare gain of infinitely repeated one-period early resolution of uncertainty, it is easier to adopt a homogenous of degree one normalization of recursive preference by defining $U = V^{\frac{1}{1-\frac{1}{\psi}}}$, where V is defined by the recursion of the form in

Equation (26). The recursion for U is given by:

$$U = \left\{ \frac{1}{1-\frac{1}{\psi}} C^{1-\frac{1}{\psi}} + \beta m \left(U'\right)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}, \tag{C11}$$

where $m(U) = \left(E\left[U^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}$ is a constant returns to scale aggregator. In the above formulation, utility is homogenous of degree one in consumption units.

Suppose a one-time early resolution of uncertainty is equivalent to raising permanent consumption by λ times. Let T be the operator that computes the welfare gain of a one-time early resolution of uncertainty.[23] Under our assumption, $TU = \lambda U$. Let T^n denote the operator that computes welfare gain for n-time repeated early resolution of uncertainty. We assume the welfare gain of n-time early resolution of uncertainty to be $\lambda(n)$ and write $T^n U = \lambda(n) \times U$. Below, we derive a recursion for $\lambda(n)$.

By the definition of the T operator,

$$T^{n+1}U = \lambda \left\{ \frac{1}{1-\frac{1}{\psi}} C^{1-\frac{1}{\psi}} + \beta m \left(T^n U'\right)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}.$$

Because m is constant returns to scale, we have $m(T^n U') = m(\lambda(n) U') = \lambda(n) m(U')$. We can use the above equation to write a recursion for $\lambda(n)$:

$$T^{n+1}U = \lambda(n+1) U = \lambda \left\{ \frac{1}{1-\frac{1}{\psi}} C^{1-\frac{1}{\psi}} + \beta \lambda(n)^{1-\frac{1}{\psi}} m \left(U'\right)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}.$$

By recursion (C11), $\frac{\frac{1}{1-\frac{1}{\psi}} C^{1-\frac{1}{\psi}}}{U} + \frac{\beta m(U')^{1-\frac{1}{\psi}}}{U} = 1$. We denote $\delta = \frac{\frac{1}{1-\frac{1}{\psi}} C^{1-\frac{1}{\psi}}}{U}$. This allows us to divide the above equation by U and obtain:

$$\lambda(n+1) = \lambda \left\{ \delta + (1-\delta) \lambda(n)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}.$$

Clearly, if $\lambda^* = \lim_{n\to\infty} \lambda(n)$ exists, it must satisfy:

$$\lambda^* = \lambda \left\{ \delta + (1-\delta) \lambda^{*1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}},$$

[23] In our setup, T can be written as: $TV = \left(E\left\{ \frac{1}{1-\frac{1}{\psi}} C^{1-\frac{1}{\psi}} + \beta \left(E\left[U^{1-\gamma} \middle| s\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1-\gamma}{1-\frac{1}{\psi}}} \right)^{\frac{1}{1-\gamma}}$, where s represents the information revealed by early resolution.

which allows us to obtain the welfare gain of infinite-horizon repeated one-period early resolution, λ^*, as a function of λ. In our estimation, $\lambda = 1.000028$. In a standard long-run risk model,[24] $\delta = 0.0029$ with a horizon of 1.5 months,[25] which yields a value of $\lambda^* = 1.0098$.

Data Appendix

D1. Constructing the variance portfolios

Our FOMC announcement dates come from the Federal Reserve's website (Federal Reserve System, Board of Governors 2020). VIX futures data come from Bloomberg (Bloomberg 2022). The Fama-French 3 factors were downloaded from Ken French's website (Fama and French 1993). S&P 500 close prices come from Yahoo Finance (Yahoo Finance 2023). Data on Fed-related news come from RavenPack Analytics (RavenPack Analytics 2022). VIX data come from the CBOE's website (Chicago Board Options Exchange 2020), and option price data come from OptionMetrics (OptionMetrics 2020). While other data are straightforward to use, the handling of the option data is more involved. Below we describe our data construction process in detail.

We first get the bid and ask data of the options to the underlying-expiration-strike price-put/call-day level. That is, for a put or call option on a certain underlying with a given expiration date and strike price, there is one price per day.[26] Having a panel at the underlying-expiration-strike price-put/call-day level, we take the average of bid and ask to get the option price. This price can be missing, however, even for large underlyings such as the S&P 500 index. This is because price inquiries can be rare for deeply in-the-money or out-of-money options. In the event that a price becomes missing and reappear on a future date, we forward fill the price, assuming a return of zero. If the price becomes missing without reappearing, we replace the first missing price with zero if the option is a call and the last available call price is less than the put price of the same strike and expiration, and with the last available price if the last available call price is greater than or equal to the put price. Similarly, if the option is a put, we replace the missing price with 0 if its price is less than the call with the same strike and expiration, and with the last available price if it is greater than the put price. This logic roughly imputes a zero final return if the option is in the money, and a return of -100% if it is out of money. While this operation is conceptually importantly, our results are robust to alternative imputation methods such as assuming all final returns are 0.

Having the prices, we construct the synthetic variance portfolios using these S&P 500 options and compute their returns. We construct these variance portfolios following the formulas in Bakshi, Kapadia and Madan (2003), with additional data cleaning procedures

[24] $\gamma = 10$, $\psi = 1.5$, $\beta = 0.998$, $\sigma = 0.0078$, $a = 0.979$ and $\varphi = 0.044$ following Epstein, Farhi and Strzalecki (2014).

[25] δ is mainly driven by β and is roughly $1.5 \times (1 - \beta)$.

[26] In the raw data, cases exist where there are two prices per day. Such cases occur because there are two types of options, e.g., standard monthly options and weekly options, that happen to share the same underlying, expiration, strike price, and put/call, and are both outstanding on the same day. In those cases, we take the average of the two available prices.

taken from the construction of the VIX index to avoid unreliable data. These procedures are available in the VIX white paper on CBOE's website, and we also describe our methodology in detail below.

Overall, the portfolio on any given day consists of out-of-money options, which are call options with strike prices higher than the previous close price of the underlying, and put options with strike prices lower than that close price. Out-of-money options with zero bid prices are excluded from the portfolio. Also, those with two consecutive zero bids between them and the at-the-money strike price are also excluded. For instance, suppose a call with strike 100 has a non-zero bid price, and on that day the at-the-money strike price is 30. Suppose the two strike prices immediately lower than 100 are 95 and 90, and calls with those two strike prices both have zero bids. Then the call option with strike price of 100 will be excluded even though it is an out-of-money option with a non-zero bid. This data exclusion logic is adopted from the CBOE's methodology in constructing the VIX index. Additionally, we require that an equal number of puts and calls are included in our variance-paying portfolio. This is to ensure that the portfolios are balanced between puts and calls and do not have strong directional delta exposure. We then manually examine these prices to censor obvious data errors that violate the monotonicity of option prices with respect to strike prices.[27]

Having the sample, we now discuss the weight of each option in the variance swap portfolio. Say an option has a strike price of K, and the two nearby strike prices flanking K for that underlying-expiration-day are K^- and K^+. Let the underlying's close price on the previous trading day be S. For the second-moment portfolio, the relative weight on the option with strike K is $\frac{(K^+-K^-)}{2}\frac{1-\log(K/S)}{K^2}$. If the strike price is the highest or the lowest for that underlying-expiration-day, the weight is then $\frac{(K-K^-)}{2}\frac{1-\log(K/S)}{K^2}$ or $\frac{(K^+-K)}{2}\frac{1-\log(K/S)}{K^2}$, respectively.

We then rescale these relative weights so that they add up to 1 for each underlying-expiration-day. Weighted-returns on these portfolios are then computed at the daily level, and then aggregated to the weekly level, where the weekly periods are defined relative to FOMC announcements as described in Section III.C.

D2. Estimating prices of the components of the variance portfolios

To estimate the relevant quantities in Corollary 1 needed to construct $\bar{\eta}_{PER}$, let $\sigma^{*2}_{t,T}$ be the time-t, per-day risk-neutral expectation of the variance between t and T. By definition,

$$\sigma^{*2}_{t,T} = \frac{1}{T-t}\left\{\sum_{j=t}^{T} E^*_t\left[Z_j\right] I_{j\in FOMC} + \left(\sum I_{j\notin FOMC}\right) Y\right\},$$

where $Z_j = Var^*\left[\ln W_1 \middle| s_0\right]$ is the implied variance for an announcement-day return, which is assumed to follow a Gamma distribution. $I_{j\in FOMC}$ is the indicator function that takes

[27] There are only 3 cases of those errors.

the value of 1 if j is an FOMC announcement day. Because Z_j is i.i.d. and the non-announcement-day implied variance Y is constant, following Johannes, Kaeck and Seeger (2023), we rewrite the above equation as:

$$\sigma_{t,T}^{*2} = Frac_{t,T}^{FOMC} \times E_t^* [Z] + \left(1 - Frac_{t,T}^{FOMC}\right) Y, \tag{D1}$$

where $Frac_{t,T}^{FOMC} = \frac{\sum_{j=t}^{T} I_{j \in FOMC}}{T-t}$ is the fraction of FOMC days during the period between t and T.

Because the application of Corollary 1 requires estimation of the pre-ROI risk-neutral expectation of implied variance, $PV_{-1}(Z)$, and the expectation of risk-neutral implied variance, $E[Z]$, we use the following panel regression to estimate the implied variance in Equation (D1):

$$\sigma_{t,T}^{*2} = \alpha + \beta_1 Frac_{t,T}^{FOMC} \cdot I_t^{ROIStart} + \beta_2 Frac_{t,T}^{FOMC} \cdot I_t^{ROIEnd} + \epsilon_{t,T}. \tag{D2}$$

Here, $I_t^{ROIStart}$ is a dummy variable taking the value of 1 if t is at the beginning of a period of ROI (6 weekdays before an FOMC announcement), and I_t^{ROIEnd} takes the value of 1 if t is at the end of a period of ROI (1 weekday before an FOMC announcement). Under our assumptions, the regression coefficient α is the average implied variance on non-announcement days, $\alpha + \beta_1$ is the pre-ROI present value of FOMC-day implied variance, $PV_{-1}(Z)$, and $\alpha + \beta_2$ is interpreted as the average level of FOMC-day implied variance, $E[Z]$. Like those in Table 4, this regression also equally weights each t.

In our estimation, $\hat{\alpha}$, $\hat{\alpha} + \hat{\beta}_1$, and $\hat{\alpha} + \hat{\beta}_2$ are 1.14e-4, 1.40e-4, and 2.64e-4, respectively. These estimates allow us to compute λ_{-1} and λ_0, the average market share of the announcement variance claim before and after the period of ROI. Given that in our sample, the average variance portfolio expires 25 weekdays away, these estimates translate into a λ_{-1} of $\frac{1.40}{1.40+24\times1.14} = 0.048$, and a λ_0 of $\frac{2.64}{2.64+24\times1.14} = 0.088$, which then lead to a $\frac{\lambda_0}{\lambda_{-1}}$ of 1.81 for the average variance portfolio.

The components $E[X]$ and $Var[X]$ in Corollary 1 are the mean and variance of the price of the variance portfolios and can also be estimated in this setting. Specifically, $E[X] = (2.64 + 24 \times 1.14)$e-4 = 29.67e-4. $Var[X]$ is estimated by running regression (D1) cross section by cross section (i.e., for each t), computing estimates $\hat{\alpha}_t + \hat{\beta}_t$ for each t that is at the end of a period of ROI, and then computing their variance across these cross sections. This yields a variance of 9.18e-6.

D3. A simultaneous hypothesis test

Our identification of PER requires four steps: variations in informativeness, predictability of informativeness, a period of resolution of informativeness, and the existence of a PER premium. In each step, we use the conventional t-statistics to individually test a corresponding null hypothesis. However, the identification of PER requires all four steps; therefore, a simultaneous test for all four hypotheses is more appropriate. In this section, we compute a simple upper bound for the family-wise error rate (FWER), the statistical concept established for multiple hypothesis testing problems. We then implement the classic sequentially rejective test developed by Holm (1979). Both procedures indicate that our four estimates are simultaneously significant at the 5% level.[28]

We denote the four null hypotheses as H_1, H_2, H_3, and H_4:

- H_1: $\xi = 0$ in Column (3) of Table 1, i.e., no IV reduction on FOMC days.
- H_2: $\xi_3 = 0$ in Column (6) of Table 2, i.e., inverse slope does not predict IV reduction on FOMC days.
- H_3: β in Column "[-5, 1]" equals β in Column "All t" of Table 3, i.e., attention on the Fed does not differentially correlate with changes of inverse slope in the periods of ROI.
- H_4: $\beta = 0$ in Column (1) of Table 4, i.e., variance portfolios have zero PER premium.

Let $\mathcal{H} \subseteq \{H_1, H_2, H_3, H_4\}$ be the set of true null hypotheses among H_1, H_2, H_3, and H_4. Given $\mathcal{H}$, the family-wise error rate (FWER) is defined as the probability of falsely rejecting *any* of the true null hypotheses in $\mathcal{H}$:

$$FWER = Pr\left(\text{Reject any } H_j \in \mathcal{H} \middle|\ H_i \text{ is true}, \forall H_i \in \mathcal{H}\right).$$

Below, we show that FWER is bounded above by the sum of the type I error rates of the four individual hypothesis tests. Because the probability of the union is less than the sum of the probabilities,

$$\begin{aligned} Pr\left(\text{Reject any } H_j \in \mathcal{H} \middle|\ H_i \text{ is true}, \forall H_i \in \mathcal{H}\right) &\leq \sum_{H_j \in \mathcal{H}} Pr\left(\text{Reject } H_j \middle|\ H_i \text{ is true}, \forall H_i \in \mathcal{H}\right), \\ &= \sum_{H_j \in \mathcal{H}} Pr\left(\text{Reject } H_j \middle|\ H_j \text{ is true}\right). \end{aligned}$$

The second equality above is due to the fact that the sampling distribution of each t-statistic used for an individual hypothesis test depends only on the parameter of interest under that hypothesis.[29] This implies that $FWER$ is bounded above by the sum of the four individual type-one error rates:

$$FWER \leq \sum_{H_j \in \mathcal{H}} Pr\left(\text{Reject } H_j \middle|\ H_j \text{ is true}\right) \leq \sum_{j=1}^{4} Pr\left(\text{Reject } H_j \middle|\ H_j \text{ is true}\right).$$

This relation holds for any $\mathcal{H}$. The individual type-one error rates (i.e., p-values) in our four tests are: 2.91e-05, 0.0271, 2.44e-05, 0.0046, which add up to 0.0318. Given the relation

[28] See Miller (1981) and Hochberg and Tamhane (2009) for textbooks on such procedures.

[29] This is known as the pivotal property of t-statistics.

above, we know that the FWER is less than or equal to 3.18%, and thus the simultaneous test is significant at the 5% level for any $\mathcal{H}$.[30]

Although the summation approach above is straightforward, Holm (1979)'s classic step-down procedure is considerably more powerful. It orders the p-values of the individual tests from lowest to highest as $p_{(1)}, p_{(2)}, \cdots, p_{(m)}$, and rejects hypotheses $H_{(1)}, H_{(2)}, \cdots H_{(k-1)}$ where k is the minimal index such that $p_{(k)} > \frac{\alpha}{m-k+1}$, and α is the desired significance level. Holm (1979) shows that the FWER across these m tests is less than or equal to α for free combinations. In our case, because $p_{(1)} = 2.44e-05 \leq 0.05/4$, $p_{(2)} = 2.91e-05 \leq 0.05/3$, $p_{(3)} = 0.0046 \leq 0.05/2$, $p_{(4)} = 0.0271 \leq 0.05$, all null hypotheses are again simultaneously rejected at the 5% level.

[30]In statistics, this absence of dependence on a specific $\mathcal{H}$ is often referred to as "for free combinations."