Online Appendix: Competitive bottlenecks and platform spillovers

Tat-How Teh[1] and Julian Wright[2]

In the following sections we provide additional workings and results referred to but not included in the main paper.

A Other microfoundations

In this section, we first describe Examples 1 - 3 that are similar to the *leading example* in the main paper in that they assume a continuum of atomistic product categories, with a single (monopoly) seller in each. We will then describe Example 4, which presents a microfoundation with sellers that behave oligopolistically.

Examples 1 - 3 are written in a way to accommodate each platform i choosing a multi-dimensional instrument vector a_i. To recover the single-dimensional case, we can simply fix all but one component of the vector for all platforms. Online Appendix B explains in our general baseline model how the analysis and results extend to this multi-dimensional setting.

As will be shown below, the functions U_i and R_i that correspond to Examples 1 - 3, also satisfy the special functional form (17) imposed in Section 3.2 when holding all but one of the multi-dimensional instruments as fixed. Therefore, the results in (Proposition 3 and Corollary 1) hold for these examples, as claimed in Section 3.2.

In Examples 1 - 3, we impose a simplifying assumption of $c = 0$. This means that seller's optimal price $p(r_i)$, the resulting consumer demand $q(r_i)$, and the corresponding utility $v(r_i)$ are all independent of commission rate r_i, and so we denote them as p^m, q^m, and v^m respectively. Then, denote $\pi(r_i) = (1 - r_i)\,\pi^m \equiv (1 - r_i)\,p^m q^m$.

□ **Example 1 (First-party entry and self-preferencing).** Continue from the *leading example*, but suppose now each platform chooses $a_i = (r_i, e_i, l_i)$, where $e_i \in \{0, 1\}$ indicates whether platform i operates as a dual-mode marketplace or not and $l_i \in \{0, 1\}$ indicates whether platform i engages in self-preferencing or not.[3] When it operates in dual mode, it introduces a first-party product whenever a third-party seller has entered in any product category.

With probability $1 - \alpha$, the first-party entry fails, and the third-party seller (in the relevant category) remains a monopolist (with corresponding gross profit π^m and buyer utility v^m). With probability α, the first-party entry succeeds. The resulting duopolistic competition results in two possible outcomes. When the platform doesn't engage in self-preferencing, the first-party profit is π^{fp} and the third-party seller profit is $(1 - r_i)\pi^d$, where $0 < \pi^d < \pi^m$, while the corresponding buyer utility is $v^d > v^m$. When the platform engages in self-preferencing, the first-party profit is $\pi^{sp} > \pi^{fp}$ and, for expositional simplicity, the third-party seller profit is set to zero, while the corresponding buyer utility is v^{sp}, where $v^{sp} < v^d$. We assume that first-party products do not "cross-list" on rival platforms.

Following the same steps in our *leading example*, we have

$$\bar{k}_i \equiv (1 - r_i)(\pi^m - \alpha e_i(\pi^m - (1 - l_i)\,\pi^d))s_i,$$

and

$$U_i = (v^m + \alpha e_i(l_i v^{sp} + (1 - l_i)\,v^d - v^m))G\left(\bar{k}_i\right)$$
$$R_i = (r_i \pi^m + \alpha e_i(l_i \pi^{sp} + (1 - l_i)\left(r_i \pi^d + \pi^{fp}\right) - r_i \pi^m))G\left(\bar{k}_i\right)s_i.$$

[1] Division of Economics, Nanyang Technological University

[2] Department of Economics, National University of Singapore

[3] A literature has recently emerged to address whether the choice of dual-mode marketplace is desirable in the context of a single platform, either absent the possibility of self-preferencing (see, for example, Etro (2021)) or also allowing for the possibility of self-preferencing (see, for example, Hagiu et al. (2022) and Anderson and Bedre-Defolie (2024)).

Here, e_i and l_i directly affect buyers' utility on platform i, as well as indirectly via how many sellers participate on platform i. Finally, seller surplus is $SS_i = \int_{k_{\min}}^{k_{\max}} \max\{\bar{k}_i - k_i, 0\} dG(k_i)$, where $\bar{k}_i$ is clearly decreasing in r_i, e_i, and l_i.

By Proposition 2, if we restrict platforms to choose only one of the instruments from (r_i, e_i, l_i) (while exogenously fixing the remaining two instruments), then each of the following holds in isolation: $r^* = r^{SE} \geq r^W$, $e^* = e^{SE} \geq e^W$, and $l^* = l^{SE} \geq l^W$. Moreover, as shown in Online Appendix B, under a relatively mild quasi-supermodularity condition, we can establish similar results even when platforms choose all instruments together, so that $(r^*, e^*, l^*) = (r^{SE}, e^{SE}, l^{SE}) \geq (r^W, e^W, l^W)$. That is, the equilibrium levels of commission, first-party entry, and self-preferencing are excessive.

□ **Example 2 (Preventing disintermediation).** Suppose sellers continue to be monopolists as in our *leading example*, but now they also have direct sales channels (e.g., their own websites). In order for buyers to transact on a seller's direct channel, buyers must first discover them through a platform. A direct channel allows a seller to avoid a platform's transaction-based fees if buyers switch from the platform to purchase from the seller through their direct channel, which we call disintermediation.[4] A fraction $\lambda_i \geq 0$ of buyers are unaware of this option to buy from the seller directly, with the remaining fraction $1 - \lambda_i$ aware of the option. Buyers have heterogeneous costs to switch to the direct channel. Specifically, with probability ζ buyers who are aware of the direct channel are assumed to be able to costlessly switch (and so buy from whichever channel is cheapest), while with probability $1 - \zeta$ buyers face a sufficiently high switching cost such they will never use the direct channel regardless of the price difference. Buyers realize which situation they are in after participating on a platform.

Each platform chooses $a_i = (r_i, \lambda_i)$, where $\lambda_i \in [\lambda_{\min}, \lambda_{\max}]$ reflects that the platform can influence the probability any given buyer will be aware of a seller's direct-channel option via its design choices. For example, a platform could take steps to prevent communication by sellers on the platform which would make it more difficult for them to inform buyers of their direct channel.

Participating sellers set prices p_i (on platforms $i = 1, ..., m$) and p_d (their price when selling directly). Buyers on platform i who are informed and able to switch would buy directly if and only if $p_i \geq p_d$. Moreover, given $r_i \geq 0$, each seller would always want to induce disintermediation. Therefore, a seller that joins a non-empty set of platform(s) $\phi \subseteq \{1, 2, ..., m\}$ sets its prices to maximize

$$\sum_{i \in \phi} \left[(1 - r_i) p_i D(p_i)(1 - (1 - \lambda_i)\zeta) + p_d D(p_d)(1 - \lambda_i)\zeta\right] s_i$$
$$\text{subject to } p_d \leq p_i, i \in \phi.$$

Given the pricing problem across channels is additively separable, the optimal price is

$$p_d = p_i = \arg\max_{p_i} \{p_i q(p_i)\} \equiv p^m$$

for all $i \in \phi$, so the standard profit and utility terms π^m and v^m still apply in this case (given $c = 0$). Each seller participates on platform i if and only if

$$k_i \leq (1 - r_i + (1 - \lambda_i)\zeta r_i) \pi^m s_i \equiv \bar{k}_i,$$

so

$$U_i = G(\bar{k}_i) v^m$$
$$R_i = (1 - (1 - \lambda_i)\zeta) r_i G(\bar{k}_i) \pi^m s_i.$$

Finally, seller surplus is $SS_i = \int_{k_{\min}}^{k_{\max}} \max\{\bar{k}_i - k_i, 0\} dG(k_i)$, where $\bar{k}_i$ is clearly decreasing in r_i and λ_i.

[4] Hagiu and Wright (2023) study disintermediation (or leakage in their terminology) in the case of a monopoly platform.

By Proposition 2, if we restrict platforms to choose only one of the instruments from (r_i, λ_i) (while exogenously fixing the remaining instrument), then each of the following holds in isolation: $r^* = r^{SE} \geq r^W$ and $\lambda^* = \lambda^{SE} \geq \lambda^W$. Moreover, as shown in Online Appendix B, under a relatively mild quasi-supermodularity condition, we can establish similar results even when platforms choose both instruments together, so that $(r^*, \lambda^*) = (r^{SE}, \lambda^{SE}) \geq (r^W, \lambda^W)$. That is, the equilibrium levels of commission and disintermediation-prevention effort are excessive.

□ **Example 3 (App tracking restriction).** Similar to Example 2, buyers must first discover sellers through a platform before transacting. Buyers on platform i can obtain (e.g., unlock) q units of content from sellers by either: (i) paying the seller price p_i per unit; or (ii) watching ads, which results in ad disutility z per unit to buyers and generates per-unit ad revenue $\pi_a (1 - \kappa_i) > 0$ to sellers. Here $\kappa_i \in [0, \kappa_{\max}]$ with $\kappa_{\max} < 1$ measures how restrictive platform i's app tracking policy is, which can limit the ad revenue of sellers, which is at most π_a. Suppose seller's revenue from (i) can be taxed by the platform through its commission r_i, while its ad revenue in (ii) cannot. We assume $z \geq 0$ is IID across buyers and sellers, drawn from the weakly log-concave CDF H.

Each platform chooses $a_i = (r_i, \kappa_i)$. Then, a typical seller that joins a non-empty set of platform(s) $\phi \subseteq \{1, 2, ..., m\}$ sets its prices to maximize its profit[5]

$$\sum_{i \in \phi} \left((1 - r_i)\, p_i q(p_i)(1 - H(p_i)) + \pi_a (1 - \kappa_i) \int_0^{p_i} q(z) dH(z) \right) s_i.$$

Observe that the pricing problems are separable, and so each seller's optimal price p on platform i is independent of the (r_j, κ_j) (when holding s_i) constant. Each seller would participate on i if and only if

$$k_i \leq \left((1 - r_i)\, pq(p)(1 - H(p)) + \pi_a (1 - \kappa_i) \int_0^{p} q(z) dH(z) \right) s_i \equiv \bar{k}_i,$$

and so

$$\begin{aligned} U_i &= \left(\int_0^\infty v(q(\min(p, z)) - \min(p, z) q(\min(p, z)) dH(z) \right) G(\bar{k}_i) \\ R_i &= r_i pq(p)(1 - H(p_i^*)) s_i G\left(\bar{k}_i\right). \end{aligned}$$

Finally, seller surplus is $SS_i = \int_{k_{\min}}^{k_{\max}} \max\{\bar{k}_i - k_i, 0\} dG(k_i)$, where $\bar{k}_i$ is clearly decreasing in r_i and κ_i.

By Proposition 2, if we restrict platforms to choose only one of the instruments from (r_i, κ_i) (while exogenously fixing the remaining instrument), then each of the following holds in isolation: $r^* = r^{SE} \geq r^W$ and $\kappa^* = \kappa^{SE} \geq \kappa^W$. Moreover, as shown in Online Appendix B, when all sellers have zero participation costs $k_i = 0$ (i.e., the distribution G is degenerate), we can establish similar results even when platforms choose both instruments together, so that $(r^*, \kappa^*) = (r^{SE}, \kappa^{SE}) \geq (r^W, \kappa^W)$. That is, the equilibrium levels of commission and app-tracking restriction effort are excessive.

□ **Example 4 (Demand-side heterogeneity and competing sellers).** This example is constructed independently of our leading example and those above (and so CDF $G(\cdot)$ has a different interpretation here). Each platform chooses its commission $a_i = r_i$. There is a continuum of product categories with mass 1 indexed by the buyers' interaction benefit parameter V, where V is drawn from some distribution G on $[0, V_{\max}]$. There are $n \geq 1$ potential competing sellers in each product category. A representative buyer's gross utility function for purchasing q_l units from each seller $k = 1, .., n$ in a particular product category is

$$u(q_1, ..., q_n) = V \sum_{k=1}^{n} q_k - \frac{n}{2} \left((1 - \theta) \sum_{k=1}^{n} q_k^2 + \frac{\theta}{n} \left(\sum_{k=1}^{n} q_k \right)^2 \right),$$

[5]We assume the profit function is strictly quasiconcave, a sufficient condition for which is that $q(p_i)$ has an elasticity (in magnitude) that is non-decreasing and is no lower than one over the relevant range.

and $\theta \in [0,1)$ is a measure of seller differentiation within the category. This is the model by Shubik and Levitan (1980). Then, buyer demand for seller k in category V is

$$D_{V,k} = \frac{1}{n}\left(V - \frac{p_k}{1-\theta} + \frac{\theta}{1-\theta}\sum_{k'=1}^{n}\frac{p_{k'}}{n}\right).$$

We assume sellers face no fixed costs of participating on a platform, but face a positive marginal cost per unit of sales $c > 0$.

Solving for the symmetric equilibrium between sellers yields the equilibrium price on platform i, which is denoted $p_V(r_i)$,

$$p_V(r_i) = \frac{(1-\theta)\,nV}{2n - \theta\,(n+1)} + \frac{(n-\theta)\,c}{(2n-\theta\,(n+1))\,(1-r_i)}.$$

This is increasing in V, and in r_i because $c > 0$. The demand and profit an individual seller gets in product category V from a representative buyer is denoted $q_V(r_i) = \frac{1}{n}\,(V - p_V(r_i))$ and

$$\begin{aligned}\pi_V(r_i) &= ((1-r_i)\,p_V(r_i) - c)\,q_V(r_i)\\ &= (1-r_i)\,\frac{(1-\theta)\,(n-\theta)}{(2n-\theta\,(n+1))^2}\left(V - \frac{c}{1-r_i}\right).\end{aligned}$$

The corresponding per-buyer utility in product category V is $v_V(r_i) = \frac{n^2}{2}q_V(r_i)^2$. Once it has joined platform i, each participating seller in product category V sets the price $p_V(r_i)$ on platform i and transacts with each buyer on that platform once, with the representative buyer consuming $q_V(r_i)$ units from each such seller.

Notice there is an equilibrium where the n sellers of type V can operate (make positive sales) and obtain a profit if and only if $(1-r_i)\,p_V(r_i) > c$. But the highest price that sellers can charge and obtain positive demand is V. Therefore, in the absence of any seller fixed costs of participation, if $r_i < 1 - \frac{c}{V}$, all n sellers in category V participate on platform i and make positive sales; while if $r_i \geq 1 - \frac{c}{V}$, none of them participate on platform i since in equilibrium they would not make a profit while making positive sales. The measure of product categories where sellers participate on platform i is $1 - G\left(\frac{c}{1-r_i}\right)$. Therefore,

$$\begin{aligned}U_i &= \int_{\frac{c}{1-r_i}}^{V_{\max}} v_V(r_i)dG(V)\\ R_i &= s_i r_i n \int_{\frac{c}{1-r_i}}^{V_{\max}} p_V(r_i)q_V(r_i)dG(V).\end{aligned}$$

Finally, seller surplus is

$$SS_i = \int_{\frac{c}{1-r_i}}^{V_{\max}} \pi_V(r_i)dG(V),$$

where $\pi_V(r_i)$ is clearly decreasing in r_i. By Proposition 2, we conclude $r^* = r^{SE} \geq r^W$. That is, the equilibrium level of commission is excessive in this oligopolistic seller model.

B Multi-dimensional instruments

We now extend the baseline model in Section 2 by allowing each platform's instrument choice $a_i \in \mathcal{A} \subseteq \mathbb{R}^N$ be a multi-dimensional vector, where $N \geq 1$. Our ordering that a higher a_i corresponds to a lower seller surplus means that $SS_i(a_i; s_i)$ is *decreasing in every dimension* of a_i, holding s_i constant, and denote $SS(a) = mSS_i(a; 1/m)$. The analysis below admits the possibility of non-unique equilibrium instruments a^* and non-unique solutions to welfare benchmarks a^{SE} and a^W (where we denote the sets of solutions a^{SE} and a^W as $\mathcal{A}^{SE}$ and $\mathcal{A}^W$ respectively.).

It is straightforward to verify that the analysis in Section 2 holds as it is. In particular, the definition of the equilibrium object (11) always applies regardless of whether a_i is single-dimensional or multi-dimensional. Denote set

$$\mathcal{A}^* = \arg\max_{a_i \in \mathcal{A}} \left\{ \frac{1}{m} U_i(a_i; \frac{1}{m}) + R_i(a_i; \frac{1}{m}) \right\}$$

We focus on the case where every $a^* \in \mathcal{A}^*$ constitute an equilibrium (this is true in, e.g., our leading example in Section 3). Then, given that $\mathcal{A}^{SE}$ is defined by the exact same condition, we have $\mathcal{A}^{SE} = \mathcal{A}^*$.

Lemma 1 now requires additional conditions. One well-known complication of multi-dimensional comparative statics is the cross-dimension effects, whereby distortions in one of the dimensions may reinforce or diminish distortions in other dimensions. To proceed, we define the following concepts as introduced by Milgrom and Shannon (1994):

- **Quasi-supermodularity** (QSM). A function $W : \mathcal{A} \to \mathbb{R}$ is *quasi-supermodular* in its argument $a_i \in \mathcal{A}$ if, for any pair of vectors $a'_i \in \mathcal{A}$ and $a''_i \in \mathcal{A}$, we have

  $$W(a') - W(a' \wedge a'') \geq (>) 0 \Rightarrow W(a' \vee a'') - W(a'') \geq (>) 0.$$

  Here, $a' \vee a''$ is the dimension-wise maxima of the two vectors and $a' \wedge a''$ is the dimension-wise minima of the two vectors.

Intuitively, quasi-supermodularity expresses a weak kind of complementarity between each dimension of vector a. That is, if an increase in some dimensions has a positive marginal return at some level of the remaining dimensions, then the marginal return will also be positive at any higher level of those remaining dimensions. Clearly, it is implied by the standard weak supermodularity condition. More generally, by Milgrom and Shannon (1994), there are several easy-to-check sufficient conditions for $W(a_i)$ to be QSM: (i) $W(a_i)$ is monotone in a_i; (ii) if a is one-dimensional then QSM trivially holds; (iii) if a is two-dimensional, then QSM is equivalent to $W(a)$ obeying single-crossing difference in a pairwise manner.[6]

To compare $\mathcal{A}^{SE}$ and $\mathcal{A}^W$, we adopt the following notion by Milgrom and Shannon (1994):

- **Strong set order**. A set $\mathcal{A}''$ is higher than set $\mathcal{A}'$ in *strong set order* (denoted as $\mathcal{A}'' \geq_{sso} \mathcal{A}'$) if for any pairs of vectors $a' \in \mathcal{A}'$ and $a'' \in \mathcal{A}''$, we have $a' \vee a'' \in \mathcal{A}''$ and $a' \wedge a'' \in \mathcal{A}'$.

Then, the following is analogous to Lemma 1 and Proposition 2. It shows that the baseline distortion persists under multi-dimensional platform instruments.

Proposition OA.1 *Suppose function $W(a)$ (or $W^{SE}(a)$) is quasi-supermodular. The seller-excluded benchmark exceeds the total-welfare benchmark ($\mathcal{A}^{SE} \geq_{sso} \mathcal{A}^W$), indicating that the seller-excluded benchmark level of instrument is excessive. Consequently, $\mathcal{A}^* = \mathcal{A}^{SE} \geq_{sso} \mathcal{A}^W$.*

Proof. (Proposition OA.1). We first verify that $W(a)$ single-crossing dominates $W^{SE}(a)$: for any $a'' > a'$, whenever $W(a'') - W(a') \geq (>)0$ holds, we must have

$$\begin{aligned} & W^{SE}(a'') - W^{SE}(a') \\ = \quad & W(a'') - W(a') + \underbrace{SS(a') - SS(a'')}_{\geq 0} \geq (>) 0 \end{aligned}$$

because $SS(\cdot)$ is decreasing. Then, we apply Theorem 1 of Amir and Rietzke (2025), which implies $\mathcal{A}^{SE} \geq_{sso} \mathcal{A}^W$, as required. ■

[6]That is, if we assume continuous choice and differentiability, and let $N = 2$ so that a platform's instrument vector is $a_i = (z_1, z_2) \in \mathbb{R}^2$, then this is equivalent to $\partial \hat{W} / \partial z_k$ being single-crossing in z_l for each dimension $k \neq l$, $k = 1, 2$. That is, if $\partial \hat{W} / \partial z_k \geq (>) 0$ at $z_l = z'_l$, then $\partial \hat{W} / \partial z_k \geq (>) 0$ for all $z_l > z'_l$.

It is useful to verify that QSM holds in Examples 1-3 presented in Section A, all of which involve multi-dimensional instruments (Example 4 has a single-dimensional instruments and so QSM trivially holds).

□ **Example 1 (First-party entry and self-preferencing).** Dropping the redundant constant terms, the total welfare objective function is

$$W(r,e,l) = \left(v^m + \pi^m + \alpha e\left(l\Delta^{sp} + (1-l)\,\Delta^{fp}\right)\right) G\left(\bar{k}_i\right) - m\int_{k_{\min}}^{\bar{k}} kdG\left(k\right),$$

where $\bar{k} = (1-r)(\pi^m - \alpha e(\pi^m - (1-l)\,\pi^d))\frac{1}{m}$. Observe that $\bar{k}$ is decreasing in r, e, and l.

Define $\Delta^{sp} = \pi^{sp} + v^{sp} - \pi^m - v^m$ and $\Delta^{fp} = \pi^{fp} + \pi^d + v^d - \pi^m - v^m$ as the ex-post efficiency gain from first-party entry with and without self-preferencing. Suppose $\Delta^{fp} > \Delta^{sp}$. Then W is decreasing in r, decreasing in e regardless of l provided Δ^{fp} is not too large, and decreasing in l:

$$\frac{dW}{dr} = \underbrace{(v^m + \pi^m + \alpha e\left(l\Delta^{sp} + (1-l)\,\Delta^{fp}\right) - m\bar{k})}_{>0 \text{ because } m\bar{k}_i<(1-r_i)\pi^*} g\left(\bar{k}\right)\frac{d\bar{k}}{dr} < 0;$$

$$\frac{dW}{dl} = \left(v^m + \pi^m + \alpha e\left(l\Delta^{sp} + (1-l)\,\Delta^{fp}\right) - m\bar{k}\right) g\left(\bar{k}\right)\frac{d\bar{k}}{dl} + \alpha e_i\left(\Delta^{sp} - \Delta^{fp}\right) G\left(\bar{k}\right) < 0$$

$$\frac{dW}{de} = \left(v^m + \pi^m + \alpha e\left(l\Delta^{sp} + (1-l)\,\Delta^{fp}\right) - m\bar{k}\right) g\left(\bar{k}\right)\frac{d\bar{k}}{de} + \alpha\left(l\Delta^{sp} + (1-l)\,\Delta^{fp}\right) G\left(\bar{k}\right) < 0$$

As such, W is QSM when these conditions hold. Proposition OA.1 then implies $(r^*, e^*, l^*) = (r^{SE}, e^{SE}, l^{SE}) \geq (r^W, e^W, l^W)$.

□ **Example 2 (Preventing disintermediation).** Dropping the redundant constant terms,

$$W(r,\lambda) = (v+\pi)G\left(\bar{k}\right) - m\int_{k_{\min}}^{\bar{k}} kdG\left(k\right),$$

where $\bar{k} = (1 - r + (1-\lambda)\zeta r)\,\frac{\pi}{m}$. Clearly, $W(r,\lambda)$ is decreasing in platform fee r and disintermediation prevention effort λ by the standard deadweight loss logic (a higher λ can be seen as amplifying the effective fees paid by sellers). Thus, $W(r,\lambda)$ is QSM, and Proposition OA.1 then implies $(r^*, \lambda^*) = (r^{SE}, \lambda^{SE}) \geq (r^W, \lambda^W)$.

□ **Example 3 (App tracking).** Assuming the seller objective function is strictly quasiconcave, then by additive separability, the optimal price p satisfies the first-order condition (FOC)

$$p = \frac{\pi_a\left(1-\kappa_i\right)}{1-r_i} + \left(1 + p\frac{q'(p)}{q(p)}\right)\frac{1-H(p)}{h(p)}.$$

Observe that p is an increasing function of $\frac{1-\kappa_i}{1-r_i}$. That is, sellers set a higher price for their apps (to divert buyers to watch ads) when ads becomes more profitable relative to their share of transaction revenue $1-r_i$. To check strict quasiconcavity of the seller objective function, notice $d\pi/dp_i$ has the same sign as

$$-p_i + \frac{\pi_a\left(1-\kappa_i\right)}{1-r_i} + \left(1+e_q\right)\frac{1-H(p_i)}{h(p_i)}, \tag{22}$$

where $e_q \equiv p_i\frac{q'(p_i)}{q(p_i)} < 0$ is elasticity of $q(.)$. By standard results, e_q is weakly decreasing in p_i if $q(.)$ is weakly log-concave or admits constant-elasticity. Therefore, as long as $(1+e_q) > 0$ then we know $(1+e_q)\,\frac{1-H(p_i)}{h(p_i)}$ is decreasing in p_i by log-concavity of $1-H$, and so (22) is always decreasing in p_i, which establishes strict-quasiconcavity.

Imposing symmetry and dropping constant terms, the total welfare objective function that is relevant for determining (r^W, κ^W) is

$$W(r,\kappa) = U_0(p)G\left(\bar{k}\right) + r_i R_0(p) G\left(\bar{k}\right) + m\int_0^{\bar{k}} (\bar{k} - k_i)dG,$$

where

$$\begin{aligned} U_0(p) &= \int_0^p u(q(z)) - zq(z)dH(z) + \int_p^\infty u(q(p)) - pq(p)dH(z) \\ R_0(p) &= pq(p)(1-H(p)) \\ \bar{k} &= \frac{(1-r)}{m} pq(p)(1-H(p)) + \frac{\pi_a(1-\kappa)}{m}\int_0^p q(z)dH(z). \end{aligned}$$

To establish quasi-supermodularity, we reframe the maximization problem as choosing $a = (r, -p)$, where

$$\kappa = \kappa(r,p) = 1 + \psi(p)\left(\frac{1-r}{\pi_a}\right)$$

and

$$\psi(p) \equiv (1+e_q)\frac{1-H(p)}{h(p)} - p < 0$$

is strictly decreasing in p by the properties on (22) as established above. Then

$$\frac{1}{G\left(\bar{k}\right)}\frac{dW}{dr} = \left(U_0(p) + r_i R_0(p)\right)\frac{g\left(\bar{k}\right)}{G\left(\bar{k}\right)}\frac{d\bar{k}}{dr} + m\frac{d\bar{k}}{dr} < 0$$

for all p because $\frac{d\bar{k}}{dr} = -\frac{1}{1-r}\bar{k} < 0$. Thus, dW/dr is single-crossing in p, as required. Likewise,

$$\frac{1}{G\left(\bar{k}\right)}\frac{dW}{dp} = \left(\frac{dU_0}{dp} + \frac{dR_0}{dp}r\right) + \left(U_0(p) + rR_0(p)\right)\varphi\left(\bar{k}\right)\frac{d\bar{k}/dp}{\bar{k}} + m\frac{d\bar{k}}{dp},$$

where $\varphi(x) \equiv \frac{xg(x)}{G(x)}$ is the elasticity of G with respect to its argument. If we impose constant-elasticity $G(k) = \left(\frac{k}{k_{\max}}\right)^\varphi$ on $[0, k_{\max}]$, and let $\varphi \to 0$, then

$$\frac{1}{G\left(\bar{k}_i\right)}\frac{d^2W}{dpdr} \to \frac{dR_0}{dp} + m\frac{d^2\bar{k}}{dpdr} < 0$$

because $\frac{dR_0}{dp} < 0$ by (22), and

$$\frac{d^2\bar{k}}{dpdr} = -\frac{1}{1-r}\frac{d\bar{k}}{dp} = \frac{1}{m}\psi'(p)\int_0^p q(z)dH(z) < 0.$$

Thus, dW/dp is single-crossing in r, as required. Proposition OA.1 implies $(r^*, -p^*) = (r^{SE}, -p^{SE}) \geq (r^W, -p^W)$. Given p is an increasing function of $\frac{1-\kappa_i}{1-r_i}$, we conclude that $p^{SE} \leq p^W$ and $r^{SE} \geq r^W$ together imply $\kappa^{SE} \geq \kappa^W$. Hence, $(r^*, \kappa^*) = (r^{SE}, \kappa^{SE}) \geq (r^W, \kappa^W)$.

C Advertising on the buyer-side

Suppose instead of setting lump-sum prices on the buyer side, each platform i chooses its advertising intensity A_i and gets an associated payoff A_i per buyer. At the same time, buyers incur an associated disutility of γA_i, where $\gamma > 0$ captures a nuisance cost. Here, $\gamma = 1$ implies that raising advertising intensity reduces buyer utility by the same amount it increases platform revenue — just like a lump-sum price. More generally though, ad monetization may be more efficient than using lump-sum prices (i.e., one dollar of extra revenue can be extracted from a buyer with less than a one dollar reduction in utility, so $\gamma < 1$), or less efficient ($\gamma > 1$).

To understand the new welfare distortion in this setting, consider the case of inefficient ad monetization ($\gamma > 1$) and consider a decrease in commission r below r^*. Fixing the level of ad monetization, the decrease in r leads to higher seller-excluded welfare because the inefficient revenue extraction means that platforms do not fully internalize buyer utility in their choice of r^*, resulting in an excessive equilibrium commission. In our *leading example*, an incomplete pass-through argument shows that this direct effect dominates any feedback effect from platforms reoptimizing their level of ad monetization. Formally, we get:

Proposition OA.2 *Consider the above model with advertising on the buyer side. Suppose $\gamma > (<)1$ so that advertising is inefficient (efficient). Then $r^* \geq (\leq) r^{SE}$, strictly so for interior solutions.*

Proof. (Proposition OA.2). We first state the equilibrium in this case without invoking the *leading example.* By the same reframing technique used to establish Proposition 1, we get

$$a^* = \arg\max_{a_i \in \mathcal{A}} \left\{ \frac{1}{\gamma m} U_i \left(a_i; \frac{1}{m} \right) + R_i \left(a_i; \frac{1}{m} \right) \right\}.$$

Meanwhile for each given instrument, we first solve for the equilibrium ad intensity $A(a)$. Following the steps in the proof of Proposition 1, solving for the symmetric FOCs gives

$$A(a) = \frac{1/m}{\Phi'(0)} - \frac{1}{m-1} \frac{\partial U_i(a; \frac{1}{m})}{\partial s_i} - \frac{\partial R_i(a; \frac{1}{m})}{\partial s_i}.$$

Therefore, when the (common) instrument a changes, we have

$$A'(a) = -(\frac{1}{m-1}) \frac{\partial^2 U_i(a; \frac{1}{m})}{\partial s_i \partial a_i} - \frac{\partial^2 R_i(a; \frac{1}{m})}{\partial s_i \partial a_i}.$$

We now specialize the expressions above to the *leading example*, where we know

$$r^* = \arg\max_{r_i \in [0, \overline{r}]} \left\{ \frac{\gamma}{m} U_i \left(r_i; \frac{1}{m} \right) + R_i \left(r_i; \frac{1}{m} \right) \right\},$$

the FOC of which is

$$\frac{1}{\gamma m} \frac{\partial U_i \left(r^*; \frac{1}{m} \right)}{\partial r_i} + \frac{\partial R_i \left(r^*; \frac{1}{m} \right)}{\partial r_i} = 0.$$

Meanwhile, using

$$\begin{aligned} U_i(r_i; s_i) &= v(r_i) \left(\frac{\pi(r_i)}{k_{\max}} \right)^{\varphi} s_i^{\varphi} \\ R_i(r_i; s_i) &= r_i p(r_i) q(r_i) \left(\frac{\pi(r_i)}{k_{\max}} \right)^{\varphi} s_i^{1+\varphi}, \end{aligned}$$

it is clear that $\frac{\partial U_i}{\partial s_i} = \frac{\varphi}{s_i} U_i$, and so $\frac{\partial^2 U_i}{\partial s_i \partial r_i} = \frac{\varphi}{s_i} \frac{\partial U_i}{\partial r_i}$; likewise, $\frac{\partial R_i}{\partial s_i} = \frac{1+\varphi}{s_i} R_i$, and so $\frac{\partial^2 R_i}{\partial s_i \partial r_i} = \frac{1+\varphi}{s_i} \frac{\partial R_i}{\partial r_i}$. Then

$$A'(r) = -\left(\frac{m}{m-1} \right) \frac{\varphi}{\gamma} \frac{\partial U_i \left(r; \frac{1}{m} \right)}{\partial r_i} - (1+\varphi) m \frac{\partial R_i \left(r; \frac{1}{m} \right)}{\partial r_i}.$$

Evaluating this at $r = r^*$ and using the FOC associated with r^*, we get

$$\begin{aligned} A'(r^*) &= -\left(\frac{m}{m-1}\right)\frac{\varphi}{\gamma}\frac{\partial U_i}{\partial r_i} + \frac{1+\varphi}{\gamma}\frac{\partial U_i}{\partial r_i} \\ &= \left(\frac{m-1-\varphi}{m-1}\right)\frac{1}{\gamma}\frac{\partial U_i}{\partial r_i}. \end{aligned}$$

From the expression of the seller-excluded welfare, we have

$$\frac{dW^{SE}(r)}{dr} = \frac{\partial U_i}{\partial r_i} + m\frac{\partial R_i}{\partial r_i} + (1-\gamma)A'(r).$$

Evaluating the above at $r = r^*$, we have

$$\begin{aligned} \frac{dW^{SE}(r^*)}{dr} &= -\left(\frac{1-\gamma}{\gamma}\right)\frac{\partial U_i}{\partial r_i} + (1-\gamma)A'(r^*) \\ &= -\left(\frac{1-\gamma}{\gamma}\right)\frac{\partial U_i}{\partial r_i} + \left(\frac{m-1-\varphi}{m-1}\right)\left(\frac{1-\gamma}{\gamma}\right)\frac{\partial U_i}{\partial r_i} \\ &= -\left(\frac{1-\gamma}{\gamma}\frac{\varphi}{m-1}\right)\frac{\partial U_i}{\partial r_i}, \end{aligned}$$

which is negative if and only if $\gamma > 1$ (because $\frac{\partial U_i}{\partial r_i} < 0$). ∎

D Details for Section 3

D.1 The leading example

□ ***Leading example* with Hotelling competition**. We first check our claim on global concavity: for any given r_i, if (16) holds, Π_i is concave in $s_i \in [0,1]$. Let $z(r_i) = v(r_i) + r_i p(r_i) q(r_i)$. Then we can rewrite (13) as:

$$\Pi_i = \left(P^{B*} + z(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi} s_i^{\varphi} - v(r^*)\left(\frac{\pi(r^*)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi} - (2s_i-1)t\right)s_i.$$

The derivatives are

$$\begin{aligned} \frac{d\Pi_i}{ds_i} &= P^{B*} + (1+\varphi)z(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi} s_i^{\varphi} \\ &\quad - v(r^*)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi-1}\left[1 - s_i - \varphi s_i\right] - 4ts_i + t \\ \frac{d^2\Pi_i}{ds_i^2} &= \varphi(1+\varphi)z(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi} s_i^{\varphi-1} + 2\varphi v(r^*)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi-1} \\ &\quad - (\varphi-1)v(r^*)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi-2}\varphi s_i - 4t. \end{aligned}$$

Among the terms in $d^2\Pi_i/ds_i^2$, only the first two components are positive, and we note $s_i^{\varphi-1}$ and $(1-s_i)^{\varphi-1}$ are both bounded below one given $\varphi \geq 1$. Recalling from (14) that $r^* = \arg\max_{r_i \in [0,\bar{r}]} \{z(r_i)\pi(r_i)^{\varphi}\}$, a sufficient condition for $d^2\Pi_i/ds_i^2 < 0$ to hold for any s_i and r_i is

$$2t > \varphi(1+\varphi)z(r^*)\left(\frac{\pi(r^*)}{k_{\max}}\right)^{\varphi}, \tag{23}$$

which coincides with the condition in (16). Notice this condition implies $2t \geq 2\varphi v(r^*)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}$ because $\varphi \geq 1$ and $z(r^*) \geq v(r^*)$. Meanwhile, the condition for there to be a unique fixed-point in (8) is equivalent to requiring (9) to be strictly decreasing in s_i, i.e.,

$$\frac{dP_i^B}{ds_i} = \varphi z(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi} s_i^{\varphi-1} + \varphi v\left(r^*\right)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}(1-s_1)^{\varphi-1} - 2t < 0,$$

which holds given (16).

Next, we provide two sets of conditions under which the objective function in (14) is strictly quasiconcave (hence has a unique maximizer).

One condition is to impose $c = 0$, which recall means $\overline{r} = 1$. Then using the same notation as in Section A, we have

$$\left(z(r_i)\pi(r_i)^{\varphi}\right)|_{c=0} = (v^m + r_i\pi^m)((1-r_i)\pi^m)^{\varphi}.$$

The derivative with respect to r_i has the same sign as

$$\left(\pi^m - \frac{\varphi\left(v^m + r_i\pi^m\right)}{1-r_i}\right)(1-r_i)^{\varphi}.$$

Observe the expression in the first (large) brackets is monotonically decreasing in r_i, and so there exists a (possibly negative) threshold $\hat{r} < 1$ such that the expression is strictly negative if and only if $r_i > \hat{r}$. Hence, $z(r_i)\pi(r_i)^{\varphi}$ is strictly single-peaked and so strictly quasiconcave.

Suppose instead $c > 0$. Then another set of conditions is $\varphi = 1$ and a linear-quadratic utility specification

$$u\left(q\right) = Vq - \frac{1}{2}q^2, \text{ such that } D(p_i) = V - p_i,$$

with $V > c$. This implies $q(r_i) = \frac{1}{2}\left(V - \frac{c}{1-r_i}\right)$, $p(r_i) = \frac{1}{2}\left(V + \frac{c}{1-r_i}\right)$, $\pi(r_i) = \frac{1-r_i}{4}\left(V - \frac{c}{1-r_i}\right)^2$, and $v(r_i) = \frac{1}{8}\left(V - \frac{c}{1-r_i}\right)^2$, where recall $\overline{r} = 1 - \frac{c}{V} < 1$. Then, the objective function defining r^* can be rewritten

$$z(r_i)\pi(r_i) = \frac{1}{32}\left(V - \frac{c}{1-r_i}\right)^3 B(r_i)$$

for $r_i \in [0, \overline{r}]$, where $V - \frac{c}{1-r_i}$ and $B(r_i) = V - c + (V + 2c)\, r_i - 2r_i^2 V$ which are both strictly concave and positive on $r_i \in [0, \overline{r})$ and $V - \frac{c}{1-\overline{r}} = 0$. This implies the maximum must occur on $[0, \overline{r})$. Within this range, the objective $z(r_i)\pi(r_i)$ can therefore be written as the product of positive and strictly concave functions, so must itself be strictly quasiconcave.

□ **Leading example with logit specification**. Suppose we impose the logit demand system $\Phi(x) = \frac{1}{1+(m-1)\exp\{-x/\mu\}}$, with scale parameter $\mu > 0$. Then

$$\Phi^{-1}\left(s_i\right) = \mu \ln\left(\frac{(m-1)s_i}{1-s_i}\right)$$

and $\frac{\partial}{\partial s_i}\Phi^{-1}\left(s_i\right) = \frac{\mu}{(1-s_i)s_i}$. By the same calculation as before

$$\begin{aligned}\frac{d\Pi_i}{ds_i} &= P^{B*} + (1+\varphi)z(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi} s_i^{\varphi} \\ &\quad - v\left(r^*\right)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi-1}\left[1 - s_i - \varphi s_i\right] - \left[\frac{\mu}{1-s_i} + \mu\ln\left(\frac{(m-1)s_i}{1-s_i}\right)\right]\end{aligned}$$

$$\begin{aligned}\frac{d^2\Pi_i}{ds_i^2} &= \varphi(1+\varphi)z(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi} s_i^{\varphi-1} + 2\varphi v\left(r^*\right)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi-1} \\ &\quad - (\varphi-1)v\left(r^*\right)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi-2}\varphi s_i - \left[\frac{\mu}{(1-s_i)^2 s_i}\right].\end{aligned}$$

We know $\max_{s_i\in[0,1]}(1-s_i)^2 s_i = 4/27$. Hence, using the equivalent condition to (23), a sufficient condition for strict concavity is

$$\mu > \frac{8}{27}\varphi(1+\varphi)z(r^*)\left(\frac{\pi(r^*)}{k_{\max}}\right)^{\varphi}. \tag{24}$$

This condition also ensures (9) is strictly decreasing in s_i. To see this, note

$$\begin{aligned}\frac{dP_i^B}{ds_i} &= \varphi z(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi} s_i^{\varphi-1} + \varphi v(r^*)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi-1} - \frac{\mu}{(1-s_i)s_i}\\ &\leq \varphi z(r^*)\left(\frac{\pi(r^*)}{k_{\max}}\right)^{\varphi} s_i^{\varphi-1} + \varphi z(r^*)\left(\frac{\pi(r^*)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi-1} - \frac{\mu}{(1-s_i)s_i}\\ &\leq 2\varphi z(r^*)\left(\frac{\pi(r^*)}{k_{\max}}\right)^{\varphi} - 4\mu\\ &< 0,\end{aligned}$$

where the first inequality is due to the definitions of $z(r_i)$ and r^*; the second inequality is due to $s_i^{\varphi-1}\leq 1$, $(1-s_i)^{\varphi-1}\leq 1$, and $\max_{s_i\in[0,1]}(1-s_i)s_i = 1/4$; and the last inequality is due to the stated sufficient condition and $\varphi \geq 1$.

□ **Buyer surplus**. Buyer surplus is given by $BS(r) = U(r;\frac{1}{m}) - P^B(r)$. Continuing from Proposition 1, we have

$$P^B(r) = \frac{1}{m\Phi'(0)} - \left(\frac{m\varphi v(r)}{m-1} + (1+\varphi)rp(r)q(r)\right)\left(\frac{\pi(r)}{mk_{\max}}\right)^{\varphi}$$

so that

$$BS(r) = \left(v(r) + \frac{m\varphi v(r)}{m-1} + (1+\varphi)rp(r)q(r)\right)\left(\frac{\pi(r)}{mk_{\max}}\right)^{\varphi} - \frac{1}{m\Phi'(0)}.$$

Hence, the maximizer can be simplified as

$$r^{BS} = \arg\max_{r\in[0,\overline{r}]}\left(\frac{m-1+m\varphi}{m-1+m\varphi-\varphi}v(r) + rp(r)q(r)\right)\pi(r)^{\varphi},$$

which we now compare with $r^* = r^{SE} = \arg\max_{r\in[0,\overline{r}]}(v(r)+rp(r)q(r))\pi(r_i)^{\varphi}$. Using the observations that $\frac{m-1+m\varphi}{m-1+m\varphi-\varphi} > 1$ and that $v(r) > 0$ is decreasing, it follows that $r^{BS} \leq r^{SE} = r^*$, with strictly inequality for interior solutions.

To see the more general pass-through logic discussed in the text, in what follows we assume that $BS(r)$ is strictly quasiconcave in r and that r^* is an interior solution. We note $\frac{d}{dr}BS(r) = \frac{\partial}{\partial r}U(r;\frac{1}{m}) - \frac{d}{dr}P^B(r)$. Using the equilibrium condition for r^*, this becomes

$$\frac{d}{dr}BS(r^*) = -m\frac{\partial R_i(r^*;\frac{1}{m})}{\partial r} - \frac{dP^B(r^*)}{dr}$$

and so $\frac{d}{dr}BS(r^*)(\leq) < 0$ if and only if

$$\frac{dP^B(r^*)}{dr}(\geq) > -m\frac{\partial R_i(r^*;\frac{1}{m})}{\partial r}. \tag{25}$$

That is, starting from the equilibrium value r^*, for any increase in r that raises the per-buyer revenue $\frac{1}{1/m}R_i$ by one unit, the per-buyer price P^B does not decrease by more than one unit. We now prove that inequality (25) always hold in strict inequality in our leading example. Continuing from Proposition 1,

$$\frac{dP^B(r)}{dr} = -\left(\frac{1}{m-1}\right)\frac{\partial U_i(r;\frac{1}{m})}{\partial r\partial s_i} - \frac{\partial R_i(r;\frac{1}{m})}{\partial r\partial s_i},$$

where the constant-elasticity $G(\cdot)$ in the leading example means

$$\frac{\partial R_i(r; \frac{1}{m})}{\partial s_i \partial r} = m(1+\varphi)\frac{\partial R_i(r; \frac{1}{m})}{\partial r} \quad \text{and} \quad \frac{\partial U_i(r; \frac{1}{m})}{\partial s_i \partial r} = m\varphi \frac{\partial U_i(r; \frac{1}{m})}{\partial r}$$

and so

$$\frac{dP^B(r)}{dr} = -\left(\frac{m\varphi}{m-1}\right)\frac{\partial U_i(r; \frac{1}{m})}{\partial r} - m(1+\varphi)\frac{\partial R_i(r; \frac{1}{m})}{\partial r}.$$

Recall the first order condition of r^* implies

$$m\frac{\partial R_i(r^*; \frac{1}{m})}{\partial r_i} = -\frac{\partial U_i(r^*; \frac{1}{m})}{\partial r_i} > 0,$$

where the inequality is due to $U_i = v(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}(1-s_i)^{\varphi}$ decreasing in r_i. So,

$$\frac{dP^B(r^*)}{dr} = \left(\frac{\varphi}{m-1} - 1\right) m\frac{\partial R_i(r^*; \frac{1}{m})}{\partial r} > -m\frac{\partial R_i(r^*; \frac{1}{m})}{\partial r}.$$

For a general platform instrument a_i beyond the leading example, the same pass-through logic of inequality (25) works. Specifically, suppose the equilibrium outcome a^* is an interior solution. Then, we get $\frac{d}{dr}BS(a^*) \leq 0$ if and only if

$$\frac{dP^B(a^*)}{da} \geq -m\frac{\partial R_i(a^*; \frac{1}{m})}{\partial a}.$$

That is, the pass-through rate of extra per-buyer revenue from a higher level of a_i onto a lower buyer-side price is no more than one. When this is true, we can conclude $a^{BS} \leq a^*$ if $BS(a)$ is quasiconcave (strictly so, if the inequality above is strict), and $a^{TUS} \equiv \arg\max_{a \in \mathcal{A}} BS(a) + mSS_i(a; \frac{1}{m}) < a^*$ if $BS(a) + mSS_i(a; \frac{1}{m})$ is quasiconcave.

D.2 The leading example with myopic buyers

Suppose buyers' perceived U_i when making their platform choice is discounted by δ, where $0 \leq \delta < 1$. We first characterize the equilibrium before specializing to the *leading example*. By the same steps that establish Proposition 1, we get

$$a^* = \arg\max_{a_i \in \mathcal{A}} \left\{\frac{\delta}{m}U_i\left(a_i; \frac{1}{m}\right) + R_i\left(a_i; \frac{1}{m}\right)\right\}$$

and $P^{B*} = P^B(a)$, where

$$P^B(a) = \frac{1}{m\Phi'(0)} - \left(\frac{\delta}{m-1}\right)\frac{\partial U_i(a; \frac{1}{m})}{\partial s_i} - \frac{\partial R_i(a; \frac{1}{m})}{\partial s_i}.$$

Meanwhile, the seller-excluded benchmark remains the same as in the baseline model because the planner takes into account the actual utility of buyers, so

$$a^{SE} = \arg\max_{a_i \in \mathcal{A}} \left\{\frac{1}{m}U_i\left(a_i; \frac{1}{m}\right) + R_i\left(a_i; \frac{1}{m}\right)\right\}.$$

Finally, $a^{BS} = \arg\max_{a_i \in \mathcal{A}} \left\{U(a_i; \frac{1}{m}) - P^B(a_i)\right\}$.

We now specialize to the *leading example*, where $a_i = r_i$. We claim that $r^* \geq r^{SE}$ and $r^* \geq r^{BS}$. In the leading example $U_i(r_i) = v(r_i)\left(\frac{\pi(r_i)}{mk_{\max}}\right)^{\varphi}$, so

$$\begin{aligned} r^* &= \arg\max_{r\in[0,\overline{r}]} \left(\delta v(r) + rp(r)q(r)\right)\pi(r)^{\varphi} \\ r^{SE} &= \arg\max_{r\in[0,\overline{r}]} \left(v(r) + rp\left(r\right)q(r)\right)\pi(r)^{\varphi} \\ r^{BS} &= \arg\max_{r\in[0,\overline{r}]} \left(\frac{m-1+m\delta\varphi}{(m-1)\left(1+\varphi\right)}v(r) + rp(r)q\left(r\right)\right)\pi(r)^{\varphi}, \end{aligned}$$

where we used

$$BS(r) = \left(v(r) + \frac{\delta m\varphi v(r)}{m-1} + (1+\varphi)rp\left(r\right)q(r)\right)\left(\frac{\pi\left(r\right)}{mk_{\max}}\right)^{\varphi} - \frac{1}{m\Phi'(0)}.$$

Given $v(r) > 0$ is decreasing, it follows that $\delta < 1$ implies $r^* \geq r^{SE}$. If r^* or r^{SE} is pinned down as an interior solution, the associated FOC then implies $r^* > r^{SE}$. Likewise,

$$\frac{m-1+m\delta\varphi}{(m-1)\left(1+\varphi\right)} > \delta \implies r^{BS} \leq r^*,$$

with strictly inequality holds for interior solutions. Moreover, $\frac{m+m\delta\varphi-1}{(m-1)(1+\varphi)} > 1$ is equivalent to $(m-1)(1-\delta) + \delta\varphi > 0$, which always holds by assumption.

D.3 Seller-side lump-sum fees

Suppose we interpret instrument a_i as a seller-side lump-sum fee ($a_i = P_i^S$). Then, $U_i = vG(\pi s_i - a_i)$ and $R_i = a_i G(vs_i - a_i)$, where the interaction benefits of buyers and sellers, v and π, are independent of the level a_i on sellers. Applying Proposition 1, in the equilibrium

$$a^* = \arg\max_{a_i\in\mathcal{A}}\left\{(\frac{v}{m} + a_i)G(\frac{\pi}{m} - a_i)\right\}.$$

Given G is log-concave, the objective function is quasiconcave and so the FOC gives

$$\begin{aligned} a^* &= -\frac{v}{m} + \frac{G(\frac{\pi}{m} - a^*)}{g(\frac{\pi}{m} - a^*)} \\ &= -\frac{v}{m} + \frac{\frac{\pi}{m} - a^*}{\varphi(\frac{\pi}{m} - a^*)}. \end{aligned} \tag{26}$$

Log-concavity implies $\frac{x}{\varphi(x)}$ is an increasing function in $x \geq 0$, i.e., $\varphi(x) - x\varphi'(x) > 0$. Totally differentiating

$$\frac{da^*}{dm} = \frac{1}{m^2}\left(v - \frac{\pi}{\varphi(x)} - \frac{\pi x}{\varphi(x)^2}\varphi'(x)\right)_{x=\frac{\pi}{m}-a^*}.$$

Therefore,

$$\frac{da^*}{dm} < 0 \Leftrightarrow \frac{v}{\pi} < \left(\frac{1}{\varphi(x)} + \frac{x\varphi'(x)}{\varphi(x)^2}\right)_{x=\frac{\pi}{m}-a^*}.$$

In the special case of constant-elasticity G, $\varphi'(\cdot) = 0$, and so

$$\frac{da^*}{dm} < 0 \Leftrightarrow \frac{v}{\pi} < \frac{1}{\varphi}.$$

Intuitively, a higher m means sellers get a lower surplus from joining each individual platform, which induces platforms to set a lower seller-side fee a^*; at the same time, a higher m also means that each platform extracts less buyer utility, and so platforms are less incentivized to attract sellers and raise buyer utility. The fee-decreasing (fee-increasing) effect dominates when the elasticity of seller participation φ is relatively large (small).

Nonetheless, if we denote $\bar{k}^* \equiv \frac{\pi}{m} - a^*$ as the marginal participating seller in the equilibrium, then (26) becomes

$$\bar{k}^* + \frac{G(\bar{k}^*)}{g(\bar{k}^*)} = \frac{\pi + v}{m}.$$

So, $\bar{k}^*$ is always decreasing in m. That is, an increase in the number of platforms results in less sellers participating on platforms in equilibrium, although each seller that does participate, participates on a greater number of platforms.

E Without symmetry and full coverage

In this Online Appendix, we extend our baseline model to an environment with possibly asymmetric platforms and a partially covered buyer-side market. Let $U_0 = 0$ be the exogenous net utility of the buyers' outside option (of not joining any platform). We first establish the equilibrium in this environment, and then revisit the comparative statics with respect to the number of platforms.

E.1 Equilibrium characterization

For any given instrument profile $(a_1, ..., a_m)$ and buyer price profile $(P_1^B, ..., P_m^B)$, the buyer-side market share profile $\boldsymbol{s} = (s_1, s_2, ..., s_m)$ is pinned down by the simultaneous fixed-point equation system:

$$s_i = \Pr\left(U_i(a_i, s_i) - P_i^B + \epsilon_i \geq \max_{j \neq i} \left\{U_j(a_j, s_j) - P_j^B + \epsilon_j, 0\right\}\right) \text{ for } i = 1, ..., m, \tag{27}$$

where the probability is taken with respect to $(\epsilon_1, ..., \epsilon_m)$ that follows some underlying distribution $F(\cdot)$. As in the baseline model, we assume a unique fixed point $\boldsymbol{s} = (s_1, s_2, ..., s_m)$ to (27) always exists.

To derive the equilibrium outcome, denote the equilibrium buyer price profile as $(P_1^{B*}, ..., P_m^{B*})$, the equilibrium instrument profile as $(a_1^*, ..., a_m^*) \in \mathcal{A}^m$, and the equilibrium buyer-side market share profile as $(s_1^*, ..., s_m^*) \in [0, 1]^m$.

□ **Reframing the maximization problem.** Consider the maximization problem of platform i. It chooses $\left(a_i, P_i^B\right)$ to maximize profit

$$\Pi_i = P_i^B s_i + R_i\left(a_i; s_i\right),$$

taking as given the choices of other platforms $\{(a_j^*, P_j^{B*})\}_{j \neq i}$. We can frame the problem as platform i directly choosing (a_i, s_i), where P_i^B is then set to implement the target market share s_i. Specifically, continuing from (27), for given instrument choice a_i, the target market share s_i can be implemented by setting

$$P_i^B = U_i\left(a_i; s_i\right) - \xi_i(s_i; \{(a_j^*, P_j^{B*})\}_{j \neq i}),$$

where the function $\xi_i(\cdot; \cdot)$ is the scalar solution ξ to the following system of equations:

$$\begin{aligned} s_i &= \Pr\left(\xi + \epsilon_i \geq \max_{j \neq i} \left\{U_j(a_j, s_j) - P_j^{B*} + \epsilon_j, 0\right\}\right) \\ s_j &= \Pr\left(U_j(a_j, s_j) - P_j^{B*} + \epsilon_i \geq \max_{k \neq j} \left\{\xi + \epsilon_i, U_k(a_k, s_k) - P_k^{B*} + \epsilon_k, 0\right\}\right) \text{ for } j \neq i, \end{aligned}$$

where the existence of the solution is guaranteed given the existence of a unique fixed point to (27). Importantly, observe the system of equations that defines $\xi_i(\cdot; \cdot)$ is independent of a_i, reflecting that the market share of s_i depends on a_i only indirectly via $U_i(a_i; s_i)$.

□ **Optimal instrument choices and equilibrium.** Therefore, platform i's problem now becomes choosing (a_i, s_i) to maximize

$$U_i(a_i; s_i)\, s_i + \xi_i(s_i; \{(a_j^*, P_j^{B*})\}_{j\neq i}) s_i + R_i(a_i; s_i),$$

which we assume to be globally strictly quasiconcave in $(a_i; s_i)$. By the principle of optimality, in any equilibrium, platform i's optimal choices necessarily satisfy

$$a_i^* = \arg\max_{a_i \in \mathcal{A}} \{U_i(a_i; s_i^*)\, s_i^* + R_i(a_i; s_i^*)\} \tag{28}$$

and

$$s_i^* = \arg\max_{s_i \in [0,1]} \{U_i(a_i^*; s_i)\, s_i + \xi_i(s_i; \{(a_j^*, P_j^{B*})\}_{j\neq i}) s_i + R_i(a_i^*; s_i))\}. \tag{29}$$

Summarizing, the equilibrium is pinned down by a system of $3m$-equations-and-$3m$-variables:

Proposition OA.3 *In any equilibrium with all m platforms active in the market, the equilibrium outcome is described by $(a_1^*, ..., a_m^*)$, $(P_1^{B*}, ..., P_m^{B*})$, and $(s_1^*, ..., s_m^*)$ that solve the (i) optimality conditions in (28) and (29) for $i = 1, ..., m$, and (ii) the consistency requirement:*

$$P_i^{B*} = U_i(a_i^*; s_i^*) - \xi_i(s_i^*; \{(a_j^*, P_j^{B*})\}_{j\neq i}) \quad \text{for } i = 1, ..., m.$$

Proposition OA.3 generalizes the baseline equilibrium characterization in Proposition 1. In this environment, the equilibrium instrument and buyer-side prices do not generally have closed-form expressions given $s_i^* \neq 1/n$ in general.

E.2 An increase in the number of platforms

We now specialize the functional forms of U_i and R_i as in Section 3.2 of the main text:

$$\begin{array}{c} U_i(a_i; s_i) = v_i(a_i) G(\pi_i(a_i) s_i) \\ R_i(a_i; s_i) = w_i(a_i) s_i G(\pi_i(a_i) s_i) \end{array}.$$

Note we allow functions v_i, w_i, and π_i and distribution G_i to be different across platforms. Then, the equilibrium instrument in (28) becomes

$$a_i^* = \arg\max_{a_i \in \mathcal{A}} \{(v_i(a_i) + w_i(a_i)) G_i(\pi_i(a_i) s_i^*)\},$$

where the equilibrium market share s_i^* is held fixed in the optimization problem. Assuming differentiability, the associated first-order condition (the functional arguments are omitted for notational simplicity) is:

$$\underbrace{\left[\frac{\partial v_i}{\partial a_i} + \frac{\partial w_i}{\partial a_i}\right]_{a_i = a_i^*}}_{\text{additional revenue per inframarginal seller}} + \underbrace{\left[(v_i + w_i)\, \varphi_i(\pi_i s_i^*) \frac{\partial \pi_i / \partial a_i}{\pi_i}\right]_{a_i = a_i^*}}_{\text{loss in seller participation}} = 0,$$

where recall $\partial\pi/\partial a_i < 0$, whereas $\varphi(k) \equiv \frac{kg(k)}{G(k)} \geq 0$ is seller participation elasticity. Utilizing the same proof that establishes Proposition 3, we conclude:

Proposition OA.4 *Suppose the equilibrium described in Proposition OA.3 exists. In this equilibrium,*

$$\frac{da_i^*}{dm} \text{ has the same sign as } -\varphi_i' \times \frac{ds_i^*}{dm}.$$

In the case of constant elasticity $\varphi_i' = 0$, then a_i^* is always independent of m.

Proposition OA.4 generalizes Proposition 3 to environments with possibly asymmetric platforms and partially covered buyer-side market, assuming that the equilibrium described in Proposition OA.3 exists (which requires quasiconcavity of platform profit functions). We make three important observations here.

First, we see that seller participation elasticity remains a key factor (as captured by the term $-\varphi_i'$ in the proposition statement). Buyer participation elasticity matters only to the extent that it influences the sign of ds_i^*/dm.

Second, suppose that $ds_i^*/dm < 0$ holds (i.e., the equilibrium buyer-side market share of platform i shrinks with the number of platforms m). Then, Proposition OA.4 immediately implies

$$\frac{da_i^*}{dm} \text{ has the same sign as } \varphi',$$

which is exactly the same condition as Proposition 3. This result is a consequence of the competitive bottlenecks logic: platforms face no real competition with respect to the seller side, and so a higher m affects a_i^* only indirectly through changes in the buyer-side market share s_i^*.

Third, by standard logic of increased competitiveness, $ds_i^*/dm < 0$ is a reasonable property, but it may not always hold in environments with asymmetric platforms and strong enough network effects. Verifying $ds_i^*/dm < 0$ requires total differentiation of the system of equations that pins down $(s_1^*, ..., s_m^*)$ in Proposition OA.3, which is analytically challenging. Nonetheless, in the special case of constant elasticity $\varphi_i' = 0$, then a_i^* is always independent of m, regardless of the sign of ds_i^*/dm.

F Details for Section 4

F.1 Heterogeneous interaction benefits

□ **Preliminaries.** We first state the equilibrium pricing by the sellers. Facing the commission rate r_i, a seller's optimal price on platform i is then

$$p(r_i) = \arg\max_{p_i} \left\{ ((1 - r_i)\, p_i - c)(V - p_i) \left(s_i\theta_{reg} + \frac{\lambda}{m}\theta_{loyal} \right) \right\},$$

where $s_i\theta_{reg} + \frac{\lambda}{m}\theta_{loyal}$ is the sum of buyers on platform i (weighted according to their interaction value). Then, define $q(r_i) = V - p(r_i)$. The linear demand form implies $q(r_i) > 0$ for all $r_i < \bar{r} = 1 - \frac{c}{V}$ and $q(r_i) = 0$ otherwise. Seller total profit from platform i is $\left(s_i\theta_{reg} + \frac{\lambda}{m}\theta_{loyal}\right)\pi(r_i)$, where $\pi(r_i) = ((1 - r_i)\, p(r_i) - c)(V - p(r_i))$, and the per-seller surplus of the buyer is

$$\begin{aligned} v_\tau(r_i) &= V\theta_\tau q(r_i) - \frac{\theta_\tau^2}{2\theta_\tau} q(r_i)^2 - p(r_i)\theta_\tau q(r_i) \\ &= \frac{\theta_\tau}{2}(V - p(r_i))^2. \end{aligned}$$

We have

$$\begin{aligned} &U_i^\tau = v_\tau(r_i) \left(\frac{\pi(r_i)\left(s_i\theta_{reg} + \frac{\lambda}{m}\theta_{loyal}\right)}{k_{\max}} \right)^\varphi \\ &R_i = r_i p(r_i) q(r_i) \left(s_i\theta_{reg} + \frac{\lambda}{m}\theta_{loyal}\right) \left(\frac{\pi(r_i)\left(s_i\theta_{reg} + \frac{\lambda}{m}\theta_{loyal}\right)}{k_{\max}} \right)^\varphi . \end{aligned}$$

Platform profit is $\left(\frac{\lambda}{m} + s_i\right) P_i^B + R_i$.

□ **Equilibrium existence.** We now use the *leading example with Hotelling competition* to demonstrate the conditions for equilibrium existence. Recall that loyal buyers have no transportation costs for their preferred platform and infinite transportation costs for the other platform, and their outside option is valued at zero.

Clearly, if $\lambda = 0$, the model reduces to the leading example with Hotelling competition, in which case the existence condition (16) immediately applies. Hence, our strategy here is to show equilibrium existence for sufficiently small $\lambda \to 0$. We focus on $\theta_{reg} = 0$.

Each platform i chooses r_i and s_i to maximize

$$\begin{aligned}\Pi_i &= \left(P^{B*} + U_i^{reg}(r_i; s_i) - U_{-i}^{reg}(r^*; 1 - s_i) - (2s_i - 1)t\right)\left(\frac{\lambda}{2} + s_i\right) + R_i(a_i; s_i) \\ &= \left(P^{B*} - (2s_i - 1)t\right)\left(\frac{\lambda}{2} + s_i\right) + r_i p(r_i) q(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}\left(\frac{\lambda}{2}\theta_{loyal}\right)^{1+\varphi},\end{aligned}$$

where we used $\theta_{reg} = 0$. Observe that r^* is determined by a single-variable maximization, regardless of the value of s_i:

$$r^* = \arg\max_{r_i \in [0, \bar{r}]} r_i p(r_i) q(r_i)) \pi(r_i)^{\varphi}.$$

Meanwhile, maximization with respect to s_i is a standard Hotelling problem and so local concavity holds and the FOC gives $P^{B*} = (1 + \lambda)\, t$. The equilibrium profit is

$$\Pi^{eqm} = \frac{t}{2}(1 + \lambda)^2 + r^* p(r^*)\, q(r^*)\left(\frac{\pi(r^*)}{k_{\max}}\right)^{\varphi}\left(\frac{\lambda}{2}\theta_{loyal}\right)^{1+\varphi}.$$

Full coverage of the regular type requires

$$b > \left(\frac{3}{2} + \lambda\right) t,$$

which also ensures full coverage of the loyal type.

It remains to rule out a global deviation where each platform just fully exploits its loyal buyers by setting

$$P^{dev} = b + v_{loyal}\left(r^{dev}\right)\left(\frac{\pi\left(r^{dev}\right)}{k_{\max}}\right)^{\varphi}\left(\frac{\lambda}{2}\theta_{loyal}\right)^{\varphi} > (1 + \lambda)\, t = P^{B*}$$

together with the optimal deviation commission r^{dev} that is the maximizer of

$$\Pi^{dev} = \max_{r_i \in [0, \bar{r}]}\left\{\left(b + v_{loyal}(r_i)\left(\frac{\pi(r_i)\frac{\lambda}{2}\theta_{loyal}}{k_{\max}}\right)^{\varphi}\right)\left(\frac{\lambda}{2} + s_i^{dev}\right) + r_i p(r_i) q(r_i)\left(\frac{\pi(r_i)}{k_{\max}}\right)^{\varphi}\left(\frac{\lambda}{2}\theta_{loyal}\right)^{1+\varphi}\right\},$$

where

$$s_i^{dev} = \frac{1}{2} + \frac{1}{2t}\left((1 + \lambda)\, t - b - v_{loyal}(r_i)\left(\frac{\pi(r_i)\frac{\lambda}{2}\theta_{loyal}}{k_{\max}}\right)^{\varphi}\right).$$

Using an envelope theorem argument, it is easy to verify that $\lim_{\lambda \to 0} \Pi^{dev} < \Pi^{eqm}$ by definition. Hence, the equilibrium exists for λ sufficiently small.

□ **Proof of Proposition 4.** Notice that only regular buyers are marginal because loyal buyers always purchase from their respective preferred platform. We apply the same reframing technique used in Proposition 1: each platform's optimal r_i (for given s_i) maximizes

$$\begin{aligned}&\left(\frac{\lambda}{m} + s_i\right) U_i^{reg} + R_i \\ =\ &\left(v_{reg}(r_i)\left(\frac{\lambda}{m} + s_i\right) + r_i p(r_i) q(r_i)\left(s_i\theta_{reg} + \frac{\lambda}{m}\theta_{loyal}\right)\right)\left(\frac{\pi(r_i)\left(s_i\theta_{reg} + \frac{\lambda}{m}\theta_{loyal}\right)}{k_{\max}}\right)^{\varphi}.\end{aligned}$$

After imposing symmetry and removing the multiplicative coefficients that are irrelevant for the maximization problem, we conclude that in the equilibrium r^* is

$$r^* = \arg \max_{r_i \in [0,\bar{r}]} \left\{ \left(v_{reg}(r) \left(1+\lambda\right) + rp(r)q(r) \left(\theta_{reg} + \lambda\theta_{loyal}\right) \right) \pi \left(r\right)^{\varphi} \right\},$$

whereas

$$P^{B*} = \frac{1}{m\Phi'(0)} - \left(\frac{m}{m-1} \varphi v_{reg}(r^*) + (1+\varphi) r^* p(r^*) q(r^*) \left(\theta_{reg} + \lambda\theta_{loyal}\right) \right) \left(\frac{\pi(r^*) \left(\theta_{reg} + \lambda\theta_{loyal}\right)}{mk_{\max}} \right)^{\varphi}.$$

Now consider W^{SE}:

$$\begin{aligned} W^{SE}(r) &= U_i^{reg} + mR_i + \lambda \left(b + U_i^{loyal} \right) + \int_{\underline{\epsilon}}^{\bar{\epsilon}} \hat{\epsilon} d\hat{F}(\hat{\epsilon}) \\ &= \lambda b + (1+\lambda) U_i^{reg} + mR_i + \lambda \left(U_i^{loyal} - U_i^{reg} \right) + \int_{\underline{\epsilon}}^{\bar{\epsilon}} \hat{\epsilon} d\hat{F}(\hat{\epsilon}), \end{aligned}$$

where $(1+\lambda) U_i^{reg} + mR_i$ is proportional to the objective of r^* and so it is maximized at r^*. Meanwhile,

$$\begin{aligned} U_i^{loyal} - U_i^{reg} &= \left(v_{loyal}(r) - v_{reg}(r) \right) \left(\frac{\pi(r) \left(\theta_{reg} + \lambda\theta_{loyal}\right)}{mk_{\max}} \right)^{\varphi} \\ &= \frac{\theta_{loyal} - \theta_{reg}}{2} (V - p(r))^2 \left(\frac{\pi(r) \left(\theta_{reg} + \lambda\theta_{loyal}\right)}{mk_{\max}} \right)^{\varphi}, \end{aligned}$$

which is monotonically decreasing (increasing) in r if $\theta_{loyal} > (<) \theta_{reg}$. A simple proof by contradiction then shows $r^{SE} \leq (\geq) r^*$ if $\theta_{loyal} > (<) \theta_{reg}$, thus completing the proof.

□ **General demand specification.** We now consider a more general demand specification rather than the linear-quadratic specification in the main text. We denote seller total profit on i as

$$\bar{\pi}(r_i; s_i) = \max_{p_i} \left\{ \left((1-r_i) p_i - c \right) \left[\frac{\lambda}{m} D_{loyal}(p_i) + s_i D_{reg}(p_i) \right] \right\},$$

where $p(r_i)$ is its maximizer. The corresponding total transaction quantity is

$$\bar{q}(r_i; s_i) = \frac{\lambda}{m} D_{loyal}(p(r_i; s_i)) + s_i D_{reg}(p(r_i; s_i)),$$

while the per-seller surplus of the buyer is $v_\tau(r_i; s_i) = u_\tau \left(D_\tau \left(p\left(r_i; s_i\right) \right) \right) - p\left(r_i; s_i\right) D_\tau \left(p\left(r_i; s_i\right) \right)$ for each type $\tau \in \{reg, loyal\}$. We have

$$\begin{aligned} U_i^{\theta} &= v_\theta(r_i; s_i) \left(\tfrac{\bar{\pi}(r_i; s_i)}{k_{\max}} \right)^{\varphi} \\ R_i &= r_i p(r_i) \bar{q}(r_i; s_i) \left(\tfrac{\bar{\pi}(r_i; s_i)}{k_{\max}} \right)^{\varphi}. \end{aligned}$$

Platform profit is $\left(\frac{\lambda}{m} + s_i \right) P_i^B + R_i$. By the same reframing technique used in Proposition 1, each platform's optimal r_i (for given $s_i = 1/m$) maximizes

$$\begin{aligned} r^* &= \arg \max_{r_i \in [0,\bar{r}]} \left\{ \frac{1+\lambda}{m} U_i^{reg} + R_i \right\} \\ &= \arg \max_{r_i \in [0,\bar{r}]} \left\{ \left(v_{reg}(r; \frac{1}{m}) \frac{1+\lambda}{m} + rp(r; \frac{1}{m}) \bar{q}(r; \frac{1}{m}) \right) \bar{\pi}(r; \frac{1}{m})^{\varphi} \right\}, \end{aligned}$$

while

$$W^{SE}(r) = \lambda b + (1+\lambda) U_i^{reg} + mR_i + \lambda \left(U_i^{loyal} - U_i^{reg} \right) + \int_{\underline{\epsilon}}^{\bar{\epsilon}} \hat{\epsilon} d\hat{F}(\hat{\epsilon}),$$

where $(1+\lambda)\,U_i^{reg}+mR_i$ is proportional to the objective of r^* and so it is maximized at r^*. Meanwhile,

$$U_i^{loyal}-U_i^{reg}=\left(v_{loyal}(r;\frac{1}{m})-v_{reg}(r;\frac{1}{m})\right)\left(\frac{\bar{\pi}(r;\frac{1}{m})}{k_{\max}}\right)^{\varphi},$$

and so $r^{SE}\leq(\geq)r^*$ holds if $\frac{d}{dr}(U_i^{loyal}-U_i^{reg})\leq(\geq)0$.

Suppose the utility function satisfies the standard Spence-Mirrlees single-crossing condition $\frac{d}{dq}u_{loyal}>\frac{d}{dq}u_{reg}$ (for all $q\geq 0$) and boundary condition $u_{loyal}(0)=u_{reg}(0)$. It implies $v_{loyal}(r;\frac{1}{m})>v_{reg}(r;\frac{1}{m})$ and $\frac{d}{dr}\left[v_{loyal}(r;\frac{1}{m})-v_{reg}(r;\frac{1}{m})\right]<0$, so that $r^{SE}\leq r^*$. In the opposite case of $\frac{d}{dq}u_{loyal}<\frac{d}{dq}u_{reg}$, the same reasoning implies $r^{SE}\geq r^*$.

F.2 Partial market coverage

□ **Preliminaries.** In this setting, the interaction benefit U_i of all buyers on platform i is the same, and it just depends on the total measure s_i of (regular and loyal) buyers on platform i. Hence, we can employ our standard technique for solving for the equilibrium. Recall, in our *leading example*, the solution for the equilibrium commission r^* is determined by a single-variable maximization, regardless of the value of s_i, as shown in (14). Thus, the determination of r^* remains unchanged even when the market is only partially covered.

Denote the total mass of buyers (both regulars and loyals) on platform i as $s_i=s_i^{reg}+s_i^{loyal}$. Continuing from the *leading example with Hotelling competition*, we know the market shares of regular buyers (i.e., those between the Hotelling line)

$$s_1^{reg}=\frac{1}{2}+\frac{U_1-U_2+P_2^B-P_1^B}{2t},$$

with $s_2^{reg}=1-s_1^{reg}$; whereas the market shares of loyal buyers (i.e., those in the hinterlands) is

$$s_i^{loyal}=\frac{b+U_i-P_i^B}{L\cdot t_L}.$$

Combining,

$$\begin{aligned}
s_1 &= \frac{1}{2}+\frac{U_1-U_2+P_2^B-P_1^B}{2t}+\frac{b+U_1-P_1^B}{L\cdot t_L}\\
s_2 &= \frac{1}{2}-\frac{U_1-U_2+P_2^B-P_1^B}{2t}+\frac{b+U_2-P_2^B}{L\cdot t_L}.
\end{aligned}$$

Without loss of generality, we normalize $L=1$ (by rescaling t_L accordingly).

It is useful to define

$$\begin{aligned}
y(r) &\equiv v(r)\frac{\pi(r)}{k_{\max}}\\
z(r) &\equiv \frac{1}{k_{\max}}\left(v(r)+rp(r)q(r)\right)\pi(r),
\end{aligned}$$

where we note $z(r)>y(r)$ for all $r\in[0,\bar{r}]$. Throughout, we assume

$$\min\{t,t_L\}>\max_{r\in[0,\bar{r}]}\{2z(r)\}\equiv 2z(r^*). \tag{30}$$

As will be shown below, condition (30) ensures that the market share expressions below are well-behaved, and that the second-order conditions for the platform's profit-maximizing pricing choices hold.

To express s_1 and s_2 explicitly in terms of prices, we substitute $U_i = y(r_i)s_i$ (given $\varphi = 1$) to get

$$\begin{aligned} s_1 &= \frac{1}{2} + \frac{y(r_1)s_1 - y(r_2)s_2 + P_2^B - P_1^B}{2t} + \frac{b + y(r_1)s_1 - P_1^B}{t_L} \\ s_2 &= \frac{1}{2} - \frac{y(r_1)s_1 - y(r_2)s_2 + P_2^B - P_1^B}{2t} + \frac{b + y(r_2)s_2 - P_2^B}{t_L}, \end{aligned}$$

which upon solving implies

$$s_1 = \frac{1}{2} + \frac{(2P_2^B - 2P_1^B + y(r_1) - y(r_2))t_L^2 + 2(tt_L - ty(r_2) - t_L y(r_2))\left(2b - 2P_1^B + y(r_1)\right)}{4tt_L^2 + 4(t + t_L)y(r_1)y(r_2) - (2t_L^2 + 4tt_L)(y(r_1) + y(r_2))}.$$

The denominator of s_1 is positive because

$$\begin{aligned} & 4tt_L^2 + 4(t + t_L)y_1y_2 - (2t_L^2 + 4tt_L)(y_1 + y_2) \\ > \; & 4tt_L^2 + 4(t + t_L)t_L^2 - 2(2t_L^2 + 4tt_L)t_L = 0, \end{aligned}$$

where the inequality uses that the denominator is decreasing in y_1 and y_2 and that $y(r) < t_L$ by (30).

Note that s_i is decreasing in P_i^B, and so the reframing technique used to establish Proposition 1 continues to apply. Then following the derivation associated with (14), we know that each platform's optimal r_i is independent of its market share s_i, and the equilibrium r^* maximizes $z(r)$. Hence, in what follows, we focus on the symmetric commission $r_1 = r_2 = r$. In this case, the market share expressions simplify to

$$\begin{aligned} s_1 &= \frac{1}{2} + \frac{t_L^2\left(P_2^B - P_1^B\right)}{2(t_L - y(r))(tt_L - (t + t_L)y(r))} + \frac{2\left(b - P_1^B\right) + y(r)}{2(t_L - y(r))} \\ s_2 &= \frac{1}{2} + \frac{t_L^2\left(P_1^B - P_2^B\right)}{2(t_L - y(r))(tt_L - (t + t_L)y(r))} + \frac{2\left(b - P_2^B\right) + y(r)}{2(t_L - y(r))}, \end{aligned} \tag{31}$$

where the denominators are positive due to (30) as noted above.

□ **Proof of Proposition 5.** Platform profit functions are $P_1^B s_1 + R_1$ and $P_2^B s_2 + R_2$ respectively, where recall

$$R_i = \frac{1}{k_{\max}} rp(r)q(r)\pi(r)s_i^2.$$

For any given r, solving the symmetric FOCs with respect to P_i^B gives

$$P^{B*} = \frac{\left(tt_L^2 + (t + t_L)y(r)(2z(r) - y(r)) - t_L(2t + t_L)z(r)\right)(t_L + 2b)}{t_L^3 + 4tt_L^2 + 4(t + t_L)y(r)z(r) - t_L(4t + 3t_L)y(r) - 2t_L(2t + t_L)z(r)}.$$

Condition (30) implies denominator of P^{B*} expression is positive given $z(r^*) > y(r)$ for all r. Substituting P^{B*} back into the expressions for s_i given by 31, the symmetric equilibrium measure of buyers on each platform will be

$$s^* = \left(b + \frac{t_L}{2}\right)\frac{2tt_L + t_L^2 - 2(t + t_L)y(r)}{t_L^3 + 4tt_L^2 + 4(t + t_L)y(r)z(r) - t_L(4t + 3t_L)y(r) - 2t_L(2t + t_L)z(r)}.$$

Note s^* is increasing in $y(r)$; and it is also increasing in $z(r)$ if $t_L(2t + t_L) > 2(t + t_L)y(r)$, which holds due to (30).

Now consider W^{SE}, which is equal to

$$W^{SE}(r) = 2bs^* + 2z(r)(s^*)^2 - \frac{t}{4} - t_L\left(s^* - \frac{1}{2}\right)^2.$$

Given $z(r)$ is maximized at r^*, at r^* a small change in r only changes W^{SE} via s^*. Then

$$\frac{dW^{SE}}{dr}|_{r=r^*} = \frac{\partial W^{SE}}{\partial s^*}\frac{ds^*}{dr}|_{r=r^*},$$

where

$$\begin{aligned}
\frac{\partial W^{SE}}{\partial s^*} &= 2b + 4z(r)s^* - 2t_L\left(s^* - \frac{1}{2}\right) \\
&= 2b + t_L - 2\left(t_L - 2z(r)\right)s^* \\
&= 2t_L\left(b + \frac{t_L}{2}\right)\frac{2t\left(t_L - y\left(r\right)\right) - t_L y(r)}{t_L^3 + 4tt_L^2 + 4\left(t+t_L\right)y(r)z(r) - t_L\left(4t+3t_L\right)y(r) - 2t_L\left(2t+t_L\right)z\left(r\right)} \\
&= 2t_L s^*\frac{2t\left(t_L - y(r)\right) - t_L y(r)}{2tt_L + t_L^2 - 2\left(t+t_L\right)y\left(r\right)} > 0,
\end{aligned}$$

where the last inequality holds given (30) as it implies $tt_L > \left(t+t_L\right)y(r)$; whereas $\frac{ds^*}{dr}|_{r=r^*}$ has the same sign as $\frac{\partial}{\partial r}y(r) < 0$. We conclude $\frac{dW^{SE}}{dr}|_{r=r^*} < 0$.

□ **Equilibrium existence.** Computing the second derivative of platform profit with respect to P_i^B, concavity holds if

$$-\left(t_L^2 + 2tt_L - 2\left(t_L+t\right)y(r)\right)\left(2tt_L^2 + 2\left(t_L+t\right)y(r)z(r) - t_L\left(2t+t_L\right)\left(y(r)+z(r)\right)\right) < 0.$$

Condition (30) implies $tt_L > \left(t+t_L\right)y(r)$, and so the first bracketed term is positive. Thus, we require

$$2tt_L^2 + 2\left(t+t_L\right)y(r)z(r) > t_L\left(2t+t_L\right)\left(y(r)+z(r)\right). \tag{32}$$

Note since the expression is linear in $y(r)$, for it to be true for all $0 \leq y(r) \leq z(r)$, it just needs to be true when $y(r) = 0$ and when $y(r) = z\left(r\right)$. When $y(r) = 0$ it requires $2tt_L > \left(2t+t_L\right)z(r)$, which is true given $tt_L > \left(t+t_L\right)z(r)$. When $y(r) = z(r)$, it requires $2tt_L^2 > 2t_L\left(2t+t_L\right)z - 2\left(t+t_L\right)z^2$, which follows from (30).

F.3 Asymmetric platforms

□ **Preliminaries.** Continuing from the *leading example with Hotelling competition*, when platform 1 offers an additional standalone benefit $\beta > 0$, we have

$$s_1 = \frac{1}{2} + \frac{U_1 - U_2 + P_2^B - P_1^B + \beta}{2t},$$

with $s_2 = 1 - s_1$. It is useful to define

$$\begin{aligned}
y(r) &\equiv v\left(r\right)\frac{\pi\left(r\right)}{k_{\max}} \\
z(r) &\equiv \frac{1}{k_{\max}}\left(v\left(r\right) + rp(r)q(r)\right)\pi\left(r\right).
\end{aligned}$$

Throughout, we assume

$$t > \max_{r\in[0,\overline{r}]}\{z(r)\} \equiv z(r^*). \tag{33}$$

As will be shown below, condition (33) ensures that the market share expression below is well-behaved, and that the second-order conditions for the platform's profit-maximizing pricing choices hold.

To express s_1 and s_2 explicitly in terms of prices, we substitute $U_i = y(r_i)s_i$ (given $\varphi = 1$) to get

$$s_1 = \frac{1}{2} + \frac{y(r_1)s_1 - y(r_2)s_2 + P_2^B - P_1^B + \beta}{2t},$$

which implies

$$s_1 = \frac{1}{2} + \frac{P_2^B - P_1^B + \beta}{(2t - y(r_1) - y(r_2))},$$

and $s_2 = 1 - s_1$. Notice the denominator is positive due to (33). Note that s_i is decreasing in P_i^B, and so the reframing technique used to establish Proposition 1 continues to apply. Then following the derivation associated with (14), we know that each platform's optimal r_i is independent of its market share s_i, and the equilibrium r^* maximizes $z(r)$.

□ **Proof of Proposition 6.** Platform profit functions are $P_1^B s_1 + R_1$ and $P_2^B s_2 + R_2$ respectively, where recall

$$R_i = \frac{1}{k_{\max}} rp(r)q(r)\pi(r)\, s_i^2.$$

For any given r, solving the FOCs gives

$$P_2^{B*} - P_1^{B*} = -2\beta \frac{t - z(r)}{3t - y(r) - 2z(r)}$$

so that

$$s_1^* = \frac{1}{2} + \frac{\beta}{6t - 2y(r) - 4z(r)}. \tag{34}$$

Since r^* is the maximizer of $z(r)$ and given $y(r)$ is decreasing in r, we have $\frac{ds_1^*}{dr}|_{r=r^*} < 0$.

Now consider W^{SE}, which is equal to

$$\begin{aligned} W^{SE} &= b + U_1 s_1^* + U_2 s_2^* + R_1 + R_2 - \frac{t}{2}(s_1^*)^2 - \frac{t}{2}(s_2^*)^2 + \beta s_1^* \\ &= b + \left(z(r) - \frac{t}{2}\right)(s_1^{*2} + s_2^{*2}) + \beta s_1^*. \end{aligned}$$

Given $z(r)$ is maximized at r^*, at r^* a small change in r only changes W^{SE} via s^*. Then

$$\frac{dW^{SE}}{dr}|_{r=r^*} = \frac{\partial W^{SE}}{\partial s_1^*}\frac{ds_1^*}{dr}|_{r=r^*} < 0$$

because

$$\begin{aligned} \frac{\partial W^{SE}}{\partial s^*} &= 2(2s_1 - 1)\left(z(r) - \frac{t}{2}\right) + \beta \\ &= \beta\left(\frac{2t - y(r)}{3t - y(r) - 2z(r)}\right) > 0, \end{aligned}$$

where the last inequality holds given (33) as it implies $t > y(r)$.

□ **Acquisitions that add to buyers' per-seller value.** Continuing from the *leading example,* suppose platform i's acquisition adds to its buyers' per-seller value by σ_i^B. This benefit is independent of the quantity of purchase. Recall that given a seller's price p_i on platform i, each buyer chooses the number of units to purchase q_i to maximize their net utility with respect to this seller: $\arg\max_{q_i}\{u(q_i) - p_i q_i + \sigma_i^B\}$. Clearly, the component σ_i^B does not change the resulting demand function. Therefore, sellers continue to solve $p(r_i) = \arg\max_{p_i}\{((1-r_i)p_i - c)D(p_i)\}$. Let $q(r_i) \equiv D(p(r_i))$. We continue to denote $v(r_i) = u(q(r_i)) - p(r_i)q(r_i)$ as the transaction value that buyers get per-seller, and $\pi(r_i) = ((1-r_i)p(r_i) - c)q(r_i)$

as each seller's per-buyer transaction profit. Then

$$U_i = (v(r_i) + \sigma_i^B)\left(\frac{\pi(r_i)s_i}{k_{\max}}\right)^{\varphi}$$

whereas $R_i = r_i p(r_i)q(r_i)s_i\left(\frac{\pi(r_i)s_i}{k_{\max}}\right)^{\varphi}$ remains the same. The equilibrium commission by the acquiring platform i is

$$r_i^* = \arg\max_{r_i\in[0,\overline{r}]}\left\{(v(r_i) + \sigma_i^B + r_i p(r_i)q(r_i))\pi(r_i)^{\varphi}\right\}.$$

Denote the objective function as $F(r_i, \sigma_i^B)$, the cross partial-derivative is

$$\frac{\partial^2}{\partial r_i \partial \sigma_i^B}F(r_i, \sigma_i^B) = \varphi\pi(r_i)^{\varphi-1}\frac{d\pi(r_i)}{dr_i} < 0$$

Therefore, by the standard monotone comparative statics argument, r_i^* is decreasing in σ_i^B. That is, platform i decreases its equilibrium commission after the buyer-side acquisition.

□ **Acquisitions that add to sellers' per-buyer value.** Continuing from the *leading example,* suppose platform i's acquisition adds to its sellers' per-buyer value by σ_i^S. There are no changes from the buyer perspective. So, we continue to denote $v(r_i) = u(q(r_i)) - p(r_i)q(r_i)$ as the transaction value that buyers get per-seller, and $\pi(r_i) = ((1 - r_i)p(r_i) - c)q(r_i)$ as each seller's per-buyer transaction profit. Then, a seller joins the platform if and only if $(\pi(r_i) + \sigma_i^S)s_i \geq k_i$. Therefore,

$$U_i = v(r_i)\left(\frac{(\pi(r_i) + \sigma_i^S)s_i}{k_{\max}}\right)^{\varphi}$$

whereas $R_i = r_i p(r_i)q(r_i)s_i\left(\frac{(\pi(r_i)+\sigma_i^S)s_i}{k_{\max}}\right)^{\varphi}$. The equilibrium commission by the acquiring platform i is

$$r_i^* = \arg\max_{r_i\in[0,\overline{r}]}\left\{(v(r_i) + r_i p(r_i)q(r_i))(\pi(r_i) + \sigma_i^S)^{\varphi}\right\}.$$

Denote the objective function as $F(r_i, \sigma_i^S)$, then

$$\frac{1}{(\pi(r_i) + \sigma_i^S)^{\varphi-1}}\frac{\partial F(r_i, \sigma_i^S)}{\partial r_i} = (\pi(r_i) + \sigma_i^S)\frac{d}{dr_i}(v(r_i) + r_i p(r_i)q(r_i)) + \varphi\frac{d\pi(r_i)}{dr_i},$$

which is single-crossing in σ_i^S: Suppose $\frac{\partial F}{\partial r_i}(\geq) > 0$, at $\sigma_i^S = \sigma'$ then

$$(\pi(r_i) + \sigma')\frac{d}{dr_i}(v(r_i) + r_i p(r_i)q(r_i)) + \varphi\frac{d\pi(r_i)}{dr_i}(\geq) > 0,$$

which means the first term is positive (because $d\pi(r_i)/dr_i < 0$). Therefore, for $\sigma'' > \sigma'$, we have

$$(\pi(r_i) + \sigma'')\frac{d}{dr_i}(v(r_i) + r_i p(r_i)q(r_i)) + \varphi\frac{d\pi(r_i)}{dr_i}(\geq) > 0.$$

Consequently, by the monotone comparative statics argument, r_i^* is increasing in σ_i^S. That is, platform i increases its equilibrium commission after the buyer-side acquisition.

G Details for Section 5

We first verify the equilibrium construction stated in the proof of Proposition 7, and then provide the omitted details corresponding to Sections 5.2 and 5.3.

G.1 Equilibrium with spillovers

To characterize any symmetric equilibrium (a^*, P^{B*}), we consider an off-path "semi-symmetric" participation equilibrium when one of the platforms (say platform $i = 1$) deviates and sets $\left(a_i, P_i^B\right) \neq (a^*, P^{B*})$, resulting in an off-equilibrium path instrument vector profile

$$\hat{\boldsymbol{a}} = (a_i, a^* \mathbf{1}_{m-1}) = (a_i, a^*, ..., a^*) \in \mathcal{A}^m,$$

buyer-side price profile $(P_i^B, P^{B*}, ..., P^{B*})$ and buyer-side market share profile:

$$(s_i, \frac{1-s_i}{m-1}\mathbf{1}_{m-1}) = \left(s_i, \frac{1-s_i}{m-1}, \cdots, \frac{1-s_i}{m-1}\right),$$

where $\mathbf{1}_{m-1}$ is a $1 \times (m-1)$ vector of ones. That is, all other $m-1$ platforms $j \neq i$ equally absorb the resulting change in market share (due to symmetry and the market being covered), resulting in $s_j = \frac{1-s_i}{m-1}$.

Then, the fixed-point definition of market share s_i in (2) becomes

$$s_i = \Phi\left(U_i(\hat{\boldsymbol{a}}; s_i, \frac{1-s_i}{m-1}\mathbf{1}_{m-1}) - U_j(\hat{\boldsymbol{a}}; s_i, \frac{1-s_i}{m-1}\mathbf{1}_{m-1}) - P_i^B + P^{B*}\right).$$

Notice we are expressing U_i and U_j as functions of $(a_1, a_2, ..., a_m; s_1, s_2, ..., s_m)$ in the exact stated order. Therefore, we let $\partial U_i/\partial s_i$ and $\partial U_j/\partial s_i$ (likewise, $\partial U_i/\partial s_j$ and $\partial U_j/\partial s_j$) denote the partial derivative of U_i and U_j with respect to their $m+i$-th argument (likewise, $m+j$-th argument). Then, the slope of the right-hand-side with respect to s_i is

$$\begin{aligned} &\Phi' \times \left(\frac{\partial U_i}{\partial s_i} - \frac{\partial U_j}{\partial s_i} - \frac{1}{m-1}(\frac{\partial U_i}{\partial s_j} - \frac{\partial U_j}{\partial s_j}) - \frac{1}{m-1}\sum_{l\neq i,j}(\frac{\partial U_i}{\partial s_l} - \frac{\partial U_j}{\partial s_l})\right) \\ < \quad & B_\Phi \times \left(\frac{m}{m-1}B_{Uown} + \frac{m}{m-1}B_{Ucross} + \frac{m-2}{m-1}2B_{Ucross}\right), \end{aligned}$$

where

$$\begin{aligned} B_\Phi &\equiv \sup_{x\in\mathbb{R}} \Phi'(x) \\ B_{Uown} &\equiv \sup_{\boldsymbol{a}\in\mathcal{A}^m} \sup_{\boldsymbol{s}\in[0,1]^m} |\frac{\partial}{\partial s_i} U_i(\boldsymbol{a}, \boldsymbol{s})| \\ B_{Ucross} &\equiv \sup_{\boldsymbol{a}\in\mathcal{A}^m} \sup_{\boldsymbol{s}\in[0,1]^m} |\frac{\partial}{\partial s_j} U_i(\boldsymbol{a}, \boldsymbol{s})|. \end{aligned}$$

Therefore, to ensure the existence of a fixed point, a formal sufficient condition is $2B_\Phi \times (B_{Uown} + 2B_{Ucross}) < 1$. Under this condition, the resulting demand system is analogous to standard discrete choice models.

Platform i chooses $\left(a_i, P_i^B\right)$ to maximize profit Π_i, taking as given (a^*, P^{B*}) set by each other platform. Following the approach of Armstrong (2006) and Tan and Zhou (2021), to solve this maximization problem, we reframe the problem as platform i directly choosing the target market share s_i implementable by its buyer-side price P_i^B, i.e., maximization with respect to (a_i, s_i). Formally, this is done by inverting (8), so that P_i^B becomes a function of (a_i, s_i) satisfying:

$$P_i^B = P^{B*} + U_i(\hat{\boldsymbol{a}}; s_i, \frac{1-s_i}{m-1}\mathbf{1}_{m-1}) - U_j(\hat{\boldsymbol{a}}; s_i, \frac{1-s_i}{m-1}\mathbf{1}_{m-1}) - \Phi^{-1}(s_i).$$

Then, platform i's problem is to choose (a_i, s_i) to maximize

$$\begin{aligned}\Pi_i\left(a_i; s_i\right) &= P_i^B s_i + R_i(a_i; s_i) \\ &= \left(P^{B*} + U_i(\hat{\boldsymbol{a}}; s_i, \frac{1-s_i}{m-1}\mathbf{1}_{m-1}) - U_j(\hat{\boldsymbol{a}}; s_i, \frac{1-s_i}{m-1}\mathbf{1}_{m-1}) - \Phi^{-1}(s_i)\right) s_i + R_i(\hat{\boldsymbol{a}}; s_i, \frac{1-s_i}{m-1}\mathbf{1}_{m-1}).\end{aligned}$$

To ensure the existence of a symmetric equilibrium, we assume that Π_i is globally strictly quasiconcave in (a_i, s_i), as in the baseline model. In any symmetric equilibrium, each platform's optimal choice of $a_i = a^*$ is a maximizer of $\Pi_i\left(a_i; s_i\right)$ while holding $s_i = 1/m$ and the instrument choices of other platforms constant at a^*. That is,

$$a^* \in \arg\max_{a_i \in \mathcal{A}} \left\{ \frac{1}{m} U_i(\hat{\boldsymbol{a}}; \frac{1}{m}\mathbf{1}) - \frac{1}{m} U_j(\hat{\boldsymbol{a}}; \frac{1}{m}\mathbf{1}) + R_i(\hat{\boldsymbol{a}}; \frac{1}{m}\mathbf{1}) \right\},$$

which is exactly (18). Meanwhile, the derivative of Π_i with respect to s_i (using $s_j = \frac{1-s_i}{m-1}$, as noted above) is

$$\begin{aligned}\frac{d\Pi_i}{ds_i} = &\left(\frac{\partial U_i}{\partial s_i} - \frac{\partial U_j}{\partial s_i} - \frac{1}{m-1}(\frac{\partial U_i}{\partial s_j} - \frac{\partial U_j}{\partial s_j}) - \frac{1}{m-1}\sum_{l\neq i,j}(\frac{\partial U_i}{\partial s_l} - \frac{\partial U_j}{\partial s_l}) - \frac{1}{\Phi'}\right) s_i + \left(\frac{\partial R_i}{\partial s_i} - \frac{1}{m-1}\sum_{l\neq i}\frac{\partial R_i}{\partial s_l}\right) \\ &+ P^{B*} + U_i - U_j - \Phi^{-1}(s_i),\end{aligned}$$

where we have omitted function arguments. Imposing symmetry, that is, $\frac{\partial U_i}{\partial s_i} = \frac{\partial U_j}{\partial s_j}$, $\frac{\partial U_i}{\partial s_j} = \frac{\partial U_j}{\partial s_i}$ for $i \neq j$ and $\frac{\partial U_i}{\partial s_l} = \frac{\partial U_j}{\partial s_l}$, and $\frac{\partial R_i}{\partial s_l} = \frac{\partial R_i}{\partial s_j}$ for $l \neq i, j$, we get

$$\frac{d\Pi_i}{ds_i} = \left(\frac{m}{m-1}(\frac{\partial U_i}{\partial s_j} - \frac{\partial U_j}{\partial s_j}) - \frac{1}{\Phi'}\right)\frac{1}{m} + (\frac{\partial R_i}{\partial s_i} - \frac{\partial R_i}{\partial s_j}) + P^{B*} - \Phi^{-1}(\frac{1}{m}).$$

So the FOC gives

$$P^{B*} = \frac{1}{m\Phi'(0)} - \frac{1}{m-1}(\frac{\partial U_i(\boldsymbol{a};\boldsymbol{s})}{\partial s_i} - \frac{\partial U_i(\boldsymbol{a};\boldsymbol{s})}{\partial s_j}) - (\frac{\partial R_i(\boldsymbol{a};\boldsymbol{s})}{\partial s_i} - \frac{\partial R_i(\boldsymbol{a};\boldsymbol{s})}{\partial s_j}), \tag{35}$$

where the derivatives are evaluated at the symmetric outcome $(\boldsymbol{a};\boldsymbol{s}) = (a^*\mathbf{1}; \frac{1}{m}\mathbf{1})$.

Meanwhile, the welfare objectives are given by

$$\begin{aligned}W^{SE}(a) &= \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[\hat{\epsilon} + U_i(a\mathbf{1}; \frac{1}{m}\mathbf{1})\right] d\hat{F}(\hat{\epsilon}) + mR_i(a\mathbf{1}; \frac{1}{m}\mathbf{1}), \\ W(a) &= W^{SE}(a) + mSS_i(a\mathbf{1}; \frac{1}{m}\mathbf{1}).\end{aligned}$$

Given SS_i is decreasing in a, it is immediately clear that $a^{SE} \geq a^W$ (as claimed in Lemma 1).

G.2 Spillovers from seller singlehoming

We continue from the *leading example* and assume that sellers' outside option is zero, and each seller is indexed by $(k_1, ..., k_m) \in [k_{\min}, k_{\max}]^m$. We add a standalone benefit b_S to seller's participation utility from joining platform i, which is now

$$b_S + \pi(r_i)s_i - k_i.$$

We assume b_S is sufficiently high to ensure full coverage of the seller-side market. Denote $\Psi(\cdot)$ as the CDF of $k_i - \max_{j\neq i}\{k_j\}$ and the corresponding derivative is denoted as $\Psi'(\cdot)$. To ensure that seller participation is well behaved, as we did on the buyer side, we assume that the extent of heterogeneity in sellers' idiosyncratic draws of participation costs $(k_1, ..., k_m)$, as measured by $1/\Psi' > 0$, is large enough.

We know from (18) that

$$r^* = \arg\max_{r_i \in [0,\bar{r}]} \left\{ \frac{1}{m} U_i - \frac{1}{m} U_j + R_i \right\}.$$

Due to the semi-symmetry structure in the off-equilibrium path when one platform i deviates, we have $n_j = \frac{1-n_i}{m-1}$ for $j \neq i$, and so

$$r^* = \arg\max_{r_i \in [0,\bar{r}]} \left\{ \frac{1}{m} \left(v(r_i) n_i - v(r^*) \frac{1-n_i}{m-1} \right) + \frac{r_i p(r_i) q(r_i)}{m} n_i \right\},$$
$$\text{where } n_i = \Psi\left(\frac{\pi(r_i) - \pi(r^*)}{m} \right).$$

It is useful to define

$$z(r_i) \equiv v(r_i) + r_i p(r_i) q(r_i).$$

Then, ignoring boundary conditions, the corresponding FOCs for r^* is

$$\left(\frac{mv(r^*)}{m-1} + r^* p(r^*) q(r^*) \right) \frac{\Psi'(0)}{m^2} \frac{d\pi(r^*)}{dr_i} + \frac{dz(r^*)}{dr_i} \frac{1}{m^2} = 0. \tag{36}$$

Meanwhile, the welfare benchmarks have

$$\begin{aligned} r^{SE} &= \arg\max_{r_i \in [0,\bar{r}]} \{ v(r_i) + r_i p(r_i) q(r_i) \} = \arg\max_{r_i \in [0,\bar{r}]} z(r_i) \\ r^W &= \arg\max_{r_i \in [0,\bar{r}]} \{ v(r_i) + (p(r_i) - c) q(r_i) \} = 0. \end{aligned}$$

Next, given $d\pi/dr_i < 0$, it is clear that $r^{SE} \geq r^*$, with strict inequality if r^* or r^{SE} is an interior solution, or if $r^* = 0$ and $r^{SE} = \bar{r}$.

□ **A closed-form solution.** To proceed further, suppose seller marginal cost is $c = 0$, so that $v(r_i) = v$, and $p(r_i)q(r_i) = pq$ are now constants that are independent of the commission rate r_i. Then, $dz/dr_i = -d\pi/dr_i = pq > 0$, and so (36) simplifies to

$$-\left(\frac{vm}{m-1} + r^* pq \right) \Psi'(0) + 1 = 0$$

$$\Longrightarrow r^* = \frac{1}{pq} \left(\frac{1}{\Psi'(0)} - \frac{vm}{m-1} \right),$$

whereas $r^{SE} = \bar{r}$ (where $\bar{r} = 1$ due to $c = 0$). Therefore, we have

$$r^* < r^{SE} = \bar{r} \quad \text{if } \frac{vm}{m-1} > \frac{1}{\Psi'(0)} - pq$$

and

$$r^* > r^W = 0 \;\; \text{if } \frac{vm}{m-1} < \frac{1}{\Psi'(0)}.$$

In particular, in the equilibrium the baseline distortion is completely mitigated (i.e., $r^* = r^W < r^{SE}$) if $\frac{vm}{m-1} \geq \frac{1}{\Psi'(0)}$. This holds when the extent of heterogeneity in sellers' idiosyncratic draws of participation costs $(k_1, ..., k_m)$ is low (provided the symmetric equilibrium still exists — see, e.g., the two-sided Hotelling specification below). If we allow platforms to choose negative commissions $r_i < 0$, then it is straightforward to show that $r^W = 0$ continues to hold, so that $\frac{vm}{m-1} > \frac{1}{\Psi'(0)}$ implies a reversion of the sign of distortion in equilibrium (i.e., $r^* < r^W$). Intuitively, the reversion reflects that platforms are overly focused on attracting sellers and thus subsidize sellers by too much relative to the socially optimal level.

As an illustration, we consider the following *two-sided Hotelling specification* with $m = 2$ platforms. That is, the buyer-side participation demand is $\Phi(x) = \frac{1}{2} + \frac{x}{2t_B}$ whereas the seller-side participation demand is $\Psi(x) = \frac{1}{2} + \frac{x}{2t_S}$, where t_B and t_S are the respective mismatch cost parameters. Then,

$$r^* = \frac{2}{pq}(t_S - v).$$

Meanwhile, the pricing equation (35) and

$$\begin{aligned} U_i &= v\left(\tfrac{1}{2} + \tfrac{(1-r_i)pqs_i - (1-r_j)pqs_j}{2t_S}\right) \\ R_i &= r_i pqs_i \left(\tfrac{1}{2} + \tfrac{(1-r_i)pqs_i - (1-r_j)pqs_j}{2t_S}\right), \end{aligned}$$

imply

$$P^{B*} = t_B - \left(v + \frac{r^* pq}{2}\right)\frac{(1-r^*)pq}{t_S} - \frac{r^* pq}{2}.$$

Suppose $p = q = 1$, and $t_B = t_S = 2$. We can verify that the symmetric equilibrium exists for v in the range $[1.5, 2]$ which maps out $r^* = 1$ down to $r^* = 0$. Note that at $v = 2$, we have $r^* = 0$, illustrating that the outcome of $r^* = r^W < r^{SE}$ does not necessarily violate equilibrium existence.

□ **Seller-side lump-sum fees**. We can apply our formula (18) to the case of seller-side lump-sum fees P_i^S considered by Armstrong (2006) and Tan and Zhou (2021). Given the absence of commissions, we can drop the function arguments in v and π. By the same analysis as above, P^{S*} is the maximizer of

$$\begin{aligned} P^{S*} &= \arg\max_{P_i^S}\left\{\frac{1}{m}U_i - \frac{1}{m}U_j + R_i\right\} \\ &= \arg\max_{P_i^S}\left\{\frac{v}{m}\left(n_i - \frac{1-n_i}{m-1}\right) + P_i^S n_i\right\}, \end{aligned}$$

where $n_i = \Psi\left(P^{S*} - P_i^S\right)$. Note we do not need the domain of feasible P_i^S to be compact for this maximization problem to be well-defined. The corresponding FOC is

$$P^{S*} = \underbrace{\frac{1/m}{\Psi'(0)}}_{\text{market power}} - \underbrace{\frac{v}{m-1}}_{\text{cross-subsidization due to benefits enjoyed by buyers}},$$

which is a special micro-founded case of the equilibrium pricing formula obtained by Tan and Zhou (2021).

G.3 Spillovers from seller-side post-participation decisions

Throughout this subsection, we assume all sellers have zero fixed costs and zero participation costs $k_i = 0$ (i.e., the distribution G is degenerate) in order to show spillovers can arise absent any fixed participation cost.

□ **Price coherence.** We first prove the claim on $p(r^{avg})\, q(r^{avg})$ being decreasing in r^{avg}. Whenever a seller is subjected to price coherence, the seller chooses its common price p to maximize

$$\left(\sum\nolimits_{i\in\phi} s_i\left((1-r_i)p - c\right)\right) D(p),$$

which can be rewritten as

$$\left((1-r^{avg})p - c\right) D(p) \sum\nolimits_{i\in\phi} s_i,$$

where $r^{avg} = \frac{1}{\sum_{i\in\phi} s_i}\sum_{i\in\phi} s_i r_i$. We denote the optimal price as $p(r^{avg})$. Given that $D(p)$ is strictly log-concave, $p(r^{avg})$ is given by the FOC:

$$\begin{aligned} p &= \frac{c}{1-r^{avg}} - \frac{D(p)}{D'(p)} \\ &\implies pD'(p) < -D(p). \end{aligned}$$

The last inequality implies

$$\begin{aligned} \frac{d}{dr^{avg}} p\left(r^{avg}\right) q(r^{avg}) &= \frac{d}{dr^{avg}} p\left(r^{avg}\right) D(p(r^{avg})) \\ &= \underbrace{(D(p) + pD'(p))}_{<0} \underbrace{\frac{dp}{dr^{avg}}}_{>0} < 0. \end{aligned}$$

Similar to the *leading example*, we denote $\pi(r) = \max_p \left((1-r)p - c\right) D(p)$. Then, the seller has joined a set ϕ of platforms (and subjected to price coherence) earns profit $\sum_{i\in\phi} s_i \pi(r^{avg})$.

Next, we verify the claims that all sellers will multihome on all platforms as long as the commission difference $\max_{j\neq i} |r_i - r_j|$ is not too large, and that the platforms have no incentive to deviate and induce large commission differences if β is small enough. Without loss of generality, it suffices to focus on the case where platform i sets $r_i \leq r^*$ while all other platforms $j \neq i$ set $r_j = r^*$. Consider an individual seller's decision on whether to multihome. Clearly, all sellers who are not subjected to price coherence would prefer to multihome. For the sellers subjected to price coherence, multihoming on all platforms is always better than joining only the higher-commission platforms (platform $j \neq i$) because

$$\begin{aligned} \pi_{all} &= \pi(r^*(1-s_i) + r_i s_i) \\ &> \pi(r^*) \\ &> \pi(r^*)(1-s_i) = \pi_{j \text{ only}}, \end{aligned}$$

since $\pi(.)$ is a decreasing function. Meanwhile, multihoming is better than singlehoming on the lower-commission platform (platform i) if and only if

$$\pi_{all} = \pi(r^*(1-s_i) + r_i s_i) \geq \pi(r_i) s_i,$$

which holds if and only if the commission difference $r^* - r_i$ is small enough.

We now verify that platforms have no incentive to set a large difference in commission as long as ω is sufficiently small. Let us pin down the equilibrium commission level r^*. Recall

$$\begin{aligned} U_i &= \omega v(r^{avg}) + (1-\omega) v(r_i) \\ R_i &= r_i \left(\omega p\left(r^{avg}\right) q(r^{avg}) + (1-\omega) p(r_i) q(r_i)\right) s_i. \end{aligned}$$

Assuming all sellers multihome on all platforms in the equilibrium, the FOC satisfies:

$$\left(\frac{\partial U_i}{\partial r_i} - \frac{\partial U_{-i}}{\partial r_i}\right)\frac{1}{m} + \frac{\partial R_i}{\partial r_i} = 0$$

$$\iff \frac{(1-\omega)v'(r^*)}{m} + \frac{p\left(r^*\right) q(r^*)}{m} + \frac{r^*}{m}\left(\frac{\omega}{m} + 1 - \omega\right)\left(p'\left(r^*\right) q(r^*) + p\left(r^*\right) q'(r^*)\right) = 0.$$

Observe that r^* is increasing in ω because the derivative of the left-hand-side with respect to ω is

$$-\frac{v'(r^*)}{m} - \frac{r^*}{m}\left(\frac{m-1}{m}\right)\underbrace{\left(p'\left(r^*\right) q(r^*) + p\left(r^*\right) q'(r^*)\right)}_{<0} > 0.$$

Suppose platform i wants to deviate by choosing $(r_i, P_i^B) \neq (r^*, P^{B*})$ to induce some sellers to single-home. Recall this necessarily requires $r_i < r^*$. This is applicable only to the mass ω of sellers that are subjected to price coherence. A successful deviation requires

$$\pi(s_i r_i + (1-s_i)r^*) < \pi(r_i)s_i.$$

Let us denote the maximum deviation fee as r^{dev}, which we know is *strictly* below r^* as long as $s_i < 1$ (i.e., buyer-side heterogeneity is not too small), for any $\omega \geq 0$. With this undercutting strategy, buyers expect utility difference

$$U_i - U_{-i} = v(r^{dev}) - (1-\omega)v(r^*) + P_i^B - P^{B*}$$

and the deviation platform profit is

$$\Pi^{dev} = \max_{P_i^B; r_i \leq r^{dev}} \left\{ \begin{array}{c} (P_i^B + r_i p(r_i) q(r_i)) \\ \times \Phi(v(r^{dev}) - (1-\omega)v(r^*) + P_i^B - P^{B*}) \end{array} \right\}.$$

Furthermore, observe that the equilibrium platform profit can be expressed as

$$\begin{aligned} \Pi^* &= (P^{B*} + r^* p(r^*) q(r^*)) \frac{1}{m} \\ &= \max_{P_i^B; r_i} \left\{ \begin{array}{c} (P_i^B + \omega r_i p(r^{avg}) q(r^{avg}) + (1-\omega) r_i p(r_i) q(r_i)) \\ \times \Phi((1-\omega)(v(r_i) - v(r^*)) + P_i^B - P^{B*}) \end{array} \right\}. \end{aligned}$$

Therefore, if $\omega \to 0$, then the two objective functions coincide. Therefore, the constraint of $r^{dev} < r^*$ implies $\Pi^{dev} < \Pi^*$.

□ **Seller investment that applies to all platforms.** Consider our *leading example.* Suppose in addition to setting prices, sellers can choose how much to invest to raise their product demand in ways that are not platform-specific (e.g., this could include investments in broad marketing efforts or quality improvements).

Specifically, each buyer chooses the number of units to purchase q_i to maximize their net utility; i.e., $\arg\max_{q_i} \{u(q_i)B(I_s) - p_i q_i\}$, where $B(I_s) > 0$ indicates the utility enhancement due to seller investment and I_s is a seller's investment level. We assume $B(\cdot)$ is differentiable, and the derivative $B'(\cdot) > 0$. We assume sellers face the associated corresponding cost function $K(I_s)$, where K is increasing and strictly convex, with boundary conditions $\lim_{I_s \to \infty} K'(I_s) = \infty$ and $K'(0) = 0$. Sellers are assumed to set I_s at the same time as their prices on the different platforms. All sellers participate given the absence of participation fixed cost.

Suppose each platform chooses $r_i \in [0, \bar{r}]$. We let $c = 0$ to simplify seller pricing. Then we define a seller's quality-adjusted price $\hat{p}_i = \frac{p_i}{B(I_s)}$, and denote the optimal quality-adjusted price as

$$\hat{p} \equiv \arg\max_{\hat{p}_i} (1-r_i) B(I_s) \hat{p}_i D_i(\hat{p}_i),$$

which does not depend on either r_i or I_s. The per-buyer gross profit (not including investment costs) of each seller is $(1-r_i) B(I_s) \pi^m$ and the per-seller surplus of the buyer is $B(I_s) v^m$, where $\pi^m = \hat{p} D(\hat{p})$ and $v^m = u(D(\hat{p})) - \hat{p} D(\hat{p})$.

Each seller's optimal investment maximizes

$$\sum_{i=1}^m (1-r_i) B(I_s) s_i \pi^m - K(I_s).$$

The above conditions ensures a seller's optimal investment I_s^* is uniquely defined, strictly positive, and satisfies the FOC

$$\sum\nolimits_{i=1}^{m} (1-r_i)\, B'(I_s^*)\, s_i \pi^m = K'(I_s^*).$$

Moreover, I_s^* is decreasing in r_i on each platform i. As a result, both $U_i = B(I_s^*)\, v^m$ and $R_i = r_i B(I_s^*)\, \pi^m s_i$ are decreasing in r_j for $j \neq i$. Therefore, there are negative spillovers and $r^* \geq r^{SE} \geq r^W$.

□ **Platform and seller investment.** Continue from the setting immediately above (which we refer to as the *seller-only investment* application) and suppose now that each platform chooses $a_i = -I_i$, where I_i is platform i's level of investment with associated convex cost $C(I_i)$. We keep the commission rate $r_i = r \in [0, \bar{r}]$ fixed and equal across all platforms $i = 1, ..., m$. Note that we define the platform instrument in terms of the negative of I_i to maintain the order of a_i, which recall was defined so that a higher a_i corresponds to a lower seller surplus.

Platform i's investment I_i scales up the buyer's gross utility obtained from transacting with any seller. The gross utility of buyers is now $u(q_i)B(I_s, I_i)$, where I_s is a seller's investment with the corresponding cost function $K(I_s)$, with the properties defined in the *seller-only investment* application above. We assume B is differentiable and increasing in both its arguments, with $B_1(I_s, I_i) > 0$ when evaluated at $I_s = 0$, and $B_1(I_s, I_i)$ weakly decreasing in I_s. This combination of assumptions ensures that each seller's optimal investment is unique, strictly positive, and finite. We say the two types of investments are complements (substitutes) if $B_1(I_s, I_i)$ is everywhere increasing (decreasing) in I_i. The timing is that platforms set their investments first (at the same time as their prices to buyers), before sellers set their investments and prices.

Defining the seller's quality-adjusted price

$$\hat{p}_i = \frac{p_i}{B(I_s, I_i)}$$

each seller sets $\hat{p}_i$ to maximize $(1-r)\, B(I_s, I_i)\, \hat{p}_i q_i(\hat{p}_i)$. Let the resulting profit maximizing price be denoted $\hat{p}$, which does not depend on either r, I_s or I_i. The per-buyer gross profit (not including investment costs) of each seller is $(1-r)\, B(I_s, I_i)\, \pi^m$ and the per-seller surplus of the buyer is $B(I_s, I_i)v^m$, where π^m and v^m are defined in the *seller-only investment* application above.

Each seller's optimal investment maximizes

$$\sum\nolimits_{i=1}^{m} (1-r)\, B(I_s, I_i) s_i \pi^m - K(I_s).$$

The above conditions ensures a seller's optimal investment I_s^* is uniquely defined, strictly positive, and satisfies the FOC

$$\sum\nolimits_{i=1}^{m} (1-r)\, B_1(I_s^*, I_i)\, s_i \pi^m = K'(I_s^*).$$

Moreover, I_s^* is decreasing (increasing) in $a_i = -I_i$ on each platform i if the two types of investments are complements (substitutes). As a result, both $U_i = B(I_s^*, I_i)\, v^m$ and $R_i = rB(I_s^*, I_i)\, \pi^m s_i - C(I_i)$ are decreasing (increasing) in $a_j = -I_j$ for $j \neq i$ if the two types of investments are complements (substitutes).

Therefore, there are negative spillovers and $I^* \leq I^{SE} \leq I^W$ (since $a^* \geq a^{SE} \geq a^W$) if the two types of investments are complements, and there are positive spillovers and $I^* \geq I^{SE}$ (since $a^* \leq a^{SE}$) which mitigates the baseline distortion that $I^{SE} \leq I^W$ if the two types of investments are substitutes.

□ **Promotion of sellers' direct channel.** We continue from Example 2 in Online Appendix A and modify it by allowing sellers to promote their direct channels. Specifically, suppose each seller chooses the amount to spend on promoting their direct channel (say spending on an advertising campaign on it), denoted as κ. Then, each buyer will become aware of the seller's direct channel with some positive probability $0 \leq Y(\kappa) \leq 1$, where $Y(0) = 0$, $Y(\infty) = 1$, $Y' > 0$ and $Y'' < 0$. Thus, if λ_i of a seller's buyers on platform i are initially uninformed of its direct channel, after promoting its direct channel, only $\lambda_i (1 - Y(\kappa))$ of its buyers on platform i will remain uninformed.

Given $G(\cdot)$ is degenerate, we know all sellers will always choose to multihome on all platforms due to the fact that sellers do not face any restrictions in setting the on-platform prices, face no other costs, and still keep a fraction of their revenues. Meanwhile, their pricing problem remains the same as Example 2. Therefore,

$$U_i = v^m.$$

Meanwhile, a seller's total profit is $\sum_{i=1}^{m}(1 - r_i + (1 - \lambda_i (1 - Y(\kappa)))\zeta r_i)\pi^m s_i - \kappa$, and the maximization with respect to κ leads to the optimal promotion spending κ^* satisfying

$$\zeta\pi^* \sum_{i=1}^{m} \lambda_i r_i s_i = \frac{1}{Y'(\kappa^*)},$$

where κ^* is increasing in $\sum_{i=1}^{m} \lambda_i r_i s_i$ given $Y'' < 0$. Moreover,

$$R_i = (1 - (1 - \lambda_i (1 - Y(\kappa^*)))\zeta) r_i \pi^m s_i.$$

Observe that R_i decreases when the "disintermediation-adjusted effective commission" $r_j\lambda_j$ on platform j increases, because a higher effective commission on platform j induces more sellers to invest in promoting their direct channels, i.e., a higher κ^*. Therefore, this direct channel mechanism results in negative spillovers in platform fees r_j and disintermediation prevention efforts λ_j through platform i revenues. We can immediately conclude from Proposition 7 that $r^* \geq r^{SE} \geq r^W$ or $\lambda^* \geq \lambda^{SE} \geq \lambda^W$.

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