ONLINE APPENDIX

Fragile New Economy: Intangible Capital, Corporate Savings Glut, and Financial Instability

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A. Proofs and Solution Algorithm

A1. Proofs

Ruling out self-financing. If entrepreneurs' investment projects can be self-financed, entrepreneurs do not need to hold liquidity for investment and the liquidity premium is zero. The equilibrium value of tangible capital is the production value, i.e., $1/(\rho+\delta+\lambda)$. If Assumption 1 holds, then even if entrepreneurs set the intangible share of investment, θ_t, to zero, the external financing capacity, $\kappa^T q_t^T = \kappa^T \left(\frac{1}{\rho+\delta+\lambda}\right)$ is still below 1, which is the cost of investment. This contradicts that investment is self-financed. Therefore, under Assumption 1, the investment project cannot be self-financed.

Proof of Proposition 3. First, I show that there exists an upper bound $\overline{\eta}(t)$ such that $\eta_t \leq \overline{\eta}(t)$. Note that $q_t^B \geq 1$ in equilibrium because if $q_t^B < 1$, bankers are better off consuming (worth 1) than retaining wealth (worth q_t^B). As will be shown later, q_t^B is a bivariate function, $q_t^B = q^B(\eta_t, t)$. Fixing t, let $\overline{\eta}(t)$ denote that lowest value of η_t where bankers consume. Therefore, $q^B(\overline{\eta}(t), t) = 1$ and $q_t^B > 1$ at $\eta_t < \overline{\eta}(t)$. Suppose there exists $\eta' > \overline{\eta}(t)$ such that η_t reaches η'. This leads to a contradiction – it is no longer optimal for bankers to consume at $\overline{\eta}(t)$ because their marginal value of wealth will surely increase: at $\overline{\eta}(t)$, if η_t increases, q_t^B will not decline because $q_t^B \geq 1$, and if η_t decreases, q_t^B will surely increase because, by definition of $\overline{\eta}(t)$, $q_t^B > 1$ for $\eta_t < \overline{\eta}(t)$. Therefore, η_t cannot increase beyond $\overline{\eta}(t)$, the upper boundary given by bankers' consumption optimality.

Next, I derive the law of motion of η_t in $(0, \overline{\eta}(t))$. According to (10), bankers' wealth satisfies the following law of motion in the region where bankers' consumption is zero, i.e., $\eta_t \in (0, \overline{\eta}(t))$:

$$\frac{dN_t^B}{N_t^B} = \mu_t^N dt + \sigma_t^N dZ_t \,, \tag{A.1}$$

where

$$\mu_t^N = r_t + x_t^B \left(\mathbb{E}_t \left[dr_t^T\right] - r_t\right) \,, \tag{A.2}$$

and

$$\sigma_t^N = x_t^B \left(\sigma_t^T + \sigma\right) \,. \tag{A.3}$$

The expression of expected return of tangible capital holdings, $\mathbb{E}_t\left[dr_t^T\right]$, can be obtained from (9). By Itô's lemma, the law of motion of η_t is given by

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^\eta dZ\,, \tag{A.4}$$

where

$$\mu_t^\eta = \mu_t^N - \mu_t^{KT} - \sigma_t^N\sigma + \sigma^2\,, \tag{A.5}$$

(where μ_t^{KT} is the expected instantaneous growth rate of K_t^T) and

$$\sigma_t^\eta = x_t^B\left(\sigma_t^T + \sigma\right) - \sigma\,. \tag{A.6}$$

According to (26), the expected instantaneous growth rate of K_t^T is given by

(A.7)
$$\begin{aligned}\mu^{KT} &= \frac{\left(\frac{1}{1-q_t^T\kappa^T(1-\theta_t)}\right)\left[\left(x_t^B-1\right)N_t^B - M_t^H\right](1-\theta_t)\,\kappa^T\lambda}{K_t^T} - \delta \\ &= \left(\frac{1}{1-q_t^T\kappa^T\left(1-\theta_t\right)}\right)\left[\left(x_t^B-1\right)\eta_t - \alpha\left(\frac{\rho-r_t}{\beta\left(t\right)}\right)^{-\frac{1}{\xi}}\right]\left(1-\theta_t\right)\kappa^T\lambda - \delta\,,\end{aligned}$$

where the second equation uses the definition of η_t and households' aggregate deposit demand given by (22). In A.2, q_t^T, r_t, x_t^B, θ_t, $\mathbb{E}_t\left[dr_t^T\right]$, σ_t^T, and the rest of variables in Proposition 3 are shown to be bivariate functions of η_t and t.

Proof of Proposition 1. First, I solve the investment problem of entrepreneurs who are hit by the Poisson shocks, and then embed the solution to the entrepreneurs' dynamic optimization. An investing entrepreneur solves the problem summarized by the Lagrange function (11):
(A.8)
$$\mathcal{L} = \max_{\{i_t,\theta_t\}}\left[q^I\kappa_t^I\theta_t + q_t^T\kappa^T\left(1-\theta_t\right) - F\left(\theta_t\right)\right]i_t - i_t + \pi_t\left[m_t^E + q_t^T\kappa^T i_t\left(1-\theta_t\right) - i_t\right]\,.$$

Given κ_t, q_t^T, m_t^E, q^I and κ^T, the entrepreneur chooses θ_t and i_t. The first-order condition (F.O.C.) for θ_t is

$$q^I\kappa_t^I - q_t^T\kappa^T\left(1+\pi_t\right) - F'\left(\theta_t\right) = 0\,, \tag{A.9}$$

and the F.O.C. for i_t is (i.e., (13) in the main text)

$$\pi_t = \left\{\left[q^I\kappa_t^I\theta_t + q_t^T\kappa^T\left(1-\theta_t\right) - F\left(\theta_t\right)\right] - 1\right\}\left(\frac{1}{1-q_t^T\kappa^T\left(1-\theta_t\right)}\right) \tag{A.10}$$

The F.O.C. for θ_t equates the marginal value of investing in intangibles and the marginal value of investing in tangibles (which includes both the value of tangible capital and the shadow value from relaxing the liquidity constraint). The F.O.C. for i_t solves the marginal value of liquidity as equal to the net profits of investment multiplied by the leverage on liquidity holdings. The liquidity constraint binds so the total investment is given by

$$i_t = \frac{m_t^E}{1 - (1-\theta_t)\,\kappa^T q_t^T}\,. \tag{A.11}$$

Next, I prove that θ_t is increasing in κ^I. First, note that, from (A.10),

$$\begin{aligned}\frac{\partial \pi_t}{\partial \theta_t} &= \left[q^I \kappa_t^I - q_t^T \kappa^T - F'(\theta_t)\right]\left(\frac{1}{1 - q_t^T \kappa^T (1-\theta_t)}\right) \\ &\quad - \left\{\left[q^I \kappa_t^I \theta_t + q_t^T \kappa^T (1-\theta_t) - F(\theta_t)\right] - 1\right\}\frac{q_t^T \kappa^T}{\left[1 - q_t^T \kappa^T (1-\theta_t)\right]^2} \\ &= \frac{q_t^T \kappa^T \pi_t}{1 - q_t^T \kappa^T (1-\theta_t)} - \frac{q_t^T \kappa^T \pi_t}{1 - q_t^T \kappa^T (1-\theta_t)} = 0\,,\end{aligned} \tag{A.12}$$

where the second equation follows from (A.9) and (A.10). Differentiating (A.9) with respect to (w.r.t.) κ_t^I, I obtain

$$q^I - q_t^T \kappa^T \frac{\partial \pi_t}{\partial \theta_t}\frac{\partial \theta_t}{\partial \kappa_t^I} - q_t^T \kappa^T \frac{\partial \pi_t}{\partial \kappa_t^I} - F''(\theta_t)\frac{\partial \theta_t}{\partial \kappa_t^I} = 0\,. \tag{A.13}$$

Rearranging the equation and using (A.12), I solve

$$\frac{\partial \theta_t}{\partial \kappa_t^I} = \frac{q^I - q_t^T \kappa^T \frac{\partial \pi_t}{\partial \kappa_t^I}}{F''(\theta_t)}\,. \tag{A.14}$$

According to (A.10), the partial derivative of π_t w.r.t. κ_t^I is

$$\frac{\partial \pi_t}{\partial \kappa_t^I} = \frac{q^I \theta_t}{1 - q_t^T \kappa^T (1-\theta_t)}\,. \tag{A.15}$$

Using this equation to substitute out $\frac{\partial \pi_t}{\partial \kappa_t^I}$ in (A.14), I obtain

$$\frac{\partial \theta_t}{\partial \kappa_t^I} = \frac{1}{F''(\theta_t)}\left[q^I - q_t^T \kappa^T \frac{q^I \theta_t}{1 - q_t^T \kappa^T (1-\theta_t)}\right] = \frac{q^I\left(1 - q_t^T \kappa^T\right)}{F''(\theta_t)\left[1 - q_t^T \kappa^T (1-\theta_t)\right]}\,. \tag{A.16}$$

In equilibrium, $q_t^T \kappa^T$ must be smaller than 1, because otherwise the entrepreneur sets $\theta_t = 0$ (i.e., investing all in tangible capital) and self-finances the project to achieve infinite profits. Therefore, the right side of (A.16) is positive, i.e., θ_t is increasing in κ_t^I.

The right side of (A.15) is positive, so π_t is increasing in κ_t^I. Finally, I prove that π_t is increasing in q_t^T. Differentiating (A.10) w.r.t. q_t^T, I obtain

$$\begin{aligned}\frac{\partial \pi_t}{\partial q_t^T} =& \frac{\kappa^T(1-\theta_t)}{1-q_t^T\kappa^T(1-\theta_t)} - \left\{\left[q^I\kappa_t^I\theta_t + q_t^T\kappa^T(1-\theta_t) - F(\theta_t)\right] - 1\right\} \frac{\left[-\kappa^T(1-\theta_t)\right]}{\left[1-q_t^T\kappa^T(1-\theta_t)\right]^2} \\ =& \frac{\kappa^T(1-\theta_t)}{1-q_t^T\kappa^T(1-\theta_t)} + \pi_t\frac{\kappa^T(1-\theta_t)}{1-q_t^T\kappa^T(1-\theta_t)} = \frac{\kappa^T(1-\theta_t)}{1-q_t^T\kappa^T(1-\theta_t)}(1+\pi_t) > 0\,.\end{aligned} \tag{A.17}$$

Next, I solve (15), i.e., the optimality condition for entrepreneurs' optimal liquidity holdings. Entrepreneurs maximize the life-time utility, $\mathbb{E}\left[\int_{t=0}^{+\infty} e^{-\rho t} dc_t^E\right]$ given the following law of motion of wealth:

$$dw_t^E = -dc_t^E + \mu_t^w w_t^E dt + \sigma_t^w w_t^E dZ_t + \left(\widehat{w}_t^E - w_t^E\right) dN_t,$$

$\mu_t^w w_t^E$ and $\sigma_t^w w_t^E$ are the drift and diffusion terms that depend on choices of tangible capital and deposit holdings and will be elaborated later. dN_t is the increment of the idiosyncratic counting (Poisson) process. At the Poisson time, an entrepreneur's wealth jumps by the total profits from investment minus the value of lost tangible capital holdings (denoted by k^{TE}),

$$\begin{aligned}\widehat{w}_t^E - w_t^E =& \left\{\left[q^I\kappa_t^I\theta_t + q_t^T\kappa^T(1-\theta_t) - F(\theta_t)\right] - 1\right\}\left(\frac{1}{1-q_t^T\kappa^T(1-\theta_t)}\right) m_t^E - q_t^T k_t^{TE} \\ =& \pi_t m_t^E - q_t^T k_t^{TE}\,.\end{aligned} \tag{A.18}$$

Note that w_t^E does not contain the existing stock of intangible capital, because when analyzing entrepreneurs' decisions, the production flows from intangible capital can be treated as goods that are directly consumed, given entrepreneurs' indifference in the timing of consumption.

I conjecture that the value function is linear in wealth w_t^E: $v_t^E = \zeta_t^E w_t^E + v^I$, where ζ_t^E is the marginal value of liquid wealth (i.e., without counting the value of intangible capital), and v^I is the present value of consumption from intangible capital. Consider a generic equilibrium diffusion process for ζ_t^E:

$$d\zeta_t^E = \zeta_t^E \mu_t^\zeta dt + \zeta_t^E \sigma_t^\zeta dZ_t,$$

where $\zeta_t^E \mu_t^\zeta$ and $\zeta_t^E \sigma_t^\zeta$ are the drift and diffusion terms, respectively. Entrepreneurs' marginal value of wealth, ζ_t^E, is a summary statistic of their investment opportunity set, which depends on the overall industry dynamics, so it does not jump when an individual is hit by the Poisson shocks.

Under this conjecture, the Hamilton-Jacobi-Bellman (HJB) equation is

$$\rho\zeta_t^E w_t^E dt = \max_{dc^E, k_t^{TE}, m_t^E} dc_t^E - \zeta_t^E dc_t^E + \{w_t^E \zeta_t^E \mu_t^\zeta + w_t^E \zeta_t^E \mu_t^w + w_t \zeta_t^E \sigma_t^\zeta \sigma_t^w + \lambda \zeta_t^E [\widehat{w}_t - w_t]\} dt\,.$$

Note that the consumption flow from intangible capital and $\rho v^I dt$ cancel each other out, because, by definition, v^I is the ρ-discounted present value of consumption flow.

Entrepreneurs can choose any $dc_t^E \in \mathbb{R}$, so ζ_t^E must be equal to one, and thus, I have also confirmed the value function conjecture. Since ζ_t^E is a constant equal to one, μ_t^ζ and σ_t^ζ are both zero. The HJB equation can be simplified:

$$\rho\zeta_t^E w_t^E dt = \max_{k_t^{TE} \geq 0, m_t^E \geq 0} \mu_t^w w_t^E dt + \lambda dt \left(\pi_t m_t^E - q_t^T k_t^{TE}\right). \tag{A.19}$$

Wealth drift includes production, the value change of tangible capital holdings, and the deposit return:

$$\mu_t^w w_t^E dt = \underbrace{k_t^{TE} dt + \mathbb{E}_t \left(q_{t+dt}^T k_{t+dt}^{TE} - q_t^T k_t^{TE}\right)}_{\mathbb{E}_t\left[dr_t^T\right] q_t^T k_t^{TE}} + r_t m_t^E dt.$$

Let $d\psi_t^E$ denote the Lagrange multiplier of the budget constraint, $q_t^T k_t^{TE} + m_t^E \leq w_t^E$. The first-order condition (F.O.C.) for optimal deposit holdings per unit of capital is: $m_t^E \geq 0$, and

$$m_t^E \left(r_t dt + \pi_t \lambda dt - d\psi_t^E\right) = 0.$$

The F.O.C. for optimal tangible capital holdings is : $k_t^{TE} \geq 0$, and

$$k_t^{TE} \left(-\mathbb{E}_t \left[dr_t^T\right] + d\psi_t^E\right) = 0.$$

Substituting these optimality conditions into the HJB equation, we have

$$\rho v_t^E dt = w_t^E d\psi_t^E.$$

Because $\zeta_t^E = 1$, $v_t^E = w_t^E$, and $d\psi_t^E = \rho dt$. Substituting $d\psi_t^E = \rho dt$ into the F.O.C. for m_t^E, we have

$$\rho - r_t = \lambda \pi_t.$$

Substituting $d\psi_t = \rho dt$ into the F.O.C. for k_t^{TE} and rearranging the equation, we have

$$\mathbb{E}_t \left[dr_t^T\right] = \rho dt\,,$$

that is, when entrepreneurs hold tangible capital, they require an expected return of ρ.

Binding liquidity constraint. Consider the following inequalities:

$$\max_{\theta_t}\left[q^I\kappa_t^I\theta_t+q_t^T\kappa^T\left(1-\theta_t\right)-F\left(\theta_t\right)\right]\geq q^I\kappa_t^I-F\left(1\right)\geq q^I\kappa_0^I-F\left(1\right)\,,$$

where the first step follows $q_t^T \geq q^I$ (due to the additional liquidity value of tangible capital) and the optimality of θ_t, and the second step follows from $\kappa_t^I \geq \kappa_0^I$. Therefore, as long as

$$q^I\kappa_0^I-F\left(1\right)>1\,, \tag{A.20}$$

we have

$$\begin{aligned}\pi_t=&\max_{\theta_t}\left\{\left[q^I\kappa_t^I\theta_t+q_t^T\kappa^T\left(1-\theta_t\right)-F\left(\theta_t\right)\right]-1\right\}\left(\frac{1}{1-q_t^T\kappa^T\left(1-\theta_t\right)}\right)\\ &\geq\left[q^I\kappa_0^I-F\left(1\right)-1\right]\left(\frac{1}{1-q_t^T\kappa^T\left(1-\theta_t\right)}\right)>0\end{aligned}$$

and the liquidity constraint binds. Note that $\left(\frac{1}{1-q_t^T\kappa^T(1-\theta_t)}\right)>0$ from Assumption 1. The calibrated parameter values satisfy the condition given by (A.20).

Proof of Proposition 2. Conjecture that the bank's value function takes the linear form: $v_t^B = q_t^B n_t^B$. Consider the following generic equilibrium diffusion process for q_t^B,

$$dq_t^B=q_t^B\mu_t^Bdt-q_t^B\gamma_t^BdZ_t.$$

Define $dy_t^B = dc_t^B/n_t^B$, the consumption-to-wealth ratio of bankers. Under the conjectured functional form, the HJB equation is

$$\begin{aligned}\rho v_t^Bdt\;\;=\;\;&\max_{dy_t^B}\left\{\left(1-q_t^B\right)\mathbb{I}_{\{dy_t^B>0\}}n_t^Bdy_t^B\right\}+\mu_t^Bq_t^Bn_t^B+\\ &\max_{x_t^B}\left\{r_t+x_t^B\left(\mathbb{E}_t\left[dr_t^T\right]-r_t\right)-x_t^B\gamma_t^B\left(\sigma_t^T+\sigma\right)\right\}q_t^Bn_t^B,\end{aligned}$$

Dividing both sides by $q_t^B n_t^B$, n_t^B is eliminated, which confirms the homogeneity property,

$$\rho=\max_{dy_t^B}\left\{\frac{\left(1-q_t^B\right)}{q_t^B}\mathbb{I}_{\{dy_t^B>0\}}dy_t^B\right\}+\mu_t^B+\max_{x_t^B}\left\{r_t+x_t^B\left(\mathbb{E}_t\left[dr_t^T\right]-r_t\right)-x_t^B\gamma_t^B\left(\sigma_t^T+\sigma\right)\right\}, \tag{A.21}$$

and the conjecture of linear value function. The indifference condition for x_t^B is

$$\mathbb{E}_t\left[dr_t^T\right]=r_t+\gamma_t^B\left(\sigma_t^T+\sigma\right)\,. \tag{A.22}$$

Substituting the expression of $\mathbb{E}_t\left[dr_t^T\right]$ given by (9) and using (15), I obtain (18).

Substituting the optimality conditions into the HJB equation, I obtain

$$\mu_t^B = \rho - r_t \,. \tag{A.23}$$

The result that $\gamma_t^B = 0$ when bankers consume is given by the smooth-pasting condition, $\partial q^B (\eta_t, t) / \partial \eta_t = 0$ (so by Itô's lemma, $\gamma_t^B = 0$), which is discussed in more details in A.2. The upper boundary $\overline{\eta}(t)$ is given by the value-matching condition of bankers' consumption, $q^B (\overline{\eta}(t), t) = 1$, and is jointly determined with the function $q_t^B = q^B (\eta_t, t)$ in the solution of PDEs of $q^B (\eta_t, t)$ and $q^T (\eta_t, t)$ in A.2.

Conditional stationary distribution of η_t. Following Brunnermeier and Sannikov (2014), I derive the conditional stationary probability density of η_t. Fixing κ_t^I and β_t, the probability density of η_t at time t, $p(\eta, t)$, satisfies the Kolmogorov forward equation

$$\frac{\partial}{\partial t} p(\eta, t) = -\frac{\partial}{\partial \eta} \left(\eta \mu^\eta (\eta) p(\eta, t)\right) + \frac{1}{2} \frac{\partial^2}{\partial \eta^2} \left(\eta^2 \sigma^\eta (\eta)^2 p(\eta, t)\right).$$

Note that, fixing κ_t^I and β_t, μ_t^η and σ_t^η are functions of η_t as shown in A.2. A stationary density is a solution to the forward equation that does not vary with time (i.e. $\frac{\partial}{\partial t} p(\eta, t) = 0$). So I suppress the time variable, and denote stationary density as $p(\eta)$. Integrating the forward equation over η, $p(\eta)$ solves the following first-order ordinary differential equation within the reflecting boundary:

$$0 = C - \eta \mu^\eta (\eta) p(\eta) + \frac{1}{2} \frac{d}{d\eta} \left(\eta^2 \sigma^\eta (\eta)^2 p(\eta)\right), \quad \eta \in (0, \overline{\eta}] \,.$$

The integration constant C is zero because of the reflecting boundary. The boundary condition for the equation is the requirement that probability density is integrated to one (i.e. $\int_{\underline{\eta}}^{\overline{\eta}} p(\eta) d\eta = 1$).

A2. Solution Algorithm

The full solution of the model consists of two parts: first, the laws of motion of state variables, and, second, the endogenous variables as functions of state variables, for example, $q_t^T = q^T (\eta_t, t)$. The Markov equilibrium has four state variable: time, η_t, K_t^I, and K_t^T. As shown in the main text, time has an exogenous and autonomous law of motion, while the last three variables' laws of motions depend on the endogenous variables that are functions of these state variables. To simplify the notation, I suppress the time subscripts in the following.

I construct a mapping from η, t, $q^B (\eta, t)$, $q^T (\eta, t)$, $\partial q^B (\eta, t) / \partial \eta$, $\partial q^T (\eta, t) / \partial \eta$, $\partial q^B (\eta, t) / \partial t$ and $\partial q^T (\eta, t) / \partial t$ to the second-order derivatives with respect to η, $\partial^2 q^B (\eta, t) / \partial \eta^2$ and $\partial^2 q^T (\eta, t) / \partial \eta^2$, i.e., a system of second-order partial differential equations for $q^B (\eta, t)$ and $q^T (\eta, t)$. Once I solved these two functions, the rest of the price variables and K^T-scaled aggregate quantities can be solved as they will be shown to depend only on η, t, the levels and derivatives of $q^B(\eta, t)$ and $q^T(\eta, t)$. This confirms the statement in Proposition 3 that these variables are bivariate functions of η and t. After solving the price variables and K^T-scaled aggregate quantities, the laws of motion of K_t^I, K_t^T, and η_t are given by (25), (26), and (28), respectively.

Constructing PDEs for $q^B(\eta, t)$ and $q^T(\eta, t)$. Inputs are η, t (and thus, $\kappa^I(t)$ and $\beta(t)$), the levels and first derivatives of $q^B(\eta, t)$ and $q^T(\eta, t)$. Outputs are $\partial^2 q^B(\eta, t)/\partial\eta^2$ and $\partial^2 q^T(\eta, t)/\partial\eta^2$. It is convenient to define the following notations of elasticities:

$$\epsilon^T \equiv \frac{\partial q^T/q^T}{\partial\eta/\eta} \text{ and } \epsilon^B \equiv \frac{\partial q^B/q^B}{\partial\eta/\eta}.$$

Step 1: Calculate σ^η, σ^T, γ^B, x^B, and r.

Proposition 1 solves the optimal intangible share of investment, θ, and the marginal value of liquidity, π, that entrepreneurs assign to deposits, as functions of q^I (constant, see (2)), q^T, and $\kappa^I(t)$ and the parameters. Given $F(\theta_t) = \frac{\phi}{2}\theta^2$, (A.9) implies a quadratic equation for θ when π is is substituted out using (A.10). Once θ is solved, (A.10) solves π. In the following, I will discuss different cases, but the values of these variables will not change across different cases.

First, consider the case where entrepreneurs do not hold any deposits. With $M^E = 0$, the deposit-market clearing condition (24) is

$$\text{(A.24)} \qquad \left(x^B - 1\right)\eta = M^H/K^T = \alpha\left(\frac{\rho - r}{\beta(t)}\right)^{-\frac{1}{\xi}},$$

where the second equation is obtained from households' aggregate deposit demand (22). Within this case, there are two scenarios. First, bankers hold all tangible capital, so $q^T K^T = x^B N^B$, i.e.,

$$\text{(A.25)} \qquad x^B = q^T/\eta,$$

and then from (A.24), r is calculated. If $r > \rho - \lambda\pi$, then entrepreneurs prefer to hold deposits, and I switch a different case where entrepreneurs hold deposits (to be discussed below). If $r \le \rho - \lambda\pi$, I proceed to calculate σ^η, σ^T, and γ^B. Jointly using $\sigma^\eta = x^B\left(\sigma^T + \sigma\right) - \sigma$ from (A.6) and $\sigma^T = \epsilon^T\sigma^\eta$ from Itô's lemma, I obtain σ^η and σ^T. Using Itô's lemma again, I obtain $\gamma^B = -\epsilon^B\sigma^\eta$. Now the bankers' discount rate is given by $r + \gamma^B\left(\sigma^T + \sigma\right)$. If $\rho < r + \gamma^B\left(\sigma^T + \sigma\right)$, then the rest of the economy has a lower discount rate than bankers, so bankers cannot hold all tangible capital, and I switch to the scenario where entrepreneurs do not hold deposits and bankers do not hold all tangible capital (to be discussed in the next paragraph). If $\rho > r + \gamma^B \left(\sigma^T + \sigma\right)$, this scenario is internally consistent and I proceed to Step 2.

Now consider the scenario where entrepreneurs do not hold deposits and bankers do not hold all tangible capital. In this scenario, x^B is calculated as follows. Given that the rest of the economy holds tangible capital, the expected return on tangible capital is ρ, and from Proposition 2,

$$\rho = r + \gamma^B \left(\sigma^T + \sigma\right) . \tag{A.26}$$

By Itô's lemma,

$$\sigma^T = \epsilon^T \sigma^\eta \text{ and } \gamma^B = -\epsilon^B \sigma^\eta. \tag{A.27}$$

I substitute these expressions of σ^T and γ^B into (A.26) to obtain a quadratic equation of σ^η, and the roots are

$$\sigma^\eta = \frac{-\epsilon^B \sigma \pm \sqrt{\left(\epsilon^B \sigma\right)^2 - 4\epsilon^B \epsilon^T \left(\rho - r\right)}}{2\epsilon^B \epsilon^T}.$$

I study a Markov equilibrium where $\epsilon^B \leq 0$ (i.e., bankers' marginal value of wealth declines in η_t), $\epsilon^T \geq 0$ (i.e., the value of tangible capital increases in η_t), and $\rho - r \geq 0$, so the only positive root is

$$\sigma^\eta = \frac{-\epsilon^B \sigma - \sqrt{\left(\epsilon^B \sigma\right)^2 - 4\epsilon^B \epsilon^T \left(\rho - r\right)}}{2\epsilon^B \epsilon^T} . \tag{A.28}$$

A positive root is selected because bankers have levered positions in tangible capital, so the shock impact is greater on N^B than on K^T, and thus, η responds positively to the Brownian shock. Using $\sigma^\eta = x^B \left(\sigma^T + \sigma\right) - \sigma$ from (A.6), I obtain

$$x^B = \frac{\sigma^\eta + \sigma}{\sigma^\eta \epsilon^T + \sigma} . \tag{A.29}$$

Using (A.29) to substitute out x^B in (A.24), I obtain

$$\left(\frac{\sigma^\eta + \sigma}{\sigma^\eta \epsilon^T + \sigma} - 1\right) \eta = \alpha \left(\frac{\rho - r}{\beta\left(t\right)}\right)^{-\frac{1}{\xi}} . \tag{A.30}$$

Using (A.28) to substitute out σ^η on the left side of (A.30), I obtain an equation for r. Once r is solved, I use (A.28) to solve σ^η, use (A.29) to solve x^B, and use (A.27) to solve σ^T and γ^B. Proceed to Step 2.

Finally, consider the case where entrepreneurs hold deposits. From Proposition

1, the equilibrium deposit rate is given by

$$r = \rho - \lambda \pi \,. \tag{A.31}$$

Given r, the deposit demand of households (scaled by K^T) is given by (22), and I obtain the aggregate deposit demand, $\left(M^E + M^H\right)/K^T$. Next, consider the scenario where bankers hold all tangible capital, i.e., $x^B = q^T/\eta$. From (A.29), I solve σ^η, and from (A.27), I solve σ^T and γ^B. Now the bankers' discount rate is given by $r + \gamma^B\left(\sigma^T + \sigma\right)$. If $\rho < r + \gamma^B\left(\sigma^T + \sigma\right)$, then the rest of economy have lower discount rate than bankers, so bankers cannot hold all tangible capital and I switch to the scenario where entrepreneurs hold deposits and bankers do not hold all tangible capital. If $\rho \geq r + \gamma^B\left(\sigma^T + \sigma\right)$, this scenario is internally consistent and I proceed to Step 2.

Now consider the scenario where entrepreneurs hold deposits and bankers do not hold all tangible capital. The expected return on tangible capital is ρ, so from Proposition 2,

$$\rho = r + \gamma^B\left(\sigma^T + \sigma\right) \,. \tag{A.32}$$

Using (A.31) to substitute r with $\rho - \lambda\pi$, I obtain

$$\lambda \pi = \gamma^B\left(\sigma^T + \sigma\right) \,. \tag{A.33}$$

Using Itô's lemma, i.e., (A.27), I substitute σ^T and γ^B out with $\epsilon^T\sigma^\eta$ and $-\epsilon^B\sigma^\eta$ respectively to obtain a quadratic equation of σ^η, and the roots are

$$\sigma^\eta = \frac{-\epsilon^B\sigma \pm \sqrt{\left(\epsilon^B\sigma\right)^2 - 4\epsilon^B\epsilon^T\lambda\pi}}{2\epsilon^B\epsilon^T}.$$

I study a Markov equilibrium where $\epsilon^B \leq 0$ (i.e., bankers' marginal value of wealth declines in η_t), $\epsilon^T \geq 0$ (i.e., the value of tangible capital increases in η_t), and, as the shadow price of funding constraint on investment, $\pi \geq 0$, so the only positive root is

$$\sigma^\eta = \frac{-\epsilon^B\sigma - \sqrt{\left(\epsilon^B\sigma\right)^2 - 4\epsilon^B\epsilon^T\lambda\pi}}{2\epsilon^B\epsilon^T}. \tag{A.34}$$

Using Itô's lemma again, i.e., (A.27), I solve σ^T and γ^B. Using $\sigma^\eta = x^B\left(\sigma^T + \sigma\right) - \sigma$ from (A.6), I solve x^B. Proceed to Step 2.

Step 2: Calculating the Second-Order Derivatives

The drift and diffusion of η are given in the proof of Proposition 3. Given q^T, π, γ^B, and σ^T, (18) solves μ^T. The following equation, obtained by Itô's lemma, solves $\frac{\partial^2 q^T}{\partial \eta^2}$:

$$\mu^T q^T = \frac{\partial q^T}{\partial t} + \frac{\partial q^T}{\partial \eta}\mu^\eta \eta + \frac{1}{2}\frac{\partial^2 q^T}{\partial \eta^2}\left(\sigma^\eta \eta\right)^2 . \tag{A.35}$$

According to (A.23), $\mu_t^B = \rho - r_t$, so the following equation, obtained by Itô's lemma, solves $\frac{\partial^2 q^B}{\partial \eta^2}$:

$$\mu^B q^B = \frac{\partial q^B}{\partial t} + \frac{\partial q^B}{\partial \eta}\mu^\eta \eta + \frac{1}{2}\frac{\partial^2 q^B}{\partial \eta^2}\left(\sigma^\eta \eta\right)^2 . \tag{A.36}$$

Boundary conditions for PDEs for $q^B(\eta, t)$ and $q^T(\eta, t)$. Tangible capital has constant cash flow, one unit of goods per unit of time, so what causes its price to vary is the discount-rate changes. Close to $\eta = 0$, an absorbing state, the banking sector is extremely small, so the discount rate (expected return) is fixed at ρ to induce the rest of economy to own tangible capital and clear the market. Thus, q^T should not vary as η approaches zero:

$$\lim_{\eta \to 0} \frac{\partial q^T(\eta, t)}{\partial \eta} = 0 . \tag{A.37}$$

Moreover, when bankers are extremely undercapitalized, their marginal value of wealth approaches infinity,

$$\lim_{\eta \to 0} q^B(\eta, t) = +\infty , \tag{A.38}$$

because q^B is the present value of one unit of equity, and it increases when the banking sector shrinks, widening the return spread between holding tangible capital and issuing deposits.

The upper boundary of η, $\overline{\eta}$, where bankers consume, is a reflecting boundary, so to rule out arbitrage (i.e., perfectly predictable variation of asset price),

$$\frac{\partial q^T(\overline{\eta}, t)}{\partial \eta} = 0 . \tag{A.39}$$

For consumption to be optimal at $\overline{\eta}$, bankers' marginal value of wealth, q^B, satisfies the value-matching condition,

$$q^B(\overline{\eta}, t) = 1 , \tag{A.40}$$

and the smooth-pasting condition

$$\frac{\partial q^B(\overline{\eta},t)}{\partial \eta}=0\,. \tag{A.41}$$

Finally, it is assumed that the linear trends of κ^I and β end at $t=\bar{t}$. When solving the model, I map $\bar{t}$ to 2010 in the data. When t reaches $\bar{t}$ and κ^I and β no longer vary, the economy converges to a time-homogeneous Markov equilibrium where the price variables and K^T-scaled quantities are functions of η_t only. Therefore, the boundary condition on the time dimension for $q^B(\eta,t)$ and $q^T(\eta,t)$ is the convergence to $\overline{q}^B(\eta)$ and $\overline{q}^T(\eta)$ of the time-homogeneous Markov equilibrium.

The functions, $\overline{q}^B(\eta)$ and $\overline{q}^T(\eta)$, of the time-homogeneous Markov equilibrium at $\bar{t}$ can be solved by a system of ordinary differential equations (ODEs) that are constructed following the same aforementioned procedure, except that at the very last step, by Itô's lemma, the second-order derivatives are solved by

$$\mu^B\overline{q}^B=\frac{d\overline{q}^B}{d\eta}\mu^\eta\eta+\frac{1}{2}\frac{d^2\overline{q}^B}{d\eta^2}\left(\sigma^\eta\eta\right)^2\,. \tag{A.42}$$

and

$$\mu^T\overline{q}^T=\frac{d\overline{q}^T}{d\eta}\mu^\eta\eta+\frac{1}{2}\frac{d^2\overline{q}^T}{d\eta^2}\left(\sigma^\eta\eta\right)^2\,. \tag{A.43}$$

The ODEs have the following conditions in analogy to (A.37) to (A.41):

- As η approaches zero: (1) $\lim_{\eta\to 0}\frac{d\overline{q}^T(\eta)}{d\eta}=0$; (2) $\lim_{\eta\to 0}\overline{q}^B(\eta)=+\infty$.
- At the upper reflecting boundary, $\overline{\eta}$: (3) $\frac{d\overline{q}^T(\eta)}{d\eta}=0$; (4) $\overline{q}^B(\overline{\eta})=1$; (5) $\frac{d\overline{q}^B(\eta)}{d\eta}=0$.

Prices and K^T-scaled quantities in Proposition 3. The solution procedure has solved q_t^T, r_t, x_t^B, θ_t as bivariate functions of η_t and t because they only depend on η_t, t, q_t^T, ϵ^T, and ϵ^B. From (22), households' aggregate deposit holdings, M_t^H/K_t^T, is $\alpha\left(\frac{\rho-r_t}{\beta(t)}\right)^{-\frac{1}{\xi}}$. Entrepreneurs' aggregate deposit holdings (scaled by K_t^T), M_t^E/K_t^T, is given by

$$\frac{M_t^E}{K_t^T}=\frac{\left(x_t^B-1\right)N_t^B-M_t^H}{K_t^T}=\left(x_t^B-1\right)\eta_t-\alpha\left(\frac{\rho-r_t}{\beta\left(t\right)}\right)^{-\frac{1}{\xi}}\,. \tag{A.44}$$

The aggregate intangible investment (scaled by K_t^T) is $\theta_t M_t^E/K_t^T$ and the aggregate tangible investment (scaled by K_t^T) is $\left(1-\theta_t\right)M_t^E/K_t^T$. Now it has been proven that the price variables and K_t^T-scaled aggregate quantities listed in Proposition 3 are bivariate functions of η_t and t.

The hierarchy of state variables. Time has its autonomous law of motion. The law of motion of η_t in the proof of Proposition 3 only depends on η_t and time t. The law of motion of K_t^T (i.e., (26) in the main text) only depends on η_t, time t, and K_t^T: using (A.7), I obtain

$$\text{(A.45)} \quad \frac{dK_t^T}{K_t^T} = \left[\left(\frac{\left(x_t^B - 1\right)\eta_t - \alpha \left(\frac{\rho - r_t}{\beta(t)}\right)^{-\frac{1}{\xi}}}{1 - q_t^T \kappa^T (1-\theta_t)} \right) \kappa^T (1-\theta_t)\lambda - \delta \right] dt + \sigma dZ_t \,,$$

where the drift is solved in the proof of Proposition 3 and the endogenous variables on the right side are bivariate functions of η_t and t. Finally, rewriting (25) from the main text, I obtain the law of motion of K_t^I, which depends on all four state variables,

$$\frac{dK_t^I}{K_t^I} = \frac{K_t^T}{K_t^I} \left[\left(x_t^B - 1\right) \frac{N_t^B}{K_t^T} - \frac{M_t^H}{K_t^T} \right] \left(\frac{1}{1 - q_t^T \kappa^T (1-\theta_t)} \right) \theta_t \kappa^I(t) \lambda dt - (\delta dt - \sigma dZ_t)$$

(A.46)

$$= \left[\frac{K_t^T}{K_t^I} \left(\frac{\left(x_t^B - 1\right)\eta_t - \alpha \left(\frac{\rho - r_t}{\beta(t)}\right)^{-\frac{1}{\xi}}}{1 - q_t^T \kappa^T (1-\theta_t)} \right) \kappa^I(t)\theta_t \lambda - \delta \right] dt + \sigma dZ_t \,.$$

Solving the model with tradable intangibles. Allowing χ fraction of intangible capital to be tradable among entrepreneurs and households only change the optimality conditions for θ and i. The rest of solution algorithm is the same as that of the main model. The F.O.C. for θ_t is

$$\text{(A.47)} \qquad q^I \kappa^I (1 + \chi\pi) - q^T \kappa^T (1+\pi) - F'(\theta) = 0 \,.$$

In contrast to (A.9), the marginal benefit of creating intangible capital has an additional component $q^I \kappa^I \chi \pi$ from relaxing the financial constraint. The F.O.C. for i is

(A.48)

$$\pi = \left\{ \left[q^I \kappa^I \theta + q^T \kappa^T (1-\theta) - F(\theta) \right] - 1 \right\} \left(\frac{1}{1 - q^T \kappa^T (1-\theta) - \chi q^I \kappa^I \theta} \right)$$

Given that $F(\theta) = \frac{\phi}{2}\theta^2$, (A.47) implies a quadratic equation for θ when π is substituted out by (A.48). Once θ is solved, (A.48) solves π.

A3. Matlab Solution Verification

Matlab versions. The goal of "main.m" in the replication package is to solve the system of differential equations (A.35) and (A.36) in Appendix A.2. It requires Matlab 2017a. Runing the code in other versions of Matlab may generate errors. Below I explain the reasons by describing how the numerical solution is computed.

The differential equations (A.35) and (A.36) are for $q^B(\eta, t)$ and $q^T(\eta, t)$. These equations are obtained from the equilibrium conditions in the main text. The procedure is explained in Appendix A.2, following closely Brunnermeier and Sannikov (2014). Solving differential equations (A.35) and (A.36) is a boundary value problem with the boundary conditions (A.37) to (A.41) in Appendix A.2. Given time t, the problem is akin to that in Brunnermeier and Sannikov (2014), and I adopt their shooting method, which involves searching for the initial conditions at the left (lower) boundary of η such that the conditions at the right (upper) boundary are satisfied.[66] When implementing this method, ode45 of Matlab R2017a is used to integrate from the left boundary to the right boundary.

As in Brunnermeier and Sannikov (2014), the right boundary of η, denoted by $\overline{\eta}$, is a free boundary that is determined by equilibrium conditions. There are three right-boundary conditions (A.39), (A.40) and (A.41). One boundary condition, for example, (A.39), can be used to pin down $\overline{\eta}$, i.e., the rightmost point or endpoint of integration (this endpoint of integration is specified in the "tspan" argument of ode45), while the other two right-boundary conditions are satisfied by adjusting the initial (left-) boundary conditions of the shooting method.[67]

Numerical integration by ode45 of Matlab R2017a starts from the leftmost point of η and ends at the rightmost point $\overline{\eta}$. However, when "main.m" is run on other versions of Matlab, the numerical integration may be interrupted by errors. For example,there may not exist a real-number solution (a real root) of the quadratic equation for σ^η (see the solution for σ^η below eq.(A.33) in Appendix A.2).

In the following, I will discuss two issues: First, why the numerical integration in Matlab 2017a and numerical integration in other versions of Matlab can differ even though the integration starts from the same initial conditions. Second, I will use the behavior of numerical integration in Matlab R2021b as an example.

What causes the numerical integration in Matlab R2017a and that in other versions to behave differently? The way that ode45 computes numerical integration changes across Matlab versions. One of such changes is how integration step sizes are determined.[68] The difference in step size across versions is reflected

[66]The shooting method is a method for solving a boundary value problem by reducing it to an initial value problem. It involves finding solutions to the initial value problem for different initial conditions until one finds the solution that also satisfies the boundary conditions of the boundary value problem.

[67]The left-boundary conditions (A.37) and (A.38) pin down a subset of initial conditions for the numerical integration. The rest of initial conditions are adjusted, so that, when the numerical integration ends at the rightmost point $\overline{\eta}$, all the right-boundary conditions are satisfied.

[68]Note that ode45 does not allow the users to specify the step size of numerical integration. The ode45 solver uses its own internal procedure to determine the step size (see the ode45 user manual on the official website of Matlab at https://www.mathworks.com/help/matlab/ref/ode45.) This internal procedure can

directly in the difference in how the interval of η is discretized. For example, $t = 20$ (the terminal date of the model), ode45 of Matlab R2017a discretizes the interval (specified in the "tspan" argument of ode45 in the codes) into 609 values, while ode45 of Matlab R2021b discretizes the same interval into 621 values. The difference in the integration step size affects the numerical solution: the solver integrates from the left to right with the output of every step being an input for the next step, so the step size, which is associated with numerical approximation error (i.e., the solution being too high or too low relative to the true value), affects the solver's output.[69] Therefore, since the step size of ode45 differs across different versions of Matlab, the computation of numerical integration can differ.

Next, I use Matlab R2021b as an example to discuss the error message reported when "main.m" is run on Matlab versions other than R2017a. In Matlab R2017a, the solution has first derivative $q^T_\eta(\eta, t)$ falling to zero (from positive values) as the integration moves towards the endpoint $\overline{\eta}$, consistent with the boundary condition (A.39). As previously discussed, when the codes were developed in Matlab R2017a, all the right-boundary conditions, including the condition (A.39) (i.e., $q^T_\eta(\overline{\eta}, t) = 0$), are satisfied at $\overline{\eta}$. However, when the code is run in other versions of Matlab, as the integration moves towards $\overline{\eta}$, the numerical solution may have $q^T_\eta(\eta, t)$ dip slightly below zero. A negative value of $q^T_\eta(\eta, t)$ is inconsistent with the equilibrium path and thus can cause a failure of solving σ^η using the equilibrium conditions (there may not exist a real-number solution to the quadratic equation that solves σ^η; see the solution for σ^η under eq.(A.33) in Appendix A.2).

In general, as ode45 computes numerical integration from the left to right boundaries of η, it generates values of $q^B(\eta, t)$ and $q^T(\eta, t)$ and their first-derivatives step-by-step, with output of each step being input of the next step. The output of each step contains numerical errors that depend on the integration step size and can cause the solutions to be too high or too low relative to the true values. Such numerical errors may cause the ode45-computed values of $q^B(\eta, t)$ and $q^T(\eta, t)$ and their first-derivatives to deviate from the true equilibrium values in certain steps and thereby cause failure of computing certain variables in the next step (which interrupts the numerical integration and generates error messages).

Theoretically, this can happen to any Matlab version. It didn't happen in Matlab R2017a, because the initial conditions for the shooting method (ode45)

vary across different Matlab versions, leading to different integration step sizes. Specifically, the "tspan" argument specifies the starting and end points of integration but it cannot specify the steps of numerical computation—when a user specifies more than two values in "tspan", the points given by "tspan" are where the solution is reported, not where the numerical integration is computed. The solution is first computed by ode45 using an internal adaptive method to determine step size of numerical integration, and then, the already computed solution is evaluated at the points specified in "tspan".

[69]Numerical methods for solving differential equations, for example, the Runge-Kutta method used in ode45, conduct piece-wise approximation of an integration. Each step introduces an approximation error, which can be too high or too low relative to the true value. As the method progresses through many steps (in case of ode45, the integration moves from the left to the right boundary of the integration interval specified in the "tspan" argument), the errors introduced at each step accumulate. The way these errors add up can vary significantly with the step size. Changes in step size can change the pattern of how and where numerical approximations deviate from the true solution.

were adjusted to make sure that the boundary conditions are satisfied and, at the same time, the numerical integration can run from the left to right without being disrupted by failure of computing certain endogenous variables. Such adjustment can be done for other versions of Matlab, but any adjustment would be version-specific, as the numerical errors that cause the values of $q^B(\eta, t)$ and $q^T(\eta, t)$ and their first-derivatives to deviate from their true equilibrium values are version-specific, dependent on integration size and more generally the internal integration procedure of ode45 that can vary across versions. Therefore, the meaningful exercise is to check whether the solution computed in Matlab R2017a indeed satisfies the equilibrium conditions to a high precision, which is done by "solution_check.m" in the replication package and will be discussed in details.

One may argue that if the solution (including the initial conditions) computed in Matlab R2017a is indeed the true solution, it should not be specific to Matlab versions, and starting from these true initial conditions, numerical integration by ode45 in other versions of Matlab should be able to run through smoothly and finishes at $\overline{\eta}$ just as the numerical integration does in Matlab R2017a. However, this argument is flawed for two reasons. First, it can never be guaranteed that the numerical solution (including the initial conditions) founded in Matlab R2017a is exactly the true solution. A numerical solution is an approximation. As previously emphasized, the meaningful exercise is to check whether the solution computed in Matlab R2017a indeed satisfies the equilibrium conditions to a high precision. Second, even if the solution and initial conditions computed in Matlab R2017a happen to be exactly true without any approximation error, numerical integration in other versions of Matlab still suffers from numerical errors in every step of integration. Such errors depend on the integration step size, which can be versions-specific, so, numerical integration may still encounter disruption in other versions even though it delivers the exactly true solution in R2017a.

So far, I have explained why running "main.m" using other versions of Matlab can generate error messages. Next, I will explain how to verify that the solution computed by "main.m" in Matlab R2017a satisfies the equilibrium conditions to a high precision. The goal of my paper is to characterize the equilibrium of the economy specified in Section III. Once the equilibrium is computed and this solution is shown to satisfy the equilibrium conditions, the equilibrium is found. The verification procedure below supports the use of all versions of Matlab.

Verifying the solution. The solution generated by Matlab R2017a is saved in the data file "solution.mat" in the replication package. It contains the functions, $q^B(\eta, t)$ and $q^T(\eta, t)$, and their first derivatives. To verify that the solution indeed solves the differential equations (A.35) and (A.36) in Appendix A.2. I provide the code "solution_check.m" that can be run using any version of Matlab. In the following, I explain the rationale behind the procedure of solution verification in "solution_check.m" and the output of this program.

The system of second-order differential equations, given by equations (A.35) and (A.36) in Appendix A.2, takes as inputs the level of the functions, $q^B(\eta, t)$

and $q^T(\eta,t)$, and their first-order derivatives to generate the second derivatives with respect to η, i.e., $q^B_{\eta\eta}(\eta,t)$ and $q^T_{\eta\eta}(\eta,t)$.[70] The solution to this system of differential equations contains the levels and first derivatives of $q^B(\eta,t)$ and $q^T(\eta,t)$. Mathematically, the solution is a fixed point of the following mapping (functional), denoted by "$\mathcal{T}$": first, the levels and first derivatives generate the second derivatives (via the system of differential equations); second, these second derivatives can be integrated to obtain the levels and first derivatives. Formally, this two-step procedure reformulates a system of differential equations into an equivalent system of integral equations. The mapping, $\mathcal{T}$, is this system of integral equations.

The program, "solution_check.m", checks whether the solution—the functions—$q^B(\eta,t)$ and $q^T(\eta,t)$, and their first derivatives in the file "solution.mat"—indeed constitutes the fixed point of this mapping, i.e., $y = \mathcal{T}(y)$, where y represents the functions, $q^B(\eta,t)$ and $q^T(\eta,t)$ and first derivatives.

First, the program takes as inputs the functions, $q^B(\eta,t)$ and $q^T(\eta,t)$, and their first derivatives from the solution file, denoted by "y_0", and computes the second derivatives using the system of differential equations (A.35) and (A.36). Second, using these second derivatives, the program computes the numerical integral of $q^B_{\eta\eta}(\eta,t)$ and $q^T_{\eta\eta}(\eta,t)$ to obtain the first derivatives, and then it computes the numerical integral of these first derivatives to obtain the levels of functions $q^B(\eta,t)$ and $q^T(\eta,t)$. Let "y_1" denote these levels and first derivatives obtained from numerically integrating the output, $q^B_{\eta\eta}(\eta,t)$ and $q^T_{\eta\eta}(\eta,t)$, from the first step. Therefore, we have $y_1 = \mathcal{T}(y_0)$.

Finally, the program, "solution_check.m", generates figures that compare y_1 and y_0. These figures are saved in PDF formats in the Matlab working directory. Each PDF file corresponds a time t, and the figures plot over η the levels and first derivatives of $q^B(\eta,t)$ and $q^T(\eta,t)$. Since $q^B(\eta,t)$—marginal value of wealth of financial intermediaries—and its first derivative have values several magnitudes larger when η is low (close to zero) than when η is high, the program reports their logarithm values in the PDF figures.

In each figure, the blue solid lines represent y_1 (labeled as "Equilibrium Mapping Image" in the legend), and the black dashed lines represents y_0 (labeled as "Matlab 2017a Solution" in the legend). When running the program, Matlab command window reports the correlation between y_0 and y_1. Overall, when y_0 and y_1 are close, the program verifies that the functions, $q^B(\eta,t)$ and $q^T(\eta,t)$, and their first derivatives in the data file "solution.mat" constitutes a fixed point of the mapping $\mathcal{T}$, and thus are indeed the solution to the system of differential equations given by equations (A.35) and (A.36) in Appendix A.2. Figure A.3, A.2, and A.1 illustrate the PDF figures at $t = 20$, 10, and 0 generated by Matlab R2021b, and Figure A.4 illustrates the command-window output.

In summary, the program "main.m" solves differential equations (A.35) and (A.36) in Appendix A.2. This program requires Matlab R2017a. The solution—

[70] As previously discussed, this system is constructed from the equilibrium conditions in the main text. The procedure is explained in Appendix A.2 and follows that in Brunnermeier and Sannikov (2014).

the functions, $q^B(\eta, t)$ and $q^T(\eta, t)$, and their first derivatives—is saved in the data file "solution.mat". The program "solution_check.m" verifies that the solution is indeed a fixed point of the equilibrium mapping, i.e., the system of integral equations equivalent to the system of differential equations (A.35) and (A.36) in Appendix A.2. Solving the equilibrium and verifying the equilibrium takes different procedures, but both "main.m" and "solution_check.m" are based upon the same set of equilibrium conditions.

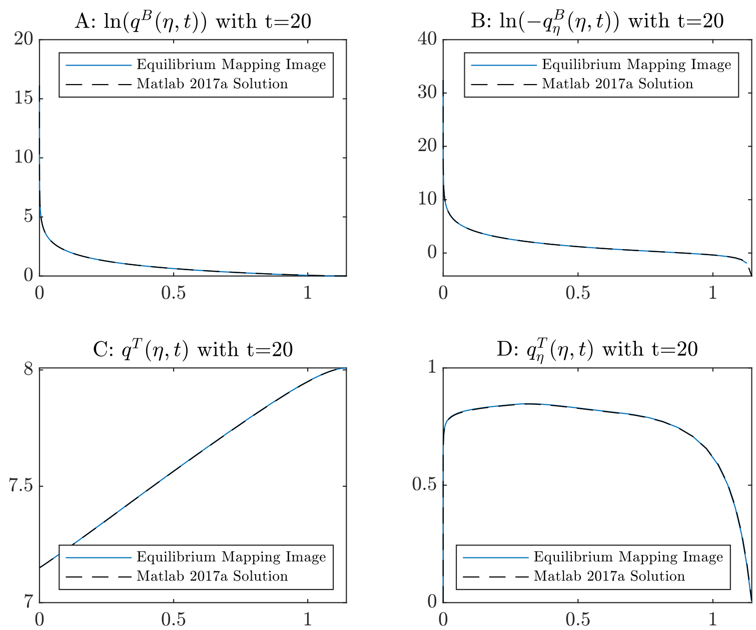


Figure A.1. : PDF Figure Output of solution_check.m for $t = 20$.

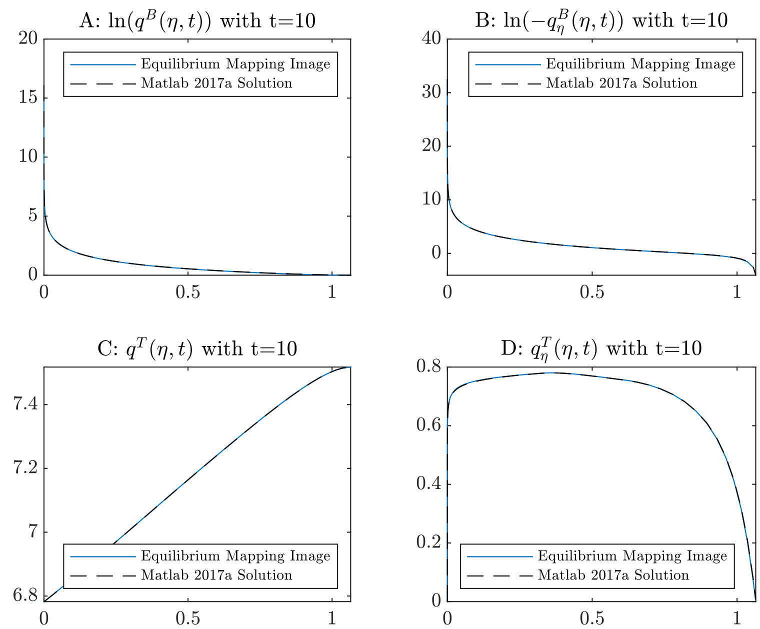


Figure A.2. : PDF Figure Output of solution_check.m for $t = 10$.

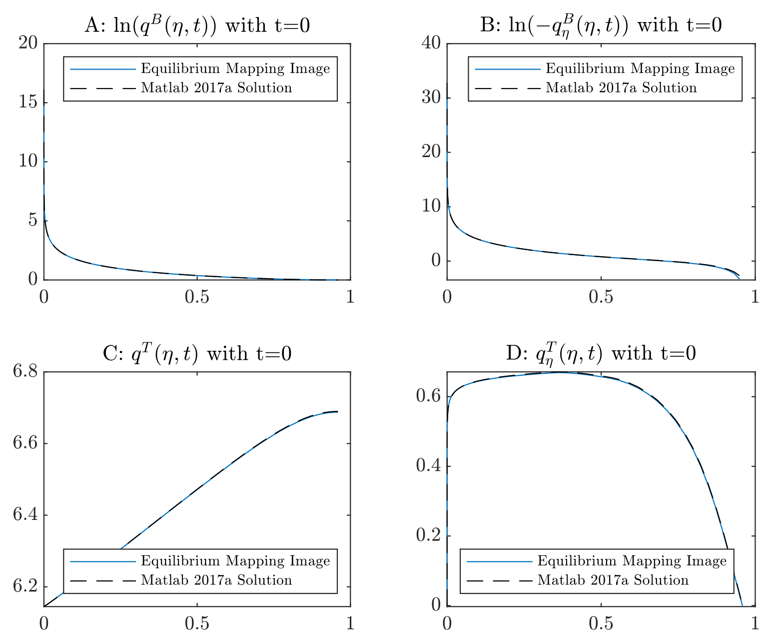
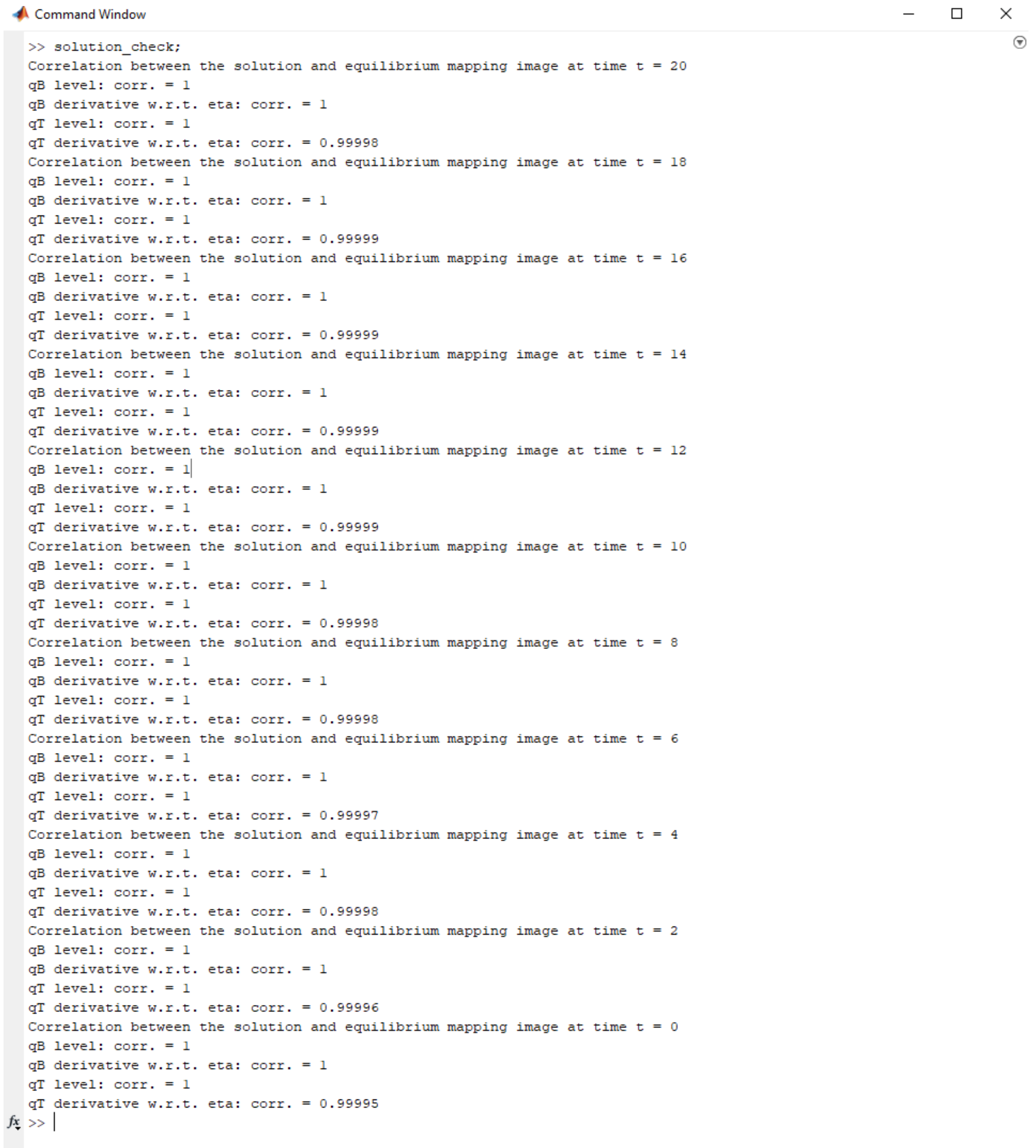


Figure A.3. : PDF Figure Output of solution_check.m for $t = 0$.



```
Command Window
>> solution_check;
Correlation between the solution and equilibrium mapping image at time t = 20
qB level: corr. = 1
qB derivative w.r.t. eta: corr. = 1
qT level: corr. = 1
qT derivative w.r.t. eta: corr. = 0.99998
Correlation between the solution and equilibrium mapping image at time t = 18
qB level: corr. = 1
qB derivative w.r.t. eta: corr. = 1
qT level: corr. = 1
qT derivative w.r.t. eta: corr. = 0.99999
Correlation between the solution and equilibrium mapping image at time t = 16
qB level: corr. = 1
qB derivative w.r.t. eta: corr. = 1
qT level: corr. = 1
qT derivative w.r.t. eta: corr. = 0.99999
Correlation between the solution and equilibrium mapping image at time t = 14
qB level: corr. = 1
qB derivative w.r.t. eta: corr. = 1
qT level: corr. = 1
qT derivative w.r.t. eta: corr. = 0.99999
Correlation between the solution and equilibrium mapping image at time t = 12
qB level: corr. = 1
qB derivative w.r.t. eta: corr. = 1
qT level: corr. = 1
qT derivative w.r.t. eta: corr. = 0.99999
Correlation between the solution and equilibrium mapping image at time t = 10
qB level: corr. = 1
qB derivative w.r.t. eta: corr. = 1
qT level: corr. = 1
qT derivative w.r.t. eta: corr. = 0.99998
Correlation between the solution and equilibrium mapping image at time t = 8
qB level: corr. = 1
qB derivative w.r.t. eta: corr. = 1
qT level: corr. = 1
qT derivative w.r.t. eta: corr. = 0.99998
Correlation between the solution and equilibrium mapping image at time t = 6
qB level: corr. = 1
qB derivative w.r.t. eta: corr. = 1
qT level: corr. = 1
qT derivative w.r.t. eta: corr. = 0.99997
Correlation between the solution and equilibrium mapping image at time t = 4
qB level: corr. = 1
qB derivative w.r.t. eta: corr. = 1
qT level: corr. = 1
qT derivative w.r.t. eta: corr. = 0.99998
Correlation between the solution and equilibrium mapping image at time t = 2
qB level: corr. = 1
qB derivative w.r.t. eta: corr. = 1
qT level: corr. = 1
qT derivative w.r.t. eta: corr. = 0.99996
Correlation between the solution and equilibrium mapping image at time t = 0
qB level: corr. = 1
qB derivative w.r.t. eta: corr. = 1
qT level: corr. = 1
qT derivative w.r.t. eta: corr. = 0.99995
>>
```

Figure A.4. : Command Window Output of solution_check.m

B. Risk Aversion and Finite EIS

In this section, I extend the model to incorporate risk-averse preferences and finite EIS (elasticity of intertemporal substitution) showing that the solution of the extended model can be achieved by making two modifications to the solution of the model in the main text. First, the functions of endogenous variables, such as $q^T(\eta_t, t)$, can be derived by the procedure in A.2 with the time discount rate ρ replaced by a function $\rho(\eta_t, t)$. The functional form of $\rho(\eta_t, t)$ depends on the risk-averse utility functions in the extended model.

Second, the laws of motion of state variables in the solution of the main model become the laws of motion under the risk-neutral measure in the extended model with risk aversion. To characterize the dynamics under the physical measure (probability measure of data generating process), a change of measure shall be performed using Girsanov's Theorem. The Markov equilibrium has four state variable: time, η_t, K_t^I, and K_t^T. A change of measure affects the laws of motion of the last three by adjusting their drifts. The adjustments depend on (1) the state variables' loadings of the aggregate shock (i.e., their diffusions) and (2) the consumers' price of risk implied by the risk-averse utility functions and the equilibrium process of aggregate consumption. This method of incorporating risk-averse preferences can be applied to other macro-finance models with risk-neutral preferences, (e.g., Brunnermeier and Sannikov, 2014).

After establishing these results, I characterize the conditions under which the equilibrium of the main (risk-neutral) model serves as an adequate approximation to the equilibrium of the extended model. The model solution has two parts, first, the endogenous variables as functions of state variables, for example, $q_t^T = q^T(\eta_t, t)$, and, second, the laws of motion of state variables. I show that the first part of the solution is an adequate approximation if the expected growth rate of consumption is stable, which holds in the model and is consistent with the theories and evidence on long-run consumption risk (Bansal and Yaron, 2004; Hansen, Heaton, and Li, 2008). I also show that ignoring risk aversion has little impact on the laws of motion of state variables under the standard risk aversion parameter in the asset pricing literature.

Incorporating preferences with risk aversion and finite EIS. Next, I introduce risk-averse preferences to the household sector. Entrepreneurs and bankers are reinterpreted as firms that maximize the present value of payouts to household shareholders, so households are the ultimate consumers in this economy. It is assumed that households face a complete market. For households, the relevant shock is the aggregate Brownian shock dZ_t. The market is complete if households can trade tangible capital and risk-free assets.[71] No arbitrage and complete mar-

[71] For risk-free assets, it is assumed that households can lend to and borrow from each other (in equilibrium, at the risk-free rate ρ_t), i.e., the negative drift of SDF in (B.1), and unlike deposits, such risk-free assets do not bring deposit-in-utility for households or relax entrepreneurs' liquidity constraints. They may represent personal IOUs.

kets imply the existence of a unique stochastic discount factor (SDF), denoted by Λ_t, which, in equilibrium, is determined by the households' marginal value of wealth (Duffie, 2001). The following analysis takes the equilibrium process of Λ_t as given,

$$\frac{d\Lambda_t}{\Lambda_t} = -\rho_t dt - \gamma_t^H d\widehat{Z}_t \,, \tag{B.1}$$

where ρ_t is the households' time discount rate in equilibrium and γ_t^H is the households' price of risk. The endogenous discount rate ρ_t replaces the parameter ρ in the main text. After analyzing the entrepreneurs' and banks' problems, I specify the households' preferences and solve Λ_t. The stochastic process $\widehat{Z}_t$ is the cumulative aggregate shock under the physical measure.

By Girsanov's Theorem, we know the following connection between the aggregate shock to capital stock, dZ_t, under the risk-neutral measure and $d\widehat{Z}_t$, the shock under the physical measure,

$$dZ_t = d\widehat{Z}_t + \gamma_t^H dt \,. \tag{B.2}$$

The idiosyncratic Poisson shocks do not affect the change of measure because they are not priced in the SDF. Entrepreneurs' information filtration under the physical measure is generated by $\widehat{Z}_t$ and the idiosyncratic Poisson shocks that trigger investment needs. Their information filtration under the risk-neutral measure is generated by Z_t and the same idiosyncratic Poisson shocks. For bankers, idiosyncratic risks are diversified away, so the relevant information filtration is generated by Z_t under the risk-neutral measure and $\widehat{Z}_t$ under the physical measure.

Girsanov's Theorem implies a connection between objective functions under the physical and risk-neutral measures: a representative entrepreneur i maximize

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-\rho_t t} dc_{i,t}^E\right] = \widehat{\mathbb{E}}\left[\int_0^{\infty} \frac{\Lambda_t}{\Lambda_0} dc_{i,t}^E\right] \,, \tag{B.3}$$

and

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-\rho_t t} dc_{i,t}^B\right] = \widehat{\mathbb{E}}\left[\int_0^{\infty} \frac{\Lambda_t}{\Lambda_0} dc_{i,t}^B\right] \,, \tag{B.4}$$

where $\widehat{\mathbb{E}}[\cdot]$ is the rational-expectation operator under the physical measure, distinguished from $\mathbb{E}[\cdot]$, the rational-expectation operator under the risk-neutral measure, and, following the notations in Appendix A, $c_{i,t}^E$ and $c_{j,t}^B$ denotes the cumulative payout of non-financial firms and banks.

The full solution of the model consists of two parts: first, the endogenous variables as functions of state variables, for example, $q_t^T = q^T(\eta_t, t)$, and, second, the laws of motion of state variables. The next proposition states the connection between an extended model with power utility and the model in the main text.

The proof is at the end of this section. This method of incorporating risk-averse preferences into an originally risk-neutral model applies to any utility function. I use power (CRRA) utility as an example.

PROPOSITION B.1: *Households have power utility over consumption and deposit-in-utility introduced in (19) and maximize*

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-\delta_H t}\left(\frac{\left(c_t^H\right)^{1-\underline{\gamma}_H}}{1-\underline{\gamma}_H}dt+\beta_t\frac{\left(m_t^H/w_t^H\right)^{1-\xi}}{1-\xi}\right)\right], \tag{B.5}$$

where δ_H and $\underline{\gamma}_H$ are the parameters for discount factor and relative risk aversion, respectively, and c_t^H denote the rate of consumption (instead of cumulative consumption).

The solutions of endogenous variables as functions of state variables can be obtained by the procedure in A.2 with the parameter ρ replaced by the following function:

$$\begin{aligned}\rho\left(\eta_t,t\right)=\underline{\delta}_H+\underline{\gamma}_H\mu^{KT}\left(\eta_t,t\right)+\underline{\gamma}_H\epsilon^{\widetilde{C}1}\left(\eta_t,t\right)\left[\mu^{\eta}\left(\eta_t,t\right)+\frac{1}{2}\epsilon^{\widetilde{C}2}\left(\eta_t,t\right)\sigma^{\eta}\left(\eta_t,t\right)^2+\sigma^{\eta}\left(\eta_t,t\right)\sigma\right]\\+\frac{1}{2}\left(\underline{\gamma}_H^2-\underline{\gamma}_H\right)\left[\epsilon^{\widetilde{C}1}\left(\eta_t,t\right)\sigma^{\eta}\left(\eta_t,t\right)+\sigma\right]^2,\end{aligned} \tag{B.6}$$

where $\mu^{\eta}(\eta_t,t)$, $\sigma^{\eta}(\eta_t,t)$, and $\mu^{KT}(\eta_t,t)$ are given by (A.5), (A.6), and (A.7) respectively in A.1, and $\epsilon^{\widetilde{C}1}(\eta_t,t)$ is the elasticity of K_t^T-scaled aggregate consumption, $\widetilde{C}_t^H\equiv C_t^H/K_t^T$, to η_t,

$$\epsilon^{\widetilde{C}1}\left(\eta_t,t\right)\equiv\frac{\partial\widetilde{C}^H\left(\eta_t,t\right)}{\partial\eta_t}\frac{\eta_t}{\widetilde{C}^H\left(\eta_t,t\right)}, \tag{B.7}$$

and $\epsilon^{\widetilde{C}2}(\eta_t,t)$ is the elasticity of $\frac{\partial\widetilde{C}^H(\eta_t,t)}{\partial\eta_t}$ to η_t,

$$\epsilon^{\widetilde{C}2}\left(\eta_t,t\right)\equiv\frac{\partial^2\widetilde{C}^H\left(\eta_t,t\right)}{\partial\eta_t^2}\frac{\eta_t}{\left(\frac{\partial\widetilde{C}^H(\eta_t,t)}{\partial\eta_t}\right)}. \tag{B.8}$$

By Girsanov's Theorem, the laws of motion of η_t, K_t^T, and K_t^I are given by (A.4), (A.45), and (A.46) respectively with dZ_t, the Brownian shock under the risk-neutral measure, replaced by

$$d\widehat{Z}_t+\gamma^H\left(\eta_t,t\right)dt \tag{B.9}$$

where $d\widehat{Z}_t$ is the Brownian shock under the physical measure, and $\gamma^H(\eta_t,t)$ is *given by*

$$\gamma^H(\eta_t, t) = \underline{\gamma}_H \left[\epsilon^{\widetilde{C}1}(\eta_t, t)\,\sigma^\eta(\eta_t, t) + \sigma\right], \tag{B.10}$$

which is the households' price of risk in equilibrium.

Comparing risk-neutral and risk-averse models. In the comparison between the risk-neutral and risk-averse models, a key object is $\epsilon^{\widetilde{C}1}(\eta_t, t)$, the the elasticity of K_t^T-scaled aggregate consumption, $\widetilde{C}_t^H \equiv C_t^H / K_t^T$, to η_t. Given $C_t^H = \widetilde{C}_t^H K_t^T$, by Itô's lemma, the volatility of consumption growth, σ_t^C is given by

$$\sigma_t^C = \epsilon^{\widetilde{C}1}(\eta_t, t)\,\sigma^\eta(\eta_t, t) + \sigma\,. \tag{B.11}$$

The constant return-to-scale technology implies that the volatility of capital growth, σ, is the volatility of output growth. Empirically, consumption growth is less volatile in data than output growth (e.g., Blanchard and Simon, 2001). Therefore, if the preference parameters are calibrated to match consumption volatility (as typically done in the asset-pricing literature (Cochrane, 2005a)), we have

$$\epsilon^{\widetilde{C}1}(\eta_t, t) < 0\,. \tag{B.12}$$

The model solution has two parts: first, the endogenous variables as functions of state variables, for example, $q_t^T = q^T(\eta_t, t)$, and, second, the laws of motion of state variables. Therefore, according to Proposition B.1, a potential misspecification from ignoring risk aversion has two consequences. First, in the algorithm that solves the functions of endogenous variables in A.2, ρ should be replaced by $\rho(\eta_t, t)$. Second, the laws of motions of state variables are in fact risk-neutral dynamics. The dynamics under the physical measure require an adjustment of drifts by replacing dZ_t with $d\widehat{Z}_t + \gamma^H(\eta_t, t)\,dt$ (see B.2).

To analyze the impact of ignoring risk aversion on the functions of endogenous variables, I examine whether $\rho(\eta_t, t)$ can be approximated by a constant. The expression of $\rho(\eta_t, t)$ in (B.6) can be simplified with the consumption growth volatility in (B.11):

$$\begin{aligned}\rho(\eta_t, t) = {} & \underline{\delta}_H + \underline{\gamma}_H \mu^{KT}(\eta_t, t) + \underline{\gamma}_H \epsilon^{\widetilde{C}1}(\eta_t, t)\left[\mu^\eta(\eta_t, t) + \frac{1}{2}\epsilon^{\widetilde{C}2}(\eta_t, t)\,\sigma^\eta(\eta_t, t)^2 + \sigma^\eta(\eta_t, t)\,\sigma\right] \\ & + \frac{1}{2}\left(\underline{\gamma}_H^2 - \underline{\gamma}_H\right)\left(\sigma_t^C\right)^2.\end{aligned} \tag{B.13}$$

Let $O(\sigma^2)$ denote the terms that involve the squared volatilities of growth rates (which all contain σ^2). Because volatilities and expectations of growth rates are of similar magnitudes in this model where aggregate quantities are driven by geometric Brownian motions, these volatility-squared terms tend to be small. Therefore, I use the following expression

$$\rho\left(\eta_t,t\right)=\underline{\delta}_H+\underline{\gamma}_H\mu^{KT}\left(\eta_t,t\right)+\underline{\gamma}_H\epsilon^{\widetilde{C}1}\left(\eta_t,t\right)\mu^{\eta}\left(\eta_t,t\right)+O\left(\sigma^2\right)\ . \tag{B.14}$$

The first and second terms are standard in asset pricing models. Given the constant return-to-scale technology, the capital growth rate, $\mu^{KT}\left(\eta_t,t\right)$, is the growth rate of aggregate output. In an endowment economy, the aggregate output (agents' endowments) is equal to the aggregate consumption in equilibrium, so, in these consumption-based models, the equilibrium risk-free rate only contains the first two terms on the right side of (B.14) (e.g., Lucas, 1978).

The third term is unique to this model. The drift of η_t, $\mu^{\eta}\left(\eta_t,t\right)$, is the expected growth rate of the ratio of bankers' wealth to tangible capital value. Because bankers hold a leveraged position in tangible capital, and the expected return on tangible capital is positive, bankers' wealth grows faster than tangible capital in expectation, and $\mu^{\eta}\left(\eta_t,t\right)$ is positive. Given that $\epsilon^{\widetilde{C}1}\left(\eta_t,t\right)<0$ (see (B.12)), the third term on the right side of (B.14) is negative.

The economy becomes more intangible-intensive over time, and firms hold more cash, which leads to an upward trend in investment and output growth. The counteracting force is also getting stronger. As the economy becomes more intangible-intensive, the liquidity premium on deposits, ρ_t-r_t, becomes larger, which increases bankers' return on wealth, and thus, pushes up $\mu^{\eta}\left(\eta_t,t\right)$.

Assuming $\rho\left(\eta_t,t\right)$ is a constant in the main model is equivalent to assuming that these two forces, $\underline{\gamma}_H\mu^{KT}\left(\eta_t,t\right)>0$ and $\underline{\gamma}_H\epsilon^{\widetilde{C}1}\left(\eta_t,t\right)\mu^{\eta}\left(\eta_t,t\right)<0$, cancel each other out. The first force is from output growth. The second is from the fact that consumption is less volatile than output growth and, due to leverage, bankers' expected return on wealth is greater than tangible capital. Empirically, this assumption means that the *expected* consumption growth rate is stable. A stable consumption growth rate is consistent with the findings of highly persistent expected consumption growth in the literature on long-run risk (Bansal and Yaron, 2004; Hansen, Heaton, and Li, 2008). approximating $\rho\left(\eta_t,t\right)$ by a constant does not cause significant misspecification. When this approximation is adequate, the functions of endogenous variables, for example, $q_t^T=q^T\left(\eta_t,t\right)$, that are solved in A.2 and presented in Section 4.2, are adequate approximations to the functions from the risk-averse model.

Next, I examine the impact of ignoring risk-aversion on the laws of motion of state variables. According to Proposition B.1, the dynamics of capital stocks given by (A.45) and (A.46) should be adjusted by replacing the Brownian shock under the risk-neutral measure, dZ_t, by the Brownian shock under the physical measure (i.e., the real shock that drives the data generating processes), $d\widehat{Z}_t$, plus a drift adjustment $\gamma^H(\eta_t, t)\,dt$:

(B.15)

$$\frac{dK_t^T}{K_t^T} = \left[\left(\frac{\left(x_t^B - 1\right)\eta_t - \alpha\left(\frac{\rho_t - r_t}{\beta(t)}\right)^{-\frac{1}{\xi}}}{1 - q_t^T \kappa^T (1-\theta_t)}\right)\kappa^T (1-\theta_t)\lambda - \delta\right] dt + \underbrace{\sigma\gamma^H(\eta_t, t)\,dt}_{\textit{risk adjustment}} + \sigma d\widehat{Z}_t\,,$$

and

(B.16)

$$\frac{dK_t^I}{K_t^I} = \left[\frac{K_t^T}{K_t^I}\left(\frac{\left(x_t^B - 1\right)\eta_t - \alpha\left(\frac{\rho_t - r_t}{\beta(t)}\right)^{-\frac{1}{\xi}}}{1 - q_t^T \kappa^T (1-\theta_t)}\right)\kappa^I(t)\,\theta_t\lambda - \delta\right] dt + \underbrace{\sigma\gamma^H(\eta_t, t)\,dt}_{\textit{risk adjustment}} + \sigma dZ_t\,.$$

Consider a relative risk aversion $\underline{\gamma}_H = 5$, which is a common value in the asset pricing literature (Cochrane, 2005a). Given that $\epsilon^{\widetilde{C}1}(\eta_t, t) < 0$ and $\sigma^\eta(\eta_t, t) > 0$ (see (A.6)), I obtain the following upper bound on the households' price of risk: from (B.34),

$$\text{(B.17)} \qquad \gamma^H(\eta_t, t) = \underline{\gamma}_H\left[\epsilon^{\widetilde{C}1}(\eta_t, t)\,\sigma^\eta(\eta_t, t) + \sigma\right] \le \underline{\gamma}_H\sigma = 0.1\,,$$

where the last equation substitutes in the value of $\underline{\gamma}_H$ and σ. Given that $\sigma = 0.02$ and $\gamma^H(\eta_t, t) < 0.1$, the risk adjustment term is bounded above by 0.002. Therefore, ignoring risk aversion understates the expected growth rate of capital (and output), and the wedge is bounded above by 0.2%. The physical-measure dynamics feature higher growth rates than those of the risk-neutral dynamics because, when changing from the physical measure to the risk-neutral measure, probability mass shifts towards the relatively worse states of the world, i.e., the risk-averse attitude is reflected by probability re-weighting. A similar calculation can be applied to the law of motion of η_t. The risk adjustment increases the drift of η_t, and, averaging over time t and η_t on the simulated paths, such an increase is less than 6% of the drift, (i.e., $< 0.06 \times \mu_t^\eta$).

In the main text, I report the model's solutions in two ways: (1) the values of endogenous variables at different points in time, averaged over η_t (e.g., Section 4.2) and (2) endogenous variables as functions of η_t (e.g., Section 4.3). The impact of ignoring risk aversion on the laws of motion of state variable only affects (1), and (2) depends on whether replacing $\rho(\eta_t, t)$ with a constant ρ is an adequate approximation, as previously discussed.

This concludes the discussion on the consequences of model misspecification from ignoring risk aversion. Next, I derive the equations in Proposition B.1.

Proof of Proposition B.1. First, I solve the entrepreneurs' problem and the bankers' problem under the risk-neutral measure, taking as given the stochastic discount factor (SDF). After specifying the households'/consumers' risk-averse utility function, I solve the SDF and perform the change of measure to obtain the physical-measure dynamics of the extended model. This method of solving models under the risk-neutral measure and then analyzing the physical-measure dynamics by applying Girsanov's Theorem is often used in settings of complete markets (Duffie, 2001).

The entrepreneurs' investment problem stays intact as it is a static problem happening only at idiosyncratic Poisson times. Therefore, the Lagrange function defined by (11) still summarizes the investment problem, and the marginal value of liquidity for the investment projects, π_t, is given by (13). Because the time discount rate changes from ρ to ρ_t, (15) in Proposition 1 is now

$$r_t = \rho_t - \lambda \pi_t \,. \tag{B.18}$$

The rest of Proposition 1 hold.

Given the homogeneity property of the bankers' problem, their value function is still $q_t^B n_t^B$, where the marginal value of equity, $q_t^B = q^B(\eta_t, t)$, has the law of motion (16) under *the risk-neutral measure.* Proposition 2 can still be used to characterize the valuation of tangible capital and the bankers' required expected return on tangible capital holdings under the *risk-neutral* measure. Note that if bankers can also access complete markets as households can, their marginal value of wealth, q_t^B, will be pinned to one, and their price of risk, γ_t^B, to zero. Being able to freely trade the aggregate shock with households is equivalent to being able to freely raise equity from households (Di Tella, 2017). Therefore, it is assumed that bankers cannot hedge the aggregate shock.[72]

Bankers' required expected return under the risk-neutral measure is (17) in Proposition 2. Under the physical measure, by Girsanov's Theorem, it becomes

$$\widehat{\mathbb{E}}_t\left[dr_t^T\right] = r_t + \gamma_t^B\left(\sigma_t^T + \sigma\right) + \gamma_t^H\left(\sigma_t^T + \sigma\right) \,. \tag{B.19}$$

Under the physical measure, banks require risk compensations not only due to the equity issuance constraint, $\gamma_t^B\left(\sigma_t^T + \sigma\right)$, but also on behalf of the household shareholders, $\gamma_t^H\left(\sigma_t^T + \sigma\right)$.

The valuation equation (18) for tangible capital in Proposition 2 still holds. The derivation in Appendix A applies under the risk-neutral measure. Equation (18) can also be derived under the physical measure but the law of motion of q_t^T and the stochastic depreciation of capital holdings have to be adjusted by the

[72] Imperfect hedging can be easily incorporated. For example, bankers can only hedge a fraction χ^B of aggregate risk due to agency friction and the need to keep "skin in the gamme" (He and Krishnamurthy, 2013). Note that given that hedging is free and bankers are effectively risk averse, bankers will hedge as much as they can. Then bankers' risk exposure for one dollar of holdings of tangible capital is $\left(1-\chi^B\right)\left(\sigma_t^T + \sigma\right)$ in equilibrium, i.e., scaled down by χ^B fraction. Bankers' required expected return under the risk-neutral measure becomes $r_t + \gamma_t^B\left(1-\chi^B\right)\left(\sigma_t^T + \sigma\right)$. After the scaling, the same solution procedure still applies. Because the scaling reduces bankers' discount rate and increase the value of tangible capital and entrepreneurs' leverage on liquidity holdings, it amplifies the feedback mechanism.

change of measure. Under the risk-neutral measure:

$$\frac{dq_t^T}{q_t^T} = \mu_t^T dt + \sigma_t^T dZ_t\,, \tag{B.20}$$

and, given (B.2), under the physical measure

$$\frac{dq_t^T}{q_t^T} = \left(\mu_t^T + \gamma_t^H \sigma_t^T\right) dt + \sigma_t^T d\widehat{Z}_t\,, \tag{B.21}$$

where the price of risk γ_t^H is multiplied by the quantity of risk σ_t^T. When moving from (B.21) to (B.20), the drift is adjusted downward, reflecting a risk adjustment via the shift of probability mass towards relatively worse states of the model. Risk aversion is reflected in the adjustment of the probability mass. The stochastic depreciation rate of capital under the physical measure is

$$\left(\delta - \gamma_t^H \sigma\right) + \sigma d\widehat{Z}_t\,. \tag{B.22}$$

The expected depreciation rate is adjusted upward when moving from the physical measure, $\delta - \gamma_t^H \sigma$, to the risk-neutral measure, δ, as the probability mass is shifted towards relatively worse states of the world to reflect risk aversion encoded in the SDF. The expected return of tangible capital holdings consists of the dividend yield, $1/q_t^T$, the expected price appreciation, $\mu_t^T + \gamma_t^H \sigma_t^T$, the expected capital depreciation, $\left(\delta - \gamma_t^H \sigma\right) + \lambda$ (counting both the normal-time depreciation and idiosyncratic Poisson destruction), and the quadratic covariation $\sigma_t^T \sigma$ from Itô's calculus, which does not change due to the volatility-invariance property of change of measure Duffie (2001). In equilibrium, the expected return is equal to bankers' required expected return in (B.19):

$$r_t + \gamma_t^B \left(\sigma_t^T + \sigma\right) + \gamma_t^H \left(\sigma_t^T + \sigma\right) = \frac{1}{q_t^T} + \left(\mu_t^T + \gamma_t^H \sigma_t^T\right) - \left[\left(\delta - \gamma_t^H \sigma\right) + \lambda\right] + \sigma_t^T \sigma\,.$$

Note that $\gamma_t^H \left(\sigma_t^T + \sigma\right)$ shows up on both sides. Rearranging the equation, we obtain (18).

For any stochastic process, its dynamics under the risk-neutral measure can be adjusted to obtain the dynamics under the physical measure. For instance, under the risk-neutral measure,

$$\frac{dq_t^B}{q_t^B} = \mu_t^B dt - \gamma_t^B dZ_t\,, \tag{B.23}$$

so, (B.2) implies that the law of motion of q_t^B under the physical measure is given by

$$\frac{dq_t^B}{q_t^B} = \left(\mu_t^B - \gamma_t^B \gamma_t^H\right) dt - \gamma_t^B d\widehat{Z}_t \,, \tag{B.24}$$

where the diffusion stays the same (i.e., the standard diffusion-invariance result) and the drift of q_t^B is "risk-adjusted". Note that q_t^B is high in the relatively worse states of the world where banks are undercapitalized. The expected appreciation of q_t^B is adjusted upward when moving from (B.24) to (B.23) because, when changing from the physical measure to risk-neutral measure, more probability mass is shifted towards the relatively worse states of the world.

Given the function $\rho(\eta_t, t)$, the procedure in A.2 can be used to solve all the variables listed in Proposition 3, and then, the laws of motion of η_t, K_t^T, and K_t^I can be derived. These laws of motion are under the risk-neutral measure, so a change of measure needs to be performed to obtain the physical-measure laws of motion. As I have shown for q_t^T and q_t^B, change of measure simply entails substituting out the Brownian shock under the risk-neutral measure, dZ_t, using (B.2).

Using the procedure in A.2 to solve the model's dynamics under the risk-neutral measure only requires the function $\rho(\eta_t, t)$. It does not require the households' utility function. To perform the change of measure, I need to have the price of risk, γ_t^H, as a function of the state variables.

Next, I solve the SDF, linking ρ_t and γ_t^H to households' consumption (and wealth) dynamics. Specifically, I confirm that ρ_t only depends on η_t and time t, i.e., $\rho_t = \rho(\eta_t, t)$, and solve the functional form. I also solve the households' price of risk, γ_t^H, as a function of these state variables.

In the following, I consider the standard time-separable power utility as an example. In this case, the stochastic discount factor is the time-discounted marginal utility of consumption (Cochrane, 2005b):

$$\Lambda_t = e^{-\delta_H t} \left(c_t^H\right)^{-\underline{\gamma}_H} \,. \tag{B.25}$$

In equilibrium, given that there is a unit mass of households, individual consumption is equal to the aggregate consumption, C_t^H. Denote the equilibrium law of motion of aggregate consumption under the physical measure by

$$\frac{dC_t^H}{C_t^H} = \mu_t^C dt + \sigma_t^C d\widehat{Z}_t \,. \tag{B.26}$$

By Itô's lemma, the law of motion of the SDF, Λ_t, is given by

$$\frac{d\Lambda_t}{\Lambda_t} = -\left[\underline{\delta}_H + \underline{\gamma}_H \mu_t^C - \frac{1}{2}\underline{\gamma}_H \left(\underline{\gamma}_H + 1\right)\left(\sigma_t^C\right)^2\right] dt - \underline{\gamma}_H \sigma_t^C d\widehat{Z}_t \,. \tag{B.27}$$

To solve μ_t^C and σ_t^C, consider the goods market-clearing condition:

$$C_t^H dt + \frac{M_t^E}{1 - q_t^T \kappa^T (1 - \theta_t)} \lambda dt = (1 + \alpha) K_t^T dt \,. \tag{B.28}$$

The left side is the sum of households' consumption and the goods invested by the λdt measure of entrepreneurs who are hit by the Poisson shock. The right side is the goods produced by tangible capital and labor. For simplicity, the goods produced by intangibles are assumed to be consumed directly by the entrepreneurs, who run the firms, as compensation for their human capital (Hart and Moore, 1994; Bolton, Wang, and Yang, 2019). Adding intangibles' output to (B.29) expands the dimension of state variables in ρ_t from two (i.e., η_t and t) to four, because both K_t^T and K_t^I show up in (B.29), and, given their distinct laws of motion, they have to be tracked separately. Dividing both sides of (B.29) by $K_t^T dt$ and rearranging it, we have

$$\widetilde{C}_t^H = 1 + \alpha - \frac{\lambda \widetilde{M}_t^E}{1 - q_t^T \kappa^T (1 - \theta_t)} \,. \tag{B.29}$$

Following the notations in the main text, I denote K_t^T-scaled values by " $\widetilde{\cdot}$ ". The procedure in A.2 solves the endogenous variables on the right side of (B.29) as functions of η_t and time t. Therefore, K_t^T-scaled aggregate consumption,

$$\widetilde{C}_t^H = \widetilde{C}^H (\eta_t, t) \,, \tag{B.30}$$

is a known function of η_t and t. so I can obtain $\epsilon^{\widetilde{C}1} (\eta_t, t)$ and $\epsilon^{\widetilde{C}2} (\eta_t, t)$. Note that under *the risk-neutral measure*, by Itô's lemma, I obtain

(B.31)

$$\frac{dC_t^H}{C_t^H} = \frac{d\widetilde{C}^H (\eta_t, t)}{\widetilde{C}^H (\eta_t, t)} + \frac{dK_t^T}{K_t^T} + \epsilon^{\widetilde{C}} (\eta_t, t) \, \sigma^\eta (\eta_t, t) \, \sigma dt \,,$$

$$= \left\{ \epsilon^{\widetilde{C}1} (\eta_t, t) \left[\mu^\eta (\eta_t, t) + \frac{1}{2} \epsilon^{\widetilde{C}2} (\eta_t, t) \, \sigma^\eta (\eta_t, t)^2 + \sigma^\eta (\eta_t, t) \, \sigma \right] + \mu^{KT} (\eta_t, t) \right\} dt$$

(B.32)

$$+ \left[\epsilon^{\widetilde{C}1} (\eta_t, t) \, \sigma^\eta (\eta_t, t) + \sigma \right] dZ_t$$

where the risk-neutral measure dynamics, $\mu^\eta (\eta_t, t)$, $\sigma^\eta (\eta_t, t)$, and $\mu^{KT} (\eta_t, t)$ are given by (A.5), (A.6), and (A.7) respectively in A.1. To change the measure, using (B.2) to substitute dZ_t with $d\widehat{Z}_t + \gamma_t^H dt$, I obtain

$$\frac{dC_t^H}{C_t^H} = \left\{ \epsilon^{\widetilde{C}1}(\eta_t, t) \left[\mu^\eta(\eta_t, t) + \frac{1}{2}\epsilon^{\widetilde{C}2}(\eta_t, t)\,\sigma^\eta(\eta_t, t)^2 + \sigma^\eta(\eta_t, t)\,\sigma \right] + \mu^{KT}(\eta_t, t) \right\} dt$$

(B.33)

$$+ \left[\epsilon^{\widetilde{C}1}(\eta_t, t)\,\sigma^\eta(\eta_t, t) + \sigma \right] \gamma_t^H dt + \left[\epsilon^{\widetilde{C}1}(\eta_t, t)\,\sigma^\eta(\eta_t, t) + \sigma \right] d\widehat{Z}_t$$

According the law of motion of Λ_t given by (B.27), the price of risk is

$$\gamma_t^H = \underline{\gamma}_H \sigma_t^C = \underline{\gamma}_H \left[\epsilon^{\widetilde{C}1}(\eta_t, t)\,\sigma^\eta(\eta_t, t) + \sigma \right] . \tag{B.34}$$

I substitute out γ_t^H in the drift term of (B.33) with the solution (B.34) and obtain

(B.35)

$$\begin{aligned} \mu_t^C &= \mu^C(\eta_t, t) \\ &= \epsilon^{\widetilde{C}1}(\eta_t, t) \left[\mu^\eta(\eta_t, t) + \frac{1}{2}\epsilon^{\widetilde{C}2}(\eta_t, t)\,\sigma^\eta(\eta_t, t)^2 + \sigma^\eta(\eta_t, t)\,\sigma \right] + \mu^{KT}(\eta_t, t) \\ &+ \underline{\gamma}_H \left[\epsilon^{\widetilde{C}1}(\eta_t, t)\,\sigma^\eta(\eta_t, t) + \sigma \right]^2 \end{aligned}$$

Substituting the solutions of μ_t^C and σ_t^C into the drift term of (B.27), I obtain

(B.36)

$$\begin{aligned} \rho_t &= \rho(\eta_t, t) \\ &= \underline{\delta}_H + \underline{\gamma}_H \mu^{KT}(\eta_t, t) + \underline{\gamma}_H \epsilon^{\widetilde{C}1}(\eta_t, t) \left[\mu^\eta(\eta_t, t) + \frac{1}{2}\epsilon^{\widetilde{C}2}(\eta_t, t)\,\sigma^\eta(\eta_t, t)^2 + \sigma^\eta(\eta_t, t)\,\sigma \right] \\ &+ \frac{1}{2}\left(\underline{\gamma}_H^2 - \underline{\gamma}_H \right) \left[\epsilon^{\widetilde{C}1}(\eta_t, t)\,\sigma^\eta(\eta_t, t) + \sigma \right]^2 . \end{aligned}$$

C. The Implications of Zero Lower Bound

While the model does not feature any nominal variables, it is important to discuss the implications of zero lower bound and examine the robustness of results in the broader context of New Keynesian models. As shown in Table 3, the equilibrium outcome matches data well except a lower and more negative interest rate in the later sample periods. The rise of intangible capital over time increases firms' demand for liquid assets and thereby exerts downward pressure on the (natural) real rate. As discussed in Section IV, this trend widens the wedge between the natural real rate and real rate under nominal price rigidity and zero lower bound, exacerbating output loss due to the liquidity trap (Eggertsson and Woodford, 2003; Christiano, Eichenbaum, and Rebelo, 2011; Eggertsson and Krugman, 2012; Fischer, 2016; Korinek and Simsek, 2016; Caballero and Farhi, 2017; Guerrieri and Lorenzoni, 2017; Caballero and Simsek, 2020; Caballero, Farhi, and Gourinchas, 2021). In the following, I will discuss in more details how nominal frictions and zero lower bound interact in the shock amplification mechanism with a particular focus on the cyclical dynamics.

The feedback mechanism in my model emphasizes a discount rate channel of asset-price variation. The bankers have low discount rates (or cost of capital) because firms attach a liquidity premium to their debts as assets that hedge the intangible investment needs. Following positive shocks, the bankers become richer via a leveraged position in tangible capital, and as these low discount-rate agents acquire more assets, they push up asset prices. Higher asset prices imply stronger investment-driven liquidity needs, further reducing the bankers' discount rate and causing asset prices to rise more. An increasing liquidity premium means a widening discount-rate gap between the bankers and entrepreneurs. This implies that reallocation of assets away from the bankers, triggered by negative shocks, will cause a large decline in asset price.

There are two ways to think about how ZLB affects the mechanism. The first is to simply assume that r_t cannot be negative because there exists an exogenous supply of liquid assets that is perfectly elastic at $r_t = 0$. This certainly dampens the mechanism because the key to the mechanism is (liquid) asset shortage and the endogenous supply of assets by risk-taking financial intermediaries. However, where does the unlimited liquidity supply come from? This triggers the second way to think about ZLB, which is more in line with the New Keynesian tradition. I will argue that ZLB does not kill the discount-rate channel of financial instability but introduces a new asymmetric cash-flow channel at ZLB. Specifically, the mechanism in the model dampens the New Keynesian mechanism on the upside (i.e., in response to positive shocks) but does not necessarily interfere it on the downside, generating asymmetric output cycles.

Following Caballero and Simsek (2020), let us assume extremely sticky (constant) prices, so $r_t \geq 0$ because the nominal rate ($= r_t$) cannot be negative. And, let us adopt the AK technology and variable capital utilization as in Caballero and Simsek (2020). Moreover, to model the aggregate demand channel, we need to introduce a different preference and endogenize ρ_t, the agents' required return or discount rate which will depend on consumption growth in equilibrium. The wedge between ρ_t and the interest rate on liquid assets (bankers' debts in particular), r_t, is the liquidity premium. In equilibrium, $\rho_t - r_t$ is driven by households' liquidity demand and firms' liquidity demand that depends on the intangible investment productivity and asset price (present value of capitalizable output of tangible capital), just as in the main text.

Consider positive shocks at $r_t = 0$. The standard wealth effect drives up the aggregate demand and, through variable capital utilization, output increases. However, the mechanism in my model generates a counteracting force. As the bankers become richer through their leveraged position, these low discount-rate agents acquire more assets. The asset price rises and raises the liquidity premium, $\rho_t - r_t$ (see Proposition 1). Given that r_t cannot fall below zero, what has to adjust is ρ_t, the agents' required savings rate that depends on the consumption growth rate in equilibrium. ρ_t must increase and this weakens the aggregate demand. So the mechanism in my model counteracts the standard New Keynesian mechanism in response to positive shocks.

Next, consider negative shocks at $r_t = 0$. The aggregate demand and output decline through the wealth effect. The asset price decline reduces the liquidity premium $\rho_t - r_t$. Because r_t can rise above zero, a lower $\rho_t - r_t$ does not necessarily require a lower ρ_t (and a higher consumption growth), so my model does not generate a counteracting force against the standard New Keynesian mechanism in response to negative shocks. Therefore, incorporating my model into a New Keynesian setting with ZLB generates asymmetric cycles with dampened upside relative to a standard New Keynesian model but similar downside. What differs from the standard New Keynesian model is that here ZLB is applied to r_t, the interest rate on liquid assets, rather than ρ_t. Moreover, the wedge, $\rho_t - r_t$, depends on the endogenous variation in asset prices.

It is worth noting that the discount-rate channel of financial instability is still at work. The discount-rate gap between the bankers and the rest of economy still widens following positive shocks, and this implies an increasingly strong response to negative shocks that trigger asset reallocation from low discount-rate bankers to high discount-rate households/consumers. The existence of ZLB does not kill this mechanism. It simply infuses this mechanism into the aggregate demand channel of New Keynesian models through the endogenous ρ_t (consumers' discount rate or required savings return), and it does so in an asymmetric fashion by dampening the upside and but not necessarily interfering the downside. What the New Keynesian setup does to my model is to bring in a new cash-flow channel. Specifically, it makes the cash flow per unit of assets/capital (i.e., the output) variable through utilization, and, due to the asymmetry in output cycle, the shock amplification through the cash-flow channel is also asymmetric (stronger for negative shocks).

D. Additional Tables and Figures

Table D.1—: Summary Statistics for Firm Cash and Leverage Regressions

Variable	Below Median Intan./Asset			Above Median Intan./Asset		
	Mean	Median	Std.	Mean	Median	Std.
Cash/Assets (%)	12.395	5.759	16.758	24.521	15.597	24.682
Intangible Investment/Investment	0.434	0.427	0.302	0.802	0.840	0.156
Intangible Investment/Total Assets	0.043	0.042	0.029	0.238	0.178	0.270
PPE/Total Assets	0.364	0.315	0.261	0.194	0.156	0.152
Leverage (%)	29.388	27.127	22.415	18.078	12.170	20.327
Asset-backed Loans/Total Assets (%)	10.771	2.891	16.564	7.964	1.314	13.674
Cashflow-backed Loans/Total Assets (%)	20.288	15.972	23.055	12.639	0.591	24.402
Acquisitions/Total Assets	0.027	0.000	0.067	0.015	0.000	0.048
Cashflow/Total Assets	0.049	0.063	0.122	-0.056	0.051	0.287
Dividend Dummy	0.398	0.000	0.489	0.227	0.000	0.419
EBITDA/Total Assets	0.103	0.115	0.518	-0.034	0.087	0.524
Inventory/Total Assets	0.120	0.067	0.148	0.177	0.147	0.163
Net Cash Receipts/Total Assets	0.091	0.100	0.468	-0.036	0.061	0.557
Net Working Capital/Total Assets	0.071	0.053	0.182	0.107	0.113	0.229
Log Real Assets (Size)	5.827	5.789	2.109	4.538	4.407	1.967
Tobin's Q	1.452	1.241	0.746	1.961	1.595	1.180

Table D.2—: Summary Statistics for Household Liquidity Holdings Regressions

Panel A*: Summary Statistics for Time Series Regression*

Variable	Mean	Std.	p20	p40	p60	p80
Liquid Holdings/GDP	0.505	0.066	0.435	0.496	0.541	0.570
Average EV/EBITDA	10.364	2.793	7.479	9.652	11.170	13.027
Tangible EV/EBITDA	8.888	2.154	7.102	8.165	9.207	10.450
Average Tobin's Q	1.823	0.342	1.526	1.693	1.903	2.055
Tangible Tobin's Q	1.414	0.185	1.256	1.394	1.496	1.534
Price/Rent Ratio	1.27	0.129	1.177	1.201	1.250	1.333

Panel B*: Summary Statistics for Panel Data Regression*

Variable	Mean	Std.	p20	p40	p60	p80
Cash/Income	0.205	0.584	0	0.011	0.052	0.180
$\Delta\ln$ (Housing Price Index)	0.064	0.128	-0.038	0.059	0.098	0.139
Age	45.296	16.390	30	38	48	59
Couple Status	0.693	0.791	0	0	1	1
Education Level	13.124	2.657	12	12	14	16
Home ownership Status	0.551	0.497	0	0	1	1
Household Size	2.641	1.483	1	2	3	4
$\Delta\ln$ (Household Income)	0.036	1.390	-1.073	-0.286	0.364	1.144
$\Delta\ln$ (Wealth excluding Home Equity)	-0.034	6.808	-4.879	-0.870	0.862	4.623

Table D.3—: Asset Tangibility, Capital Valuation, and Corporate Cash Holdings

***Panel A**: EV/EBITDA & Intangible-Driven Corporate Cash Holdings*

$\frac{\text{Cash}}{\text{Assets}}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
PPE/Assets	-0.496	-0.223	-1.699***	-1.441***	-0.846**	-0.400	-1.909***	-1.476***
(decile)	(0.372)	(0.336)	(0.315)	(0.282)	(0.412)	(0.399)	(0.313)	(0.303)
Ave. EV/EBITDA	1.848***		1.020***					
	(0.275)		(0.244)					
PPE/Assets×	-0.259***	-0.281***	-0.136***	-0.153***				
Ave. EV/EBITDA	(0.034)	(0.032)	(0.027)	(0.025)				
Tan. EV/EBITDA					1.825***		0.882***	
					(0.331)		(0.273)	
PPE/Assets×					-0.267***	-0.307***	-0.141***	-0.174***
Tan. EV/EBITDA					(0.040)	(0.041)	(0.031)	(0.031)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
Year FE	No	Yes	No	Yes	No	Yes	No	Yes
Observations	152,801	152,801	133,632	133,632	152,801	152,801	133,632	133,632
Adjusted R^2	0.1795	0.1859	0.3096	0.3164	0.1745	0.1838	0.3076	0.3159

***Panel B**: Tobin's Q & Intangible-Driven Corporate Cash Holdings*

$\frac{\text{Cash}}{\text{Assets}}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
PPE/Assets	-0.533	0.021	-1.752***	-1.280***	0.927	1.297*	-1.125**	-0.836
(decile)	(0.632)	(0.621)	(0.415)	(0.415)	(0.837)	(0.735)	(0.555)	(0.519)
Ave. Tobin's Q	9.908***		4.836**					
	(2.587)		(1.800)					
PPE/Assets×	-1.483***	-1.729***	-0.776***	-0.961***				
Ave. Tobin's Q	(0.327)	(0.332)	(0.211)	(0.214)				
Tan. Tobin's Q					20.213***		10.620***	
					(4.560)		(3.152)	
PPE/Assets×					-2.937***	-3.136***	-1.430***	-1.555***
Tan. Tobin'sQ					(0.577)	(0.513)	(0.373)	(0.347)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
Year FE	No	Yes	No	Yes	No	Yes	No	Yes
Observations	152,801	152,801	133,632	133,632	152,801	152,801	133,632	133,632
Adjusted R^2	0.1726	0.1827	0.3072	0.3155	0.1732	0.1823	0.3077	0.3151

Firm-year clustered standard errors in parentheses
* $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$

Table D.4—: Intangible Investment, Tobin's Q, and Corporate Cash Holdings

$\frac{\text{Cash}}{\text{Assets}}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intan./Assets	-2.724	-3.372	-1.629	-2.272	-8.200***	-8.730***	-5.672**	-6.244***
(quintile)	(2.219)	(2.227)	(1.761)	(1.783)	(2.650)	(2.532)	(2.117)	(2.070)
Ave. Tobin's Q	-5.072***		-4.586***					
	(0.981)		(0.682)					
Intan./Assets×	4.993***	5.326***	3.729***	3.963***				
Ave. Tobin's Q	(1.215)	(1.235)	(0.958)	(0.983)				
Tan. Tobin's Q					-10.064***		-7.866***	
					(1.568)		(1.253)	
Intan./Assets×					10.317***	10.669***	7.648***	7.925***
Tan. Tobin's Q					(1.923)	(1.856)	(1.530)	(1.512)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
Year FE	No	Yes	No	Yes	No	Yes	No	Yes
Observations	152,826	152,826	133,632	133,632	152,826	152,826	133,632	133,632
Adjusted R^2	0.1843	0.2038	0.2671	0.2831	0.1880	0.2057	0.2699	0.2842

Firm-year clustered standard errors in parentheses
* $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$

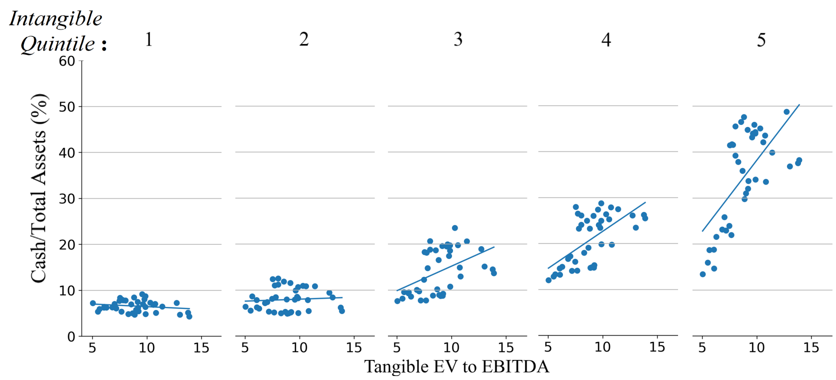


Figure D.1. : Tangible Capital Valuation and Cash Holdings by Intangibility

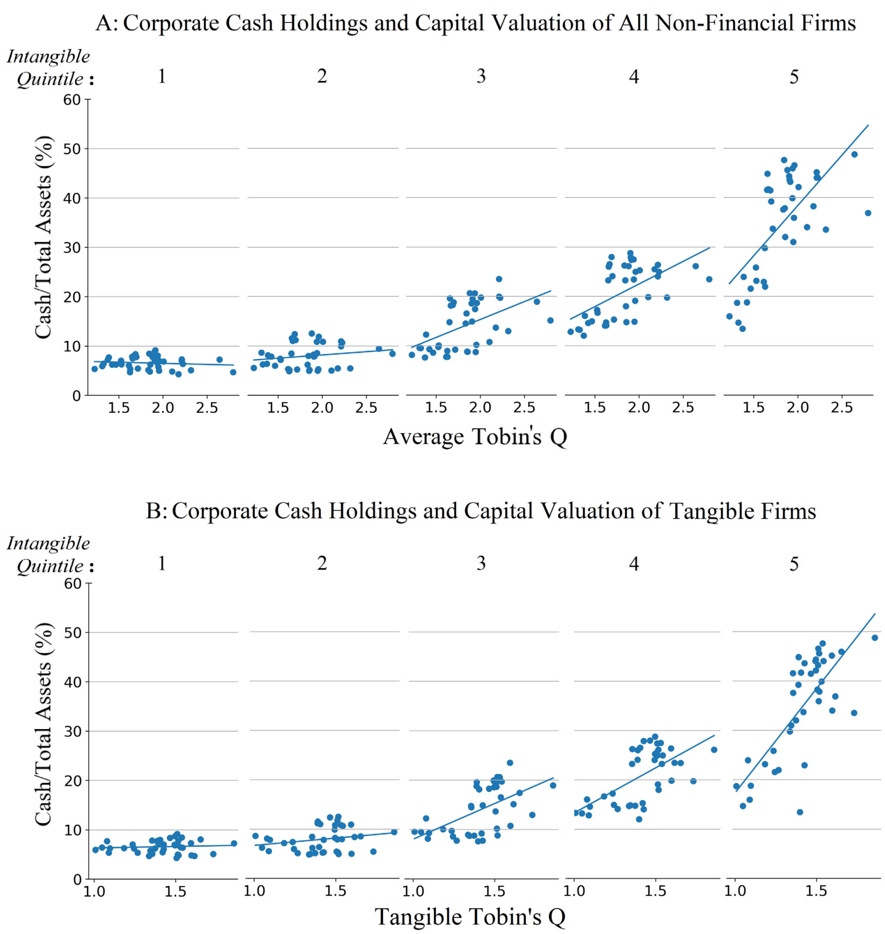


Figure D.2. : Tobin's Q and Cash Holdings by Intangibility

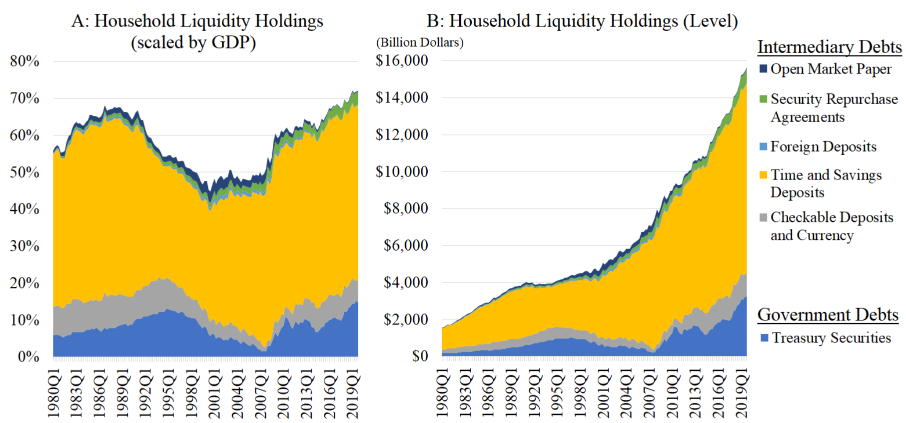


Figure D.3. : Decomposing Households' Holdings of Liquid Securities

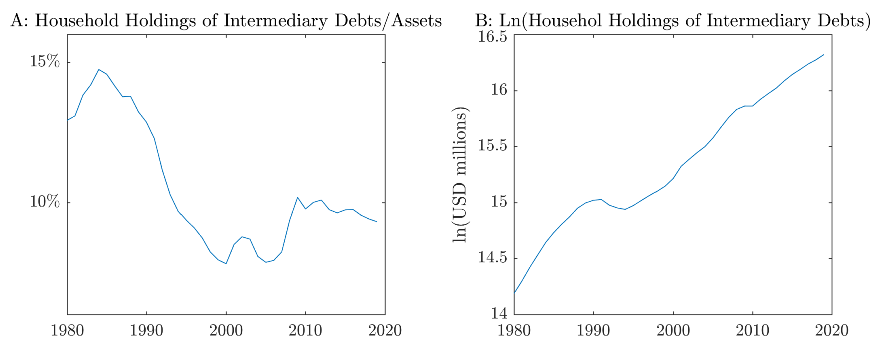


Figure D.4. : Households' Holdings of Intermediary Debts