Online Appendix: “Firm Responses and Wage Effects of Foreign Demand Shocks with Fixed Labor Costs and Monopsony”

Emmanuel Dhyne, Ayumu Ken Kikkawa, Toshiaki Komatsu,
Magne Mogstad, Felix Tintelnot

August 31, 2025

A Data appendix

A.1 Aggregating VAT identifiers into firms

As discussed in the main text, the NBB datasets are available at the level of VAT identifiers. In order to conduct our analyses at the firm level, we aggregate multiple VAT identifiers into the firm identifiers, using the same procedure as in Dhyne et al. (2021). We leverage the information from ownership filings in the annual accounts as well as the Balance of Payments survey to determine if a pair of VAT identifiers belongs to the same firm. We aggregate multiple VAT identifiers to the same firm if they are linked with at least 50 percent of ownership or if they share the same foreign parent firm that holds at least 50 percent of their shares. In order to determine the foreign parent firm of a given VAT identifier, we apply a "fuzzy string matching" method, comparing all possible pairs of foreign firms' names. Lastly, we correct for potential misreporting by linking the pair of VAT identifiers that are linked one year before and one year after.

After collecting multiple VAT identifiers that belong to the same firm, we then assign their firm identifier using the "most representative" VAT identifiers among them. The criteria for selecting such "head VAT" identifiers are explained in detail in Appendix C.4 of Dhyne et al. (2021). Once we determine the head VAT identifiers for all firms that have multiple VAT identifiers, we then sum up all the variables across VAT identifiers to the firm level. In order to avoid the double counting of transactions within the firm, we further adjust total sales and inputs by the amount of B2B sales between the pair of VAT identifiers that belong to the same head VAT identifier. For other variables such as firms' age, their primary industry, and location of their main economic activities, we take those of head VAT identifiers.

A.2 Merging NBB datasets with BCSS datasets

The BCSS datasets are available at the level of Banque Carrefour des Entreprises (Crossroads Bank for Enterprises, BCE) identifiers. All businesses in Belgium are required to register with the BCE, which assigns them the unique identifiers upon registration. Registration with the BCE is required for firms to pay VAT, so the BCE identifiers can be easily converted to VAT identifiers. In order to match the BCSS datasets with the NBB datasets, we first convert all BCE identifiers to VAT identifiers and then aggregate multiple VAT identifiers into firms, as explained in Appendix A.1.

A.3 Coverage and summary statistics on the merged sample

Table 4 reports the coverage of our main estimation sample (NBB sample) in 2012 and compares it to the official aggregate statistics obtained from Eurostat (2012). Our sample covers a large majority of the aggregate value added, gross output, labor costs, exports, and imports in the Belgian economy. We also report the coverage of the subsample of firms for which we have additional information from the worker data (merged NBB-BCSS sample for the firms with 10 or more FTE employees at least once from 2002 to 2014), which still makes up most of the total sales, inputs, and trade in the Belgian economy.

Table 4: Coverage of NBB and NBB-BCSS datasets in 2012

	Eurostat
GDP (excl. Gov.&Fin.)	248
Output (excl. Gov.&Fin.)	672
Import	310
Export	311

	NBB sample	NBB-BCSS sample
Count:		
Total	98,745	26,470
Direct exporters	11,892	7,024
Indirect exporters	74,529	18,043
Value added	164	145
Total sales	796	704
Network sales	225	190
Import	292	279
Export	292	281
Labor cost	100	90
Employment (FTE)	1,824,066	1,578,505

Notes: All numbers except for count and employment are denominated in billion euro in current prices. Belgian GDP and output are for all sectors excluding the public and financial sectors. Data for Belgian GDP, output, imports, and exports are from Eurostat (2012). Firms' value added is from the reported values from the annual accounts. Firms' sales consist of their sales to other firms in the NBB sample (network sales), sales to households at home, and direct exports to foreign markets. The counts for direct exporters and indirect exporters are mutually exclusive.

Using our main estimation sample, we also present the (direct and total) export participation and shares of firms that directly export to those that only export indirectly, building on the work of Dhyne et al. (2021). This finding motivates why our model and analysis will include indirect export through the production network to measure the firms' exposure to foreign demand.

To arrive at this conclusion, we first construct measures of the firms' total export. As in Dhyne et al. (2021), we assume the firm's composition of inputs in production does not vary across its buyers, so that we can measure the total export of a firm by the total share of output that it sells directly or indirectly to foreign markets (i.e., the total export share). Formally, the total export share of firm k, r_{kF}^{Total}, is defined as the share of revenue from direct export, r_{kF}, and the share of revenue coming from sales to other domestic firms, multiplied by the total export shares of those firms:

$$r_{kF}^{Total} = r_{kF} + \sum_{i \in W_k} r_{ki} r_{iF}^{Total}, \tag{25}$$

where W_k denotes the set of buyers of firm k, and r_{kF} and r_{ki} are the share of k's revenue that comes from direct export and from sales to domestic firm i, respectively. The denominator of the export shares is the total revenue of the firm, which consists of sales to other domestic firms, sales to households, and direct exports.

It is important to observe that the definition of the total export share is recursive. A firm's total export share is the sum of its direct export share and the share of its sales to other domestic firms multiplied by the total export shares of those firms. Thus, the total export share is high if a lot of the firm's output is exported directly to foreign markets or indirectly via sales to domestic buyers with high export shares.

Table 5: Descriptive statistics in 2012

(a) Direct and total export participation

	Exporters and non-exporters	Exporters only	Non-exporters only
Number of observations	98,745	11,892	86,853
Fraction of firms with total export participation	0.875	1.000	0.858
Average export shares:			
Total export	0.138	0.445	0.096
Direct export	0.039	0.322	0.000
Indirect export	0.100	0.122	0.096

(b) Firm characteristics

	Exporters and non-exporters	Exporters only	Non-exporters only
Log sales	13.6	15.5	13.3
Log TFP	10.7	11.3	10.6
Log value added	12.5	13.9	12.3
Log FTE employment	1.5	2.5	1.3
Log average wage	10.5	10.8	10.5

Notes: This table uses the main estimation sample of private-sector firms in Belgium in 2012 (see Section 2.3 for details). Panel (a): The total export share of firm k, r_{kF}^{Total}, is recursively defined as $r_{kF}^{Total} = r_{kF} + \sum_{i \in W_k} r_{ki} r_{iF}^{Total}$, which can be decomposed into direct export share, r_{kF}, and indirect export share, $\sum_{i \in W_k} r_{ki} r_{iF}^{Total}$. Panel (b): For each column, we report the averages of variables listed on the left for a set of firms noted at the top of the column. Firms' sales consist of their sales to other firms in the NBB sample (network sales), sales to households at home, and direct exports to foreign markets. Firms' TFPs are calculated using the estimation procedure of Wooldridge (2009). Firms' value added and FTE employment are from the reported values from the annual accounts. Firms' average wages are the reported labor costs divided by their FTE employment.

In panel (a) of Table 5, we compare the (direct and total) export participation and shares of firms that directly export to those that only export indirectly. While few Belgian firms are directly exporting, a majority of the firms are indirectly exporting through sales to domestic buyers that subsequently trade internationally. In fact, even the firms that do not directly export are, on average, selling nearly 10 percent of their output indirectly to foreign markets.

B Model appendix

B.1 General equilibrium of the model in Section 3.1

We characterize the firm-level outcomes implied by the firms' profit maximization and cost minimization problem. First, the sum of the variable and fixed costs of firm k can be written as

$$\begin{aligned} TC_k =& \phi_k^{-1} \frac{\frac{1+\varepsilon}{\varepsilon}(1-\alpha_{\ell k}) + \alpha_{\ell k}}{\left(\frac{1+\varepsilon}{\varepsilon}(1-\alpha_{\ell k})\right)^{1-\alpha_{\ell k}} \alpha_{\ell k}^{\alpha_{\ell k}}} \left(\sum_{j \in Z_k} \left(\omega_{jk}^v\right)^\sigma p_j^{1-\sigma} + \left(\omega_{Fk}^v\right)^\sigma p_{Fk}^{1-\sigma} \right)^{\frac{1-\alpha_{\ell k}}{1-\sigma}} w_k^{\alpha_{\ell k}} q_k \\ &+ \left(\sum_{j \in Z_k} \left(\omega_{jk}^f\right)^\sigma p_j^{1-\sigma} + \left(\omega_{Fk}^f\right)^\sigma p_{Fk}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \bar{q}_k^f \\ &+ w_k \bar{\ell}_k^f, \end{aligned} \tag{26}$$

where the first term represents the variable costs, the second term represents the fixed input purchases, and the last term represents the fixed labor costs. Note that firms face a common demand elasticity of σ regardless of whom they sell to; hence,

$$p_k = \mu_k c_k = \frac{\sigma}{\sigma - 1} c_k. \tag{27}$$

Taking the total derivative of the total cost with respect to output quantity, one can derive the firm's marginal cost,

$$c_k = \frac{1}{\phi_k \alpha_{\ell k}^{\alpha_{\ell k}} (1-\alpha_{\ell k})^{1-\alpha_{\ell k}}} \left(\sum_{j \in Z_k} \left(\omega_{jk}^v\right)^\sigma p_j^{1-\sigma} + \left(\omega_{Fk}^v\right)^\sigma p_{Fk}^{1-\sigma} \right)^{\frac{1-\alpha_{\ell k}}{1-\sigma}} \left(\frac{1+\varepsilon}{\varepsilon} w_k \right)^{\alpha_{\ell k}}. \tag{28}$$

The marginal cost follows the standard structure except that the firm's wage enters the cost with a wedge of $\frac{1+\varepsilon}{\varepsilon}$. One can then derive the total variable input cost of the firm—the first term in equation (26)—in terms of its sales $p_k q_k$, by substituting in equations (27) and (28):

$$w_k \ell_k^v + \sum_j p_j q_{jk}^v + p_{Fk} q_{Fk}^v = \left(\frac{\varepsilon_m}{\varepsilon_m + 1} \alpha_{\ell k} + 1 - \alpha_{\ell k} \right) \frac{\sigma - 1}{\sigma} p_k q_k. \tag{29}$$

The firm's variable labor input share out of its variable cost, $s_{\ell k}^v$, is a constant but lower than the Cobb-Douglas parameter $\alpha_{\ell k}$ as a result of the upward sloping labor supply curve:

$$s_{\ell k}^v = \frac{\frac{\varepsilon}{1+\varepsilon} \alpha_{\ell k}}{1 - \alpha_{\ell k} + \frac{\varepsilon}{1+\varepsilon} \alpha_{\ell k}}. \tag{30}$$

The share of variable inputs from firm j out of firm k's variable cost, s_{jk}^v, can be expressed as the share of variable input purchases times the share of firm j's goods out of the variable input purchases:

$$s^v_{jk} = \frac{1-\alpha_{\ell k}}{1-\alpha_{\ell k}+\frac{\varepsilon}{1+\varepsilon}\alpha_{\ell k}} \frac{\left(\omega^v_{jk}\right)^\sigma p_j^{1-\sigma}}{\sum_{j\in Z_k}\left(\omega^v_{jk}\right)^\sigma p_j^{1-\sigma}+\left(\omega^v_{Fk}\right)^\sigma p_{Fk}^{1-\sigma}}. \tag{31}$$

Analogously, the share of variable imports in variable cost is expressed as

$$s^v_{Fk} = \frac{1-\alpha_{\ell k}}{1-\alpha_{\ell k}+\frac{\varepsilon}{1+\varepsilon}\alpha_{\ell k}} \frac{\left(\omega^v_{Fk}\right)^\sigma p_{Fk}^{1-\sigma}}{\sum_{j\in Z_k}\left(\omega^v_{jk}\right)^\sigma p_j^{1-\sigma}+\left(\omega^v_{Fk}\right)^\sigma p_{Fk}^{1-\sigma}}. \tag{32}$$

Similar to the variable input purchases, one can write firm j's share and import share in firm k's total purchases of fixed intermediate inputs as follows:

$$s^f_{jk} = \frac{\left(\omega^f_{jk}\right)^\sigma p_j^{1-\sigma}}{\sum_{j\in Z_k}\left(\omega^f_{jk}\right)^\sigma p_j^{1-\sigma}+\left(\omega^f_{Fk}\right)^\sigma p_{Fk}^{1-\sigma}} \tag{33}$$

$$s^f_{Fk} = \frac{\left(\omega^f_{Fk}\right)^\sigma p_{Fk}^{1-\sigma}}{\sum_{j\in Z_k}\left(\omega^f_{jk}\right)^\sigma p_j^{1-\sigma}+\left(\omega^f_{Fk}\right)^\sigma p_{Fk}^{1-\sigma}}. \tag{34}$$

Firm-level sales consist of the sum of domestic sales to other firms as either variable or fixed inputs, domestic sales to domestic final demand, and exports. Therefore, we have the following equation for firm k's sales:

$$p_k q_k = \sum_{i\in W_k} s^v_{ki}\frac{p_i q_i}{\mu_i} + \sum_{i\in W_k} s^f_{ki} c^f_i \bar{q}^f_i + s_{kH}E_H + p_k^{1-\sigma}D_{kF}, \tag{35}$$

where W_k is the set of firm k's domestic buyers and

$$s_{kH} = \frac{\beta_{kH}^{\sigma-1}p_k^{1-\sigma}}{\sum_j \beta_{jH}^{\sigma-1}p_j^{1-\sigma}} \tag{36}$$

is firm k's share in household expenditure.

We close the model by assuming that all variable profits generated by firms are transferred back to households. We obtain the following expression for aggregate household income:

$$E_H = \sum_k w_k \ell^v_k + \sum_k \frac{\mu_k - 1}{\mu_k} p_k q_k - \sum_j \sum_k p_j q^f_{jk} - \sum_k p_{Fk} q^f_{Fk} - TB, \tag{37}$$

where TB is the aggregate trade balance. Labor market clearing implies that firms' labor demand equals the total labor supply in each labor market:

$$L_m = \sum_k \frac{1}{w_k} s^v_{\ell k} \frac{p_k q_k}{\mu_k} + \sum_k \bar{\ell}^f_k. \tag{38}$$

Definition 1 (Equilibrium) *Given the set of price of imports p_{Fk}, foreign demand shifters D_{kF}, aggregate trade balance TB, aggregate labor supply L, firms' domestic supplier sets Z_k and their importing and exporting decisions, and firms' fixed overhead input requirements $\bar{q}^f_k$ and $\bar{\ell}^f_k$, an equilibrium is the firms' wages, $\{w_k\}$, and the aggregate expenditure, E_H, such that equations* (5)–(7), (11)–(13), *and* (27)–(38) *hold.*

B.2 Derivations of equations (15) and (16)

To obtain equation (15), we take the total derivative of equation (10) while holding supply-side technology parameters fixed. From equation (10), the right-hand side of which is constant, we have

$$d \log \ell^v_k w_k(\ell_k) = d \log p_k q_k \left(\ell^v_k, \left\{q^v_{jk}\right\}, q^v_{Fk}\right).$$

Further rearranging the above using equation (8), we obtain

$$\begin{aligned} d \log \ell_k w_k(\ell_k) + d \log \left(1 - \frac{\bar{\ell}^f_k}{\ell_k}\right) &= d \log p_k q_k \left(\ell^v_k, \left\{q^v_{jk}\right\}, q^v_{Fk}\right) \\ d \log \ell_k w_k(\ell_k) + \frac{\bar{\ell}^f_k}{\ell^v_k} d \log \ell_k &= d \log p_k q_k \left(\ell^v_k, \left\{q^v_{jk}\right\}, q^v_{Fk}\right). \end{aligned}$$

We know from the labor supply curve of equation (5) that

$$\begin{aligned} d \log \ell_k &= \varepsilon d \log w_k \\ &= \frac{\varepsilon}{1+\varepsilon} d \log w_k \ell_k. \end{aligned}$$

Plugging this in, we have

$$\left(1 + \frac{\bar{\ell}^f_k}{\ell^v_k} \frac{\varepsilon}{1+\varepsilon}\right) d \log w_k \ell_k = d \log p_k q_k \left(\ell^v_k, \left\{q^v_{jk}\right\}, q^v_{Fk}\right),$$

and hence

$$\frac{d \log w_k \ell_k}{d \log p_k q_k \left(\ell^v_k, \left\{q^v_{jk}\right\}, q^v_{Fk}\right)} = \frac{\ell^v_k}{\ell_k} \frac{1+\varepsilon}{\frac{\ell^v_k}{\ell_k} + \varepsilon}.$$

We take a similar approach in deriving equation (16). The output elasticity in equation (14) can be written as

$$\frac{\partial \log q_k \left(\ell_k^v, \left\{q_{jk}^v\right\}, q_{Fk}^v\right)}{\partial \log q_{jk}^v} = \left(1 - \alpha_{\ell k}\right) \omega_{jk}^v \left(\frac{q_{jk}^v}{q_k^v}\right)^{\frac{\sigma-1}{\sigma}},$$

where q_k^v is the CES bundle of variable intermediate inputs. The term $\frac{q_{jk}^v}{q_k^v}$ depends only on the relative prices of firm k's suppliers, which we assume to be constant. Then one can write the total derivative of equation (14) as

$$d \log p_j q_{jk}^v = d \log p_k q_k \left(\ell_k^v, \left\{q_{jk}^v\right\}, q_{Fk}^v\right).$$

Further rearranging using equation (8), we obtain

$$d \log p_j q_{jk} + d \log \left(1 - \frac{q_{jk}^f}{q_{jk}}\right) = d \log p_k q_k \left(\ell_k^v, \left\{q_{jk}^v\right\}, q_{Fk}^v\right).$$

The fixed input purchases from firm j are given by equation (12) and only depend on the prices of firm j and the prices of other suppliers of firm k, which are all taken as fixed. Hence, one can further rearrange and obtain the following:

$$\begin{aligned} d \log p_j q_{jk} + \frac{q_{jk}^f}{q_{jk}^v} d \log q_{jk} &= d \log p_k q_k \left(\ell_k^v, \left\{q_{jk}^v\right\}, q_{Fk}^v\right) \\ \frac{d \log p_j q_{jk}}{d \log p_k q_k \left(\ell_k^v, \left\{q_{jk}^v\right\}, q_{Fk}^v\right)} &= \frac{q_{jk}^v}{q_{jk}}. \end{aligned}$$

B.3 Non-homothetic production function

We explore how labor cost and input purchases move with sales, when one assumes non-homotheticity in production functions but with no fixed costs in production. We consider a common class of non-homothetic production function—isoelastic nonhomothetic CES production function—following Hanoch (1975), Comin et al. (2021), Matsuyama (2023), and others. With this non-homothetic production function, we illustrate how labor cost and input purchases respond to sales changes. We investigate whether the model can rationalize the empirical pattern observed in the data: the labor cost elasticity to sales is below one and the input purchase elasticity to sales is close to one. We find that under this setup, whenever the labor cost elasticity to sales is less than one, the input purchase elasticity to sales should be larger than one. This leads to our conclusion that a non-homothetic production function is not sufficient to explain our main empirical findings.

To ease exposition and to build basic intuition, we first characterize the elasticities by assuming that firms take as given the prices of labor and input purchases. We then illustrate how the results change once one incorporates upward sloping labor supply curve. As in the main text, firm k uses labor ℓ_k^v and input purchases q_k^v for production. We assume that the production function takes the following implicit form:

$$1 = \left(\beta_\ell^{\frac{1}{\sigma}} q_k^{\frac{\epsilon_\ell - \sigma}{\sigma}} \left(\ell_k^v\right)^{\frac{\sigma-1}{\sigma}} + \beta_n^{\frac{1}{\sigma}} q_k^{\frac{\epsilon_n - \sigma}{\sigma}} \left(q_k^v\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

where the two non-homotheticity parameters ϵ_ℓ and ϵ_n are related to the elasticity of input usage with respect to output, as we will show below. The input with a larger value of this non-homotheticity parameter requires less amount of input to achieve an increase in output. And as the firm grows, the firm shifts its input usage towards the input with a larger non-homotheticity parameter. Because we consider a case in which the labor cost elasticity to sales is less than one, we assume throughout that $\epsilon_\ell < \epsilon_n$. On the output side, we assume that firms face a demand elasticity of $\sigma > 1$ and charge a constant markup of $\frac{\sigma}{\sigma-1}$. Moreover, we assume $\frac{\epsilon_\ell - \sigma}{1-\sigma} > 0$ and $\frac{\epsilon_n - \sigma}{1-\sigma} > 0$ to ensure global monotonicity where the input expenditure is increasing in the output quantity (as we show below), which effectively creates an upper bound on the two parameters, $0 < \epsilon_\ell < \epsilon_n < \sigma$.

B.3.1 Price taking firms

We first assume that firms take as given input prices, w_k and p_k^I. The cost minimization problem of the firm gives the firm's average cost index, c_k, of

$$c_k = \left(\beta_\ell q_k^{\epsilon_\ell - 1} w_k^{1-\sigma} + \beta_n q_k^{\epsilon_n - 1} \left(p_k^I\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}},$$

with expenditure shares of

$$\begin{aligned} s_{\ell k} =& \frac{\beta_\ell w_k^{1-\sigma} \left(E_k / c_k\right)^{\epsilon_\ell - 1}}{c_k^{1-\sigma}} \\ s_{Ik} =& \frac{\beta_n \left(p_k^I\right)^{1-\sigma} \left(E_k / c_k\right)^{\epsilon_n - 1}}{c_k^{1-\sigma}}. \end{aligned}$$

The firm's expenditure on inputs can be written as $E_k = q_k c_k$ and demand for inputs can be expressed as

$$\begin{aligned} \ell_k^v =& \beta_\ell \frac{w_k^{-\sigma}}{c_k^{1-\sigma}} q_k^{\epsilon_\ell - 1} E_k = \beta_\ell \frac{w_k^{-\sigma}}{c_k^{-\sigma} q_k^{-\sigma}} q_k^{\epsilon_\ell - \sigma} \\ q_k^v =& \beta_n \frac{\left(p_k^I\right)^{-\sigma}}{c_k^{1-\sigma}} q_k^{\epsilon_n - 1} E_k = \beta_n \frac{\left(p_k^I\right)^{-\sigma}}{c_k^{-\sigma} q_k^{-\sigma}} q_k^{\epsilon_n - \sigma}. \end{aligned}$$

From the average cost index, we have the following relationship between output quantity and average cost,

$$\frac{d \ln c_k}{d \ln q_k} = \frac{\bar{\epsilon}_k - 1}{1 - \sigma},$$

where $\bar{\epsilon}_k$ is the weighted average of the two non-homotheticity parameters, $\bar{\epsilon}_k = \epsilon_\ell s_{\ell k} + \epsilon_n s_{Ik}$. The firm's marginal cost can be expressed as

$$\begin{aligned} MC_k =& \frac{dE_k}{dq_k} = \left(1 + \frac{d \ln c_k}{d \ln q_k}\right) \frac{E_k}{q_k} \\ =& \frac{\bar{\epsilon}_k - \sigma}{1 - \sigma} c_k, \end{aligned}$$

with

$$\frac{d \ln E_k}{d \ln q_k} = \frac{\bar{\epsilon}_k - \sigma}{1 - \sigma}.$$

The output price is the markup times the marginal cost,

$$p_k = \frac{\sigma}{\sigma - 1} MC_k.$$

Putting these together, the relationship between expenditure and sales can be written as

$$E_k = c_k q_k = \frac{\sigma - 1}{\sigma} \frac{1 - \sigma}{\bar{\epsilon}_k - \sigma} p_k q_k.$$

Using the above, we derive the following elasticities of input usage with respect to a sales change:

$$\begin{aligned} \frac{d \ln w_k \ell_k^v}{d \ln p_k q_k} =& \underbrace{\left(\sigma - (\sigma - 1) \frac{\sigma - \epsilon_\ell}{\sigma - \bar{\epsilon}_k}\right)}_{\frac{d \ln w_k \ell_k^v}{d \ln E_k}} \frac{d \ln E_k}{d \ln p_k q_k} \\ \frac{d \ln p_k^I q_k^v}{d \ln p_k q_k} =& \underbrace{\left(\sigma - (\sigma - 1) \frac{\sigma - \epsilon_n}{\sigma - \bar{\epsilon}_k}\right)}_{\frac{d \ln p_k^I q_k^v}{d \ln E_k}} \frac{d \ln E_k}{d \ln p_k q_k}. \end{aligned} \tag{39}$$

Equation (39) implies that when $\epsilon_\ell < \epsilon_n$, the elasticity of labor cost to expenditure, $\frac{d \ln w_k \ell_k^v}{d \ln E_k}$, is less than one while the elasticity of purchases to expenditure, $\frac{d \ln p_k^I q_k^v}{d \ln E_k}$, is larger than one. To reconcile with our main empirical results where the labor cost elasticity to sales is less than one and the input purchase elasticity to sales is close to one, one would need the expenditure elasticity to sales, $\frac{d \ln E_k}{d \ln p_k q_k}$, to be less than one.

We first write this expenditure elasticity to sales as

$$\frac{d\ln E_k}{d\ln p_k q_k} = \frac{d\ln \frac{\sigma-1}{\sigma}\frac{1-\sigma}{\bar{\epsilon}_k-\sigma} p_k q_k}{d\ln p_k q_k}$$
$$= 1 + \frac{\bar{\epsilon}_k}{\sigma - \bar{\epsilon}_k}\frac{d\ln \bar{\epsilon}_k}{d\ln p_k q_k}.$$

As seen from above, how the expenditure changes in response to a change in sales can be summarized by how the average non-homotheticity parameter, $\bar{\epsilon}_k$, changes with sales. Recall that the non-homotheticity parameters govern how input expenditures respond to scale. And if $\bar{\epsilon}_k$ increases with scale, then the firm's overall expenditure will be less responsive when it scales up. One can show that

$$\frac{d\ln \bar{\epsilon}_k}{d\ln q_k} = \frac{\epsilon_\ell}{\bar{\epsilon}_k}\epsilon_\ell s_{\ell k} + \frac{\epsilon_n}{\bar{\epsilon}_k}\epsilon_n s_{Ik} - \bar{\epsilon}_k > 0,$$

meaning that as the firm scales up, it uses more inputs with higher non-homotheticity parameter, and hence $\bar{\epsilon}_k$ increases. Using this we derive the elasticity $\frac{d\ln \bar{\epsilon}_k}{d\ln p_k q_k}$ to find:

$$\frac{d\ln \bar{\epsilon}_k}{d\ln p_k q_k} = \underbrace{\left(\frac{\epsilon_\ell}{\bar{\epsilon}_k}\epsilon_\ell s_{\ell k} + \frac{\epsilon_n}{\bar{\epsilon}_k}\epsilon_n s_{Ik} - \bar{\epsilon}_k\right)}_{\frac{d\ln \bar{\epsilon}_k}{d\ln q_k}} \underbrace{\frac{\sigma-1}{\sigma-\bar{\epsilon}_k}}_{\frac{d\ln q_k}{d\ln E_k}>0} \frac{d\ln E_k}{d\ln p_k q_k},$$

and therefore $\frac{d\ln E_k}{d\ln p_k q_k}$ to be larger than one:

$$\frac{d\ln E_k}{d\ln p_k q_k} = \frac{1}{1 - \frac{\bar{\epsilon}_k}{\sigma-\bar{\epsilon}_k}\left(\frac{\epsilon_\ell}{\bar{\epsilon}_k}\epsilon_\ell s_{\ell k} + \frac{\epsilon_n}{\bar{\epsilon}_k}\epsilon_n s_{Ik} - \bar{\epsilon}_k\right)\frac{\sigma-1}{\sigma-\bar{\epsilon}_k}} > 1. \tag{40}$$

The intuition is as follows. As the firms scales up, it shifts its input usage towards the input that require less for a given increase in output, implying that its returns to scale increases as it scales up. This leads to marginal cost falling faster than the average cost, making its sales, $p_k q_k$, less responsive than its expenditure, $E_k = c_k q_k$, to a given increase in output. Therefore, the total input expenditure grows at a faster rate than its sales.

Taken together, equations (39) and (40) imply that this nonhomothetic production function is not able to reconcile the empirical finding where the input purchase elasticity to sales is one while the labor cost elasticity to sales is less than one.

B.3.2 Upward sloping labor supply curve

We now incorporate upward sloping labor supply curve, represented by the following:

$$\ell_k = A_k w_k^{\varepsilon}.$$

The firm's cost minimization problem now yields the following average cost index and the expenditure shares.

$$\begin{aligned}
c_k &= \left(\beta_\ell q_k^{\epsilon_\ell - 1}\left(\frac{\varepsilon}{\varepsilon+1}\right)^{\sigma} w_k^{1-\sigma} + \beta_n \left(p_k^I\right)^{1-\sigma} q_k^{\epsilon_n - 1}\right)^{\frac{1}{1-\sigma}} \\
s_{\ell k} &= \frac{\beta_\ell q_k^{\epsilon_\ell - 1}\left(\frac{\varepsilon}{\varepsilon+1}\right)^{\sigma} w_k^{1-\sigma}}{c_k^{1-\sigma}} \\
s_{Ik} &= \frac{\beta_n \left(p_k^I\right)^{1-\sigma} q_k^{\epsilon_n - 1}}{c_k^{1-\sigma}}.
\end{aligned}$$

The average cost index and the labor supply equation yield the following relationships:

$$\begin{aligned}
(1-\sigma)\, d\ln c_k &= (\bar{\epsilon}_k - 1)\, d\ln q_k + s_{\ell k}(1-\sigma)\, d\ln w_k \\
d\ln \ell_k^v &= -\sigma d\ln w_k + \sigma d\ln c_k + \epsilon_\ell d\ln q_k.
\end{aligned}$$

Combining these two together, we have the elasticity of average cost to scale written as:

$$\frac{d\ln c_k}{d\ln q_k} = \frac{\bar{\epsilon}_k - 1}{1-\sigma} + s_{\ell k}\frac{d\ln w_k}{d\ln q_k}.$$

We assume that wage increases with scale, $\frac{d\ln w_k}{d\ln q_k} > 0$.[27]

These relationships yield the following elasticities

$$\begin{aligned}
\frac{d\ln E_k}{d\ln q_k} &= \frac{\sigma - \bar{\epsilon}_k}{\sigma - 1} + s_{\ell k}\frac{d\ln w_k}{d\ln q_k} \\
\frac{d\ln E_k}{d\ln c_k} &= \frac{\frac{\sigma-\bar{\epsilon}_k}{\sigma-1} + s_{\ell k}\frac{d\ln w_k}{d\ln q_k}}{\frac{1-\bar{\epsilon}_k}{\sigma-1} + s_{\ell k}\frac{d\ln w_k}{d\ln q_k}},
\end{aligned}$$

[27] Combining with the labor supply curve equation, we obtain $\frac{d\ln w_k}{d\ln q_k} = \frac{\frac{\sigma}{\varepsilon+\sigma}\frac{\sigma-\bar{\epsilon}_k}{\sigma-1} - \frac{\sigma-\epsilon_\ell}{\varepsilon+\sigma}}{1 - s_{\ell k}\frac{\sigma}{\varepsilon+\sigma}}$. We assume $\frac{\sigma}{\sigma-1} > \frac{\sigma-\epsilon_\ell}{\sigma-\bar{\epsilon}_k}$ to ensure that wage increases with scale.

which we use to derive the following input elasticities to sales:

$$
\begin{aligned}
\frac{d\ln w_k \ell_k^v}{d\ln p_k q_k} &= \left(\sigma - (\sigma-1)\,\frac{\sigma-\epsilon_\ell}{\sigma-\bar{\epsilon}_k}\frac{1}{1+\frac{\sigma-1}{\sigma-\bar{\epsilon}_k}s_{\ell k}\frac{d\ln w_k}{d\ln q_k}} - (\sigma-1)\,\frac{d\ln w_k}{d\ln E_k}\right)\frac{d\ln E_k}{d\ln p_k q_k} \\
\frac{d\ln p_k^I q_k^v}{d\ln p_k q_k} &= \left(\sigma - (\sigma-1)\,\frac{\sigma-\epsilon_n}{\sigma-\bar{\epsilon}_k}\frac{1}{1+\frac{\sigma-1}{\sigma-\bar{\epsilon}_k}s_{\ell k}\frac{d\ln w_k}{d\ln q_k}}\right)\frac{d\ln E_k}{d\ln p_k q_k}
\end{aligned}
\tag{41}
$$

Comparing equation (41) with equation (39), one can observe that as the firm scales up, the firm faces a higher wage. This results in a lower labor cost elasticity to sales, as captured by the term $(\sigma-1)\,\frac{d\ln w_k}{d\ln E_k}$. To ensure that the weighted sum of the input share elasticities sum up to one, $1 = s_{\ell k}\frac{d\ln w_k \ell_k^v}{d\ln E_k} + s_{Ik}\frac{d\ln p_k^I q_k^v}{d\ln E_k}$, the two input elasticities to scale are adjusted upwards with the term $1/\left(1+\frac{\sigma-1}{\sigma-\bar{\epsilon}_k}s_{\ell k}\frac{d\ln w_k}{d\ln q_k}\right)$. Ignoring so far how the expenditure elasticity $\frac{d\ln E_k}{d\ln p_k q_k}$ is affected, we see that the input purchase elasticity to sales increases further by incorporating upward sloping labor supply curve.

As the last step, we focus on how the expenditure responds to sales change, $\frac{d\ln E_k}{d\ln p_k q_k}$. We have

$$
\begin{aligned}
\frac{d\ln E_k}{d\ln p_k q_k} = 1 + \Bigg(& \frac{\frac{\sigma-\bar{\epsilon}_k}{\sigma-1}}{\frac{\sigma-\bar{\epsilon}_k}{\sigma-1} + s_{\ell k}\frac{d\ln w_k}{d\ln q_k}}\frac{\bar{\epsilon}_k}{\sigma-\bar{\epsilon}_k}\frac{d\ln\bar{\epsilon}_k}{d\ln q_k} \\
& - \frac{s_{\ell k}\frac{d\ln w_k}{d\ln q_k}}{\frac{\sigma-\bar{\epsilon}_k}{\sigma-1} + s_{\ell k}\frac{d\ln w_k}{d\ln q_k}}\left(\frac{d\ln s_{\ell k}}{d\ln q_k} + \frac{d\ln\left(\frac{d\ln w_k}{d\ln q_k}\right)}{d\ln q_k}\right)\Bigg)\frac{d\ln q_k}{d\ln E_k}\frac{d\ln E_k}{d\ln p_k q_k}.
\end{aligned}
$$

This equation shows that the expenditure elasticity to sales depends on how the average non-homotheticity parameter, labor cost share, and the wage elasticity to scale, responds to scale ($\frac{d\ln\bar{\epsilon}_k}{d\ln q_k}$, $\frac{d\ln s_{\ell k}}{d\ln q_k}$, and $\frac{d\ln\left(\frac{d\ln w_k}{d\ln q_k}\right)}{d\ln q_k}$). We show that the average non-homotheticity parameter increases with scale at a faster rate when wage is increasing,

$$
\frac{d\ln\bar{\epsilon}_k}{d\ln q_k} = \frac{\epsilon_\ell}{\bar{\epsilon}_k}\epsilon_\ell s_{\ell k} + \frac{\epsilon_n}{\bar{\epsilon}_k}\epsilon_n s_{Ik} - \bar{\epsilon}_k + s_{\ell k}\left(1-\frac{\epsilon_\ell}{\bar{\epsilon}_k}\right)(\sigma-1)\,\frac{d\ln w_k}{d\ln q_k} > 0.
$$

We also show that the labor share is decreasing with scale at a faster rate when wage is increasing,

$$
\frac{d\ln s_{\ell k}}{d\ln q_k} = \epsilon_\ell - \bar{\epsilon}_k - (\sigma-1)\,(1-s_{\ell k})\,\frac{d\ln w_k}{d\ln q_k} < 0.
$$

Moreover, we can show that the elasticity of wage to scale is decreasing with scale, $\frac{d\ln\left(\frac{d\ln w_k}{d\ln q_k}\right)}{d\ln q_k} < 0$, under the parametric space that ensures $\frac{d\ln w_k}{d\ln q_k} > 0$. Taken together, these results lead to the expenditure elasticity to sales being greater than one, $\frac{d\ln E_k}{d\ln p_k q_k} > 1$.

In conclusion, accounting for upward sloping labor supply curve would further increase the input purchase elasticity to sales, pushing it further away from one. Therefore, we maintain the result that this nonhomothetic production function is not able to reconcile the empirical finding where the input purchase elasticity to sales is one while the labor cost elasticity to sales is less than one.

B.4 Adjustment frictions in labor costs

In this appendix we illustrate how adjustment costs in variable labor inputs affect the labor cost elasticity with respect to sales. In particular, we consider a simplified version of the model outlined in Section 3.1, where the production function is defined as in equation (7). We consider two alternative types of labor adjustment frictions: a quadratic adjustment cost, and labor adjustment à la Calvo (1983). With these setups, we investigate the long run effect of a permanent demand shock. We show that the ratio of variable labor cost to sales remains a constant, even when adjustment frictions are accounted for. As a result, fixed costs in labor can be identified by examining the long run response of total labor costs to changes in sales induced by a permanent demand shock.

B.4.1 Quadratic adjustment cost in labor inputs

We first consider quadratic adjustment costs in variable labor inputs. To focus solely on the role of the adjustment cost in variable labor inputs, we assume that the firm's productivity and the price of input purchases $\left(p_k^I\right)$ are fixed. The firm chooses its variable labor cost one period in advance, and its output price and variable input purchases contemporaneously. The firm's profit maximizing problem is written as follows:

$$
\begin{aligned}
\max_{p_{kt},\ell^v_{kt+1},q^v_{kt}} \sum_{t=0}^{\infty} \beta^t & \left(p_{kt}q_{kt} - w_{kt}\ell^v_{kt} - p^I_k q^v_{kt} - w_{kt}\bar{\ell}^f_k - p^I_{kt}\bar{q}^f_k - \frac{\psi}{2}\left(\frac{\ell^v_{kt+1}-\ell^v_{kt}}{\ell^v_{kt}}\right)^2 \ell^v_{kt} \right) \\
s.t.\ q_{kt} &= p_{kt}^{-\sigma} D_{kt} \\
q_{kt} &= \phi_k \left(q^v_{kt}\right)^{1-\alpha_{\ell k}} \left(\ell^v_{kt}\right)^{\alpha_{\ell k}} \\
w_{kt} &= A_{kt}^{-\frac{1}{\varepsilon}} \left(\ell^v_{kt} + \bar{\ell}^f_k\right)^{\frac{1}{\varepsilon}},
\end{aligned}
$$

where the first constraint is the demand curve the firm faces, the second is the firm's production function, and the last constraint is the labor supply curve.[28] We illustrate the solution to this dynamic problem where there are i.i.d. shocks on the firm-level demand shifter, D_{kt}, such that $E\left[D_{k,t+1}|D_{k,t}\right] = E\left[D_k\right]$. Consistent with the assumption made in the main text,

[28] Note that when there are no adjustment costs, $\psi = 0$, then the problem becomes identical to the static problem.

we assume that firms are infinitesimal in the labor market. Therefore the only state variables are the demand shifter and the labor input.

The Bellman equation can be written as:

$$
\begin{aligned}
V\left(\ell_k^v, D_k\right) = & \max_{\ell_k^{v'}, q_k^v} \phi_k^{\frac{\sigma-1}{\sigma}} \left(q_k^v\right)^{\frac{\sigma-1}{\sigma}(1-\alpha_{\ell k})} \left(\ell_k^v\right)^{\frac{\sigma-1}{\sigma}\alpha_{\ell k}} D_k^{\frac{1}{\sigma}} - A_k^{-\frac{1}{\varepsilon}} \left(\ell_k^v + \bar{\ell}_k^f\right)^{\frac{\varepsilon+1}{\varepsilon}} - p_k^I q_k^v - p_k^I \bar{q}_k^f \\
& - \frac{\psi}{2} \left(\frac{\ell_k^{v'} - \ell_k^v}{\ell_k^v}\right)^2 \ell_k^v + \beta E\left[V\left(\ell_k^{v'}, D_k^{'} \mid D_k\right)\right].
\end{aligned}
$$

The first order condition with respect to the input purchases gives the same relationship between contemporaneous variable input purchases and sales,

$$
\frac{p_k^I q_k^v}{p_k q_k} = \frac{\sigma - 1}{\sigma} \left(1 - \alpha_{\ell k}\right).
$$

The first order condition with respect to the variable labor inputs using the above solution for input purchases can be written as follows:

$$
\begin{aligned}
\psi \left(\frac{\ell_k^{v'} - \ell_k^v}{\ell_k^v}\right) = & \beta \frac{d}{d\ell_k^{v'}} E\left[V\left(\ell_k^{v'}, D_k^{'}\right)\right] \\
\psi \frac{\ell_k^{v'}}{\ell_k^v} - \psi = & \beta \frac{\sigma-1}{\sigma} \phi_k^{\frac{\sigma-1}{\sigma}} \left(q_k^{v'}\right)^{\frac{\sigma-1}{\sigma}(1-\alpha_{\ell k})} \left(\ell_k^{v'}\right)^{\frac{\sigma-1}{\sigma}\alpha_{\ell k}} E\left[D_k^{\frac{1}{\sigma}}\right] \left(\ell_k^{v'}\right)^{-1} \alpha_{\ell k} \\
& - \beta \frac{\varepsilon+1}{\varepsilon} A_k^{-\frac{1}{\varepsilon}} \left(\ell_k^{v'} + \bar{\ell}_k^f\right)^{\frac{1}{\varepsilon}} + \beta \frac{\psi}{2} \left(\left(\frac{\ell_k^{v''}}{\ell_k^{v'}}\right)^2 - 1\right).
\end{aligned}
$$

We focus on the labor inputs under steady state, namely where $\ell_k^{v'} = \ell_k^v$. Hence, we have

$$
E\left[\frac{w_k^{'} \ell_k^{v'}}{p_k^{'} q_k^{'}}\right] = \alpha_{\ell k} \frac{\varepsilon}{1+\varepsilon} \left(\frac{\sigma}{\sigma - 1}\right)^{-1},
$$

implying a constant relationship between variable labor cost and sales, as in equation (10) of the static model.

In the long run steady state, we find that the ratios of both variable labor cost and variable input purchases to sales are constant even under the presence of quadratic adjustment costs in variable labor inputs. These imply that the long run elasticities of labor cost and input purchases to sales change that is driven by an idiosyncratic shock on the firm's demand shifter would be the same as in equations (15) and (16). Furthermore, if one assumes away fixed overhead costs in labor inputs, then the ratio of total labor cost to sales becomes a constant. In this case, the long run elasticity of labor cost to sales becomes one.

B.4.2 Labor adjustment à la Calvo (1983)

We now consider the firm facing labor adjustment frictions à la Calvo (1983). Suppose in each period, the firm cannot adjust labor inputs with probability θ, while intermediate inputs can be adjusted freely every period. The value function then becomes:

$$\begin{aligned} V\left(\ell_k^v, D_k\right) = & \max_{q_k^v} \phi_k^{\frac{\sigma-1}{\sigma}} \left(q_k^v\right)^{\frac{\sigma-1}{\sigma}(1-\alpha_{\ell k})} \left(\ell_k^v\right)^{\frac{\sigma-1}{\sigma}\alpha_{\ell k}} D_k^{\frac{1}{\sigma}} - A_k^{-\frac{1}{\varepsilon}} \left(\ell_k^v + \bar{\ell}_k^f\right)^{\frac{\varepsilon+1}{\varepsilon}} - p_k^I q_k^v - p_k^I \bar{q}_k^f \\ & + \beta\theta E\left[V\left(\ell_k^v, D_k^{'} \mid D_k\right)\right] + \beta\left(1-\theta\right) E\left[\max_{\ell_k^{v'}} V\left(\ell_k^{v'}, D_k^{'} \mid D_k\right)\right]. \end{aligned}$$

Note that if firms could adjust their labor inputs freely, i.e., $\theta = 0$, then the problem becomes identical to the static problem.

Given ℓ_k^v and D_k, the optimal amount of variable input purchases in any period can be derived from the first order condition and is the same as the static problem:

$$\frac{p_k^I q_k^v}{p_k q_k} = \frac{\sigma-1}{\sigma}\left(1-\alpha_{\ell k}\right). \tag{42}$$

The first order condition with respect to labor inputs can be written as:

$$\begin{aligned} \frac{\partial V\left(\ell_k^v, D_k\right)}{\partial \ell_k^v} = 0 = & \frac{\sigma-1}{\sigma}\alpha_{\ell k}\phi_k^{\frac{\sigma-1}{\sigma}} \left(q_k^v\right)^{\frac{\sigma-1}{\sigma}(1-\alpha_{\ell k})} \left(\ell_k^v\right)^{\frac{\sigma-1}{\sigma}\alpha_{\ell k}} D_k^{\frac{1}{\sigma}} \left(\ell_k^v\right)^{-1} - \frac{\varepsilon+1}{\varepsilon} w_k \\ & + \beta\theta E\left[\frac{\partial}{\partial \ell_k^v}\left(\phi_k^{\frac{\sigma-1}{\sigma}} \left(q_k^{v'}\right)^{\frac{\sigma-1}{\sigma}(1-\alpha_{\ell k})} \left(\ell_k^v\right)^{\frac{\sigma-1}{\sigma}\alpha_{\ell k}} \left(D_k^{'}\right)^{\frac{1}{\sigma}}\right)\right] - \beta\theta\frac{\varepsilon+1}{\varepsilon} w_k \\ & + \cdots. \end{aligned} \tag{43}$$

In the second line of equation (43), the firm takes into account that the amount of intermediate input purchases is an endogenous object that satisfy equation (42). Arranging further, we obtain the following ratio of variable labor cost to sales:

$$\frac{w_k \ell_k^v}{p_k q_k} = \alpha_{\ell k}\frac{\varepsilon}{1+\varepsilon}\left(\frac{\sigma}{\sigma-1}\right)^{-1} \frac{1+\sum_{s=1}^{\infty}\left(\frac{\beta\theta}{1-\frac{\sigma-1}{\sigma}(1-\alpha_{\ell k})}\right)^s E\left[\left(\frac{D_{k,t+s}}{D_{k,t}}\right)^{\frac{1}{\sigma}\frac{1}{1-\frac{\sigma-1}{\sigma}(1-\alpha_{\ell k})}}\right]}{1+\sum_{s=1}^{\infty}\left(\beta\theta\right)^s}. \tag{44}$$

Equation (44) reveals that the share of variable labor cost to sales can be different from the optimal share in the static problem, $\alpha_{\ell k}\frac{\varepsilon}{1+\varepsilon}\left(\frac{\sigma}{\sigma-1}\right)^{-1}$. When the firm expects a larger demand in the future (larger expectation of $\frac{D_{k,t+s}}{D_{k,t}}$), then the firm hires more labor today because with some probability, it would not be able to adjust the amount of labor when larger demand is realized. However, as long as the series of expected future demand are fixed, the variable labor cost to sales ratio is constant. Moreover, the equation implies that in response to a permanent demand shock where current and future demand is shifted by a constant multiplier, the variable labor cost to sales share before and after the shock is unaffected, as the ratio of $\frac{D_{k,t+s}}{D_{k,t}}$ remains the same. This is true for one firm conditional on being able to change its labor, and also for the aggregate once all firms have the opportunity to change their labor. Therefore, we find that when firms face frictions in labor cost adjustment à la Calvo (1983), the long run elasticity of labor cost to a sales change that is driven by a permanent demand shock is the same as that in equation (15).

B.5 System of counterfactual changes in variables

B.5.1 Setup of the counterfactual exercise

In our main counterfactual exercise in Section 6, we assume that firms charge a common markup of $\frac{\sigma}{\sigma-1}$ and that firms face upward sloping labor supply curve with elasticity ε. By having firms set a common markup of $\frac{\sigma}{\sigma-1}$, we have a discrepancy between a firm's variable input cost inferred from its sales and markup, $\left(\frac{\varepsilon}{\varepsilon+1}\alpha_{\ell k}+1-\alpha_{\ell k}\right)\frac{\sigma-1}{\sigma}p_k q_k$, and its variable input cost inferred from the estimated variable cost shares and observed input expenditure, $varinput_k = w_k \ell_k^v + \sum_j p_j q_{jk}^v + p_{Fk} q_{Fk}^v$. We denote these firm-level discrepancies by adj_k:

$$adj_k = \underbrace{\left(\frac{\varepsilon}{\varepsilon+1}\alpha_{\ell k}+1-\alpha_{\ell k}\right)\frac{\sigma-1}{\sigma}p_k q_k}_{\text{theory-implied input}} - \underbrace{varinput_k}_{\text{observed input}} .$$

One interpretation of this term adj_k is the usage of firm k's inventories. If $adj_k > 0$, then the firm is purchasing fewer variable inputs than what is implied from its sales and markup and hence is using the past inventory of inputs to produce. If $adj_k < 0$, then the firm is purchasing more variable inputs than what is implied from its sales and markup and hence is accumulating inventory for future use.

In the counterfactual exercise, we follow Dhyne et al. (2022) and assume that the ratios of adj_k relative to the firm's two different variable inputs are fixed. This is consistent with an interpretation in which the fraction of how much inventory the firm uses (or accumulates) relative to its inputs and sales does not change in response to foreign shocks. With this assumption, we have the following relationship:

$$\widehat{varinput}_k = \frac{\frac{\sigma-1}{\sigma}p_k q_k}{varinput_k}\widehat{p_k q_k} - \frac{adj_k}{varinput_k}\widehat{adj}_k,$$

where we have $\widehat{varinput}_k = \widehat{p_k q_k} = \widehat{adj}_k$.

This treatment of the differences in variable input costs is isomorphic to assuming that firms charge firm-specific markups of $\mu_k = \frac{p_k q_k}{w_k \ell_k^v + \sum_j p_j q_{jk}^v + p_{Fk} q_{Fk}^v}$, which can be read from the data. To see this, we refer to equation (45) in Appendix B.5.2, which illustrates how the change in aggregate income is affected by changes in firms' variable profits $\left(\frac{\pi_k^v p_k q_k}{E_H}\widehat{p_k q_k}\right)$ and the changes in the discrepancy terms $\left(\frac{adj_k}{E_H}\widehat{adj}_k\right)$. If one assumes that firms charge markups of μ_k, then the effect of the changes in their variable profits on aggregate income can be summarized by $\frac{\frac{\mu_k - 1}{\mu_k} p_k q_k}{E_H}\widehat{p_k q_k}$. With the assumption that firm sales and the discrepancy terms move in tandem $\left(\widehat{adj}_k = \widehat{p_k q_k}\right)$, the effects on aggregate income are isomorphic to each other.

B.5.2 Counterfactual changes in response to import price and foreign demand shocks

The steps to solve for the counterfactual outcomes are as follows:

1. Guess the changes in firm-level wages $\hat{w}_k$. If $\varepsilon = \infty$, then the guess is common across all firms.

2. Compute the firm-level changes in total and variable labor inputs, $\hat{\ell}_k$ and $\hat{\ell}_k^v$, using equations (5) and (8):

$$\hat{\ell}_k = \frac{\hat{w}_k^\varepsilon}{\sum_j \ell_j \hat{w}_j^\varepsilon} L$$
$$\hat{\ell}_k^v = \frac{\ell_k}{\ell_k^v}\hat{\ell}_k - \frac{\bar{\ell}_k^f}{\ell_k^v}.$$

   Skip this step if $\varepsilon = \infty$.

3. Solve for $\hat{c}_k$ using equation (28):

$$\hat{c}_k = \left(\sum_{j \in Z_k} \frac{s_{jk}^v}{1 - s_{\ell k}^v}\hat{c}_j^{1-\sigma} + \frac{s_{Fk}^v}{1 - s_{\ell k}^v}\hat{p}_{Fk}^{1-\sigma}\right)^{\frac{1-\alpha_{\ell k}}{1-\sigma}} \hat{w}_k^{\alpha_{\ell k}}.$$

4. Compute the change in shares and prices using equations (31), (32), (33), (34), and

(36):

$$
\begin{aligned}
\hat{s}^v_{jk} &= \frac{\hat{c}_j^{1-\sigma}}{\sum_{j\in Z_k} \frac{s^v_{jk}}{1-s^v_{\ell k}}\hat{c}_j^{1-\sigma} + \frac{s^v_{Fk}}{1-s^v_{\ell k}}\hat{p}_{Fk}^{1-\sigma}} \\
\hat{s}^v_{Fk} &= \frac{\hat{p}_{Fk}^{1-\sigma}}{\sum_{j\in Z_k} \frac{s^v_{jk}}{1-s^v_{\ell k}}\hat{c}_j^{1-\sigma} + \frac{s^v_{Fk}}{1-s^v_{\ell k}}\hat{p}_{Fk}^{1-\sigma}} \\
\hat{s}^f_{jk} &= \frac{\hat{c}_j^{1-\sigma}}{\left(\hat{c}^f_k\right)^{1-\sigma}} \\
\hat{s}^f_{Fk} &= \frac{\hat{p}_{Fk}^{1-\sigma}}{\left(\hat{c}^f_k\right)^{1-\sigma}} \\
\left(\hat{c}^f_k\right)^{1-\sigma} &= \sum_{j\in Z_k} s^f_{jk}\hat{c}_j^{1-\sigma} + s^f_{Fk}\hat{p}_{Fk}^{1-\sigma} \\
\hat{s}_{kH} &= \frac{\hat{c}_k^{1-\sigma}}{\sum_j s_{jH}\hat{c}_j^{1-\sigma}}.
\end{aligned}
$$

5. Solve for $\widehat{p_k q_k}$ using equation (35):

$$
\widehat{p_k q_k} = \sum_{i\in W_k} r^v_{ki}\hat{s}^v_{ki}\widehat{p_i q_i} + \sum_{i\in W_k} r^f_{ki}\hat{s}^f_{ki}\hat{c}^f_i + r_{kH}\hat{s}_{kH}\hat{E}_H + r_{kF}\hat{c}_k^{1-\sigma}\hat{D}_{kF},
$$

where we have the following revenue shares of firm k:

$$
r^v_{ki} = \frac{s^v_{ki}p_i q_i}{p_k q_k \mu_i},\ r^f_{ki} = \frac{p_k q^f_{ki}}{p_k q_k},\ r_{kH} = \frac{s_{kH}E_H}{p_k q_k},\ r_{kF} = \frac{p_k^{1-\sigma}D_{kF}}{p_k q_k}.
$$

The change in aggregate household expenditure is written as

$$
\begin{aligned}
\hat{E}_H =& \sum_k \frac{w_k \ell_k}{E_H}\hat{w}_k + \sum_k \frac{\pi^v_k}{E_H}\widehat{p_k q_k} - \sum_j\sum_k \frac{p_j q^f_{jk}}{E_H}\hat{s}^f_{jk}\hat{c}^f_k - \sum_k \frac{p^f_{Fk}q^f_{Fk}}{E_H}\hat{s}^f_{Fk}\hat{c}^f_k \\
&- \sum_k \frac{w_k \bar{\ell}^f_k}{E_H}\hat{w}_k - \frac{TB}{E_H} + \sum_k \frac{adj_k}{E_H}\widehat{adj}_k,
\end{aligned} \tag{45}
$$

where $\widehat{adj}_k = \widehat{p_k q_k}$ and $\pi^v_k = \left(1 - \left(\frac{\varepsilon_m}{\varepsilon_m+1}\alpha_{\ell k} + 1 - \alpha_{\ell k}\right)\frac{\sigma-1}{\sigma}\right)p_k q_k$.

6. Update $\hat{w}_k$ with the following and iterate from Step 2 until $\hat{w}_k$ converges:

$$\hat{w}_k^{new} = \frac{\widehat{p_k q_k}}{\hat{\ell}_k^v}$$
$$\hat{w}_k = d\hat{w}_k^{new} + (1-d)\,\hat{w}_k.$$

If $\varepsilon = \infty$, then use the following to update the common guess of wage change:

$$\hat{w}_k^{new} = \frac{\sum_k w_k \ell_k^v \widehat{p_k q_k} + \sum_k w_k \bar{\ell}_k^f \hat{w}_k}{\sum_k w_k \ell_k}.$$

7. Finally, check that the trade balance holds (i.e., the exogenous TB is unchanged).

B.5.3 Counterfactual changes in response to firm productivity shocks

The system of counterfactual changes in variables when one considers changes in firms' productivities is similar to that presented in Appendix B.5.2. Instead of the changes in import price $\hat{p}_{Fk}$, we consider changes in productivities, $\hat{\phi}_k$. Hence, we replace Step 3 in Appendix B.5.2 with the following equation that solves for $\hat{c}_k$ given the shocks $\hat{\phi}_k$ and guess of $\hat{w}_k$:

$$\hat{c}_k = \hat{\phi}_k^{-1} \left(\sum_{j \in Z_k} \frac{s_{jk}^v}{1 - s_{\ell k}^v} \hat{c}_j^{1-\sigma} \right)^{\frac{1-\alpha_{\ell k}}{1-\sigma}} \hat{w}_k^{\alpha_{\ell k}}.$$

B.6 Fixed costs and aggregate labor demand

To gain intuition about how accounting for fixed labor inputs alter wage responses to foreign trade shocks, we work with a simple framework of aggregate labor supply and demand. In this stylized framework we consider one representative firm with no input-output linkages. We also abstract away from upward sloping labor supply curve, as in the aggregate, labor supply is fixed. To obtain the aggregate labor demand curve, we start with the output demand curve,

$$q = p^{-\sigma} D,$$

where p is the output price with price being the product of markup and marginal cost, $p = \mu c$. The demand shifter D contains demand from domestic final demand and export demand, $D = \frac{E}{p^{1-\sigma}} + D_F$, where E is the domestic household income. We consider a net production function that combines the two primary factors, labor and imports, with a Cobb-Douglas aggregater. Denoting variable labor inputs and imports by ℓ^v and I respectively, and prices of labor and imports by w and p^I respectively, we have

$$q = \phi \left(\ell^v\right)^{\alpha} I^{1-\alpha}$$
$$c = A\phi^{-1} w^{\alpha} \left(p^I\right)^{1-\alpha},$$

where $A = \alpha^{-\alpha}\left(1-\alpha\right)^{-(1-\alpha)}$ is a constant, ϕ is the aggregate productivity term, and α is the share of labor inputs in variable costs. Finally, we have the following relationships for the aggregate income and trade balance,

$$E = w\ell^v + \frac{\mu-1}{\mu}E + \frac{\mu-1}{\mu}p^{1-\sigma}D_F$$
$$p^{1-\sigma}D_F = p^I I.$$

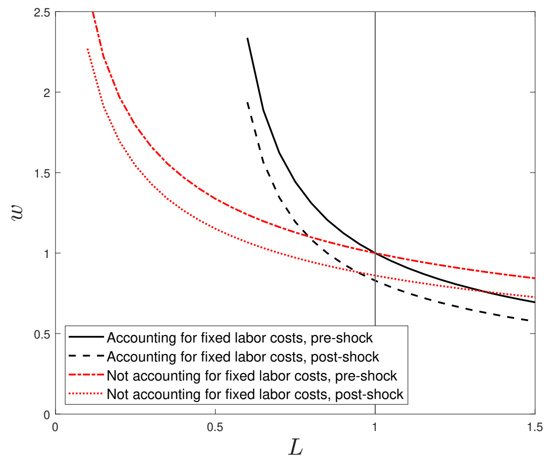
The above relationships implicitly yield a downwards sloping aggregate labor demand curve, where wage w is written as a function of total labor input (the sum of variable and fixed labor inputs, $L = \ell^v + \ell^f$) and foreign price and demand shifters, p^I and D_F.

Accounting for fixed labor inputs alter the aggregate labor demand curve in two ways. First, it rotates the aggregate labor demand curve clockwise around the initial equilibrium, as variable labor input is now less than L at $L - \ell^f$. Intuitively, when fixed labor inputs are accounted for, the amount of labor inputs that contribute to production decreases as the total labor supply is fixed. This effectively moves the economy to the more inelastic part of the aggregate labor demand curve. Second, accounting for fixed labor inputs alters the measurement of the labor cost share in variable costs, α, which governs the elasticities of wage with respect to foreign price p^I and demand D_F. Because there are labor inputs that are used as fixed overhead costs, the share of labor cost in variable costs, α, is lower than when one reads it off from the total cost shares. Therefore, for a given increase in foreign price (an increase in p^I) or for a given reduction in foreign demand (a reduction in D_F), wage is predicted to decrease more by accounting for fixed labor costs. Intuitively, when labor cost account for a smaller share in variable cost, output price is less responsive to wage. With aggregate labor supply fixed, the economy responds to a negative demand by lowering the price through lower wage, and to an increase in the foreign price by countering the price increase through a lower wage. In both cases, wage has to decrease with a larger magnitude when fixed labor costs are accounted for.

To visually convey how the aggregate labor demand curve changes when fixed labor inputs are accounted for, we numerically solve for the aggregate labor demand curves under different parametrizations, and plot them in Figure 7. We consider an economy where the aggregate labor supply is fixed at $L = 1$ (the vertical line). The two black lines use the labor cost share parameter of $\alpha = 0.3$ and account for fixed labor costs with $\ell^f = 0.5$. The solid black lines compute the labor demand when $D_F = 1$ and the dashed black lines compute the labor demand when the foreign demand decreases by 30 percent, at $D_F = 0.7$. Their values are normalized with the level of wage when foreign demand is $D_F = 1$ and the labor supply is $L = 1$. The two lines show that when the foreign demand decreases by 30 percent, the aggregate labor demand curve shifts to the left, implying that the wage decreases given a fixed aggregate labor supply.

In contrast, the two red lines, also normalized with the level of wage when the foreign demand is $D_F = 1$ and the labor supply is $L = 1$, do not account for fixed labor inputs. The dash-dotted red line uses $D_F = 1$ as the foreign demand, and the dotted red line uses $D_F = 0.7$. The economy in the dash-dotted red line has the same share of total labor expenditure in total costs as in the solid black line when $L = 1$. As discussed above, not accounting for fixed labor inputs should increase the measurement of α. We compute the share of labor costs in total inputs for the solid black line with labor supply of $L = 1$, and use $\alpha = 0.46 (= 0.3 * 2/(1 + 0.3))$ for the two red lines. These two sets of lines confirm our arguments above: Accounting for fixed labor inputs makes the aggregate labor demand less elastic, and increases the magnitudes of the downward shifts in the labor demand upon negative foreign trade shocks.

Figure 7: Aggregate labor demand



Notes: This figure plots the aggregate labor demand curves computed with the following parameter values: $\sigma = 4$, $\phi = 1$, $p^I = 1$. We consider two economies that have the same share of labor expenditure in total costs. The two black lines use the variable labor cost share of $\alpha = 0.3$ and depict the labor demand when fixed labor inputs are accounted for with $\ell^f = 0.5$. The solid black line uses $D_F = 1$ as the foreign demand, and the dashed black line uses $D_F = 0.7$. The values of the two black lines are normalized to the wage under $D_F = 1$ and $L = 1$. The two red lines use the variable labor cost share of $\alpha = 0.46$ and depict the labor demand when fixed labor inputs are not accounted for, $\ell^f = 0$. 0.46 is the share of total labor cost out of total inputs when $\ell^f = 0.5$, $D_F = 1$, and $L = 1$. The dash-dotted red line uses $D_F = 1$ as the foreign demand, and the dotted red line uses $D_F = 0.7$. The values of the two red lines are normalized to the wage under $D_F = 1$ and $L = 1$.

C Additional empirical results

C.1 Additional results on labor costs and sales

In Figure 1 of Section 2.4, we explore the relationship between firms' changes in labor costs and sales, that are indicative of how much labor are used as fixed overhead inputs. In this section, we examine different cuts of the data to explore whether any group of firms have significantly lower correlation between changes in labor cost and sales than other firms. Table 6 reports the regression coefficients on changes in labor costs on changes in sales for different sets of firms. We find that groups of firms that are often associated with having larger fixed costs, such as exporters and importers, have significantly lower correlation between changes in labor cost and sales, suggesting that fixed labor cost shares are higher for those firms.

However, as we discussed in Section 2.4, one needs to be cautious in drawing conclusions from Table 6 because input costs and sales are simultaneously determined and may be affected by both measurement error and many omitted variables. The IV estimates reported in Appendix C.9 show a broadly comparable relationship between changes in labor costs and sales for firms that trade internationally and those that do not. This suggests that firms that trade internationally have higher fixed costs (as they are larger), but comparable fixed share of costs.

Table 6: Relationship between firm-level sales and labor costs

	Sales and labor costs (four-year change)	
	Slope of labor costs	Difference between groups
Exporters	0.471***	
	(0.00941)	-0.119***
Non-exporters	0.590***	(0.0189)
	(0.0201)	
Importers	0.503***	
	(0.00926)	-0.0846***
Non-importers	0.587***	(0.0208)
	(0.0217)	
Exporters or importers	0.499***	
	(0.00981)	-0.0951***
Domestic firms	0.594***	(0.0209)
	(0.0224)	

Notes: This table uses the main estimation sample of private-sector firms in Belgium (see Section 2.3 for details). In each panel, we report the regression coefficients of four-year changes in firms' labor costs on changes in their sales, for different groups of firms. Industry-fixed effects are included at the NACE four-digit level. Standard errors in parentheses are clustered at the NACE four-digit level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

C.2 Wage differentials and firm effects

In Figure 1 of Section 2.4, we show that changes in average wage are correlated with the changes in sales, suggesting that firms face upward sloping labor supply curve. Using the firm and worker data, in this section we show that i) firms that are more exposed to foreign markets are larger, more productive, and pay higher wages, and that ii) these wage differentials cannot be entirely explained by observed or unobserved differences across workers. These findings further motivate why we will depart from the canonical model of a competitive labor market where wages depend only on the marginal product of workers and not the firm that employs them.

A large body of previous work has documented that firms that export look very different from non-exporters along a number of important dimensions. This is also true in the Belgian data: the descriptive statistics reported in Table 2 of Dhyne et al. (2021) show that the direct exporters not only are more productive and have higher sales but also have more employees and pay higher wages than other firms. This pattern in the data is consistent with an imperfectly competitive labor market where each individual firm faces an upward-sloping labor supply curve, implying that wages are an increasing function of firm size and productivity. However, several alternative explanations exist.

One alternative explanation is that workers could be paid differentially because of unobserved skill differences, not imperfect competition (see, e.g., Abowd et al., 1999, Gibbons et al., 2005). To investigate this possibility, we run a set of wage regressions on a sample of workers who switch firms (and have at least four years of tenure at both the origin and destination firms, to ensure that we can accurately measure their wages both before and after the move). This sample is based on the subset of firms for which we have additional information from the worker data (see Section 2.3 for details). The results are presented in Table 7. In the first column, we regress the log wages of workers on a dummy variable for being employed in a firm that directly exports, controlling only for calendar year effects. In the second and third columns, we add controls for observable worker characteristics and sector fixed effects, respectively. In the final column, we use the panel dimension of the data to add controls for worker fixed effects. By including these fixed effects, we control for any time-invariant (observed or unobserved) worker heterogeneity. Since aggregate shocks are absorbed by the time fixed effects, identification is achieved from a common trend assumption in the workers' wages in the absence of moving to firms that directly export.

To empirically assess the common trend assumption in the workers' wages in the absence of moving to direct exporters, we also perform the following movers analysis. We consider a sample of workers who switch their main jobs between $t-1$ and t and have tenures of no fewer than four years at both origin and destination firms. We then use the balanced panel of movers from $t-4$ to $t+3$ and estimate the effects of moving from non-exporters to

Table 7: Wage regressions on the sample of movers

	(1)	(2)	(3)	(4)
Exporter dummy	0.229***	0.131***	0.0639***	0.0258***
	(0.00375)	(0.00307)	(0.00361)	(0.00288)
Number of workers	10,179	10,179	10,179	10,179
Number of firms	7,101	7,101	7,101	7,101
Calendar year FE	Yes	Yes	Yes	Yes
Worker characteristics		Yes	Yes	Yes
Industry FE			Yes	Yes
Worker FE				Yes

Notes: This table uses the subsample of firms for which we have additional information from the worker data (see Section 2.3 for details). For each column, we run a worker-level regression of log FTE wage on the sample of movers between any firms. Movers in year t are defined as workers who are employed by the origin firms at no later than $t-4$, switch their jobs between $t-1$ and t, and stay at their destination firms at least until $t+3$. The sample is balanced from $t-4$ to $t+3$. Observations in years $t-1$ and t are dropped from the regressions, to ensure we only use full-year employment spells in a given firm. Worker characteristics include worker class (blue collar or white collar)—which can vary across employers for the same worker—gender, and age bin-year effects. Industry fixed effects are included at the NACE four-digit level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

exporters by running the following regression:

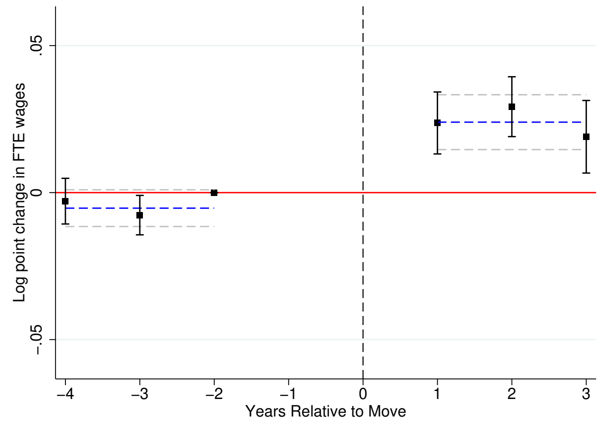
$$\log w_{n,s} = \sum_{\kappa=-4}^{3} \eta_\kappa \mathbf{1}[s=\kappa] + \sum_{k=-4}^{3} \chi_\kappa \mathbf{1}[s=\kappa, T(n)=1] + \zeta_n + \xi_{n,s}, \tag{46}$$

where $\log w_{n,s}$ denotes mover n's log wage in year s (relative to the year of move), $T(n)$ is an indicator for the move from non-exporters to exporters, and ζ_n is a worker fixed effect. In order to ensure that we only use full-year employment spells in a given firm, we drop the observations in years $t-1$ and t. We also pool all movers in the regression and assume that the effects of moving from exporters to non-exporters are symmetric.

Figure 8 presents a graphical representation of the exporter wage premium. In this figure, we report the estimated coefficients χ_κ in equation (46) for κ from -4 to 3 and normalize the estimates by setting $\chi_{-2} = 0$. As in Table 7, we additionally control for calendar year effects, observable time-varying worker characteristics, and sector fixed effects. Our findings support common trends prior to the move, suggesting that the wage growth of workers moving to a firm that does not directly export can be a valid counterfactual for those moving to a firm that directly exports.

Taken together, the results in Table 7 show that controlling for (observed and unobserved) worker characteristics significantly reduces the differences in wages between workers in firms that do and do not directly export, highlighting the benefits of using panel data in our setting. Nevertheless, even after controlling for worker characteristics, workers in firms that directly export still earn about 2.6 percent more than workers in firms that do not directly

Figure 8: Graphical representation of exporter wage premium from movers analysis



Notes: This figure uses the subsample of firms for which we have additional information from the worker data (see Section 2.3 for details). We run a worker-level regression based on equation (46) and report the estimated coefficients $\{\chi_\kappa\}_{\kappa=-4}^{3}$. We define movers in year t as workers who are employed by the origin firms at no later than $t-4$, switch their jobs between $t-1$ and t, and stay at their destination firms at least until $t+3$. The sample of movers is balanced from $t-4$ to $t+3$. Observations in years $t-1$ and t are dropped from the regression to ensure that we only use full-year employment spells in a given firm. The estimates are normalized by setting $\chi_{-2}=0$. The assignment to the exporter or non-exporter category is made based on firms' export participation status at $t-2$ for both origin and destination firms. Industry fixed effects are included at the NACE four-digit level.

export, consistent with imperfect competition in the labor market.

An alternative explanation to imperfect competition for why (even the same) workers are paid differently across firms is that observed wages may not necessarily reflect the full compensation that individuals receive from working in a given firm. Indeed, both survey data (e.g., Hamermesh, 1999, Pierce, 2001, Maestas et al., 2018) and experimental studies (e.g., Mas and Pallais, 2017, Wiswall and Zafar, 2018, Chen et al., 2020) suggest that workers may be willing to sacrifice higher wages for better non-wage job characteristics or amenities when choosing an employer. Thus, the wage premia in firms that directly export could reflect compensating differentials for unfavorable amenities, not imperfect competition. To distinguish between compensating differentials and imperfect competition as sources of wage differentials, we will, in Section 5, exploit changes in employment and wages within firms in response to plausibly exogenous foreign demand shocks.

C.3 Firm-level output prices, wages, and input prices

In Section 2.4, we discuss a set of empirical facts on the relationships in our data between firm-level sales, labor costs, and intermediate input purchases. In this section, we present another set of facts describing the differential pass-through rates of changes in wages and input prices on firm-level output prices. In a simple model where output markups are fixed and input markets are competitive, the pass-through rates of changes in input prices to the output price are informative of the shares of those inputs in total variable costs. We regress firm-level output price changes on changes in average wage and the price of input purchases, and find that changes in output prices are more correlated with the changes in input prices than with the changes in average wage. This fact is suggestive of labor cost having a smaller share than input purchases in firms' variable costs. We also run the same regression but where we weigh the wage change and the input price change with their observed share in total costs in the previous period. Again, we find that the coefficient on average wage change is smaller than that on input price change, consistent with labor being used more intensively in fixed costs than in variable costs.

In this exercise, we use information on unit values provided by Duprez and Magerman (2018), which is based on Prodcom data that contains output values and quantities at the 8-digit level for large firms in the mining and quarrying, and manufacturing sectors.[29] For the set of firms that we have additional information on unit values, we first compute firm-level output price changes by taking the log differences of unit values for each firm. For multiple product firms, we compute the weighted average price change by using the products' output value at the initial period as weights. Firm-level input price changes are computed by aggregating these output price changes of each firm's suppliers. Since Prodcom only covers a small subset of firms in the Belgian economy (less than 1 percent of firms in 2012), for many firms, not all suppliers are in this subsample.[30] Therefore, we compute the input price change by taking the weighted average of the output price changes among Prodcom suppliers. We use these suppliers' initial period sales share in the firm's input purchases from all Prodcom suppliers as weights.[31] For the set of importers, we use the customs records and incorporate import price changes. We do so by taking the changes in the import prices for each firm-product pair, and then aggregate up to the firm-level using the initial period

[29]To be in the Prodcom sample, firms need to meet certain criteria that includes having 20 or more employees or sales above 4 million Euro. For details see Appendix A of Duprez and Magerman (2018). For mapping product codes across multiple years, we build on the year-to-year concordance of product codes provided by Duprez and Magerman (2022).

[30]However, since the sample of analysis is also the sample covered in Prodcom, we find that there is at least one supplier that is in Prodcom for more than 90 percent of the firms in Prodcom.

[31]This implicitly assumes that the input price changes from suppliers that are not in Prodcom are the same as those from suppliers that are in Prodcom. We discuss the implications of this potential measurement error below.

import values as weights.

To explore how changes in wages and input prices translate to changes in output prices, we regress firm-level four-year output price changes on both firm-level four-year wage changes and average input price changes. To account for differences across industries, in some specifications we control for industry-year fixed effects. We report the results in Columns (1) and (2) in Table 8. We first note that across these columns, the values of the coefficients on both inputs are far below one and do not add up to one. It is worth mentioning that one needs to be cautious in directly interpreting these coefficients as shares of these inputs in variable costs for several reasons. First, input price changes are likely subject to measurement error as not all suppliers are in the Prodcom sample. Second, firms may respond to input price changes by changing their markups, which makes our coefficients biased (Amiti et al., 2019). Similarly, changes in firms' productivities or technologies may drive both the changes in input and output prices, generating omitted variable biases. Moreover, nominal rigidity in output prices may dampen the pass-through of input prices on output prices, as in Gagliardone et al. (2023). Nevertheless, the magnitudes of these coefficients are consistent with those from Baqaee et al. (2023).

Having mentioned these limitations, these results may still be useful for inferring suggestive evidence on labor cost shares in variable inputs. For example, if we assume that the biases mentioned above are the same across the two coefficients, the relative magnitudes of the coefficients are informative.[32] The changes in the prices of input purchases are more associated with the changes in output prices, than the changes in average wages. This pattern is suggestive of labor cost having a smaller share in firms' variable costs than inputs.

In Column (3), we weigh the average wage change and the input price change with their observed share in total costs in the previous period. To aid the interpretation, suppose that the biases affect both coefficients equally. Then, these coefficients reject the null hypothesis that fixed costs and variable costs have the same labor and input composition. If the composition of labor and inputs in fixed costs is the same as that in variable costs, the two coefficients should equal each other. Furthermore, as with the previous specification, the relative magnitude of the coefficients here is also informative about the labor cost share in firms' fixed costs. When the labor cost share in fixed costs is higher than the labor cost share in variable costs, then the true labor share in variable costs is lower than the observed labor share in total costs. Then, one would expect a lower coefficient for wage change than for input price changes. Therefore, these results are consistent with labor being used more intensively in fixed costs than in variable costs.

Taken together, the results in Table 8 suggest the importance of labor cost in firms'

[32] If the biases affect the estimates multiplicatively, the ratio of the estimated coefficients is informative about the ratio of the true coefficients, and if the biases enter the estimates additively, the difference between the estimated coefficients is informative about the difference in the true coefficients.

fixed costs. However, it is difficult to draw definitive conclusions under the biases that are discussed above. Therefore, as argued in Section 2.4, to fully understand the cost structure of firms we make use of an explicit model together with an exogenous firm-level shifter in demand.

Table 8: Relationship between firm-level output prices, wages, and input prices

	(1)	(2)	(3)
	Four-year output price change		
Four-year difference			
Wage change	0.0189	0.0171	0.0572
	(0.0124)	(0.0149)	(0.0512)
Average input price change	0.159***	0.112***	0.174***
	(0.0240)	(0.0192)	(0.0267)
Ind.Year FE		Yes	Yes
Weighted by $t-1$ cost shares			Yes

Notes: This table uses the subsample of 35,772 firm-year observations in Belgium from 2002 to 2014 for which we have additional information on price from Prodcom data. For each column, we run a firm-level regression of the output price change on the wage change and the average input price change. Firm-level output price changes are computed by taking the weighted average of the log differences of unit values for each firm. Log differences of unit values are trimmed at top and bottom 1 percent. We then compute the input price changes by taking the weighted average of the output price changes among Prodcom suppliers. For the set of importers, we additionally incorporate import price changes using the initial period import values as weights. Industry fixed effects are included at the NACE four-digit level. In Columns (3), wage changes and average input price changes are weighted by the cost shares of labor costs and input purchases at $t-1$, respectively. Standard errors in parentheses are clustered at the NACE four-digit level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

C.4 Export growth at the firm-country-product level

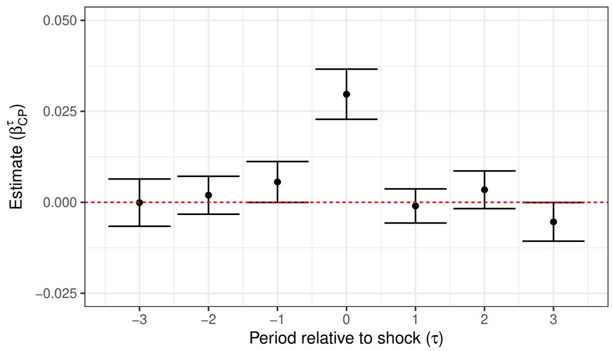
In this section, we examine how changes in $\Delta_{t-1}^{t} \log WID_{c,p}$ changes the exports of product p by firm k to country c. In doing so, we run a regression analogous to the one in equation (21) at the firm-country-product level:

$$\Delta_{t+\tau-1}^{t+\tau} \log X_{kF,c,p} = \ddot{\alpha}^{\tau} + \beta_{CP}^{\tau} \Delta_{t-1}^{t} \log WID_{c,p} + \ddot{\varphi}_{k,c,p,t}^{\tau}, \tag{47}$$

where $\Delta_{t+\tau-1}^{t+\tau} \log X_{kF,c,p}$ denotes the log change in firm k's export of product p to country c from year $t+\tau-1$ to year $t+\tau$. The estimates $\{\beta_{CP}^{\tau}\}_{\tau=-3}^{3}$ reported in Figure 9 suggest that idiosyncratic changes in the world import demand give rise to persistent and unanticipated increases in export demand.[33]

[33]We note that the sample of firms used in Figure 9 only includes direct exporters. On the other hand, the analyses in Section 4 also capture indirect trade.

Figure 9: Export growth at the firm-country-product level



Notes: This figure uses 4,443,426 firm-country-product-year observations in Belgium based on the main estimation sample from 2002 to 2014 (see Section 2.3 for details). We run seven firm-country-product-level regressions based on equation (47) for τ from -3 to 3 with exports as the outcome variable. This figure shows the point estimates as well as 95 percent confidence intervals. Variables are winsorized at the top and bottom 0.5 percentiles. Standard errors are clustered at the NACE four-digit level. All specifications include country-year fixed effects, product-year fixed effects, industry-year fixed effects, and firm fixed effects

C.5 Specification checks

In this section, we consider several alternative specifications to our main results presented in Table 1. For each alternative specification, we also report the cumulative increase in sales ($\sum_{\tau=0}^{3} \beta^{\tau}$ in equation (21)) relative to its instantaneous response to the foreign demand shock (β^0). In Table 9, we report the results in which we weight each firm by its lagged employment. Table 10 shows the sensitivity of our results to additionally controlling for location-year fixed effects. We use level 2 of the Eurostat NUTS classification as a measure of location. In these specifications, our IV estimates relative to the cumulative increase in sales are not substantially affected.

C.6 Wages and work rate of stayers

In our main IV estimates reported in Table 1 of Section 5.1, we consider all the workers in our sample. However, we could also look at the incumbent workers who stay in the same firm. An advantage of doing so is that it keeps the composition of the workforce fixed, and thus, we are not confounding increases in the wages paid to a given worker with changes in the quality of the workers. In the first column of Table 11, we report the IV estimate on wages for the sample of stayers (who stay in the same firm before and after the demand shock, from $\tau = -1$ to $\tau = 3$). This sample is based on the subset of firms for which we have additional information from the worker data (see Section 2.3 for details).[34] We find that their wages increase by 1.3 percent in response to a 10 percent increase in sales, which is similar to the impact for all workers.

[34] We note that part of the wage growth of stayer workers may be from promotions to higher ranked occupations. Occupation information is not available in the social security data.

Table 9: Worker impacts and firm responses to foreign-demand induced change in sales: weighted by employment at $t-1$

	(1) Sales	(2) Average wage	(3) FTE employment	(4) Labor cost	(5) Input purchases	(6) Domestic input purchases
Instantaneous response	1.000***	0.108**	0.0869	0.195***	1.081***	0.764***
($\gamma^{-1,0}$)		(0.0449)	(0.0582)	(0.0715)	(0.144)	(0.115)
Cumulative response	0.494***	0.0604	0.208*	0.268**	0.511***	0.383***
($\sum_{\tau=0}^{3} \gamma^{\tau-1,\tau}$, unbalanced)	(0.164)	(0.0788)	(0.119)	(0.125)	(0.117)	(0.121)
Cumulative response	0.513***	0.0650	0.147*	0.212**	0.388**	0.181
($\sum_{\tau=0}^{3} \gamma^{\tau-1,\tau}$, balanced)	(0.165)	(0.0554)	(0.0811)	(0.104)	(0.197)	(0.173)
Industry-Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: This table uses the main estimation sample of private-sector firms in Belgium from 2002 to 2014 (see Section 2.3 for details). For each outcome variable, we estimate its elasticity with respect to sales. We run four firm-level 2SLS regressions based on equation (22) for $\tau \in \{0, 1, 2, 3\}$ and $\tau' = \tau - 1$ and report the instantaneous response ($\gamma^{-1,0}$) as well as the cumulative response (the sum of four coefficients $\{\gamma^{\tau-1,\tau}\}_{\tau=0}^{3}$). In the third row, we use a balanced panel of firms that are observed for four consecutive years from $t-1$ to $t+3$. The first-stage F-statistics for excluded instruments is 142.3. Variables are winsorized at the top and bottom 0.5 percentiles. In all regressions, we weight each firm by its employment at $t-1$. Standard errors in parentheses are clustered at the NACE four-digit level, and standard errors of the cumulative responses are computed using the bootstrap method. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 10: Worker impacts and firm responses to foreign-demand induced change in sales: including location-year fixed effects

	(1) Sales	(2) Average wage	(3) FTE employment	(4) Labor cost	(5) Input purchases	(6) Domestic input purchases
Instantaneous response	1.000***	0.0881***	0.0667**	0.155***	0.942***	0.760***
($\gamma^{-1,0}$)		(0.0298)	(0.0321)	(0.0369)	(0.0765)	(0.0670)
Cumulative response	0.757***	0.109**	0.323***	0.432***	0.779***	0.597***
($\sum_{\tau=0}^{3} \gamma^{\tau-1,\tau}$, unbalanced)	(0.0771)	(0.0472)	(0.0592)	(0.0705)	(0.120)	(0.104)
Cumulative response	0.663***	0.0842**	0.283***	0.367***	0.700***	0.532***
($\sum_{\tau=0}^{3} \gamma^{\tau-1,\tau}$, balanced)	(0.0773)	(0.0413)	(0.0520)	(0.0603)	(0.117)	(0.101)
Industry-Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Location-Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: This table uses the main estimation sample of private-sector firms in Belgium from 2002 to 2014 (see Section 2.3 for details). For each outcome variable, we estimate its elasticity with respect to sales. We run four firm-level 2SLS regressions based on equation (22) for $\tau \in \{0, 1, 2, 3\}$ and $\tau' = \tau - 1$ and report the instantaneous response ($\gamma^{-1,0}$) as well as the cumulative response (the sum of four coefficients $\{\gamma^{\tau-1,\tau}\}_{\tau=0}^{3}$). In the third row, we use a balanced panel of firms that are observed for four consecutive years from $t-1$ to $t+3$. The first-stage F-statistics for excluded instruments is 138.8. Variables are winsorized at the top and bottom 0.5 percentiles. Standard errors in parentheses are clustered at the NACE four-digit level, and standard errors of the cumulative responses are computed using the bootstrap method. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Another advantage of the stayers sample is that it allows us to examine if the increase in our measure of FTE employment reflects the hiring of new workers or an increase in the working hours of incumbent workers. To do so, we report, in Table 11, the IV estimates on hours of work (as measured as the share of full-time employment) and on hourly wages for the sample of stayers. The estimated impact on hours of work is close to zero, whereas the effect on hourly wages is close to what we find for all workers. Taken together, these findings suggest that Belgian firms mostly adjust to demand shocks by hiring additional workers, not by increasing incumbent workers' hours of work.

Table 11: IV estimates on the wages and work rate of stayers

	(1) FTE employment	(2) Stayer wage	(3) Stayer hourly wage	(4) Stayer work rate
Panel A: IV estimates				
Cumulative response	0.296***	0.126***	0.127***	0.0005
($\sum_{\tau=0}^{3} \gamma^{\tau-1,\tau}$, balanced)	(0.0675)	(0.0438)	(0.0376)	(0.0166)
Panel B: Reduced form				
Reduced form	0.108***	0.0458***	0.0462***	0.0002
	(0.0264)	(0.0152)	(0.0140)	(0.0061)
Number of observations	75,849	75,849	75,849	75,849
Industry-Year FE	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes

Notes: This table uses the subsample of firms for which we have additional information from the worker data (see Section 2.3 for details). For each outcome variable, we estimate its elasticity with respect to sales. We run four firm-level 2SLS regressions based on equation (22) for $\tau \in \{0, 1, 2, 3\}$ and $\tau' = \tau - 1$ and report the cumulative response (the sum of four coefficients $\{\gamma^{\tau-1,\tau}\}_{\tau=0}^{3}$). The first-stage F-statistics for excluded instruments is 57.5. Variables are winsorized at the top and bottom 0.5 percentiles. Standard errors in parentheses are clustered at the NACE four-digit level and computed using the bootstrap method. We compute firm-level average stayer wage, stayer hourly wage, and stayer work rate based on the balanced panel of stayers from $t-1$ to $t+3$. The analysis is based on 452,025 worker-year observations of stayers, which yield 75,849 firm-year observations of private-sector firms in Belgium from 2003 to 2014. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

C.7 Testing for imperfect competition in the labor market

In Table 2, we present the IV estimates of the labor supply elasticity (ε) and the wage markdown ($1/(1+\varepsilon)$). In this section, we provide a formal testing procedure for the imperfect competition in the labor market. Perfect competition means that the firm can hire more workers in response to a foreign demand shock without bidding up wages. In the notation of equations (23) and (24), perfect competition corresponds to the null hypothesis that the instrument impacts employment but has no effect on wages ($\beta_L \neq 0$ and $\beta_W = 0$). The alternative hypothesis of imperfect competition is that the instrument impacts both employment and wages (both $\beta_L \neq 0$ and $\beta_W \neq 0$).

This is a non-standard testing problem because both the null and the alternative hypotheses includes the condition that foreign demand shock affects employment, $\beta_L \neq 0$. If employment was not affected, both perfect and imperfect competition would predict no impact on wages. To test the hypotheses, we therefore use the closure method of Marcus et al. (1976). It reduces the problem of constructing multiple test procedures which control the family wise error rates to the construction of single tests which control the usual probability of a Type 1 error (see Romano et al., 2011). Applied to our problem, the closure method considers three hypotheses:

$H_{1,2}$: The instrument has no effect on both wages and employment, i.e. $\beta_L = \beta_W = 0$
H_1: The instrument has no effect on wages, i.e. $\beta_W = 0$
H_2: The instrument has no effect on employment, i.e. $\beta_L = 0$

If we reject these three hypotheses, we can reject perfect competition and conclude there is imperfect competition, since the instrument impacts both employment and wages.

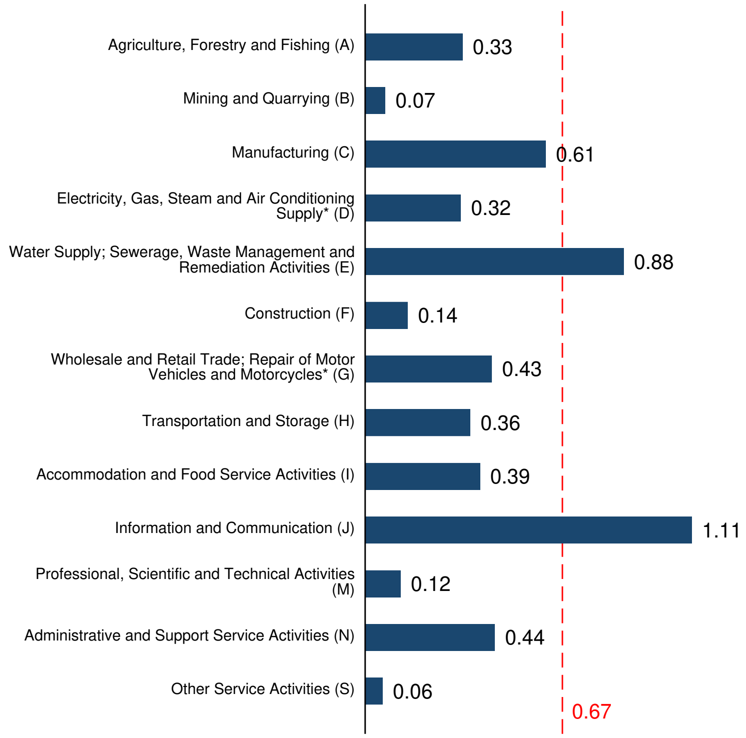
The closure method is straightforward to implement using standard statistical tests: H_1 and H_2 can be tested by considering the usual t-statistic on the instrument in the first-stage in equation (24) and the reduced form in equation (23), whereas $H_{1,2}$ can be tested by computing the F-statistic for the joint hypothesis that the instrument has no effect on wages and employment in these equations. The closure method tells us that both $H_{1,2}$, H_1, and H_2 can be rejected if the p-value for each test is less than or equal to a predefined level of significance α. When applied to our data, this means that we can reject the three hypotheses $H_{1,2}$, H_1, and H_2 at the same level of significance that we can reject H_1, since it is the test that gives us the highest p-value. We present the results in Panel D of Table 2

C.8 Elasticities of input purchases by suppliers' industries

In this section, we allow the elasticity of input purchases, and, thus, the fraction of an input that is used as a fixed factor, to vary across the types of inputs. In order to estimate the cumulative elasticities of input purchases by different types of inputs, we first categorize the purchases of inputs by the industry of supplier for domestic purchases and by the HS product code for import transactions. We then use an HS to NACE concordance to map the product-level import transactions to the industry level, so that we classify both domestic and foreign input purchases by supplying industries.

Figure 10 shows the cumulative elasticities of (domestic and foreign) input purchases at the NACE one-digit level. We report those elasticities relative to the cumulative increase in total sales of 0.67, as referenced by the dotted red line. For instance, we find that purchases from the manufacturing industry, which account for around half of all input purchases in the Belgian economy, increase by 6.7 percent when firms receive foreign demand shocks to increase their sales by 6.1 percent. On the other hand, input purchases from most of the service industry (NACE G to N one-digit sectors) do not increase as much, implying that service inputs have higher fixed input cost shares.

Figure 10: Elasticities of input purchases by suppliers' NACE one-digit industries



Notes: This figure uses the main estimation sample of private-sector firms in Belgium from 2002 to 2014 (see Section 2.3 for details). For each bar, we report the cumulative elasticities of input purchases from suppliers' respective industries. To compute the cumulative elasticities, we run four firm-level 2SLS regressions based on equation (22) for $\tau \in \{0, 1, 2, 3\}$ and $\tau' = \tau - 1$ and compute the sum of four coefficients $\{\gamma^{\tau-1,\tau}\}_{\tau=0}^{3}$. Variables are winsorized at the top and bottom 0.5 percentiles. The dotted red line corresponds to the cumulative response of sales. We report the cumulative elasticities at NACE one-digit sections. We exclude the public and financial sectors from our sample, and we drop NACE L (Real Estate Activities) because of the small sample size. (*) We include the input purchases from NACE 46.71 (Wholesale of solid, liquid and gaseous fuels and related products) and NACE 47.3 (Retail sale of automotive fuel in specialised stores) in NACE D (Electricity, Gas, Steam and Air Conditioning Supply) instead of NACE G (Wholesale and Retail Trade; Repair of Motor Vehicles and Motorcycles).

C.9 Fixed labor input shares by firm categories

In Section 5.1, we assume that the fixed share of labor inputs is homogeneous across all firms in the Belgian economy. In this section, we allow the fixed shares of labor inputs to vary across firm categories. Table 12 reports our estimates when we distinguish between firms that trade internationally and those that do not. In doing so, we first estimate the cumulative elasticities of labor cost for each firm category by interacting our IV model in equation (22) with firm categories. We then use equation (15) to solve for the fixed share of labor inputs $(1 - \ell_k^v/\ell_k)$. Our IV estimates suggest that firms that trade internationally and those that do not have comparable fixed share of costs.

In the third column, we also report the weighted averages of fixed labor input shares, weighted by the shares of aggregate sales by firm categories. We find that these weighted averages are not substantially different from our main estimate in the first row, in which the fixed shares of labor inputs are assumed to be homogeneous across all firms.

Table 12: Fixed shares of labor inputs by firm categories

	Labor cost elasticity	Fixed share of labor inputs	
		by category	weighted average
All firms	0.528	0.529	
Exporters	0.552	0.505	0.516
Non-exporters	0.503	0.554	
Importers	0.493	0.564	0.549
Non-importers	0.588	0.469	
Exporters or importers	0.535	0.522	0.523
Domestic firms	0.532	0.525	

Notes: This table uses the main estimation sample of 995,739 firm-year observations in Belgium from 2002 to 2014 (see Section 2.3 for details). For each row, we report the cumulative elasticities of labor cost and fixed share of labor inputs $(1 - \ell_k^v/\ell_k)$ by firm category. To obtain the elasticities of labor cost, we estimate the cumulative responses $(\sum_{\tau=0}^{3} \gamma^{\tau-1,\tau})$ of labor cost for each firm category by interacting our IV model in equation (22) with firm categories. We then use equation (15) to solve for the fixed share of labor inputs $(1 - \ell_k^v/\ell_k)$. The reported elasticities of labor cost are relative to the cumulative responses of sales (see Section 5.1 for details). We use the shares of aggregate sales by firm categories as weights when computing the weighted average of fixed labor input shares.

D Additional counterfactual results

D.1 Total import shares

To gain intuition on how accounting for firms' fixed inputs affects firms' and aggregate responses to foreign demand shocks, we focus on the firm-level measure of total import share, defined in Dhyne et al. (2021). Because we impose the trade balance condition, the uniform foreign demand shock that we consider in the exercises can also be seen as a shock where the prices of imports uniformly increase.[35] A firm's total import share, which measures how much of the firm's variable inputs originate directly or indirectly from abroad, is a useful statistic that captures the degree of the firm's exposure to the foreign shock.

Firm k's total import share, $s_{Fk}^{v,Total}$, is defined in a recursive manner as follows:

$$s_{Fk}^{v,Total} = s_{Fk}^{v} + \sum_{j \in Z_k} s_{jk}^{v} s_{Fj}^{v,Total}, \tag{48}$$

where s_{Fk}^{v} and s_{jk}^{v} are the shares of foreign imports and inputs from firm j in the firm's variable costs. Dhyne et al. (2021) show that firms' total import shares become relevant statistics in predicting firm-level outcomes at a first-order approximation: when the labor market is competitive, the costs of firms with higher total import shares increase more than those of firms with lower total import shares in response to a uniform increase in the price of imports. Moreover, Dhyne et al. (2023a) show that the share of import content in final consumption, which is a weighted average of firm-level total import shares, $s_F^{Total} = \sum_k s_{kH} s_{Fk}^{v,Total}$, is a relevant statistics for calculating the welfare effects of trade.

Through the measure of firms' total import shares, one can see the two main effects of fixed inputs. On the one hand, if for example a large fraction of labor costs is a fixed input, the variable cost shares of s_{Fk}^{v} and s_{jk}^{v} become larger. This will magnify any direct cost shock from an import price change and indirect cost shocks from domestic suppliers. On the other hand, some of the foreign inputs are fixed as well, which, all else equal, lowers the direct cost shock through lower values of s_{Fk}^{v}. Quantitatively, however, more than 70 percent of imports are calculated as variable inputs (based on the estimated elasticities for the NACE one-digit level classification), and since around 50 percent of labor costs are fixed, the direct foreign input share tends to be larger under fixed inputs as well.

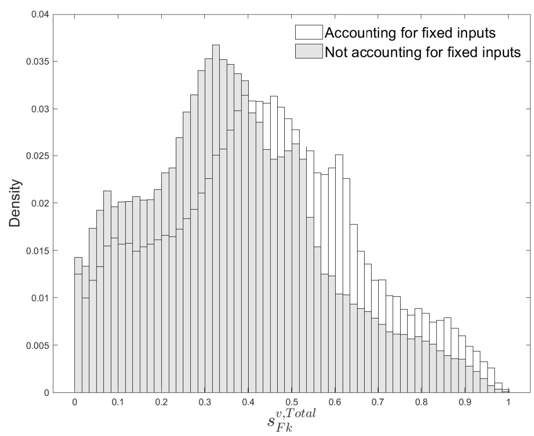
Panel (a) of Figure 11 plots the distributions of the total import shares, $s_{Fk}^{v,Total}$, one accounting for and another not accounting for fixed inputs. When one accounts for fixed inputs, the total import shares of firms in variable costs are larger (with the median firm

[35] This symmetry is called Lerner's symmetry. It implies that the outcomes from this uniform change in foreign demand can be mapped into an equivalent set of outcomes from a uniform change in import prices. In this case, the 5 percent increase in foreign tariffs on Belgian exports is equivalent to a 5 percent uniform increase in the price of Belgian imports.

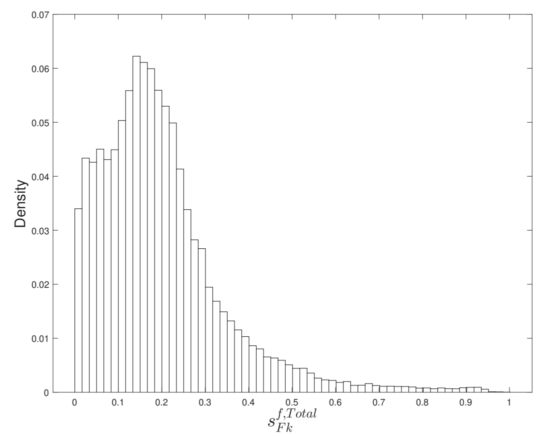
having a share of 43 percent) than the total import shares of firms when not accounting for fixed inputs (with the median firm having a share of 35 percent). This difference also aggregates up when computing the share of import content in final consumption: when accounting for fixed inputs the import content share is 61 percent while it is 55 percent when not accounting for fixed inputs. Relatedly, we compute and plot the share of how much of a firm's fixed inputs originate directly or indirectly from abroad in panel (b) of the figure. We find that these shares are generally much lower than the total import share of variable inputs: 17 percent of the median firm's fixed inputs originates from abroad.

Figure 11: Total import shares

(a) Total import share in variable input costs



(b) Total import share in fixed input costs



Notes: The left panel shows the distribution of the firm-level total import shares in variable input costs, $s_{Fk}^{v,Total}$, defined in equation (48). The white bars show the distribution of the shares when one accounts for fixed inputs, and the grey bars show the distribution of the shares when one does not account for fixed inputs. The right panel shows the distribution of the firm-level total import shares in fixed input costs, $s_{Fk}^{f,Total}$. Firms' total import shares in fixed input costs are defined recursively as in $s_{Fk}^{f,Total} = s_{Fk}^{f} + \sum_{j \in Z_k} s_{jk}^{f} s_{Fj}^{f,Total}$.

D.2 Incorporating nested logit structure in worker preference

In this appendix we consider an extension of the model where Belgium is no longer a single labor market. In our baseline model presented in the main text, Belgium was set to be a single labor market in which each firm is faced with a homogeneous iso-elastic labor supply curve, resulting in a constant markdown of wages. We consider a richer alternative, where Belgium is divided in multiple labor markets and workers have a nested structure in their preferences as in Lamadon et al. (2022). With this richer setup, we explore the sensitivity of how aggregate variables, such as the average real wage and real income, respond to foreign demand shocks.

D.2.1 Setup

We maintain the same preference structure for worker n as in equation (4). However, instead of assuming that ν_{nk} is distributed according to a Type-1 Extreme Value distribution with parameter ε, we assume a nested structure of

$$F\left(\overrightarrow{\nu}_n\right) = \exp\left[-\sum_m \left(\sum_{k\in J_m} \exp\left(-\beta\frac{\nu_{nk}}{\rho_m}\right)\right)^{\rho_m}\right].$$

As with ε, the parameter β governs the dispersion of taste shocks (smaller parameter value corresponds to larger dispersion). The parameter $\rho_m \in [0,1]$ governs the degree of independence in the taste shocks across firms within the labor market m (smaller ρ_m corresponds to more correlate taste shocks for firms in labor market m). In the special case where there are no correlation $\rho_m = 1$ and $\beta = \varepsilon$, then we converge back to the baseline case.

From the nested structure in workers' preference, we define a wage index for each labor market,

$$I_m = \left(\sum_{k\in J_m} \delta_k^{\frac{\beta}{\rho_m}} w_k^{\frac{\beta}{\rho_m}}\right)^{\frac{\rho_m}{\beta}}.$$

Each firm views itself as infinitesimal in the labor market and takes the wage indices I_m as given when choosing wages. The labor supply curve that each firm k faces hence becomes

$$\ell_k = L \times \underbrace{\frac{I_m^\beta}{\sum_{m'} I_{m'}^\beta}}_{\text{emp share of } m} \times \underbrace{\frac{\delta_k^{\frac{\beta}{\rho_m}} w_k^{\frac{\beta}{\rho_m}}}{I_m^{\frac{\beta}{\rho_m}}}}_{\text{firm } k\text{'s emp share in } m}. \tag{49}$$

Given this structure, the counterfactual exercise illustrated in Appendix B.5.2 can be operationalized through the following steps. First, define firm-level labor supply elasticity as $\varepsilon_k = \frac{\beta}{\rho_{m(k)}}$, and replace ε with ε_k. Second, the first equation in Step 2 will be replaced with

$$\hat{\ell}_k = L\frac{\left(\frac{\sum_{j\in J_{m(k)}}\ell_j}{\sum_{j\in J_{m(k)}}\ell_j\hat{w}_j^{\varepsilon_j}}\right)^{1-\rho_{m(k)}}}{\sum_{m'}\left(\frac{\sum_{j\in J_{m'}}\ell_j}{\sum_{j\in J_{m'}}\ell_j\hat{w}_j^{\varepsilon_j}}\right)^{1-\rho_{m'}}\left(\sum_{j\in J_{m'}}\ell_j\hat{w}_j^{\varepsilon_j}\right)}\hat{w}_k^{\varepsilon_k}.$$

D.2.2 Estimation

In estimating β and ρ_m, we follow Lamadon et al. (2022) and consider idiosyncratic firm-level shocks and market-level shocks. From the idiosyncratic firm-level foreign demand shocks that exogenously shift firms' sales, we obtain the following relationship that is derived from the new labor supply curve of equation (49).

$$\frac{d\log w_k\ell_k}{d\log p_kq_k}\Big/\frac{d\log \ell_k}{d\log p_kq_k}=\frac{1+\frac{\beta}{\rho_{m(k)}}}{\frac{\beta}{\rho_{m(k)}}}.$$

The above equation is analogous to equation (17), and it allows us to estimate $\frac{\beta}{\rho_m}$ by running the second stage regressions (22) for each market, where firm-level labor costs and employment are the outcome variables. The estimated labor supply elasticity, $\varepsilon_k=\frac{\beta}{\rho_{m(k)}}$, can now vary at the labor market-level. We use these estimated values to back out the variable cost shares in labor costs using equation (15), which will now also potentially vary at the market-level.

From the same labor supply curve of equation (49), we obtain the following relationship between labor market-level employment changes and the weighted average wage changes in each market (with weights being the firm's employment share in the market).

$$d\log\left(\sum_{k\in J_m}\ell_k\right)=d\log\left(\frac{L}{\sum_{m'}I_{m'}^{\beta}}\right)+\beta\left(\sum_{k\in J_m}\underbrace{\frac{\delta_k^{\frac{\beta}{\rho_m}}w_k^{\frac{\beta}{\rho_m}}}{I_m^{\frac{\beta}{\rho_m}}}}_{\text{firm } k\text{'s emp share in } m}d\log w_k\right).\tag{50}$$

We operationalize these estimating equations by defining labor markets as the interaction of NACE three-digit industries (230 industries) and NUTS level 3 regions (43 regions). We consider two alternative assumptions on how ρ_m vary across labor markets. First, we assume that ρ_m are common across all labor markets. This implies $\varepsilon=\frac{\beta}{\rho_m}\ \forall m$ and β is identified off equation (50). For each labor market, we aggregate employment changes, wage changes, and also aggregate the export demand shocks (using the same employment shares as weights), and obtain the estimate for β. We include in the regression year fixed effects and control for the weighted aggregate of firms' total household revenue shares. As reported in Table 13, we find that the estimate for β is 3.2, and because $\varepsilon=3.9$, the common parameter of ρ is calculated to be 0.82.

The second assumption we consider is that ρ_m are heterogeneous but common at a higher level of labor market aggregation. In particular, we assume that ρ_m vary across six broad markets, which are defined as the interaction of NUTS level 1 regions (Brussels, Flanders, and Wallonia) and manufacturing versus non-manufacturing sectors. We estimate the labor supply elasticity $\frac{\beta}{\rho_m}$ for each broad market, and using $\beta=3.2$ we back out the estimates for

Table 13: Market-level IV estimates of labor supply elasticity

	(1) Four-year difference of log employment
Panel A: IV estimates	
Labor supply elasticity (β)	3.171**
	(1.521)
Panel B: First stage	
First stage	0.0652**
	(0.0254)
Panel C: Reduced form	
Reduced form	0.207***
	(0.0503)
First-stage F-statistics	6.62
Implied ρ	0.82
Year FE	Yes

Notes: This table reports the second stage, first stage, and reduced form results of specification (50), where we regress employment changes on weighted average wage changes instrumented by the export demand shocks, all aggregated at the labor market level. For wage changes and export demand shocks, we use firms' employment shares as weights. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

ρ_m. The estimated values for ρ_m range from 0.64 to 1.[36]

D.2.3 Counterfactual results

Using the two alternative sets of estimates, we conduct counterfactual exercises that are analogous to those in Section 6. In particular, we consider a five percent increase in foreign tariffs on all Belgian exports and compute how the average real wage and real income respond to this shock.

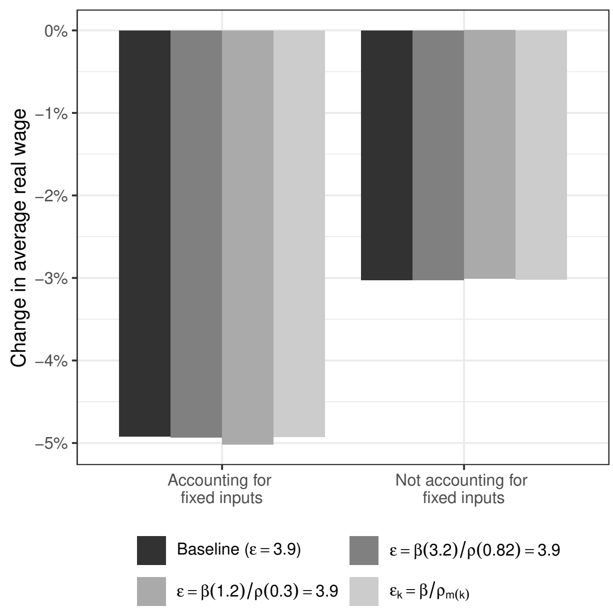
Figures 12 and 13 report the counterfactual results for the average real wage and real income, respectively. The leftmost black bars in the table reports the results in our baseline case where Belgium is treated as one labor market, and are identical to the black bars in Figures 6 and 14. The second bars from the left allow for multiple labor markets but assume a common ρ, with the estimated values of $\beta = 3.2$ and $\rho = 0.82$. Because the estimated value of ρ is close to 1 in which case the model collapses to the baseline model, the quantitative impact of allow for multiple labor markets are small. The third bars from the left still assume a common value of ρ, but use alternative values of β and ρ to explore sensitivity of the results. We consider a lower value of ρ with $\rho = 0.3$, and maintain that firms face a labor supply curve with elasticity of $\varepsilon = \frac{\beta}{\rho} = 3.9$, hence $\beta = 1.2$. We find that higher correlation of taste shocks within labor markets magnifies the responses of real wage, but

[36]Estimation results available upon request. When the point estimate for $\frac{\beta}{\rho_m}$ is smaller than 3.2, we set ρ_m to be 1 as ρ_m is bounded from above by 1. This correcting procedure was done for the two estimates involving Wallonia.

the magnitudes of the differences are quantitatively small. The rightmost bars allow for heterogeneous ρ_m that are estimated for each broad labor market. Again, we see a small impact of labor market heterogeneity as the estimated values of ρ_m are generally close to 1.

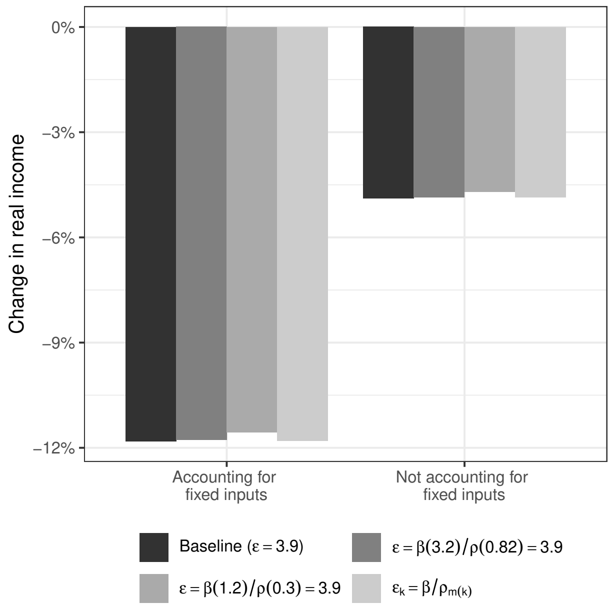
Taken together, the figures imply that allowing for a more general structure in worker preference by incorporating multiple labor markets and potentially heterogeneous markdowns on wages do not quantitatively alter the predictions of aggregate responses to foreign demand shocks.

Figure 12: Changes in average real wage in response to a 5 percent increase in foreign tariffs (alternative worker preference)



Notes: In this figure, we report the changes in average real wage, $\sum_k \frac{w_k \ell_k}{\sum_j w_j \ell_j} \hat{w}_k \hat{\ell}_k / \hat{P}$, due to a 5 percent increase in foreign tariffs on Belgian exports. Each bar represents the response under different structures of worker preference. The baseline results in the leftmost bars are identical to the black bars in Figure 6. The middle two bars use ρ that is common across labor markets, with one using the estimated values of $\beta = 3.2$ and $\rho = 0.82$, and another using $\beta = 1.2$ and $\rho = 0.3$. The rightmost bars under $\varepsilon = \frac{\beta}{\rho_m}$ assume a preference structure where ρ_m varies across six broad labor markets.

Figure 13: Changes in real income in response to a 5 percent increase in foreign tariffs (alternative worker preference)

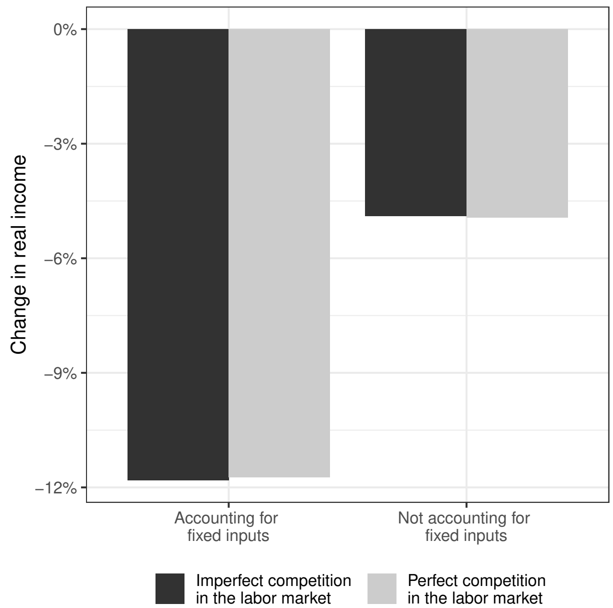


Notes: In this figure, we report the changes in real income, $\hat{E}/\hat{P}$, due to a 5 percent increase in foreign tariffs on Belgian exports. Each bar represents the response under different structures of worker preference. The baseline results in the leftmost bars are identical to the black bars in Figure 6. The middle two bars use ρ that is common across labor markets, with one using the estimated values of $\beta = 3.2$ and $\rho = 0.82$, and another using $\beta = 1.2$ and $\rho = 0.3$. The rightmost bars under $\varepsilon = \frac{\beta}{\rho_m}$ assume a preference structure where ρ_m varies across six broad labor markets.

D.3 Changes in real income

Figure 14 reports the changes in real income, $\hat{E}/\hat{P}$, in response to a 5 percent increase in foreign tariffs.

Figure 14: Changes in real income in response to a 5 percent increase in foreign tariffs

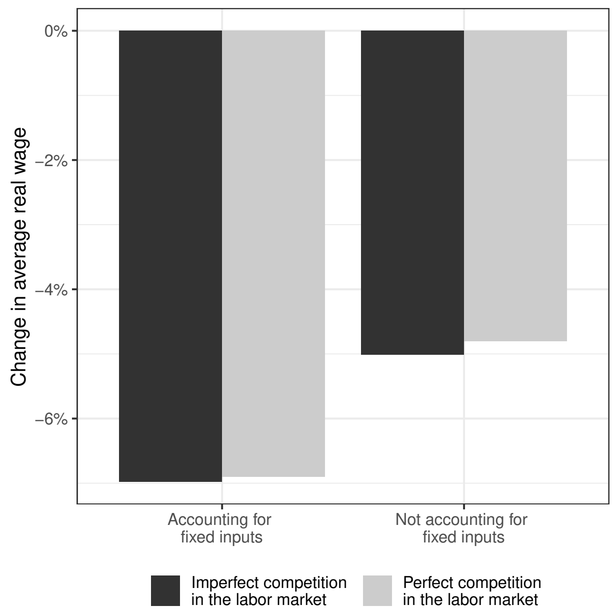


Notes: In this figure, we report the changes in real income, $\hat{E}/\hat{P}$, due to a uniform 5 percent increase in foreign tariffs on Belgian exports. Each bar represents the response under different parameterizations of the model presented in Section 3. We use our estimated labor supply elasticity $\varepsilon = 3.9$ in the counterfactual Belgian economies with upward-sloping labor supply curves. When accounting for fixed inputs, we use the fraction of fixed inputs for both labor and intermediate inputs (at NACE one-digit level) that we obtained in Section 5.

D.4 Domestic productivity shocks

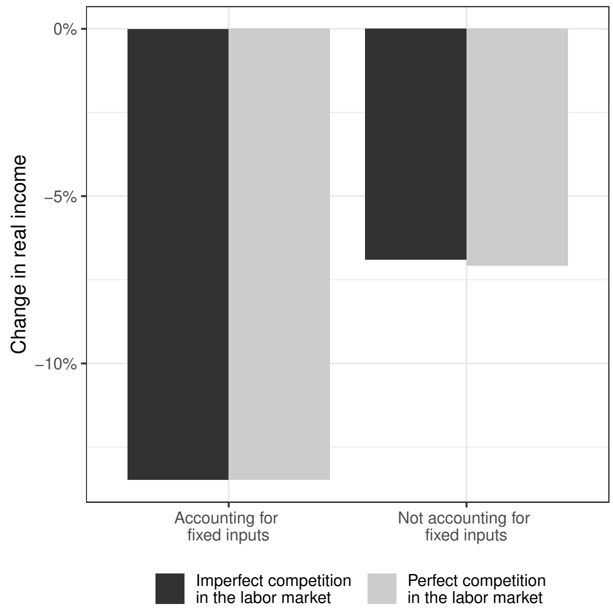
Figures 15 and 16 report the changes in average real wage and real income in response to a 5 percent reduction in productivity ϕ_k for all manufacturing firms. We outline the steps to solve for the counterfactual outcomes in Appendix B.5.3.

Figure 15: Changes in average real wage in response to a 5 percent reduction in manufacturing firms' productivity



Notes: In this figure, we report the changes in average real wage, $\sum_k \frac{w_k \ell_k}{\sum_j w_j \ell_j} \hat{w}_k \hat{\ell}_k / \hat{P}$, due to a 5 percent reduction in manufacturing firms' productivity. Each bar represents the response under different parameterizations of the model presented in Section 3. We use our estimated labor supply elasticity $\varepsilon = 3.9$ in the counterfactual Belgian economies with upward-sloping labor supply curves. When accounting for fixed inputs, we use the fraction of fixed inputs for both labor and intermediate inputs (at NACE one-digit level) that we obtained in Section 5.

Figure 16: Changes in average real income in response to a 5 percent reduction in manufacturing firms' productivity



Notes: In this figure, we report the changes in real income, $\hat{E}/\hat{P}$, due to a 5 percent reduction in manufacturing firms' productivity. Each bar represents the response under different parameterization of the model presented in Section 3. We use our estimated labor supply elasticity $\varepsilon = 3.9$ in the counterfactual Belgian economies with upward-sloping labor supply curves. When accounting for fixed inputs, we use the fraction of fixed inputs for both labor and intermediate inputs (at NACE one-digit level) that we obtained in Section 5.

References

Abowd, John M., Francis Kramarz, and David N. Margolis, "High Wage Workers and High Wage Firms," *Econometrica*, 3 1999, *67*, 251–333.

Acemoglu, Daron and Alireza Tahbaz-Salehi, "Firms, Failures, and Fluctuations: The Macroeconomics of Supply Chain Disruptions," Working Paper 27565, National Bureau of Economic Research July 2020.

Adao, Rodrigo, Michal Kolesár, and Eduardo Morales, "Shift-share designs: Theory and inference," *The Quarterly Journal of Economics*, 2019, *134* (4), 1949–2010.

_ , Paul Carrillo, Arnaud Costinot, Dave Donaldson, and Dina Pomeranz, "Imports, exports, and earnings inequality: Measures of exposure and estimates of incidence," *The Quarterly Journal of Economics*, 2022, *137* (3), 1553–1614.

Alfaro-Urena, Alonso, Isabela Manelici, and Jose P Vasquez, "The Effects of Multinationals on Workers: Evidence from Costa Rica," Working Paper 2019.

Almunia, Miguel, Pol Antras, David Lopez-Rodriguez, and Eduardo Morales, "Venting Out: Exporters during a Domestic Slump," *American Economics Review*, 2021, *111* (11), 3611–3662.

Amiti, Mary, Oleg Itskhoki, and Jozef Konings, "International Shocks, Variable Markups and Domestic Prices," *Review of Economic Studies*, 2019.

Angrist, Joshua and Michal Kolesár, "One instrument to rule them all: The bias and coverage of just-ID IV," *Journal of Econometrics*, 3 2024, *240*, 105398.

Antras, Pol, Teresa C. Fort, and Felix Tintelnot, "The Margins of Global Sourcing: Theory and Evidence from U.S. Firms," *American Economic Review*, 2017, *107* (9), 2514 – 64.

Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare, "New Trade Models, Same Old Gains?," *American Economic Review*, February 2012, *102* (1), 94–130.

_ , Federico Huneeus, and Yuhei Miyauchi, "Spatial Production Networks," Working Paper 2023.

Atalay, Enghin, "How Important Are Sectoral Shocks?," *American Economic Journal: Macroeconomics*, October 2017, *9* (4), 254–80.

Autor, David H., David Dorn, and Gordon H. Hanson, "The China Syndrome: Local Labor Market Effects of Import Competition in the United States," *American Economic Review*, 2013, *103* (6), 2121–2168.

_ , _ , _ , and Jae Song, "Trade Adjustment: Worker-Level Evidence," *The Quarterly Journal of Economics*, 11 2014, *129*, 1799–1860.

_ , _ , Lawrence F Katz, Christina Patterson, and John Van Reenen, "The fall of the labor share and the rise of superstar firms," *The Quarterly Journal of Economics*, 2020, *135* (2), 645–709.

Balke, Neele and Thibaut Lamadon, "Productivity Shocks, Long-Term Contracts, and Earnings Dynamics," *American Economic Review*, July 2022, *112* (7), 2139–77.

Baqaee, David Rezza and Emmanuel Farhi, "The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem," *Econometrica*, 2019, *87*, 1155–1203.

_ and _ , "Productivity and Misallocation in General Equilibrium," *The Quarterly Journal of Economics*, 09 2019, *135* (1), 105–163.

_ and _ , "Entry vs. Rents: Aggregation with Economies of Scale," Working Paper 27140, National Bureau of Economic Research May 2020.

_ and _ , "Networks, Barriers, and Trade," Working Paper 26108, National Bureau of Economic Research July 2022.

_ , Ariel Burstein, Cédric Duprez, and Emmanuel Farhi, "Supplier Churn and Growth: A Micro-to-Macro Analysis," Working Paper 31231, National Bureau of Economic Research May 2023.

Bartelsman, Eric, John Haltiwanger, and Stefano Scarpetta, "Cross-country differences in productivity: The role of allocation and selection," *American Economic Review*, 2013, *103* (1), 305–334.

Berger, David, Kyle Herkenhoff, and Simon Mongey, "Labor Market Power," *American Economic Review*, April 2022, *112* (4), 1147–93.

Bigio, Saki and Jennifer La'O, "Distortions in Production Networks," *The Quarterly Journal of Economics*, 11 2020, *135*.

Calvo, Guillermo A., "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, 1983, *12* (3), 383–398.

Campbell, John Y. and Pierre Perron, *Pitfalls and Opportunities: What Macroeconomists Should Know about Unit Roots*, MIT Press, January

Card, David, Ana Rute Cardoso, Joerg Heining, and Patrick Kline, "Firms and labor market inequality: Evidence and some theory," *Journal of Labor Economics*, 2018, *36* (S1), S13–S70.

Chan, Mons, Sergio Salgado, and Ming Xu, "Heterogeneous Passthrough from TFP to Wages," Working Paper 2021.

Chen, Kuan-Ming, Claire Ding, John A List, and Magne Mogstad, "Reservation Wages and Workers' Valuation of Job Flexibility: Evidence from a Natural Field Experiment," Working Paper 27807, National Bureau of Economic Research September 2020.

Comin, Diego, Danial Lashkari, and Martí Mestieri, "Structural Change With Long-Run Income and Price Effects," *Econometrica*, 2021, *89* (1), 311–374.

Costinot, Arnaud, Matti Sarvimaki, and Jonathan Vogel, "Exposure(s) to Trade and Earnings Dynamics: Evidence from the Collapse of Finnish-Soviet Trade," Working Paper 2022.

Crossroads Bank for Social Security, "Employer-Employee Dataset," 2003–2014. Confidential dataset.

De Loecker, Jan, Catherine Fuss, and Johannes Van Biesebroeck, "International competition and firm performance : Evidence from Belgium," *NBB Working Paper Series*, 2014, *269.*

_, Jan Eeckhout, and Gabriel Unger, "The Rise of Market Power and the Macroeconomic Implications," *Quarterly Journal of Economics*, 2020, *135* (2), 561–644.

Dekle, Robert, Jonathan Eaton, and Samuel Kortum, "Unbalanced Trade," *American Economic Review Papers & Proceedings*, 2007, *97* (2), 351–355.

Demidova, Svetlana, Konstantin Kucheryavyy, Takumi Naito, and Andrés Rodríguez-Clare, "The small open economy in a generalized gravity model," *Journal of International Economics*, 2024, *152*, 103997.

Demir, Banu, Ana Cecília Fieler, Daniel Yi Xu, and Kelly Kaili Yang, "O-Ring Production Networks," *Journal of Political Economy*, 1 2024, *132*, 200–247.

Dhyne, Emmanuel, Ayumu Ken Kikkawa, and Glenn Magerman, "Imperfect Competition in Firm-to-Firm Trade," *Journal of the European Economic Association*, 3 2022.

_ , _ , Magne Mogstad, and Felix Tintelnot, "Trade and Domestic Production Networks," *The Review of Economic Studies*, 3 2021, *88*, 643–668.

_ , _ , _ , and _ , "Measuring the Share of Imports in Final Consumption," *AEA Papers and Proceedings*, May 2023, *113*, 81–86.

_ , _ , Xianglong Kong, Magne Mogstad, and Felix Tintelnot, "Endogenous production networks with fixed costs," *Journal of International Economics*, 11 2023, *145*, 103841.

_ , Glenn Magerman, and Stela Rubinova, "The Belgian production network 2002-2012," *NBB Working Paper Series*, 2015, *288.*

Dix-Carneiro, Rafael, "Trade Liberalization and Labor Market Dynamics," *Econometrica*, 2014, *82* (3), 825–885.

_ and Brian K. Kovak, "Trade Liberalization and Regional Dynamics," *American Economic Review*, October 2017, *107* (10), 2908–46.

Duprez, Cedric and Glenn Magerman, "Price updating in production networks," *NBB Working Paper Series*, 2018, (352).

_ and _ , "Correspondences of EU product classifications," Technical Report 2022.

Ederhof, Merle, Venky Nagar, and Madhav Rajan, "How Economically Significant Are Unused Capacity Costs? A Large-Scale Empirical Analysis," *Management Science*, 2021, *67* (3), 1956–1974.

Egger, Hartmut, Peter Egger, Udo Kreickemeier, and Christoph Moser, "The exporter wage premium when firms and workers are heterogeneous," *European Economic Review*, 2020, *130*, 103599.

Elliott, Matt, Ben Golub, and Matt Leduc, "Supply Network Formation and Fragility," Working Paper 2021.

Eurostat, "National Accounts," 2012. https://ec.europa.eu/eurostat/web/national-accounts/data/database, accessed July 28, 2020.

_ , "Structure of earnings survey (SES)," 2014. https://ec.europa.eu/eurostat/data/database, accessed May 10, 2025.

Felix, Mayara, "Trade, Labor Market Concentration, and Wages," Working Paper 2021.

Frías, Judith A, David S Kaplan, Eric Verhoogen, and David Alfaro-Serrano, "Exports and Wage Premiums: Evidence from Mexican Employer-Employee Data," *Review of Economics and Statistics*, 2024, pp. 1–17.

Friedrich, Benjamin, Lisa Laun, Costas Meghir, and Luigi Pistaferri, "Earnings Dynamics and Firm-Level Shocks," Working Paper 25786, National Bureau of Economic Research April 2019.

Gagliardone, Luca, Mark Gertler, Simone Lenzu, and Joris Tielens, "Anatomy of the Phillips Curve: Micro Evidence and Macro Implications," Working Paper 31382, National Bureau of Economic Research June 2023.

Galle, Simon, Andrés Rodríguez-Clare, and Moises Yi, "Slicing the Pie: Quantifying the Aggregate and Distributional Effects of Trade," *The Review of Economic Studies*, 1 2023, *90*, 331–375.

Garin, Andrew and Filipe Silvério, "How Responsive Are Wages to Firm-Specific Changes in Labour Demand? Evidence from Idiosyncratic Export Demand Shocks," *Review of Economic Studies*, 5 2024, *91*, 1671–1710.

Gibbons, Robert, Lawrence F. Katz, Thomas Lemieux, and Daniel Parent, "Comparative Advantage, Learning, and Sectoral Wage Determination," *Journal of Labor Economics*, 10 2005, *23*, 681–724.

Guiso, Luigi, Luigi Pistaferri, and Fabiano Schivardi, "Insurance within the Firm," *Journal of Political Economy*, 10 2005, *113*, 1054–1087.

Gutiérrez, Agustın, "Labor market power and the pro-competitive gains from trade," *Working Paper*, 2022.

Hamermesh, D. S., "Changing Inequality in Markets for Workplace Amenities," *The Quarterly Journal of Economics*, 11 1999, *114*, 1085–1123.

Hanoch, Giora, "Production and Demand Models with Direct or Indirect Implicit Additivity," *Econometrica*, 1975, *43* (3), 395–419.

Helpman, Elhanan, Oleg Itskhoki, Marc-Andreas Muendler, and Stephen J Redding, "Trade and inequality: From theory to estimation," *The Review of Economic Studies*, 2016, *84* (1), 357–405.

Howell, Sabrina T and J. David Brown, "Do Cash Windfalls Affect Wages? Evidence from R&D Grants to Small Firms," Working Paper 26717, National Bureau of Economic Research January 2020.

Hummels, David, Rasmus Jørgensen, Jakob Munch, and Chong Xiang, "The Wage Effects of Offshoring : Evidence from Danish Matched Worker-Firm Data," *American Economic Review*, 2014, *104* (6), 1597–1629.

Huneeus, Federico, "Production network dynamics and the propagation of shocks," Working Paper 2018.

_ , Kory Kroft, and Kevin Lim, "Earnings Inequality in Production Networks," Working Paper 28424, National Bureau of Economic Research February 2021.

Irarrazabal, Alfonso, Andreas Moxnes, and Karen Helene Ulltveit-Moe, "Heterogeneous Firms or Heterogeneous Workers? Implications for Exporter Premiums and the Gains from Trade," *The Review of Economics and Statistics*, 07 2013, *95* (3), 839–849.

Juhn, Chinhui, Kristin McCue, Holly Monti, and Brooks Pierce, "Firm Performance and the Volatility of Worker Earnings," *Journal of Labor Economics*, 1 2018, *36*, S99–S131.

Kim, Ryan and Jonathan Vogel, "Trade and Welfare (Across Local Labor Markets)," Working Paper 27133, National Bureau of Economic Research May 2020.

_ and _ , "Trade Shocks and Labor Market Adjustment," *American Economic Review: Insights*, March 2021, *3* (1), 115–30.

Klenow, Peter J and Huiyu Li, "Entry Costs Rise with Growth," Working Paper 32974, National Bureau of Economic Research September 2024.

Kline, Patrick, Neviana Petkova, Heidi Williams, and Owen Zidar, "Who Profits from Patents? Rent-Sharing at Innovative Firms," *The Quarterly Journal of Economics*, 2019, *134* (3), 1343–1404.

Kovak, Brian K., "Regional Effects of Trade Reform: What Is the Correct Measure of Liberalization?," *American Economic Review*, August 2013, *103* (5), 1960–76.

Kroft, Kory, Yao Luo, Magne Mogstad, and Bradley Setzler, "Imperfect Competition and Rents in Labor and Product Markets: The Case of the Construction Industry," Working Paper 27325, National Bureau of Economic Research June 2020.

Lamadon, Thibaut, Magne Mogstad, and Bradley Setzler, "Imperfect Competition, Compensating Differentials, and Rent Sharing in the US Labor Market," *American Economic Review*, 1 2022, *112*, 169–212.

Lim, Kevin, "Endogenous Production Networks and the Business Cycle," Working Paper 2018.

MacKenzie, Gaelan, "Trade and market power in product and labor markets," Technical Report, Bank of Canada Staff Working Paper 2021.

Maestas, Nicole, Kathleen J Mullen, David Powell, Till von Wachter, and Jeffrey B Wenger, "The Value of Working Conditions in the United States and Implications for the Structure of Wages," Working Paper 25204, National Bureau of Economic Research October 2018.

Marcus, Ruth, Eric Peritz, and K. R. Gabriel, "On Closed Testing Procedures with Special Reference to Ordered Analysis of Variance," *Biometrika*, 12 1976, *63*, 655.

Mas, Alexandre and Amanda Pallais, "Valuing Alternative Work Arrangements," *American Economic Review*, 12 2017, *107*, 3722–3759.

Matsuyama, Kiminori, "Non-CES Aggregators: A Guided Tour," *Annual Review of Economics*, 2023, *15* (1), 235–265.

National Bank of Belgium, "Annual Account Filings," 2002–2014. Confidential dataset.

_ , "Business-to-Business Transactions Dataset," 2002–2014. Confidential dataset.

_ , "International Trade Dataset," 2002–2014. Confidential dataset.

Oberfield, Ezra, "A Theory of Input-Output Architecture," *Econometrica*, 2018, *86*, 559–589.

_ **and Devesh Raval**, "Micro Data and Macro Technology," *Econometrica*, 2021, *89*, 703–732.

Pierce, B., "Compensation Inequality," *The Quarterly Journal of Economics*, 11 2001, *116*, 1493–1525.

Pierce, Justin R. and Peter K. Schott, "The Surprisingly Swift Decline of US Manufacturing Employment," *American Economic Review*, July 2016, *106* (7), 1632–62.

Reenen, John Van, "The Creation and Capture of Rents: Wages and Innovation in a Panel of U.K. Companies," *The Quarterly Journal of Economics*, 1996, *111* (1), 195–226.

Romano, Joseph P., Azeem Shaikh, and Michael Wolf, "Consonance and the Closure Method in Multiple Testing," *The International Journal of Biostatistics*, 1 2011, *7*, 1–25.

Rosen, Sherwin, "Chapter 12 The theory of equalizing differences," in "Handbook of Labor Economics," Vol. 1, Elsevier, 1986, pp. 641–692.

Roussille, Nina and Benjamin Scuderi, "Bidding for Talent: A Test of Conduct in a High-Wage Labor Market," Working Paper 2024.

Sharma, Garima, "Collusion Among Employers in India," Working Paper 2024.

Statbel, "PRODCOM survey," 2002–2014. Confidential dataset.

Taschereau-Dumouchel, Mathieu, "Cascades and Fluctuations in an Economy with an Endogenous Production Network," Working Paper 2018.

Traiberman, Sharon, "Occupations and import competition: Evidence from Denmark," *American Economic Review*, 2019, *109* (12), 4260–4301.

Traina, James, "Is Aggregate Market Power Increasing? Production Trends Using Financial Statements," Working Paper 2018.

United Nations, "Comtrade Database," 2002–2014. https://comtrade.un.org/, accessed July 27, 2020.

vom Lehn, Christian and Thomas Winberry, "The Investment Network, Sectoral Comovement, and the Changing U.S. Business Cycle," *The Quarterly Journal of Economics*, 12 2021, *137*, 387–433.

Wiswall, Matthew and Basit Zafar, "Preference for the Workplace, Investment in Human Capital, and Gender," *The Quarterly Journal of Economics*, 2 2018, *133*, 457–507.

Wooldridge, Jeffery M., "On estimating firm-level production functions using proxy variables to control for unobservables," *Economics Letters*, 2009, *104*, 112–114.