Supplemental Appendix to Financial Frictions: Micro vs Macro Volatility

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This appendix contains (a) Additional results and information for Section II of the paper; (b) The details of the choice problems in Section III of the paper; (c) Details of the estimation of the household income process; (d) Further results for the baseline model; (e) Details of the three asset model and further results for this model.

Additional Results and Information for Section II

Table A1 reports descriptive statistics of the household dataset examined in Section II. Denmark entered a cyclical downturn at the onset of the financial crisis and the recovery started in 2014. The cyclical dynamics are reflected by changes in average consumption expenditures and asset values. The average ratio of net household wealth to disposable income displays considerable fluctuations over time, while the ratio of net liquid assets to disposable income (excl. housing and mortgages) is stable and close to one on average.

Table A2 reports the results of estimating Equation (3) when either capitalizing car expenditures or excluding households that purchase a car from the data in the year of the purchase. As is evident, the coefficient estimates are robust to the treatment of car spending and similar to those reported in Table 1.

Table A3 reports the results from estimating:

$$\text{(A1)} \qquad \begin{aligned} \Delta \log c_{i,t} &= \sum_j \mathbf{1}_{A_{i,t} \in A_j^{Net}} \left(\beta_{0,j} \Delta \log y_{i,t} + \beta_{1,j} R_{i,t}^S + \beta_{2,j} R_{i,t}^S \Delta \log y_{i,t} \right) \\ &+ \eta X_{i,t} + \alpha_i + \gamma_t + \varepsilon_{i,t} \end{aligned}$$

We also experiment with including or excluding a household fixed effect. The results are similar to those reported in Table 1 except for the effect of the spread on above-median wealth households when we first differenced consumption and omitted the household fixed effect.

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TABLE A1—DESCRIPTIVE STATISTICS

	(1) 2007 mean	(2) 2012 mean	(3) 2017 mean
Net wealth	747,251.61	448,838.04	567,918.01
Assets	1,188,575.63	927,978.87	1,012,632.15
Debt	441,324.02	479,140.83	444,714.13
Liquid wealth	279,290.77	248,087.95	248,530.79
Share net zero wealth	0.08	0.10	0.11
Disposable income	242,772.93	250,876.25	261,013.50
Labor income	246,299.02	235,152.34	241,692.47
Consumption	257,785.87	233,279.09	248,923.70
Age of household head	51.16	50.93	50.86
Household size	1.85	1.84	1.81
N	2,145,397	2,316,459	2,395,008

Note: Net wealth is the sum of housing wealth, portfolio wealth, bank deposits, and bank and mortgage debt, as well as some major durable goods such as cars. Assets are gross assets, liabilities are gross liabilities. Liquid assets are defined as net wealth less housing and mortgages. Unless otherwise stated, all numbers are averages and deflated to 2003 Danish Kroner.

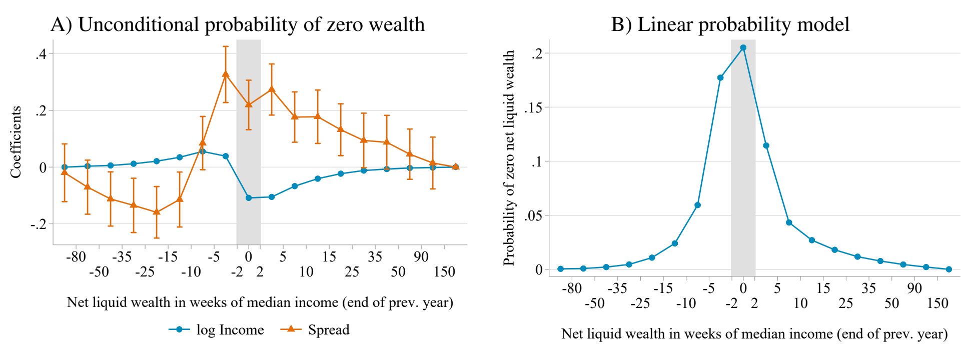

FIGURE A1. WEALTH DYNAMICS EXCLUDING HOUSING AND MORTGAGES

Note: The figure shows unconditional transition probabilities to the zero net wealth state by net wealth decile (Panel A) and the change in transition probabilities with cross-sectional changes in income and spread (Panel B), estimated from equation (2). Vertical bars are 95 percent confidence bands. The net wealth measure excludes housing assets and mortgage debt. Zero wealth is indicated by grey shading and defined as net assets within a range of plus/minus two weeks of median household income.

TABLE A2—ROBUSTNESS: DIFFERENT TREATMENT OF CAR PURCHASES

	(1)	(2)	(3)	(4)
log income	0.375***		0.358***	
	(0.00308)		(0.00325)	
Low net wealth × log Income		0.400***		0.384***
		(0.00398)		(0.00429)
High net wealth × log Income		0.339***		0.320***
		(0.00391)		(0.00402)
Spread	-0.268***		-0.251***	
	(0.0151)		(0.0145)	
Low net wealth × Spread		-0.339***		-0.286***
		(0.0177)		(0.0164)
High net wealth × Spread		-0.123***		-0.145***
		(0.0208)		(0.0204)
log income × Spread	1.131***		1.131***	
	(0.0671)		(0.0714)	
Low net wealth × log Income × Spread		1.357***		1.409***
		(0.0893)		(0.0975)
High net wealth × log Income × Spread		0.737***		0.647***
		(0.0864)		(0.0887)
#Observations	17,935,813	17,935,813	15,177,507	15,177,507
R^2	0.580	0.584	0.613	0.616
RMSE	0.269	0.268	0.234	0.233

Note: The table reports the relationship of consumption with income, consumer credit spreads and their interaction, estimated from Equation (3). High (low) net wealth denotes households above (below) the median. Standard errors clustered at the household level. In columns (1) and (2) cars are capitalized using their official tax value. (3) and (4) exclude households that have purchased a car in the current or previous year from the sample.

TABLE A3—ROBUSTNESS: RESULTS FOR FIRST-DIFFERENCE SPECIFICATION

	(1)	(2)	(3)	(4)
$\triangle$log income	0.296***		0.311***	
	(0.00440)		(0.00530)	
Low net wealth × $\triangle$log income		0.322***		0.347***
		(0.00566)		(0.00705)
High net wealth × $\triangle$log income		0.248***		0.264***
		(0.00648)		(0.00750)
Spread	-0.197***		-0.346***	
	(0.00562)		(0.0151)	
Low net wealth × spread		-0.268***		-0.519***
		(0.00804)		(0.0196)
High net wealth × spread		-0.0868***		-0.110***
		(0.00880)		(0.0205)
$\triangle$log income × spread	2.327***		2.171***	
	(0.107)		(0.128)	
Low net wealth × $\triangle$log income × spread		2.462***		2.286***
		(0.137)		(0.169)
High net wealth × $\triangle$log income × spread		1.893***		1.826***
		(0.160)		(0.183)
#Observations	17,313,355	17,313,355	16,546,917	16,546,917
R^2	0.0616	0.0634	0.124	0.133
RMSE	0.348	0.347	0.364	0.362

Note: The table reports the relationship of consumption with income, consumer credit spreads and their interaction, estimated from Equation (3). High net wealth denotes households above the median and low net wealth those below. Columns (1) and (2) for exclude household specific trends, columns (3) and (4) include household specific trends. Standard errors clustered at the household level.

Choice Problems in Baseline Model

B1. Households

The dynamic programs faced by households can be formulated as follows. First, to simplify notation, remove time-subscripts and let $\mathbf{b}_i = (b_i^G, b_i^D, b_i^L)$ denote household i's beginning of period asset portfolio, and $\mathbf{S}$ the vector of relevant aggregate state variables. Let $\mathscr{V}_i^s$ denote the value functions for a worker household ($s = w$) and for a rentier ($s = r$). A worker's Bellman equation is given as:

$$\mathscr{V}_i^w(\mathbf{b}_i, h_i, \mathbf{S}) = \max\left[\mathrm{u}(c_i, l_i) + \beta\mathbb{E}((1-\phi_w)\mathscr{V}i^w\left(\mathbf{b}_i', h_i', \mathbf{S}'\right) + \phi_w\mathscr{V}_i^r\left(\mathbf{b}_i', \mathbf{S}'\right))\right],$$

subject to (6)-(7) and to the flow budget constraint:

$$c_i + (b_i^G)' + (b_i^D)' - (b_i^L)' \leq (1-\tau_h)wh_il_i + R_S\left(b_i^G + b_i^D\right) - R_Lb_i^L,$$

where τ_h is a proportional income tax rate and a prime denotes next period. For rentiers instead:

$$\mathscr{V}_i^r(\mathbf{b}_i, 0, \mathbf{S}) = \max\left[u(c_i, l_i) + \beta\mathbb{E}(\phi_r\mathscr{V}_i^w\left(\mathbf{b}_i', 1, \mathbf{S}'\right) + (1-\phi_r)\mathscr{V}_i^r\left(\mathbf{b}_i', 0, \mathbf{S}'\right))\right],$$

subject to (6)-(7) and to the flow budget constraint:

$$c_i + (b_i^G)' + (b_i^D)' - (b_i^L)' \leq (1-\tau_h)\mathscr{F} + R_S\left(b_i^G + b_i^D\right) - R_Lb_i^L.$$

B2. Banks

Banks face the following optimization problem:

$$\mathbf{V}^b(n_t^z, S_t) = \max\mathbb{E}_t\beta\left((1-\theta)n_{t+1}^z + \theta V^b\left(n_{t+1}^z\right)\right)$$

subject to (10) and to:

$$\lambda a_t^z \leq \mathbf{V}^b(n_t^z, S_t)$$

where $a_t^z = \left(Q_tb_{F,t+1}^z + b_{D,t+1}^z\right)$ are the bank's assets. To solve this, guess that:

$$\mathbf{V}^b(n_t^z, S_t) = \rho_tn_t^z$$

Subject to this guess, (12) can be expressed as a constraint on leverage, l_t^s:

$$l_t^z = \frac{a_t^z}{n_t^z} \leq \frac{\rho_t}{\lambda}$$

Substituting (10) into (13), we can then express the bank's value as:

$$\begin{aligned}\rho_t n_t^z &= \max \mathbb{E}_t [\beta ((1-\theta)+\theta\rho_{t+1}) (R_{K,t+1} - R_{S,t+1}) a_t^z \\ &\quad + \beta ((1-\theta)+\theta\rho_{t+1}) R_{S,t+1} n_t^z]\end{aligned}$$

The first-order necessary conditions and the envelope condition are:

$$\begin{aligned}\mu_t^z \lambda &= \mathbb{E}_t [\beta ((1-\theta)+\theta\rho_{t+1}) (R_{K,t+1} - R_{S,t+1}) \\ 0 &= \mu_t^z [\rho_t n_t^z - \lambda a_t^z] \\ \rho_t &= \frac{\mathbb{E}_t \beta ((1-\theta)+\theta\rho_{t+1}) R_{S,t+1}}{1-\mu_t^z}\end{aligned}$$

where $\mu_t^z \geq 0$ is the Kuhn-Tucker multiplier on (12). When the incentive constraint binds, banks expect to earn excess returns on their investments relative to the cost of capital (the deposit rate), $\mathbb{E}_t (R_{K,t+1} - R_{S,t+1}) > 0$, otherwise they equalize. We now impose that the incentive constraint binds so that leverage is equalized across banks. Given this, the Kuhn-Tucker multiplier is identical across banks and given as:

$$\mu_t = \max \left(1 - \frac{\mathbb{E}_t \beta ((1-\theta)+\theta\rho_{t+1}) R_{S,t+1} N_t}{\lambda A_t}, 0 \right) \in (0,1)$$

where $N_t = \int n_t^z dz$, $A_t = \int a_t^z dz$. This confirms the guess on the value function and implies:

$$\begin{aligned}\rho_t &= \frac{\mathbb{E}_t \beta ((1-\theta)+\theta\rho_{t+1}) R_{S,t+1}}{1 - \mathbb{E}_t [\beta ((1-\theta)+\theta\rho_{t+1}) (R_{K,t+1} - R_{S,t+1}) / \lambda} \\ l_t &= \frac{\rho_t}{\lambda} = \frac{\mathbb{E}_t \beta ((1-\theta)+\theta\rho_{t+1}) R_{S,t+1}}{\lambda - \mathbb{E}_t [\beta ((1-\theta)+\theta\rho_{t+1}) (R_{K,t+1} - R_{S,t+1})}\end{aligned}$$

The equilibrium law of motion of an individual bank z's net worth is then:

$$n_{t+1}^z = (l_t R_{K,t+1} + (1-l_t) R_{S,t+1}) n_t^z$$

The aggregate banking sector net worth now follows from noting that l_t is independent of net worth.

B3. Goods Producers

Let $\mathbf{V}_r^F \left(P_{r,t-1}^F, S_t \right)$ denote the expected present value of real profits of a producer that charged the nominal price $P_{r,t-1}^F$ last period. Goods producers then solve the problem:

$$\mathbf{V}_r^F \left(P_{r,t-1}^F, S_t \right) = \max_{P_{r,t}^F} \left(\upsilon_{r,t}^G + \beta \mathbb{E}_t \mathbf{V}_r^F \left(P_{r,t}^F, S_{t+1} \right) \right)$$

subject to (18). The first order condition for $P^F_{r,t}$ and the envelope condition are given as:

$$\begin{aligned}
\left(1-\eta\left(1-\frac{P^m_t}{P^F_{r,t}}\right)\right)\frac{1}{P_t}y_{r,t} &= \frac{\eta}{\omega_Y}\frac{1}{P^F_{r,t}}\log\left(\frac{P^F_{r,t}}{P^F_{r,t-1}}\right)Y_t-\beta\mathbb{E}_t\frac{\partial\mathbf{V}^F_f\left(P^F_{r,t},S_{t+1}\right)}{\partial P^F_{r,t}}\\
\frac{\partial\mathbf{V}^F_r\left(P^F_{r,t-1},S_t\right)}{\partial P^F_{r,t-1}} &= \frac{\eta}{\omega_Y}\frac{1}{P^F_{r,t-1}}\log\left(\frac{P^F_{r,t}}{P^F_{r,t-1}}\right)Y_t
\end{aligned}$$

which implies that:

$$\log(\pi_{h,t})=\beta\mathbb{E}_t\log(\pi_{h,t+1})\frac{Y_{t+1}}{Y_t}+\kappa_Y\frac{p_{h,t}}{P_t}\left(\frac{P_{m,t}}{p_{h,t}}-\frac{\eta-1}{\eta}\right)\frac{y_{h,t}}{Y_t}$$

Combining these and focusing on a symmetric equilibrium gives us Equation (21).

B4. Capital Producers

Capital producers solve the following dynamic problem:

$$V^K(I_{n,t-1},S_t)=\max_{I_{n,t}}\left(\upsilon^I_t+\beta\mathbb{E}_tV^K(I_{n,t},S_{t+1})\right)$$

The first-order necessary condition for $I_{n,t}$ and the envelope condition are given as:

$$\text{(B1)}\quad (Q_t-1)+\beta\mathbb{E}_t\frac{\partial V^K(I_{n,t},S_{t+1})}{\partial I_{n,t}} = \omega_I\log\left(\frac{I_{n,t}+\psi}{I_{n,t-1}+\psi}\right)+\frac{\omega_I}{2}\left(\log\left(\frac{I_{n,t}+\psi}{I_{n,t-1}+\psi}\right)\right)^2$$

$$\text{(B2)}\quad \frac{\partial V^K(I_{n,t-1},S_t)}{\partial I_{n,t-1}} = \omega_I\left(\log\left(\frac{I_{n,t}+\psi}{I_{n,t-1}+\psi}\right)\right)\frac{I_{n,t}+\psi}{I_{n,t-1}+\psi}$$

Combining these gives us Equation (29).

ESTIMATION OF THE HOUSEHOLD INCOME PROCESS

Assume that log household income is determined as:

$$\begin{aligned}
y_{i,t} &= \delta_t+\delta_Z Z_{i,t}+\tilde{y}_{i,t}\\
\tilde{y}_{i,t} &= x_{i,t}+\varepsilon_{i,t}\\
x_{i,t} &= \rho x_{i,t-1}+e_{i,t}\\
\varepsilon_{i,t} &\sim N(0,\sigma^2_\varepsilon)\\
e_{i,t} &\sim N(0,\sigma^2_e)
\end{aligned}$$

where δ_t is a time fixed effect, $Z_{i,t}$ is a vector of household characteristics, $x_{i,t}$ is a persistent idiosyncratic income component, and $\varepsilon_{i,t}$ is a transitory income shock. The autocovariances of residualized household income of order 0-2 are given as:

$$m_{1,t} = \mathbb{E}(\tilde{y}_{i,t} \cdot \tilde{y}_{i,t}) = \frac{1}{1-\rho^2}\sigma_e^2 + \sigma_\varepsilon^2 \tag{C1}$$

$$m_{2,t} = \mathbb{E}(\tilde{y}_{i,t} \cdot \tilde{y}_{i,t-1}) = \frac{\rho}{1-\rho^2}\sigma_e^2 \tag{C2}$$

$$m_{3,t} = \mathbb{E}(\tilde{y}_{i,t} \cdot \tilde{y}_{i,t-2}) = \frac{\rho^2}{1-\rho^2}\sigma_e^2 \tag{C3}$$

These three moments identify jointly $\Gamma = (\rho, \sigma_e, \sigma_\varepsilon)$. We estimate Γ with GMM using an identity weighting matrix.

Further Results for the Baseline Model

Figure D1 decomposes the transmission of capital quality shocks to aggregate consumption into the effect of each price showing how wages and interest rates differ in the impact on the level of consumption vs. its dynamics.

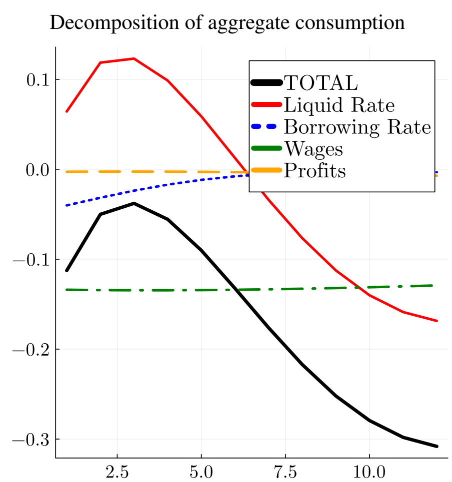

Figure D1. Transmission to Consumption: Capital quality shock

Note: The figure plots the decomposition of the response of aggregate consumption to a one percent negative capital quality shock into the effect of each price sequence by using household policy functions.

Figure D2 shows the variance decomposition of output, investment, inflation, and consumption at horizon 1, 4, and 16 quarters for the baseline model and calibration. Aggregate output is heavily influenced by TFP shocks both at short and longer forecast horizons. Aggregate investment is sensitive to capital quality shocks in the short run, more by TFP in the long run, and with some importance of monetary policy shocks in the medium run. Inflation is sensitive to all three shocks in the short run, but with a smaller importance of TFP in the longer run. Consumption of the rich is heavily dominated by TFP in the short run while the poor are sensitive to capital quality shocks (and to monetary shocks) at such short horizons. In the medium run, monetary policy shocks (which impact on savings rates) are instead important for wealthier households.

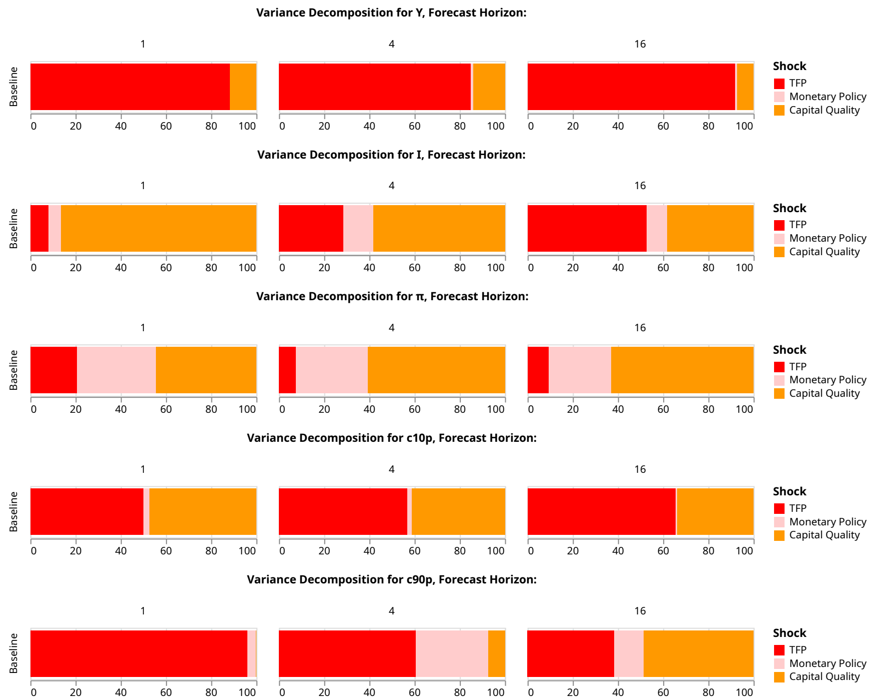

FIGURE D2. CONDITIONAL VARIANCE DECOMPOSITION

Note: Conditional variance decomposition at quarter 1,4,16 for selected variables in the baseline calibration of the model and shocks.

Figures D3 and D4 show additional impulse responses to monetary and TFP shocks. Spreads move significantly in response to monetary policy shocks but less so in the face of TFP shocks. Banking frictions and incomplete markets impact significantly on how monetary policy shocks affect the economy while incomplete markets is the key aspect for TFP shocks due to the muted dynamics of spreads.

Figures D5 presents the results of estimating Equation (2), which captures transitions to zero wealth, using model-generated data. Figure D6 shows the corresponding results for Equation (3), reflecting the consumption response. For this analysis, we solve households' optimal responses to idiosyncratic income and spread shocks, simulating 100 panels of 20,000 households each. In each panel, households are subjected to the same spread shock, mimicking a scenario where all households in that panel are served by a single bank. This approach parallels the identification strategy in the empirical section, which relies on household and time fixed effects.

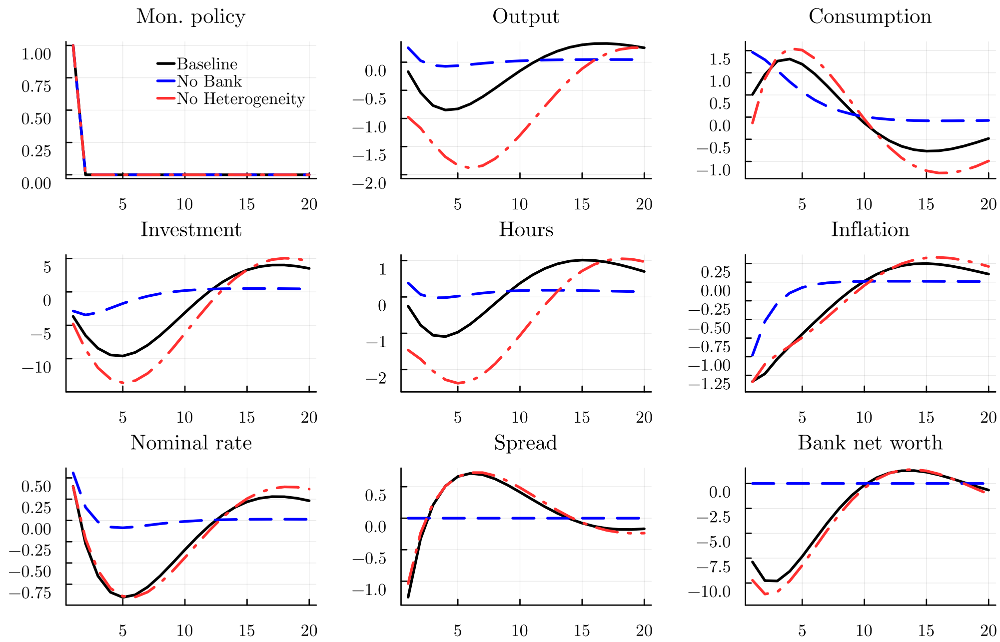

FIGURE D3. AGGREGATE EFFECTS OF A MONETARY SHOCK

Note: Impulse responses to a one percentage point positive nominal interest rate shock. See Figure 5 for legend.

The size and persistence of spread shocks are identified from Danish bank-level interest rate data. We model the loan rate ($R_{L,t}$) and deposit rate ($R_{S,t}$) as linear functions of a common factor (z_t). Specifically, the model is defined as:

$$R_{S,t} = a \cdot z_t, \quad R_{L,t} = b \cdot z_t,$$

where the spread between the loan and deposit rates is given by: $\text{Spread}_t = R_{L,t} - R_{S,t} = (b-a) \cdot z_t$. The common factor z_t follows a stationary AR(1) process:

$$z_t = \rho \cdot z_{t-1} + \eta_t, \quad \eta_t \sim \mathcal{N}(0, \sigma_\eta^2),$$

with $\rho = 0.85$ governing its persistence. The coefficient on the deposit rate is $a = 0.177$ and the coefficient on the loan rate is $b = 3.195$.

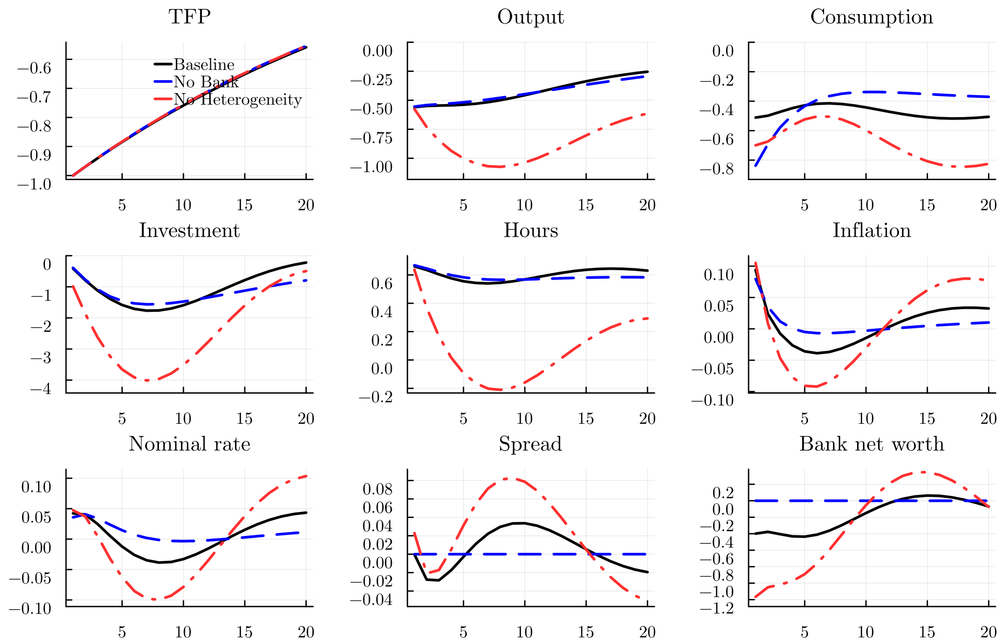

FIGURE D4. AGGREGATE EFFECTS OF A TFP SHOCK

Note: Impulse responses to a one percent negative TFP shock. See Figure 5 for legend.

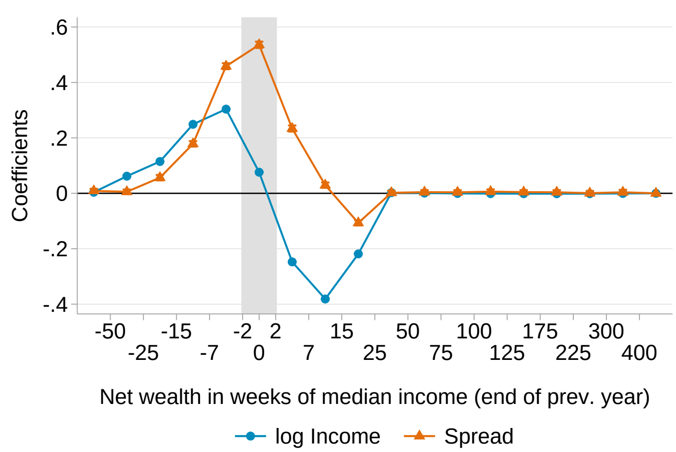

FIGURE D5. ZERO NET WEALTH DYNAMICS

Note: The figure shows the change in transition probabilities into the zero net wealth state with cross-sectional changes in income and the consumer credit spread (estimated from Equation (2)). Zero net wealth is defined as net assets within a range of plus/minus two weeks of median household income.

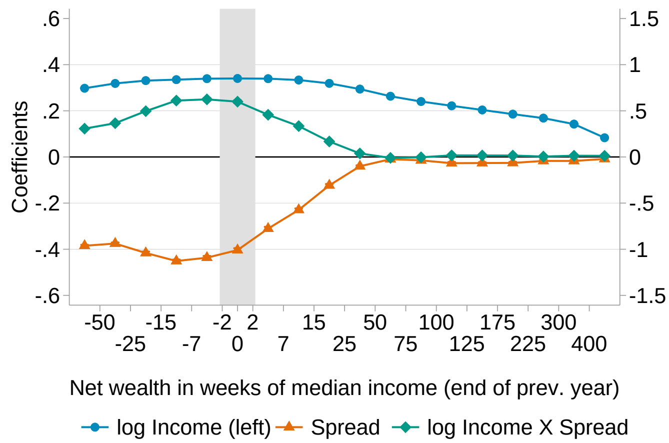

FIGURE D6. CONSUMPTION AND THE SPREAD IN THE MODEL

Note: The figure illustrates the parameters estimated from Equation (3) on model simulated data in response to idiosyncratic income and spread shocks. The underlying wealth distribution is trimmed at the 3rd and 97th percentile. The error bars illustrate 95% confidence intervals. Standard errors clustered at the household level.

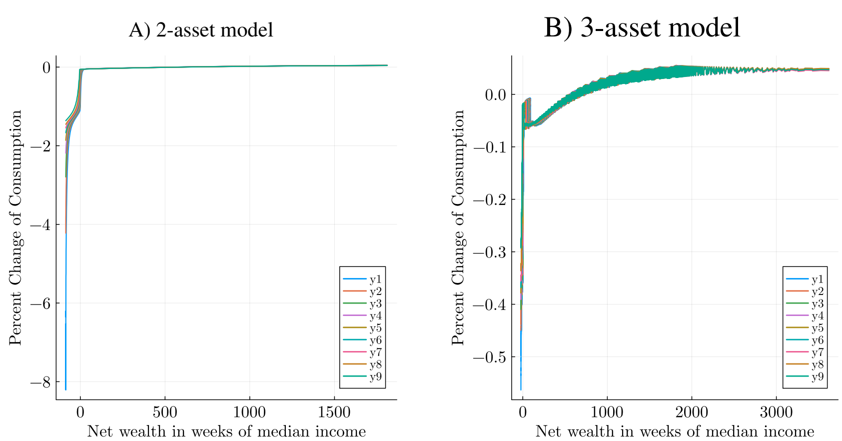

FIGURE D7. POLICY FUNCTIONS FOR CONSUMPTION

Note: The figure illustrates the impact of spreads on consumption conditional on wealth and the income state. Panel A shows the baseline model, Panel B the three-asset model.

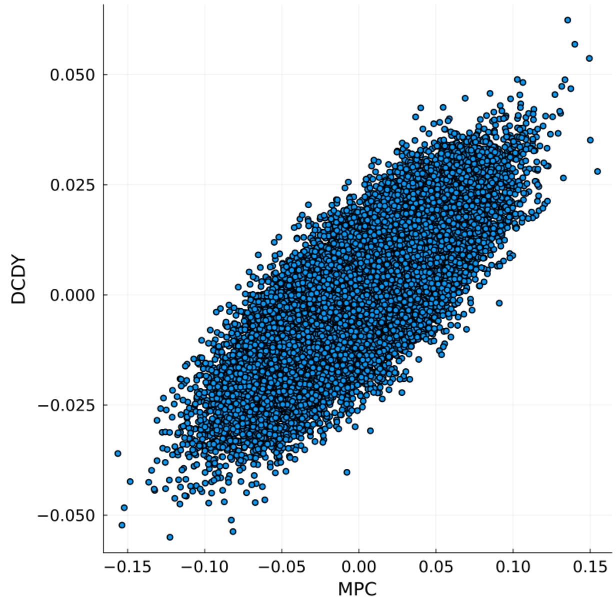

FIGURE D8. SCATTER PLOT: DCDY AND MPC

Note: Simulation of model implied average MPC and DCDY (HP-filtered) in response to TFP, monetary, and capital quality shocks. DCDY is calculated using the regression coefficients from estimating equation (3) on simulated model data. The correlation between the two series is 88 percent.

TABLE D1—BUSINESS CYCLE MOMENTS: MPC COMPARISON

Moments	Baseline	Constant Spread	No bank
σ_{MPC}/σ_Y	2.58	0.41	0.53
$corr(MPC,Y)$	-0.59	-0.33	-0.20

Note: σ_x denotes the percentage standard deviation of x, $corr(x,y)$ is the correlation of x and y. Model moments computed for HP-filtered data. Model moments are in response to TFP, monetary, and capital quality shocks. Standard deviations and correlations for the MPC are based on annual data.

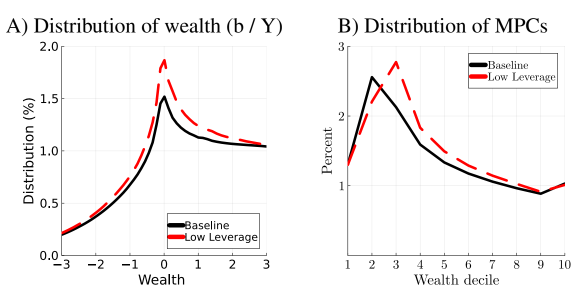

FIGURE D9. DISTRIBUTIONS: BASELINE AND RESTRICTED LEVERAGE

The Three Asset Model

Here we discuss the relevant parts of the three-asset model studied in Section VII. We focus on the elements that differ from the baseline two asset model presented in Section III.

E1. Households

In the three asset model, households can hold capital, $k_{i,t}$ which they rent directly to firms at the real capital rental rate $r_{k,t}$. Households cannot go short on the illiquid asset $k_{i,t+1} \geq 0$. They can carry out maintenance every period which corresponds to depreciation at the constant proportional rate $\delta \in (0,1)$. However, in a given period, they can adjust capital holdings *actively* only with the probability $\phi_k \in (0,1)$ which is constant across time and households. Households that actively change their capital stock, purchase new capital at the price Q_t (relative to the price of consumption). Thus, the one-period expected return on the illiquid asset is $\mathbb{E}_t R_{I,t+1} = \mathbb{E}_t(r_{K,t+1} + Q_{t+1} - \delta)/Q_t$.[1] As long as $\phi_k < 1$, households will only hold capital if $\mathbb{E}_t(R_{I,t+1} - R_{S,t+1}) > 0$.

Let $\mathbf{b}_{i,t} = (b^G_{i,t}, b^D_{i,t}, k_{i,t}, b^L_{i,t})$ denote household i's beginning of period asset portfolio, $\mathbf{S}_t$ the vector of relevant aggregate state variables, and $\mathscr{V}^{w,a}_i$ the value function for a household that can adjust its illiquid bond holding. The Bellman equation for such a household is given as:

$$
\begin{aligned}
\mathscr{V}^{w,a}_i(\mathbf{b}_{i,t}, h_{i,t}, \mathbf{S}_t) \;=\;& \max[\mathrm{u}(c_{i,t}, l_{i,t}) \\
&+\; \beta\mathbb{E}_t((1-\phi_w)(\phi_k \mathscr{V}^{w,a}_i(\mathbf{b}_{i,t+1}, h_{i,t+1}, \mathbf{S}_{t+1}) \\
&+\; (1-\phi_k)\mathscr{V}^{w,n}_i(\mathbf{b}_{i,t+1}, h_{i,t+1}, \mathbf{S}_{t+1})) \\
&+\; \phi_w(\phi_k \mathscr{V}^{r,a}_i(\mathbf{b}_{i,t+1}, \mathbf{S}_{t+1}) \\
&+\; (1-\phi_k)\mathscr{V}^{r,n}_i(\mathbf{b}_{i,t+1}, \mathbf{S}_{t+1}))))]
\end{aligned} \tag{E1}
$$

subject to (6)-(7) and to the flow budget constraint:

$$
\begin{aligned}
& c_{i,t} + b^G_{i,t+1} + b^D_{i,t+1} + Q_t(k_{i,t+1} - k_{i,t}) - b^L_{i,t+1} \leq \\
& (1-\tau_{h,t})\, w_t h_{i,t} l_{i,t} + R_{S,t}\left(b^G_{i,t} + b^D_{i,t}\right) + (r_{K,t} - \delta)k_{i,t} - R_{L,t} b^L_{i,t}
\end{aligned} \tag{E2}
$$

$\mathscr{V}^{w,n}_i$ is the value function of a household that cannot adjust illiquid assets this period,

[1] Note that $R_{I,t}$ includes a capital gain. For a household that cannot adjust its capital stock, the net-of-capital-gains return is $R_{I,t} - Q_t/Q_{t-1}$.

while $\mathscr{V}_i^{r,s}$ denotes the rentiers' value functions. $\mathscr{V}_i^{w,n}$ is given as:

$$\begin{aligned}\mathscr{V}_i^{w,n}(\mathbf{b}_{i,t},h_{i,t},\mathbf{S}_t) &= \max[\mathrm{u}(c_{i,t},l_{i,t}) \\ &+ \beta\mathbb{E}_t((1-\phi_w)(\phi_k\mathscr{V}_i^{w,a}(\mathbf{b}_{i,t+1},h_{i,t+1},\mathbf{S}_{t+1}) \\ &+ (1-\phi_k)\mathscr{V}_i^{w,n}(\mathbf{b}_{i,t+1},h_{i,t+1},\mathbf{S}_{t+1})) \\ &+ \phi_w(\phi_k\mathscr{V}_i^{r,a}(\mathbf{b}_{i,t+1},\mathbf{S}_{t+1}) \\ &+ (1-\phi_k)\mathscr{V}_i^{r,n}(\mathbf{b}_{i,t+1},\mathbf{S}_{t+1})))]\end{aligned} \tag{E3}$$

subject to (6)-(7) and to the flow budget constraint:

$$\begin{aligned}c_{i,t}+b^G_{i,t+1}+b^D_{i,t+1}-b^L_{i,t+1} &\leq \\ (1-\tau_{h,t})\,w_t h_{i,t} l_{i,t} + R_{S,t}\left(b^G_{i,t}+b^D_{i,t}\right) &+ (r_{K,t}-\delta)k_{i,t} - R_{L,t}b^L_{i,t}\end{aligned} \tag{E4}$$

The rentiers' value function is the solution to:

$$\begin{aligned}\mathscr{V}_i^{r,a}(\mathbf{b}_{i,t},h_{i,t},\mathbf{S}_t) &= \max[\mathrm{u}(c_{i,t},l_{i,t}) \\ &+ \beta\mathbb{E}_t((1-\phi_r)(\phi_k\mathscr{V}_i^{w,a}(b_{i,t+1},h_{i,t+1},S_{t+1}) \\ &+ (1-\phi_k)\mathscr{V}_i^{w,n}(b_{i,t+1},h_{i,t+1},S_{t+1})) \\ &+ \phi_r(\phi_k\mathscr{V}_i^{r,a}(b_{i,t+1},S_{t+1}) \\ &+ (1-\phi_k)\mathscr{V}_i^{r,n}(b_{i,t+1},S_{t+1})))]\end{aligned} \tag{E5}$$

subject to (6)-(7) and to the flow budget constraint:

$$\begin{aligned}c_{i,t}+b^G_{i,t+1}+b^D_{i,t+1}+b^I_{i,t+1}-b^L_{i,t+1} &\leq (1-\tau_{h,t})\mathscr{F}_t \\ &+R_{S,t}\left(b^G_{i,t}+b^D_{i,t}\right)+R_{I,t}b^I_{i,t}-R_{L,t}b^L_{i,t}\end{aligned} \tag{E6}$$

Finally, the dynamic programme of a rentier who cannot adjust their illiquid bonds is given as:

$$\begin{aligned}\mathscr{V}_i^{r,n}(\mathbf{b}_{i,t},h_{i,t},\mathbf{S}_t) &= \max[\mathrm{u}(c_{i,t},l_{i,t}) \\ &+ \beta\mathbb{E}_t((1-\phi_r)(\phi_k\mathscr{V}_i^{w,a}(\mathbf{b}_{i,t+1},h_{i,t+1},\mathbf{S}_{t+1}) \\ &+ (1-\phi_k)\mathscr{V}_i^{w,n}(\mathbf{b}_{i,t+1},h_{i,t+1},\mathbf{S}_{t+1})) \\ &+ \phi_r(\phi_k\mathscr{V}_i^{r,a}(\mathbf{b}_{i,t+1},\mathbf{S}_{t+1}) \\ &+ (1-\phi_k)\mathscr{V}_i^{r,n}(\mathbf{b}_{i,t+1},\mathbf{S}_{t+1})))]\end{aligned} \tag{E7}$$

subject to (6)-(7) and to the flow budget constraint:

$$\begin{aligned}c_{i,t}+b^G_{i,t+1}+b^D_{i,t+1}-b^L_{i,t+1} &\leq (1-\tau_{h,t})\mathscr{F}_t \\ R_{S,t}\left(b^G_{i,t}+b^D_{i,t}\right) &+ (R_{I,t}-1)b^I_{i,t}-R_{L,t}b^L_{i,t}\end{aligned} \tag{E8}$$

In this economy, households may again be constrained or not, but it is their liquid wealth that matters. First, the household may be a saver and on a "short run" Euler equation with a slope determined by the return on liquid assets. Alternatively, the household may be a borrower and not constrained by (6) and on an Euler equation with slope determined by the borrowing rate:

$$\begin{aligned}(c^{\mathrm{I}}_{i,t})^{-\vartheta_c} &= \beta\mathbb{E}_t(c^{\mathrm{I}}_{i,t+1})^{-\vartheta_c}R_{S,t+1}\\ (c^{\mathrm{II}}_{i,t})^{-\vartheta_c} &= \beta\mathbb{E}_t(c^{\mathrm{II}}_{i,t+1})^{-\vartheta_c}R_{L,t+1}\end{aligned}$$

using the same notation as in Section III. There are also two groups of *constrained* households with high marginal propensities to consume. Households may be indebted and up against the borrowing constraint, or they may hold no liquid wealth and neither want to save nor borrow. Assuming for simplicity that households were in either of these states at date $t-1$, their consumption levels are given as:

$$\begin{aligned}c^{\mathrm{III}}_{i,t} &= (1-\tau_{h,t})\,w_t h_{i,t} l_{i,t} + (r_{K,t}-\delta)k_{i,t} - (R_{L,t}-1)\underline{\mathbf{b}}\\ c^{\mathrm{IV}}_{i,t} &= (1-\tau_{h,t})\,w_t h_{i,t} l_{i,t} + (r_{K,t}-\delta)k_{i,t}\end{aligned}$$

Here there may be a substantial amount of type IV agents and such households may be wealthy due to illiquid asset holdings. When credit spreads rise, the kink exaggerates and a larger measure of agents will find themselves with no liquid assets and high MPCs.

E2. Intermediate Goods Producers

Intermediate goods producers rent part of their capital input from households. The effective capital input is given as:

$$k^e_{j,t} = \xi_t k^P_{j,t} + k^R_{j,t} \tag{E9}$$

where $k^R_{j,t}$ denotes capital rented from households. We assume that the capital quality shock, $\xi_t > 0$ impacts only equity financed capital. The demand for labor and rented capital input solve:

$$\upsilon^m_{j,t} = \max_{n_{j,t},k^R_{j,t}} \left(P^m_t m_{j,t} - w_t n_{j,t} - r_{K,t}k^R_{j,t}\right)$$

which implies that:

$$w_t = P^m_t \alpha Z_t n^{\alpha-1}_{j,t}\left(k^e_{j,t}\right)^{1-\alpha} \tag{E10}$$

$$r_{K,t} = P^m_t(1-\alpha)Z_t n^{\alpha}_{j,t}\left(k^e_{j,t}\right)^{-\alpha} \tag{E11}$$

Having paid households for the cost of rental of labor and capital, the firm pays its equity holders its profits and the market value of its capital stock net of maintenance costs:

$$\varsigma^m_{j,t} = \upsilon^m_{jt} + Q_t\xi_t k^p_{jt} - \delta\xi_t k^p_{jt}$$

where $\upsilon_{jt}^{m}=(1-\alpha)P_{t}^{m}Z_{t}n_{j,t}^{\alpha}\left(k_{j,t}^{e}\right)^{1-\alpha}(1-k_{t}^{R}/k_{t}^{e})$. Thus, the return on equity offered is:

$$R_{K,t}=\frac{(r_{K,t}+Q_{t}-\delta)\,\xi_{t}}{Q_{t-1}} \tag{E12}$$

where $r_{K,t}=(1-\alpha)P_{t}^{m}Z_{t}n_{j,t}^{\alpha}\left(k_{j,t}^{e}\right)^{-\alpha}$ is the marginal product of "effective" capital. To get Equation (E12), define the return $R_{K,t}=\varsigma_{j,t}/(Q_{t-1}k_{j,t}^{P})$ and note that $\upsilon_{j,t}^{m}=r_{K,t}(k_{j,t}^{e}-k_{j,t}^{R})=r_{K,t}\xi_{t}k_{j,t}^{P}$.

E3. Capital Goods Producers

The law of motion of aggregate capital is:

$$K_{t+1}-\left(K_{t}^{r}+\xi_{t}K_{t}^{p}\right)=I_{n,t}. \tag{E13}$$

and I_t and CI_t then follow as:

$$I_{t} = I_{n,t}+\delta\left(K_{t}^{r}+\xi_{t}K_{t}^{p}\right), \tag{E14}$$

$$CI_{t} = I_{t}+\frac{\omega_{I}}{2}\left(\log\left(\frac{I_{n,t}+\psi}{I_{n,t-1}+\psi}\right)\right)^{2}(I_{n,t}+\psi). \tag{E15}$$

where K_{t}^{r} is the aggregate amount of capital held directly by households and rented to firms, and K_{t}^{p} is the aggregate amount of capital that intermediate firms finance through equity issues.

TABLE E1—THREE ASSET MODEL PARAMETERIZATION

	Description	Value		Description	Value
Households			**Monetary and fiscal policy**		
β	Discount factor	0.9855	$\overline{\pi}$	Inflation target	1.00
χ	Disutility weight of labor	0.20	κ_π	Response to inflation	1.50
$1/\vartheta_c$	Intertemp. elasticity	2/3	κ_R	Int.rate smoothing	0.70
ϑ_l	Frisch elasticity	0.75	$\overline{G}/\overline{Y}$	Gov. spending share	0.26
ϕ_w	Transition prob. to rentier	0.001	$\overline{B}^G/\overline{Y}$	Gov. debt ratio	0.39
ϕ_r	Transition prob. to worker	0.0625	τ_h	tax rate	0.38
$\underline{\mathbf{b}}$	Borrowing constraint	$1\ \overline{Y}$	κ_G	Response of G to debt	0.20
ϕ_k	Illiquidity of capital	0.0025			
Supply side			**Stochastic shocks**		
α	Output elasticity to labor	0.67	ρ_h	Persistence of HH income shocks	0.948
δ	Depreciation rate	0.02	ρ_z	Persistence of TFP shocks	0.970
ω_I	Adjustment costs	2.3	σ_h^2	Variance of HH income shocks	0.097^2
η	Elasticity of substitution	21	σ_z^2	Variance of TFP shocks	0.0205^2
ω_Y	Price stickiness	0.10	σ_ξ^2	Variance of cap.q. shocks	0.0205^2
			σ_R^2	Variance of mon.pol. shocks	0.002^2
Banking					
λ	Divertible fract. of assets	0.38	θ	Bank survival rate	0.972
ζ	Funds new managers	0.0037	ω_b	Consumer loan cost	0.0075

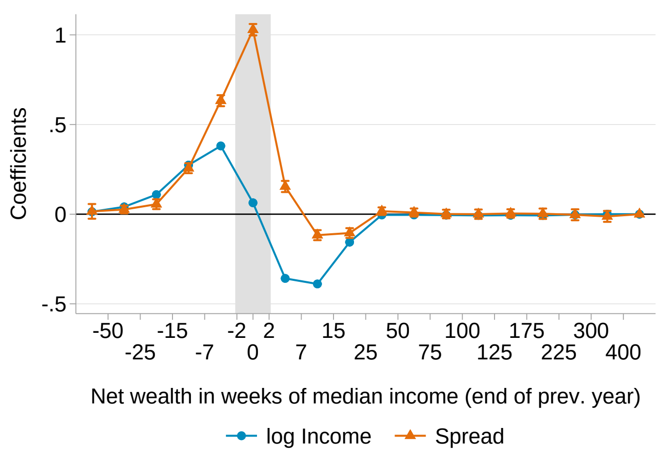

FIGURE E1. ZERO NET WEALTH DYNAMICS IN THE 3-ASSET MODEL

Note: The figure shows in the 3-asset model the change in transition probabilities into the zero net wealth state with cross-sectional changes in income and the consumer credit spread (estimated from Equation (2)). Zero net wealth is defined as net assets within a range of plus/minus two weeks of median household income.

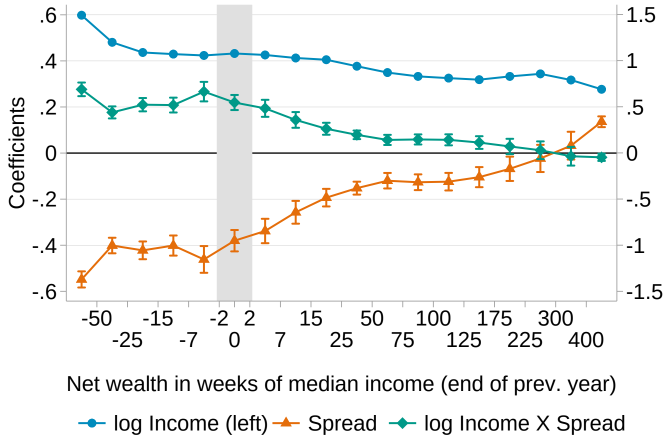

FIGURE E2. CONSUMPTION AND THE SPREAD IN THE 3-ASSET MODEL

Note: This figure illustrates the relationship in the 3-asset model between consumption and income, borrowing spreads and their interaction, estimated from Equation (3) based on model-simulated data in response to idiosyncratic income and spread shocks. We here allow the return on illiquid assets to move when credit spreads change.

TABLE E2—MOMENTS: BASELINE AND RESTRICTED LEVERAGE

	Baseline		**3-asset model**	
	Baseline	Low leverage	Baseline	Low leverage
Leverage	2.93	2.64	2.93	2.64
	Interest rates			
Return on capital (R_K, %)	4.70	4.82	4.63	4.60
Return on bonds and deposits (R_S, %)	3.82	3.54	3.26	2.70
Lending interest rate (R_L, %)	7.87	8.00	7.80	7.77
	Aggregates			
Output	4.89	4.91	4.88	4.90
Capital	49.23	48.95	49.36	49.65
Labor supply	1.54	1.55	1.45	1.46
Consumption	2.64	2.70	2.67	2.66
	Household distribution			
At kink (%)	4.38	5.33	12.47	14.58
Borrowers (%)	21.93	24.49	33.98	38.69
Gini wealth	78.21	82.79	74.34	74.71
Gini consumption	15.64	16.50	18.03	17.98
Gini income	28.61	30.16	25.23	25.19

Note: We compare the baseline steady state to one with 10% less leverage (diversion parameter λ going from 0.381 to 0.445). The last two columns do so for the model with household portfolios consisting of 3 assets.