Online Appendix for Cyclical Attention to Saving

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This appendix lays out the full mathematical details of the quantitative model in Section IV. Aside from the attention problem and banking sector, the model closely follows that of Harrison and Oomen (2010) (HO), which is in turn based on Smets and Wouters (2007), extended to an open economy as in Adolfson et al. (2007).

Households

Households maximize expected discounted utility $E_t \sum_{s=0}^{\infty} \beta^s U_{t+s}$, where instantaneous utility is given by:

$$U_t = \frac{1}{1-\frac{1}{\sigma^c}} \left(\frac{c_t}{\bar{c}_{t-1}^{\psi^{hab}}} \right)^{1-\frac{1}{\sigma^c}} - (\kappa^h)^{-\frac{1}{\sigma^h}} e^{\zeta_t^{\kappa h}} \frac{1}{1+\frac{1}{\sigma^h}} (h_t)^{1+\frac{1}{\sigma^h}} - \mu e^{\zeta_t^{\mu}} \mathcal{I}_t(i_t^e) \tag{O.1}$$

where $\beta \in (0,1)$ is the discount factor, $\sigma^c > 0$ is the elasticity of intertemporal substitution, $\sigma^h > 0$ is the elasticity of labor supply, and $\kappa^h > 0$ gives the weight of labor supply in utility. $\bar{c}_{t-1}$ is lagged aggregate consumption, taken as given by households, and so the parameter ψ^{hab} gives the degree of external habit formation. c_t is household consumption, h_t is labor supply, and $\zeta_t^{\kappa h}$ is an exogenous shock to the disutility of labor. In equilibrium $\bar{c}_t = c_t$ as all households are identical, but the households do not take this into account when making choices. Finally, $\mu > 0$ is the marginal cost of information, ζ_t^{μ} is an exogenous shock to this cost, i_t^e is the effective nominal interest experienced by the household, and $\mathcal{I}_t(i_t^e)$ is the information processing required to achieve that effective interest rate, as formalized in Section I of the paper.

The budget constraint is:

$$PC_t c_t + PI_t inv_t + B_t - (1 + i_{t-1}^e) B_{t-1} = W_t h_t + R_t k_t^s + \Pi_t^v + \Pi_t^b - PC_t \tau_t \tag{O.2}$$

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PC_t is the consumer price index. As well as consumption, household spending consists of investment inv_t at price PI_t, and asset accumulation. One-period domestic bonds B_t are subject to the attention problem studied in Section I, and thus carry an effective nominal interest rate of i_t^e. Income comes from supplying labor at nominal wage W_t, supplying capital services k_t^s at rental rate R_t, profits from firms Π_t^v, and a transfer from the banking system Π_t^b that includes both bank profits and transaction costs. There is a lump sum tax of $PC_t\tau_t$ from the government.

Consumption indices

The equations above consider composite consumption c_t, with the composite price index PC_t. c_t is a CES combination of domestically-produced goods c_t^h and foreign-produced goods (imports) c_t^m:

$$c_t \equiv \kappa^c \left((1-\psi^m)(c_t^h)^{1-\frac{1}{\sigma^m}} + \psi^m (c_t^m)^{1-\frac{1}{\sigma^m}} \right)^{\frac{\sigma^m}{\sigma^m-1}} \tag{O.3}$$

where $\kappa^c > 0$ is a parameter, $\psi^m \in (0,1)$ is the expenditure weight of imported consumption goods in aggregate consumption, and $\sigma^m > 1$ is the elasticity of substitution between domestic and foreign consumption.

The associated price index is:

$$PC_t \equiv \frac{1}{\kappa^c} \left[(1-\psi^m)^{\sigma^m} (PH_t)^{1-\sigma^m} + (\psi^m)^{\sigma^m} (PM_t)^{1-\sigma^m} \right]^{\frac{1}{1-\sigma^m}} \tag{O.4}$$

where PH_t is the price of goods produced at home and PM_t is the price of imported consumption goods. It will be convenient to express these relative to PC_t:

$$1 = \frac{1}{\kappa^c} \left[(1-\psi^m)^{\sigma^m} \left(p_t^h\right)^{1-\sigma^m} + (\psi^m)^{\sigma^m} (p_t^m)^{1-\sigma^m} \right]^{\frac{1}{1-\sigma^m}} \tag{O.5}$$

where $p_t^h = PH_t/PC_t, p_t^m = PM_t/PC_t$.

Expenditure allocation

Given c_t, the allocation of expenditure between home and foreign is:

$$c_t^h = (1-\psi^m)^{\sigma^m} (\kappa^c)^{\sigma^m - 1} (p_t^h)^{-\sigma^m} c_t \tag{O.6}$$

$$c_t^m = (\psi^m)^{\sigma^m} (\kappa^c)^{\sigma^m - 1} (p_t^m)^{-\sigma^m} c_t \tag{O.7}$$

Total consumption expenditure is:

$$c_t = p_t^h c_t^h + p_t^m c_t^m \tag{O.8}$$

Capital accumulation

Capital accumulates according to the law of motion:

$$k_t = inv_t + (1-\delta)k_{t-1} - \Delta_t^k - \Delta_t^z \tag{O.9}$$

where k_t is capital purchased in period t, which will be available for production in period $t+1$. $\delta > 0$ is the depreciation rate. Δ_t^k is a quadratic cost associated with changing the capital stock:

$$\Delta_t^k \equiv \frac{\chi^k}{2\bar{k}_{t-1}} \left[k_t - \left(\frac{\bar{k}_{t-1}}{\bar{k}_{t-2}} \right)^{\epsilon^k} k_{t-1} + \frac{\bar{k}_{t-1}}{\chi^k} \zeta_t^k \right]^2 \tag{O.10}$$

Note that these costs arise if a household's own capital accumulation deviates from the aggregate rate of capital accumulation in the previous period, as $\bar{k}_t$ denotes aggregate capital that the household takes as given. This cost is controlled by the parameters $\chi^k, \epsilon^k > 0$. ζ_t^k is an exogenous shock to the capital adjustment cost.

Δ_t^z is an additional depreciation which is increasing in capital utilization. Households rent capital services to firms, which depend on previously installed capital and utilization z_t:

$$k_t^s = z_t k_{t-1} \tag{O.11}$$

Choosing a higher z_t increases the capital services the household can supply, but implies a faster depreciation of the capital stock, through Δ_t^z:

$$\Delta_t^z \equiv \frac{\chi^z}{1+\sigma^z} \left[(z_t)^{1+\sigma^z} - 1 \right] k_{t-1} \tag{O.12}$$

where $\chi^z, \sigma^z > 0$ control the magnitude and slope of utilization-related depreciation.

First Order Conditions

The household chooses $c_t, i_t^e, k_t, inv_t, B_t, z_t$ to maximize the present discounted sum of the utility in equation (O.1) subject to equations (O.2), (O.9), (O.11), and the convex costs of increasing i^e_t. The first order conditions are:

$$\frac{1}{\bar{c}_{t-1}^{\psi^{hab}}}\left(\frac{c_t}{\bar{c}_{t-1}^{\psi^{hab}}}\right)^{-\frac{1}{\sigma^c}} = PC_t\Lambda_t \tag{O.13}$$

$$\beta E_t \Lambda_{t+1} B_t = \mu e^{\zeta_t^\mu} \mathcal{I}_t'(i_t^e) \tag{O.14}$$

$$\Theta_t\left(1+\frac{\partial \Delta_t^k}{\partial k_t}\right) = \beta E_t\left[\Lambda_{t+1}R_{t+1}z_{t+1} + \Theta_{t+1}\left(1-\delta-\frac{\partial \Delta_{t+1}^k}{\partial k_t}-\frac{\partial \Delta_{t+1}^z}{\partial k_t}\right)\right] \tag{O.15}$$

$$\Lambda_t PI_t = \Theta_t \tag{O.16}$$

$$\Lambda_t = \beta E_t(1+i_t^e)\Lambda_{t+1} \tag{O.17}$$

$$\Lambda_t R_t k_{t-1} = \Theta_t \chi^z z_t^{\sigma^z} k_{t-1} \tag{O.18}$$

where Λ_t, Θ_t are the Lagrange multipliers on (O.2) and (O.9), and:

$$\frac{\partial \Delta_t^k}{\partial k_t} = \chi^k \frac{k_t - \left(\frac{\bar{k}_{t-1}}{\bar{k}_{t-2}}\right)^{\epsilon^k} k_{t-1} + \frac{\bar{k}_{t-1}}{\chi^k}\zeta_t^k}{\bar{k}_{t-1}} \tag{O.19}$$

$$\frac{\partial \Delta_{t+1}^k}{\partial k_t} = -\chi^k\left(\frac{\bar{k}_t}{\bar{k}_{t-1}}\right)^{\epsilon^k} \frac{k_{t+1} - \left(\frac{\bar{k}_t}{\bar{k}_{t-1}}\right)^{\epsilon^k} k_t + \frac{\bar{k}_t}{\chi^k}\zeta_{t+1}^k}{\bar{k}_t} \tag{O.20}$$

$$\frac{\partial \Delta_{t+1}^z}{\partial k_t} = \chi^z \frac{z_{t+1}^{1+\sigma^z} - 1}{1+\sigma^z} \tag{O.21}$$

Combining equations (O.13) and (O.17) gives the consumption Euler equation,[1] while combining equations (O.13) and (O.14) (and transforming to be in terms of real bonds $b_t = B_t/PC_t$) gives the attention first order condition (equation 39 in the main paper).

Labor unions

Households supply labor to a continuum of unions, who in turn set wages. Rather than choosing labor supply directly, households agree to supply all labor demanded at the wage set by the union. Unions supply differentiated labor varieties $h_t(i)$ to a perfectly competitive labor packer, who combines varieties with a CES aggregator to an aggregate labor supply h_t:

$$h_t \equiv \left[\int_0^1 h_t(i)^{\frac{\sigma^w-1}{\sigma^w}} di\right]^{\frac{\sigma^w}{\sigma^w-1}} \tag{O.22}$$

where $\sigma^w > 1$ is the elasticity of substitution between labor varieties.

[1] See equation O.93 below for how the risk premium shock is incorporated into this equation.

Cost minimization implies the demand for each variety is:

$$h_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\sigma^w} h_t \tag{O.23}$$

where $W_t(i)$ is the nominal wage set by the union i, and W_t is the aggregate nominal wage:

$$W_t \equiv \left[\int_0^1 W_t(i)^{1-\sigma^w} di\right]^{\frac{1}{1-\sigma^w}} \tag{O.24}$$

Unions set wages to maximize expected discounted utility of their members, subject to a cost of wage adjustment and this demand function. The adjustment cost is quadratic in deviations from a target wage inflation rate Ξ_t^w. Their problem is therefore:

$$\begin{aligned} \max_{W_t(i)} E_t \sum_{s=0}^{\infty} \beta^s \Bigg\{ & \Lambda_{t+s} W_{t+s}(i) h_{t+s}(i) - (\kappa^h)^{-\frac{1}{\sigma^h}} e^{\zeta_{t+s}^{\kappa h}} \frac{1}{1+\frac{1}{\sigma^h}} (h_{t+s}(i))^{1+\frac{1}{\sigma^h}} \\ & - \Lambda_{t+s} \frac{\chi^w}{2} \left(\frac{W_{t+s}(i)}{W_{t+s-1}(i)\Xi_{t+s}^w} - 1\right)^2 W_{t+s} \Bigg\} \end{aligned} \tag{O.25}$$

subject to (O.23), and:

$$\Xi_t^w = \left(\frac{W_{t-1}}{W_{t-2}}\right)^{\epsilon^w} \tag{O.26}$$

where $\chi^w > 0$ controls the strength of wage adjustment costs, and $\epsilon^w \geq 0$ controls the degree of indexation to past wage inflation. If $\epsilon^w = 0$, then the cost is the standard Rotemberg-style cost, with no indexation to past wage changes. If $\epsilon^w > 0$, wages are instead partially indexed to past wage growth. Notice that while wages received and wage adjustment costs are discounted by Λ_{t+s}, the disutility of labor is not, as it is a utility cost rather than a monetary cost.

Taking the first order condition and then imposing symmetry among unions ($W_t(i) = W_t$, $h_t(i) = h_t$) yields:

$$\begin{aligned} (1-\sigma^w)h_t + (\kappa^h)^{-\frac{1}{\sigma^h}} e^{\zeta_t^{\kappa h}} \sigma^w \frac{h_t^{1+\frac{1}{\sigma^h}}}{W_t \Lambda_t} - \frac{\chi^w W_t}{W_{t-1}\Xi_t^w}\left(\frac{W_t}{W_{t-1}\Xi_t^w} - 1\right) & \\ + E_t \frac{\beta \chi^w W_{t+1}^2 \Lambda_{t+1}}{W_t^2 \Xi_{t+1}^w \Lambda_t}\left(\frac{W_{t+1}}{W_t \Xi_{t+1}^w} - 1\right) &= 0 \end{aligned} \tag{O.27}$$

Now rewrite in terms of real wages $w_t \equiv W_t/PC_t$:

$$(1-\sigma^w)h_t = -(\kappa^h)^{-\frac{1}{\sigma^h}} e^{\zeta_t^{\kappa h}} \sigma^w \frac{h_t^{1+\frac{1}{\sigma^h}}}{w_t u_{ct}} + \frac{\chi^w w_t \pi_t}{w_{t-1}\Xi_t^w}\left(\frac{w_t}{w_{t-1}\Xi_t^w}\pi_t - 1\right)$$
$$- E_t \frac{\beta \chi^w w_{t+1}^2 \pi_{t+1} u_{ct+1}}{w_t^2 \Xi_{t+1}^w u_{ct}}\left(\frac{w_{t+1}}{w_t \Xi_{t+1}^w}\pi_{t+1} - 1\right) \tag{O.28}$$

where $u_{ct} \equiv \Lambda_t PC_t$ is the marginal utility of consumption:

$$u_{ct} = \frac{1}{\bar{c}_{t-1}^{\psi^{hab}}}\left(\frac{c_t}{\bar{c}_{t-1}^{\psi^{hab}}}\right)^{-\frac{1}{\sigma^c}} \tag{O.29}$$

and we note that Ξ_t^w can be written in real terms as:

$$\Xi_t^w = \left(\frac{w_{t-1}}{w_{t-2}}\pi_{t-1}\right)^{\epsilon^w} \tag{O.30}$$

Note that in HO, they use Calvo staggered wage setting, rather than this Rotemberg-style setup. I use the quadratic adjustment cost setup to keep the exposition of the model brief. Since we consider a steady state with no trend inflation, the steady steady state and log-linearized wage Phillips curve are identical in these two setups. To map from the parameters here to the wage-resetting probability in HO, replace χ^w with:

$$\chi^w \equiv \frac{(\sigma^w - 1)(1-\psi^w)\bar{h}}{\psi^w(1-\beta(1-\psi^w))}\left(1 + \frac{\sigma^w}{\sigma^h}\right) \tag{O.31}$$

where ψ^w is the probability a union can reset W_t each period. With this substitution, equation (O.28) implies exactly the same log-linearized wage Phillips curve as HO. See Born and Pfeifer (2020) for details of how this χ^w expression is derived.

Firms

Domestic producers

There is a continuum of monopolistically competitive intermediate goods producers, who produce output for production of domestic goods y_t^{hv} and for production of export goods y_t^{xv}. Their total output, $y_t^v = y_t^{hv} + y_t^{xv}$, is given by a CES production function over labor and capital services:

$$y_t^v = tfp_t\left[(1-\alpha)(h_t)^{\frac{\sigma^y - 1}{\sigma^y}} + \alpha (k_t^s)^{\frac{\sigma^y - 1}{\sigma^y}}\right]^{\frac{\sigma^y}{\sigma^y - 1}} \tag{O.32}$$

where tfp_t is aggregate productivity, $\alpha > 0$ is the capital share, and $\sigma^y > 0$ is the elasticity of substitution between factors of production. Letting $r_t \equiv R_t/PC_t$ be the real rental rate, real total costs are $w_t h_t + r_t k_t^s$. Minimizing this cost for a given y_t^v gives:

$$\frac{w_t}{r_t} = \frac{1-\alpha}{\alpha}\left[\frac{k_t^s}{h_t}\right]^{\frac{1}{\sigma^y}} \tag{O.33}$$

$$h_t = \left(\frac{w_t}{1-\alpha}\right)^{-\sigma^y}\left[(1-\alpha)^{\sigma^y} w_t^{1-\sigma^y} + \alpha^{\sigma^y} r_t^{1-\sigma^y}\right]^{\frac{\sigma^y}{1-\sigma^y}} \frac{y_t^v}{tfp_t} \tag{O.34}$$

$$k_t^s = \left(\frac{r_t}{\alpha}\right)^{-\sigma^y}\left[(1-\alpha)^{\sigma^y} w_t^{1-\sigma^y} + \alpha^{\sigma^y} r_t^{1-\sigma^y}\right]^{\frac{\sigma^y}{1-\sigma^y}} \frac{y_t^v}{tfp_t} \tag{O.35}$$

Marginal costs are then given by:

$$mc_t = \left[(1-\alpha)^{\sigma^y} w_t^{1-\sigma^y} + \alpha^{\sigma^y} r_t^{1-\sigma^y}\right]^{\frac{1}{1-\sigma^y}} \frac{1}{tfp_t} \tag{O.36}$$

Perfectly competitive final goods producers combine intermediate goods varieties from domestic and foreign firms using a Leontief technology:

$$y_t^h = \min\{\frac{y_t^{hv}}{\kappa^{hv}}, \frac{mi_t^h}{1-\kappa^{hv}}\} \tag{O.37}$$

$$y_t^x = \min\{\frac{y_t^{xv}}{\kappa^{xv}}, \frac{mi_t^x}{1-\kappa^{xv}}\} \tag{O.38}$$

where κ^{hv}, κ^{xv} are parameters, y_t^h, y_t^x denote final goods production for the domestic and export markets respectively, and mi_t^h, mi_t^x denote imported intermediate inputs used for each final good. The indices y_t^{hv} and y_t^{xv} are CES aggregates of intermediate varieties:

$$y_t^{hv} \equiv \left[\int_0^1 y_t^{hv}(i)^{\frac{\sigma^{hb}}{-}1}\sigma^{hb} di\right]^{\frac{\sigma^{hb}}{\sigma^{hb}-1}} \tag{O.39}$$

$$y_t^{xv} \equiv \left[\int_0^1 y_t^{xv}(i)^{\frac{\sigma^{xb}-1}{\sigma^{xb}}} di\right]^{\frac{\sigma^{xb}}{\sigma^{xb}-1}} \tag{O.40}$$

(O.41)

where $\sigma^{hb}, \sigma^{xb} > 1$ are elasticities of substitution between varieties. Minimizing final good producer costs yields:

$$\frac{\kappa^{hv}}{1-\kappa^{hv}} = \frac{y_t^{hv}}{mi_t^h} \tag{O.42}$$

$$\frac{\kappa^{xv}}{1-\kappa^{xv}} = \frac{y_t^{xv}}{mi_t^x} \tag{O.43}$$

$$y_t^{hv}(i) = \left(\frac{p_t^{hv}(i)}{p_t^{hv}}\right)^{-\sigma^{hb}} y_t^{hv} \tag{O.44}$$

$$y_t^{xv}(i) = \left(\frac{p_t^{xv}(i)}{p_t^{xv}}\right)^{-\sigma^{xb}} y_t^{xv} \tag{O.45}$$

$$mc_t^h = \kappa^{hv} p_t^{hv} + (1-\kappa^{hv}) p_t^m \tag{O.46}$$

$$mc_t^x = \kappa^{xv} p_t^{xv} + (1-\kappa^{xv}) p_t^m \tag{O.47}$$

where mc_t^h, mc_t^x denote the final good producer's (real) marginal costs in the domestic and export sectors. Since final goods producers are perfectly competitive, the prices of domestic and export goods, again expressed relative to PC_t, are equal to their respective marginal costs: $p_t^h = mc_t^h$ and $p_t^x = mc_t^x$. In the expressions for individual variety demands and marginal costs, the price indices for intermediate inputs for domestic and export production (p_t^{hv} and p_t^{xv}) are defined as:

$$p_t^{hv} \equiv \left[\int_0^1 (p_t^{hv}(i))^{1-\sigma^{hb}} di\right]^{\frac{1}{1-\sigma^{hb}}} \tag{O.48}$$

$$p_t^{xv} \equiv \left[\int_0^1 (p_t^{xv}(i))^{1-\sigma^{xb}} di\right]^{\frac{1}{1-\sigma^{xb}}} \tag{O.49}$$

where $y_t^{hv}(i)$ and $y_t^{xv}(i)$ are the quantities of intermediate goods demanded from producer i for each type of production, and $p_t^{hv}(i)$ and $p_t^{xv}(i)$ are the prices set by that producer. p_t^{xv} is specifically the relative export price expressed in domestic currency, defined as:

$$p_t^{xv} \equiv \frac{PXVF_t}{PC_t ER_t} \tag{O.50}$$

where $PXVF_t$ is the price of intermediate goods in the export sector in foreign currency terms, and ER_t is the nominal exchange rate.

Price setting

Intermediate goods produces can set different prices for goods used in the production of final goods for domestic consumption and for export: i.e. different prices for $y_t^{hv}(i)$ and $y_t^{xv}(i)$. In both cases, they set prices to maximize expected discounted profits, net of quadratic price adjustment costs. Their optimization problem is therefore:

$$\max_{p_t^{hv}(i),p_t^{xv}(i)} E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s} \Bigg\{ p_{t+s}^{hv}(i) y_{t+s}^{hv}(i) + p_{t+s}^{xv} y_{t+s}^{xv}(i) - w_{t+s} h_{t+s}(i) - r_{t+s} k_{t+s}^{s}(i)$$

(O.51)

$$- \frac{\chi^{hv}}{2} \left(\frac{p_{t+s}^{hv}(i)}{p_{t+s-1}^{hv}(i)} \frac{\pi_{t+s}}{\Xi_{t+s}^{hv}} - 1 \right)^2 p_{t+s}^{hv} y_{t+s}^{hv} - \frac{\chi^{xv}}{2} \left(\frac{p_{t+s}^{xv}(i) q_{t+s}}{p_{t+s-1}^{xv}(i) q_{t+s-1}} \frac{\pi_{t+s}^{f}}{\Xi_{t+s}^{xv}} - 1 \right)^2 p_{t+s}^{xv} y_{t+s}^{xv} \Bigg\}$$

subject to the production function (O.32), and demand for domestic and export varieties (O.44) and (O.45). Note the objective function is specified here in real terms, with nominal profits and costs divided through by PC_t. This is also why inflation π_t appears in the adjustment cost for domestic-use goods: if the original costs are in the growth of prices PHV_{t+s}/PHV_{t+s-1}, expressing each price relative to PC_{t+s} transforms that ratio into $\pi_{t+s} p_{t+s}^{hv} / p_{t+s-1}^{hv}$. The same logic generates the adjustment costs for export goods, which depend on the change in those prices in foreign currency:

$$\frac{PXVF_{t+s}}{PXVF_{t+s-1}} = \frac{p_{t+s}^{xv} PC_{t+s} ER_{t+s}}{p_{t+s-1}^{xv} PC_{t+s-1} ER_{t+s-1}} = \frac{p_{t+s}^{xv} q_{t+s} \pi_{t+s}^{f}}{p_{t+s-1}^{xv} q_{t+s-1}} \tag{O.52}$$

where q_t is the real exchange rate, defined as:

$$q_t = \frac{PC_t ER_t}{PCF_t} \tag{O.53}$$

in which PCF_t is the foreign price level. $\pi_t^f \equiv PCF_t / PCF_{t-1}$ is foreign inflation.

As with wage setting, there is partial indexation of domestic and export prices to past inflation in those prices through Ξ_t^{hv}, Ξ_t^{xv}:

$$\Xi_t^{hv} = \left(\frac{p_{t-1}^{hv}}{p_{t-2}^{hv}} \pi_{t-1} \right)^{\epsilon^{hv}} \tag{O.54}$$

$$\Xi_t^{xv} = \left(\frac{p_{t-1}^{xv} q_{t-1}}{p_{t-2}^{xv} q_{t-2}} \pi_{t-1}^{f} \right)^{\epsilon^{xv}} \tag{O.55}$$

$\chi^{hv}, \chi^{xv} > 0$ control the degree of price stickiness, and $\epsilon^{hv}, \epsilon^{xv} > 0$ control the degree of price indexation.

Taking the first order conditions, and then imposing cost minimization (O.33)-(O.36) and that all intermediate goods firms are symmetric, so set the same prices in equilibrium, we obtain Phillips curves for each good:

$$
\begin{aligned}
&1-\sigma^{hb}+\frac{\sigma^{hb}mc_t}{p_t^{hv}}-\chi^{hv}\frac{p_t^{hv}\pi_t}{p_{t-1}^{hv}\Xi_t^{hv}}\left(\frac{p_t^{hv}}{p_{t-1}^{hv}}\frac{\pi_t}{\Xi_t^{hv}}-1\right)\\
&\qquad+\beta\chi^{hv}E_t\frac{\Lambda_{t+1}y_{t+1}^{hv}}{\Lambda_t y_t^{hv}}\frac{(p_{t+1}^{hv})^2\pi_{t+1}}{(p_t^{hv})^2\Xi_{t+1}^{hv}}\left(\frac{p_{t+1}^{hv}}{p_t^{hv}}\frac{\pi_{t+1}}{\Xi_{t+1}^{hv}}-1\right)=0
\end{aligned}
\tag{O.56}
$$

$$
\begin{aligned}
&1-\sigma^{xb}+\frac{\sigma^{xb}mc_t}{p_t^{xv}}-\chi^{xv}\frac{p_t^{xv}q_t\pi_t^f}{p_{t-1}^{xv}q_{t-1}\Xi_t^{xv}}\left(\frac{p_t^{xv}q_t}{p_{t-1}^{xv}q_{t-1}}\frac{\pi_t^f}{\Xi_t^{xv}}-1\right)\\
&\qquad+\beta\chi^{xv}E_t\frac{\Lambda_{t+1}y_{t+1}^{xv}}{\Lambda_t y_t^{xv}}\frac{(p_{t+1}^{xv})^2q_{t+1}\pi_{t+1}^f}{(p_t^{xv})^2q_t\Xi_{t+1}^{xv}}\left(\frac{p_{t+1}^{xv}q_{t+1}}{p_t^{xv}q_t}\frac{\pi_{t+1}^f}{\Xi_{t+1}^{xv}}-1\right)=0
\end{aligned}
\tag{O.57}
$$

Like consumption, investment goods are a CES aggregate of many investment good varieties:

$$
inv_t \equiv \left[\int_0^1 inv_t(i)^{\frac{\sigma^{hb}-1}{\sigma^{hb}}}di\right]^{\frac{\sigma^{hb}}{\sigma^{hb}-1}}
\tag{O.58}
$$

All investment goods are assumed to be produced domestically, and are produced with the same technology as the domestic consumption good c_t^h. Since the elasticity of substitution between varieties is also the same (σ^{hb}), we have that the price indices will be identical: $PI_t = PH_t$. The demand for an individual investment good variety is given by:

$$
inv_t(i)=\left(\frac{p_t^h(i)}{p_t^h}\right)^{-\sigma^{hb}}inv_t
\tag{O.59}
$$

Banks

These are described in detail in Section IV.A.2 of the paper. Equations 41, 42, 44, and 45, reproduced as (O.60)-(O.63) here, define the probability of choosing the good bank p_t^g, the effective interest rate i_t^e, and the first order conditions of good and bad banks.

$$
p_t^g=\frac{\exp(\frac{i_t^g}{\lambda_t})}{\exp(\frac{i_t^g}{\lambda_t})+\exp(\frac{i_t^b}{\lambda_t})}
\tag{O.60}
$$

$$
i_t^e=p_t^g i_t^g+(1-p_t^g)i_t^b
\tag{O.61}
$$

$$(O.62) \qquad (1 - p_t^g) \cdot (i_t^{CB} - i_t^g - \chi_0^g - \zeta_t^\chi) = \lambda_t$$

$$(O.63) \qquad p_t^g \cdot (i_t^{CB}(1 - \chi_1) - i_t^b - (\chi_0^b - \chi_1 \bar{i}^{CB}) - \zeta_t^\chi - \zeta_t^{\chi b}) = \lambda_t$$

where i_t^g, i_t^b are the nominal interest rates set by the good and bad bank respectively, λ_t is the shadow value of information, i_t^{CB} is the interest rate set by the central bank, $\chi_0^g, \chi_0^b, \chi_1$ are parameters setting the levels and responsiveness to i_t^{CB} of bank transaction costs, $\bar{i}^{CB}$ is the steady state of i_t^{CB}, and $\zeta_t^\chi, \zeta_t^{\chi b}$ are exogenous shocks to the level and dispersion of bank costs. As in Section I of the paper, the shadow value of information is related to information processing:

$$(O.64) \qquad \mathcal{I}_t'(i_t^e) = \lambda_t^{-1}$$

Government

The government budget constraint is:

$$(O.65) \qquad PC_t \tau_t = PH_t g_t + i_{t-1}^{CB} B_t^g$$

where g_t is government spending, which is spent on home goods only. Contrary to HO, we assume that the government issues a positive supply of bonds B_t^g, so alongside g_t government expenditure includes interest payments on these bonds, paid at the central bank interest rate. The lump sum tax τ_t adjusts each period to satisfy this budget constraint.

The supply of bonds is such that the real supply is constant at $B_t^g / PC_t = b$.

Monetary policy

The central bank chooses the nominal policy rate i_t^{CB} according to a Taylor rule with interest-rate smoothing determined by parameter θ^{rcb}:

$$(O.66) \qquad \frac{1 + i_t^{CB}}{1 + \bar{i}^{CB}} = \left(\frac{1 + i_{t-1}^{CB}}{1 + \bar{i}^{CB}} \right)^{\theta^{rcb}} \left\{ \pi_t^{\theta^p} \left(\frac{y_t^v}{\bar{y}^v tfp_t} \right)^{\theta^y} \right\}^{1-\theta^{rcb}} e^{\zeta_t^{rcb}}$$

where $\bar{y}^v$ is steady state output, and so $\bar{y}^v tfp_t$ is a measure of potential output. ζ_t^{rcb} is an exogenous monetary policy shock.

Market clearing

Market clearing of domestic goods and export goods requires

$$y_t^h = c_t^h + inv_t + g_t \tag{O.67}$$

$$y_t^x = x_t \tag{O.68}$$

where x_t is the quantity of exports demanded by foreign countries.

Total domestic output is equal to the total production of intermediate goods:

$$y_t^v = y_t^{hv} + y_t^{xv} = \kappa^{hv} y_t^h + \kappa^{xv} y_t^x \tag{O.69}$$

where the final equality uses final goods producer production functions ((O.37) and (O.38)) and cost minimization ((O.42) and (O.43)).

Factor market clearing requires that labor and capital services supplied by households equal labor and capital services demanded by intermediate firms. Domestic bond market clearing requires that real bonds demanded by households equal b, the constant supply of such bonds from the government.

Foreign variables

Demand for final export goods is given by:

$$x_t = \kappa^x \left(\frac{q_t p_t^x}{p_t^{xf}} \right)^{-\sigma^x} c_t^f \tag{O.70}$$

where $\kappa^x, \sigma^x > 0$ are parameters, p_t^{xf} is exogenous world export prices, expressed relative to PCF_t. c_t^f is exogenous export demand from foreign countries.

Imports prices are set in the domestic currency, and are assumed to be the same for all imports, no matter whether they are used directly for consumption, or in the domestic production of final goods for domestic use or export. Monopolistically competitive foreign firms face Rotemberg-style quadratic costs of price adjustment, partially indexed to past import good inflation. Domestic final goods producers aggregate imported goods as intermediate inputs using the CES aggregator:

$$y_t^m \equiv \left[\int_0^1 \left(y_t^m(i) \right)^{\frac{\sigma^{mb}-1}{\sigma^{mb}}} di \right]^{\frac{\sigma^{mb}}{\sigma^{mb}-1}} \tag{O.71}$$

where $\sigma^{mb} > 1$ is the elasticity of substitution between varieties of imported goods, which is the same no matter whether the imports are for consumption or use as further intermediate inputs into production. The price index is therefore given by:

$$p_t^m \equiv \left[\int_0^1 (p_t^m(i))^{1-\sigma^{mb}} di\right]^{\frac{1}{1-\sigma^{mb}}} \tag{O.72}$$

Thus, the demand facing foreign exporter i is:

$$y_t^m(i) = \left(\frac{p_t^m(i)}{p_t^m}\right)^{-\sigma^{mb}} y_t^m \tag{O.73}$$

where $y_t^m(i) \equiv c_t^m(i) + mi_t^h(i) + mi_t^x(i)$ is the total demand for imports from firm i.

The problem of foreign exporter i is therefore:[2]

(O.74)

$$\max_{p_t^m(i)} E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s}^f \left\{ p_{t+s}^m(i) y_{t+s}^m(i) - mc_{t+s}^f(i) y_{t+s}^m(i) - \frac{\chi^m}{2}\left(\frac{p_{t+s}^m \pi_{t+s}}{p_{t+s-1}^m \Xi_{t+s}^m} - 1\right)^2 p_{t+s}^m y_{t+s}^m \right\}$$

subject to demand (O.73). Λ_t^f is the marginal utility of real income to the owners of the foreign firm, $mc_t^f(i)$ is foreign exporter i's marginal cost, and Ξ_t^m captures the partial indexation to past import inflation:

$$\Xi_t^m = \left(\frac{p_{t-1}^m}{p_{t-2}^m}\pi_{t-1}\right)^{\epsilon^m} \tag{O.75}$$

To proceed, we assume that foreign producers purchase goods on world markets at the exogenous price p_t^{xf}, which implies (PC_t-deflated) marginal costs of foreign producers in domestic currency terms are:

$$mc_t^f = \frac{p_t^{xf}}{q_t} \tag{O.76}$$

Taking first order conditions and rearranging, we obtain a Phillips curve for imports:

$$\begin{aligned} &1 - \sigma^{mb} + \frac{\sigma^{mb} p_t^{xf}}{q_t p_t^m} - \chi^m \frac{p_t^m \pi_t}{p_{t-1}^m \Xi_t^m}\left(\frac{p_t^m}{p_{t-1}^m}\frac{\pi_t}{\Xi_t^m} - 1\right) \\ &\quad + \beta\chi^m E_t \frac{\Lambda_{t+1}^f y_{t+1}^m}{\Lambda_t^f y_t^m}\frac{(p_{t+1}^m)^2 \pi_{t+1}}{(p_t^m)^2 \Xi_{t+1}^m}\left(\frac{p_{t+1}^m}{p_t^m}\frac{\pi_{t+1}}{\Xi_{t+1}^m} - 1\right) = 0 \end{aligned} \tag{O.77}$$

[2]Note that domestic inflation π_t only features in this problem because the objective function has been normalized by PC_t. Adjustment costs are quadratic in deviations of PM_t/PM_{t-1} from the target rate Ξ_t^m. When we express that ratio in terms of relative import prices $p_t^m \equiv PM_t/PC_t$, it becomes $\pi_t \cdot p_t^m/p_{t-1}^m$.

Note that we do not need to specify a process for the foreign stochastic discount factor $(\Lambda^f_{t+1}/\Lambda^f_t)$. As long as it is assumed to be stationary, it cancels out in both the steady state, and when we log-linearize the model before solving.

Exchange rates and the balance of payments

Assume that foreign exchange market participants can trade in domestic and foreign bonds, but make up a negligible amount of the domestic market and so do not affect the government budget constraint. They can access domestic government bonds directly, so earn i^{CB}_t on them, not the household i^e_t. The nominal interest rate on foreign bonds is i^f_t. The real exchange rate q_t is then determined according to the UIP condition:

$$\mathbb{E}_t \frac{1+i^{CB}_t}{\pi_{t+1}} \left(1+\chi^{nfa}(nfa_t - \overline{nfa})\right) = \mathbb{E}_t \frac{1+i^f_t}{\pi^f_{t+1}} \frac{q_t}{q_{t+1}} \tag{O.78}$$

where nfa_t is the domestic country's real net foreign asset position, and $\overline{nfa}$ is the steady state of nfa_t. If the parameter $\chi^{nfa} = 0$, this reduces to the standard UIP condition. I instead calibrate χ^{nfa} to a small positive value, implying that movements in the domestic country's net foreign asset position will create a small wedge in UIP. This could come, for example, from quadratic costs in holding net foreign asset positions that deviate from the steady state position.[3] The wedge is necessary to ensure that the steady state nfa_t is determinate, as discussed in Ghironi and Melitz (2005).

The net foreign asset position evolves to satisfy the balance of payments, i.e. so that changes in the financial account balance those in the current account:

$$nfa_t = nfa_{t-1} \frac{1+i^f_{t-1}}{\pi^f_t} \frac{q_{t-1}}{q_t} + p^x_t x_t - p^m_t (c^m_t + mi^h_t + mi^x_t) \tag{O.79}$$

Additional definitions

Table 12 in the main paper lists the log-linearized model equations. To express the second (the wage Phillips curve) concisely, it is helpful to define wage inflation:

$$\pi^w_t \equiv \frac{W_t}{W_{t-1}} = \frac{w_t}{w_{t-1}} \pi_t \tag{O.80}$$

[3] Harrison and Oomen (2010) indeed have such a microfoundation for the wedge, which they include in the household budget constraint. See the discussion of departures from their model below for further details.

Similarly, other inflation rates are defined:

$$\pi_t^{hv} \equiv \frac{PHV_t}{PHV_{t-1}} = \frac{p_t^{hv}}{p_{t-1}^{hv}} \pi_t \tag{O.81}$$

$$\pi_t^{xvf} \equiv \frac{PXVF_t}{PXVF_{t-1}} = \frac{p_t^{xv}}{p_{t-1}^{xv}} \frac{q_t}{q_{t-1}} \pi_t^f \tag{O.82}$$

$$\pi_t^{m} \equiv \frac{PM_t}{PM_{t-1}} = \frac{p_t^{m}}{p_{t-1}^{m}} \pi_t \tag{O.83}$$

(O.84)

In addition, it simplifies the log-linearization to use gross rather than nominal interest rates:

$$r_t^e \equiv 1 + i_t^e \tag{O.85}$$

$$r_t^{CB} \equiv 1 + i_t^{CB} \tag{O.86}$$

$$r_t^g \equiv 1 + i_t^g \tag{O.87}$$

$$r_t^b \equiv 1 + i_t^b \tag{O.88}$$

$$r_t^f \equiv 1 + i_t^f \tag{O.89}$$

In equations (O.63) and (O.62), the shocks ζ_t^χ and $\zeta_t^{\chi b}$ are introduced as mean-zero shocks. To avoid them dropping out in the log-linearization, we substitute out for them using:

$$\zeta_t^\chi = e^{\hat{\zeta}_t^\chi} - 1 \tag{O.90}$$

$$\zeta_t^{\chi b} = e^{\hat{\zeta}_t^{\chi b}} - 1 \tag{O.91}$$

where $\hat{\zeta}_t^\chi, \hat{\zeta}_t^{\chi b}$ are also mean-zero AR(1) shocks.

In the same way, we also replace the mean-zero capital adjustment cost shock ζ_t^k with:

$$\zeta_t^k = e^{\hat{\zeta}_t^k} - 1 \tag{O.92}$$

Finally, we add two non-microfounded shocks which are common in the DSGE literature. The first is a risk premium shock ζ_t^c, which modifies the consumption Euler equation obtained by combining equation (O.17) with (O.13) and the definition of u_{ct} to:

$$u_{ct} = \beta e^{\zeta_t^c} \mathbb{E}_t \frac{(1 + i_t^e)}{\pi_{t+1}} u_{ct+1} \tag{O.93}$$

The second is a markup shock ζ_t^{hb}, which modifies the Phillips curve for domestic goods (O.56) to:

$$1 - \sigma^{hb} + \frac{\sigma^{hb} mc_t}{p_t^{hv}} - \chi^{hv} \frac{p_t^{hv}\pi_t}{p_{t-1}^{hv}\Xi_t^{hv}} \left(\frac{p_t^{hv}}{p_{t-1}^{hv}} \frac{\pi_t}{\Xi_t^{hv}} - 1 \right)$$

$$+ \beta \chi^{hv} E_t \frac{\Lambda_{t+1} y_{t+1}^{hv}}{\Lambda_t y_t^{hv}} \frac{(p_{t+1}^{hv})^2 \pi_{t+1}}{(p_t^{hv})^2 \Xi_{t+1}^{hv}} \left(\frac{p_{t+1}^{hv}}{p_t^{hv}} \frac{\pi_{t+1}}{\Xi_{t+1}^{hv}} - 1 \right) = \chi^{hv}(e^{\zeta_t^{hb}} - 1) \tag{O.94}$$

Model variables

The model can be reduced to a system of 27 endogenous variables:[4] c_t, c_t^h, c_t^m, h_t, i_t^b, i_t^{CG}, i_t^e, i_t^g, inv_t, k_t, λ_t, nfa_t, p_t^g, p_t^h, p_t^{hv}, p_t^m, p_t^x, p_t^{xv}, π_t, q_t, r_t, u_{ct}, w_t, x_t, y_t^h, y_t^v, z_t. The equilibrium conditions consist of equations (O.6), (O.7), (O.8), (O.9), (O.14), (O.15), (O.18), (O.28), (O.29), (O.32), (O.33), (O.46), (O.47), (O.57), (O.60), (O.61), (O.62), (O.63), (O.66), (O.67), (O.69), (O.70), (O.77), (O.78), (O.79), (O.93), (O.94). There are 14 exogenous shock processes: tfp_t, g_t, ζ_t^c, ζ_t^{hb}, $\zeta_t^{\kappa h}$, $\hat{\zeta}_t^k$, ζ^{rcb}, π_t^f, c_t^f, r_t^f, p_t^{xf}, $\hat{\zeta}_t^{\chi}$, $\hat{\zeta}_t^{\chi b}$, ζ_t^{μ}. These have the following processes:

$$tfp_t = \rho_{tfp} tfp_{t-1} + e_{tfpt} \tag{O.95}$$

$$g_t = \rho_g g_{t-1} + e_{gt} \tag{O.96}$$

$$\zeta_t^c = \rho_{\zeta^c} \zeta_{t-1}^c + (1 - \rho_{\zeta^c}^2)^{\frac{1}{2}} e_{\zeta^c t} \tag{O.97}$$

$$\zeta_t^{hb} = \rho_{\zeta^{hb}} \zeta_{t-1}^{hb} + (1 - \rho_{\zeta^{hb}}^2)^{\frac{1}{2}} e_{\zeta^{hb} t} \tag{O.98}$$

$$\hat{\zeta}_t^k = \rho_{\zeta^k} \hat{\zeta}_{t-1}^k + (1 - \rho_{\zeta^k}^2)^{\frac{1}{2}} e_{\zeta^k t} \tag{O.99}$$

$$\zeta_t^{\kappa h} = e_{\zeta^{\kappa h} t} \tag{O.100}$$

$$\zeta_t^{rcb} = e_{\zeta^{rcb} t} \tag{O.101}$$

$$\hat{\zeta}_t^{\chi} = \rho_{\zeta^{\chi}} \hat{\zeta}_{t-1}^{\chi} + (1 - \rho_{\zeta^{\chi}}^2)^{\frac{1}{2}} e_{\zeta^{\chi} t} \tag{O.102}$$

$$\hat{\zeta}_t^{\chi b} = \rho_{\zeta^{\chi b}} \hat{\zeta}_{t-1}^{\chi b} + (1 - \rho_{\zeta^{\chi b}}^2)^{\frac{1}{2}} e_{\zeta^{\chi b} t} \tag{O.103}$$

$$\zeta_t^{\mu} = e_{\zeta^{\mu} t} \tag{O.104}$$

with $e_{xt} \sim i.i.d.N(0, \sigma_x^2)$ for $x \in \{tfp, g, \zeta^c, \zeta^{hb}, \zeta^k, \zeta^{\kappa h}, \zeta^{rcb}, \zeta^{\chi}, \zeta^{\chi b}, \zeta^{\mu}\}$. $\pi_t^f, c_t^f, r_t^f, p_t^{xf}$ then follow the VAR(4) process detailed in Appendix E.2.2.

Steady state

For this section, $\bar{x}$ refers to the steady state of the associated variable x_t. I consider a steady state in which inflation is zero in all goods. That is, $\bar{\pi} = 1$, $\bar{\pi}^f = 1$. As a direct

[4] All variables excluded from this list are simple functions of the included variables. For example, taxes τ_t are a function of ph_t, g_t, and i_{t-1}^{CB} through the government budget constraint (O.65).

result, the indexation variables $\bar{\Xi}^{hv}$, $\bar{\Xi}^{xv}$, $\bar{\Xi}^{w}$ are all also equal to 1.

Relative prices Without loss of generality, I impose that $\bar{p}^h = \bar{p}^m$. In equation O.5 in steady state, this implies

$$\bar{p}^h = \kappa^c \left[(1 - \psi^m)^{\sigma^m} + (\psi^m)^{\sigma^m} \right]^{\frac{1}{\sigma^m - 1}} \tag{O.105}$$

I set the parameter κ^c to

$$\kappa^c = \left[(1 - \psi^m)^{\sigma^m} + (\psi^m)^{\sigma^m} \right]^{\frac{1}{1 - \sigma^m}} \tag{O.106}$$

which is a normalization that ensures $\bar{p}^h = \bar{p}^m = 1$.

Using this, and the fact that final goods producers price at marginal cost, equation O.46 in steady state is

$$1 = \kappa^{hv} \bar{p}^{hv} + 1 - \kappa^{hv} \tag{O.107}$$

which implies $\bar{p}^{hv} = 1$.

From this, equation O.56 implies

$$\overline{mc} = \frac{\sigma^{hb} - 1}{\sigma^{hb}} \tag{O.108}$$

which can be substituted into equation O.57 to give

$$\bar{p}^{xv} = \left(\frac{\sigma^{xb}}{\sigma^{xb} - 1} \right) \left(\frac{\sigma^{hb} - 1}{\sigma^{hb}} \right) \tag{O.109}$$

Equation O.47 in steady state, again using the fact that final goods producers price at marginal cost, gives

$$\bar{p}^x = 1 + \kappa^{xv} \left(\frac{\sigma^{hb} - \sigma^{xb}}{\sigma^{hb}(\sigma^{xb} - 1)} \right) \tag{O.110}$$

In all quantitative exercises, I assume as in HO that elasticities of substitution are equal across export and domestic markets (i.e. $\sigma^{xb} = \sigma^{hb}$). In these equations, the assumption implies that $\bar{p}^{xv} = \bar{p}^x = 1$.

Finally, from equation O.77 in steady state

$$\bar{p}^{xf} = \frac{\sigma^{mb} - 1}{\sigma^{mb}} \bar{q} \tag{O.111}$$

where the steady state real exchange rate $\bar{q}$ is derived below, and as with the other elasticities of substitution I set $\sigma^{mb} = \sigma^{hb}$ in all quantitative exercises.

Firms Without loss of generality, I fix the steady state of output at $\bar{y}^v = 1$. This aids the calibration of steady state government spending and investment as the relative contributions of those objects to UK GDP in the data. Given these, I then use the equations of the firm problem back out the steady state TFP required $\bar{y}^v = 1$ to hold.

Specifically, I calibrate χ^z such that steady state capital utilization $\bar{z} = 1$. Equation O.11 then implies that in steady state $\bar{k}^s = \bar{k}$. I fix steady state investment to $\overline{inv}$, which is calibrated to match the average share of investment in output in UK national accounts. Equation O.9 then implies that:

$$\bar{k} = \frac{\overline{inv}}{\delta} \tag{O.112}$$

Rearranging the firm first order condition on capital services (equation O.35) then gives the steady state of TFP:

$$\overline{tfp} = \bar{k}^{\frac{1}{\sigma^y - 1}} \left(\frac{\bar{r}}{\alpha}\right)^{\frac{\sigma^y}{\sigma^y - 1}} (\overline{mc})^{-\frac{\sigma^y}{\sigma^y - 1}} \tag{O.113}$$

From equations O.16 and O.18 we have

$$\bar{R} = \chi^z \overline{PI} \tag{O.114}$$

Using the definition $R_t = r_t \cdot PC_t$, and the result above that $PI_t = PH_t$, this rearranges to

$$\bar{r} = \bar{p}_t^h \chi^z = \chi^z \tag{O.115}$$

where the final equality uses the fact that relative prices are 1 in steady state.

Using all of these results, and equation O.108, equation O.113 becomes:

$$\overline{tfp} = \left(\frac{\chi^z \sigma^{hb}}{\alpha(\sigma^{hb} - 1)}\right)^{\frac{\sigma^y - 1}{\sigma^y}} \bar{k}^{\frac{1}{\sigma^y - 1}} \tag{O.116}$$

Next, I find the steady state of hours $\bar{h}$. Take the production function of intermediate goods producers (equation O.32), substitute out for $\bar{y}^v = 1$, and rearrange to obtain:

$$\bar{h} = \left[\frac{1}{1-\alpha} \left(\frac{1}{\overline{\overline{tfp}}} \right)^{\frac{\sigma^y-1}{\sigma^y}} - \frac{\alpha}{1-\alpha} \bar{k}^{\frac{\sigma^y-1}{\sigma^y}} \right]^{\frac{\sigma^y}{\sigma^y-1}} \tag{O.117}$$

Using equation O.33, and substituting in that $\bar{r} = \chi^z$ and $\bar{k}^s = \bar{k}$, we obtain steady state real wages

$$\bar{w} = \frac{1-\alpha}{\alpha} \chi^z \left(\frac{\bar{k}}{\bar{h}} \right)^{\frac{1}{\sigma^y}} \tag{O.118}$$

Exhange rates and balance of payments Without loss of generality, I normalize $\bar{c}^f$ to 1. Given this, and the previously derived $\bar{p}^x = 1$, equation O.70 implies

$$\bar{x} = \kappa^x \left(\frac{\bar{q}}{\bar{p}^{xf}} \right)^{-\sigma^x} \tag{O.119}$$

I set the parameter κ^x to $\bar{x}(\bar{p}^{xf})^{-\sigma^x}$, which implies $\bar{q} = 1$.

From UIP (equation O.78) we obtain:

$$\bar{i}^f = \bar{i}^{CB} \tag{O.120}$$

Next, I turn to the steady state current account balance. From market clearing (equation O.69), and cost minimization (equations O.42 and O.43), we have:

$$\bar{y}^v = \bar{y}^{hv} + \bar{y}^{xv} = \frac{\kappa^{hv}}{1-\kappa^{hv}} \overline{mi}^h + \frac{\kappa^{xv}}{1-\kappa^{xv}} \overline{mi}^x \tag{O.121}$$

which rearranges to:

$$\bar{y}^v = \frac{1}{1-\kappa^{hv}} \overline{mi}^h + \frac{1}{1-\kappa^{xv}} \overline{mi}^x - \overline{mi}^h - \overline{mi}^x \tag{O.122}$$

Substituting out for the first two terms using the production functions for final goods for home and export consumption (equations O.37 and O.38):

$$\bar{y}^v = \bar{y}^h + \bar{y}^x - \overline{mi}^h - \overline{mi}^x \tag{O.123}$$

Rearranging this, and using $\bar{y}^x = \bar{x}$ from equation O.68, we obtain:

$$\bar{x} - \overline{mi}^h - \overline{mi}^x = \bar{y}^v - \bar{y}^h = 1 - \bar{y}^h \tag{O.124}$$

Substituting out for $\bar{y}^h$ using equation O.67, this becomes

$$\bar{x} - \overline{mi}^h - \overline{mi}^x = 1 - \bar{c}^h - \overline{inv} - \bar{g} \tag{O.125}$$

Subtracting consumption of imports from both sides, and noting from equation O.8 that $\bar{c}^h + \bar{c}^m$ adds up to total consumption $\bar{c}$, we obtain an expression for the current account balance $\overline{ca}$:

$$\overline{ca} \equiv \bar{x} - \bar{c}^m - \overline{mi}^h - \overline{mi}^x = 1 - \bar{c} - \overline{inv} - \bar{g} \tag{O.126}$$

Steady state investment and government spending are calibrated externally (see Firm section above). I therefore fix $\bar{c}$ to match the average current account balance as a percentage of GDP over the period considered (since total output $\bar{y}^v$ is normalized to 1). Steady state exports adjust to ensure that this $\bar{c}$ is consistent with market clearing, as derived below.

With these results in hand, rearrange the law of motion for net foreign assets (equation O.79) in steady state to:

$$-\bar{i}^f \overline{nfa} = \bar{x} - \bar{c}^m - \overline{mi}^h - \overline{mi}^x \tag{O.127}$$

where I have used that relative prices $\bar{p}^m$ and $\bar{p}^x$ are both 1 in steady state. The right hand side of this equation is equal to $\overline{ca}$, so using equations O.126 and O.120, this becomes

$$\overline{nfa} = -\left(\frac{1 - \bar{c} - \overline{inv} - \bar{g}}{\bar{i}^{CB}}\right) \tag{O.128}$$

Households From equation O.17, we have $\bar{i}^e = \beta^{-1} - 1$.

Given the steady state consumption calibration described above, equations O.6 and O.7 then give the consumption of domestic and imported goods

$$\bar{c}^h = (1 - \psi^m)^{\sigma^m} (\kappa^c)^{\sigma^m - 1} \bar{c} \tag{O.129}$$

$$\bar{c}^m = (\psi^m)^{\sigma^m} (\kappa^c)^{\sigma^m - 1} \bar{c} \tag{O.130}$$

In addition, the steady state marginal utility of consumption comes from equation O.13

$$\bar{u}_c = \bar{c}^{-\frac{1}{\sigma^c} + \psi^{hab}(\frac{1}{\sigma^c} - 1)} \tag{O.131}$$

With the results derived here we also obtain, from equations O.67 and O.37 respectively

$$\bar{y}^h = \bar{c}^h + \overline{inv} + \bar{g} \tag{O.132}$$

$$\overline{mi}^h = (1 - \kappa^{hv})\bar{y}^h \tag{O.133}$$

Information and banks To find the steady state parameters in the attention block of the model, I first define two new steady state objects, which are calibration targets. First, $\overline{mn}$ is the spread between the policy rate and the unconditional mean interest rate available on savings:

$$\overline{mn} \equiv \bar{i}^{CB} - \frac{\bar{i}^g + \bar{i}^b}{2} \tag{O.134}$$

Second, $\overline{sd}$ as the standard deviation of available interest rates:

$$\overline{sd} \equiv \frac{\bar{i}^g - \bar{i}^b}{2} \tag{O.135}$$

Both of these are calibrated to long-run moments from the Moneyfacts data, as described in Appendix E.2.

From the attention first order condition (combining equations O.13 and O.14), we have

$$\bar{\lambda} = \frac{\mu}{\beta b \bar{u}_c} \tag{O.136}$$

Rearranging equation O.60 yields

$$\bar{p}^g = \frac{\exp\left(\frac{2\overline{sd}}{\bar{\lambda}}\right)}{\exp\left(\frac{2\overline{sd}}{\bar{\lambda}}\right) + 1} \tag{O.137}$$

Using $\bar{i}^e = \beta^{-1}$ and the definition of $\overline{sd}$, equation O.61 can be written as

$$\bar{i}^b = \beta^{-1} - 2\bar{p}^g\overline{sd} \tag{O.138}$$

Substituting this into equation O.135 gives

$$\bar{i}^g = \beta^{-1} + 2(1 - \bar{p}^g)\overline{sd} \tag{O.139}$$

Having solved for each offered interest rate, we now use equation O.134 to back out the steady state policy rate:

$$\bar{i}^{CB} = \overline{mn} + \frac{\bar{i}^g + \bar{i}^b}{2} \tag{O.140}$$

Finally, we use the bank first order conditions (equations O.62 and O.63) to back out the cost parameters χ^g and χ^b required to hit the calibration targets $\overline{mn}, \overline{sd}$ in steady state. Specifically, rearranging these first order conditions in steady state gives:

$$\bar{i}^g = \bar{i}^{CB} - \chi^g - \frac{\bar{\lambda}}{1 - \bar{p}^g} \tag{O.141}$$

$$\bar{i}^b = \bar{i}^{CB} - \chi^b - \frac{\bar{\lambda}}{\bar{p}^g} \tag{O.142}$$

Substituting these optimality conditions into the definitions of $\overline{mn}$ and $\overline{sd}$ and rearranging we obtain two conditions pinning down χ^g, χ^b. The unique solution to these conditions is

$$\chi^g = \overline{mn} - \overline{sd} - \frac{\bar{\lambda}}{1 - \bar{p}^g} \tag{O.143}$$

$$\chi^b = \overline{mn} + \overline{sd} - \frac{\bar{\lambda}}{\bar{p}^g} \tag{O.144}$$

Exports From equation O.69 we have

$$\bar{y}^v = \kappa^{hv} \bar{y}^h + \kappa^{xv} \bar{y}^x \tag{O.145}$$

Substituting out for $\bar{y}^h$ using equation O.67, using that $\bar{y}^v = 1$, and rearranging gives

$$\bar{y}^x = \frac{1 - \kappa^{hv}(\bar{c}^h + \overline{inv} + \bar{g})}{\kappa^{xv}} \tag{O.146}$$

Note that from equation O.68, $\bar{x} = \bar{y}^x$. Equation O.38 then implies

$$\overline{mi}^x = (1 - \kappa^{xv}) \bar{y}^x \tag{O.147}$$

Government I set steady state government spending $\bar{g}$ to match the share of government spending in GDP from UK national accounts. Steady state lump sum taxes are then pinned down by equation O.65, which in real terms in steady state is

$$\bar{\tau} = \bar{g} + \bar{i}^{CB} b \tag{O.148}$$

Comparison to Harrison and Oomen (2010)

Aside from the introduction of inattention to savings and a banking sector, I only make minimal changes to the model in Harrison and Oomen (2010). In the shocks, I use a risk premium shock rather than a discount factor shock, and I assume that the labor disutility shock is i.i.d. (Harrison and Oomen estimate its persistence at 0.001). In price setting, I model labor unions and foreign exporters as facing quadratic adjustment costs of price changes, rather than Calvo-style staggered contracts. This makes no difference to the log-linearized equations, but makes the exposition simpler and brings them into line with the price-setting problem of domestic intermediate goods producers. Finally, Harrison and Oomen allow households to invest in foreign bonds, subject to a quadratic cost of holding a portfolio that deviates from steady state net foreign assets. In contrast, I do not allow households to access these bonds, and instead impose UIP and the balance of payments separately. The reason for this is that, if I followed the Harrison and Oomen approach, UIP would depend on i_t^e rather than the policy rate i_t^{CB}. It is not plausible that arbitrageurs in foreign exchange markets are subject to the same information frictions as households, and so I impose UIP separately. The log-linearized versions of (O.78) and (O.79) therefore correspond exactly to those in Harrison and Oomen, but they are not derived from the household problem.

The attention and bank problems introduce 5 new variables not in the Harrison and Oomen model: i_t^e, λ_t, p_t^g, i_t^g, i_t^b. The new equations are the first order condition on attention (O.14), the choice probability rule (O.60), the definition of i_t^e (O.61), and the two bank first order conditions (O.62 and O.63). There are three new shocks, to attention (ζ_t^μ), the level of bank interest rates (ζ_t^χ) and their dispersion ($\zeta_t^{\chi b}$).

Two-agent model extension

In Appendix E.3, I introduce an extension to the quantitative model to include borrowers. This section sets out this extended model.

Households

A fraction $1 - q_d$ of households are savers. They face exactly the same utility function (equation O.1) as in the representative-agent model. Their budget constraint is also unchanged, except for a lump sum tax which will be transferred to debtor households.

The saver budget constraint is therefore:

(O.149)

$$PC_t c_t + PI_t inv_t + B_t - (1 + i^e_{t-1})B_{t-1} = W_t h_t + R_t k^s_t + \Pi^v_t + \Pi^b_t - PC_t \tau_t - PC_t \frac{q_d}{1 - q_d} \tau_0$$

where the final term is the new lump-sum tax, set to ensure the real transfer to each debtor household is equal to τ_0. Including this tax does not affect the first order conditions (equations O.13-O.18). Note that bank profit Π^b_t now also includes any profit made by banks engaged in lending, though again since this is lump sum it does not affect the first order conditions.

The remaining q_d households are debtors. Their instantaneous utility function is identical to that of savers, but they have a lower discount factor $\beta^d < \beta$. This means they will borrow in equilibrium, and will hold no capital or shares in firms. Their problem is given by

(O.150)

$$\max_{c^d_t, D_t, i^e_t} E_t \sum_{s=0}^{\infty} (\beta^d)^s \left[\frac{1}{1 - \frac{1}{\sigma^c}} \left(\frac{c^d_t}{\tilde{c}^{\psi^{hab}}_{t-1}} \right)^{1 - \frac{1}{\sigma^c}} - (\kappa^h)^{-\frac{1}{\sigma^h}} e^{\zeta^{\kappa h}_t} \frac{1}{1 + \frac{1}{\sigma^h}} (h_t)^{1 + \frac{1}{\sigma^h}} - \mu e^{\zeta^{\mu}_t} \mathcal{I}^d(i^{ed}_t) \right]$$

subject to

$$PC_t c^d_t - D_t + (1 + i^{ed}_{t-1})D_{t-1} = W_t h_t - PC_t \tau_t + PC_t \tau_0 \tag{O.151}$$

$$D_t \leq PC_t d \tag{O.152}$$

$$\mathcal{I}^{d\prime}(i^{ed}_t) < 0, \quad \mathcal{I}^{d\prime\prime}(i^{ed}_t) > 0 \tag{O.153}$$

where c^d_t is debtor consumption, D_t is nominal debt, i^{ed}_t is the effective interest rate on debt, and $\mathcal{I}^d(i^{ed}_t)$ is the information processing required to achieve that effective interest rate. Equation O.152 is the borrowing constraint: real debt cannot exceed the exogenous limit d. Note that in the budget constraint debtors have the same labor income $w_t h_t$ as savers, which is explained in the labor union section below. Habits for both savers and debtors depend on $\tilde{c}_{t-1}$, which is aggregate consumption across both household types, defined as:

$$\tilde{c}_t = (1 - q_d)c_t + q_d c^d_t \tag{O.154}$$

In real terms, the budget constraint of debtors is:

$$c_t^d - d_t + \frac{(1+i_{t-1}^{ed})}{\pi_t} d_{t-1} = w_t h_t - \tau_t + \tau_0 \tag{O.155}$$

where $d_t \equiv D_t/PC_t$ is real debt.

The first order conditions of debtors are:

$$\frac{1}{\tilde{c}_{t-1}^{\psi^{hab}}} \left(\frac{c_t^d}{\tilde{c}_{t-1}^{\psi^{hab}}} \right)^{-\frac{1}{\sigma^c}} = PC_t \Lambda_t^d \tag{O.156}$$

$$\beta^d E_t \Lambda_{t+1}^d D_t = -\mu e^{\zeta_t^\mu} \mathcal{I}_t^{d\prime}(i_t^{ed}) \tag{O.157}$$

$$\Lambda_t^d = \beta^d E_t (1+i_t^{ed}) \Lambda_{t+1}^d + \Theta_t^d \tag{O.158}$$

where Λ_t^d and Θ_t^d are the Lagrange multipliers on the budget constraint and the borrowing constraint respectively.

β^d will be set sufficiently low that in the neighbourhood of steady state, $\beta^d(1+i_t^{ed}) < 1$. Equation O.158 implies that $\Theta_t^d > 0$, i.e. that the borrowing constraint binds. This means that $d_t = d$, and debtor consumption is determined by the budget constraint (equation O.155) alone. Debtors are therefore hand-to-mouth, with a marginal propensity to consume of 1.

For both types of household, the consumption index is defined using the same CES aggregator over home and imported goods (equation O.3), so the price index remains as in equation O.4, and the allocation of expenditure between home and imported goods for savers is as in equations O.6 and O.7. For debtors, the equivalent allocation equations are

$$c_t^{hd} = (1-\psi^m)^{\sigma^m} (\kappa^c)^{\sigma^m - 1} (p_t^h)^{-\sigma^m} c_t^d \tag{O.159}$$

$$c_t^{md} = (\psi^m)^{\sigma^m} (\kappa^c)^{\sigma^m - 1} (p_t^m)^{-\sigma^m} c_t^d \tag{O.160}$$

Since the coefficients are the same for both household types, we can write expressions for aggregate domestic and imported consumption as

$$\tilde{c}_t^h = (1-\psi^m)^{\sigma^m} (\kappa^c)^{\sigma^m - 1} (p_t^h)^{-\sigma^m} \tilde{c}_t \tag{O.161}$$

$$\tilde{c}_t^m = (\psi^m)^{\sigma^m} (\kappa^c)^{\sigma^m - 1} (p_t^m)^{-\sigma^m} \tilde{c}_t \tag{O.162}$$

Individuals

Individuals within saver households are as in the representative-agent model. Individuals within the debtor households are as described in Appendix C.2.1. Specifically, since we will assume the number of lending banks is $N^d = 2$, we can denote the probability of choosing the lower interest rate lender as p_t^{gd}. Solving the individual's rational inattention problem yields:

$$p_t^{gd} = \frac{\exp\left(-\frac{i_t^{gd}}{\lambda_t^d}\right)}{\exp\left(-\frac{i_t^{gd}}{\lambda_t^d}\right) + \exp\left(-\frac{i_t^{bd}}{\lambda_t^d}\right)} \tag{O.163}$$

where i_t^{gd}, i_t^{bd} are the good (low) and bad (high) interest rates on loans offered by the two banks, and λ_t^d is the shadow value of information about borrowing.

The effective interest rate experienced by borrowers is then

$$i_t^{ed} = p_t^{gd} i_t^{gd} + (1 - p_t^{gd}) i_t^{bd} \tag{O.164}$$

Using equation C.22 from the main paper, $\mathcal{I}^{d\prime}(i_t^{ed}) = -(\lambda_t^d)^{-1}$.

Banks

Deposit-taking banks are as in the representative-agent model. Lending banks are modeled as in Appendix C.2.1, with $N^d = 2$. As in that Appendix, we have that the first order conditions for profit maximization for good and bad banks respectively are

$$\frac{dp_t^{gd}}{di_t^{gd}} \cdot (i_t^{gd} - i_t^{CB} - \chi_t^{gd}) = -p_t^{gd} \tag{O.165}$$

$$-\frac{dp_t^{gd}}{di_t^{bd}} \cdot (i_t^{bd} - i_t^{CB} - \chi_t^{bd}) = -(1 - p_t^{gd}) \tag{O.166}$$

Differentiating equation O.163 with respect to each interest rate and substituting into these first order conditions, they become

$$(1 - p_t^{gd}) \cdot (i_t^{gd} - i_t^{CB} - \chi_t^{gd}) = \lambda_t^d \tag{O.167}$$

$$p_t^{gd} \cdot (i_t^{bd} - i_t^{CB} - \chi_t^{bd}) = \lambda_t^d \tag{O.168}$$

The costs χ_t^{gd} and χ_t^{bd} are specified in Appendix E.3, and are reproduced here as equations O.169 and O.170

$$\chi_t^{gd} = \chi_0^{gd} + \zeta_t^{\chi} \tag{O.169}$$

$$\chi_t^{bd} = \chi_0^{bd} + \chi_1(i_t^{CB} - \bar{i}^{CB}) + \zeta_t^{\chi} + \zeta_t^{\chi b} \tag{O.170}$$

where $\chi_0^{gd}, \chi_0^{bd}, \chi_1$ are constants, and $\zeta_t^{\chi}, \zeta_t^{\chi b}$ are AR(1) exogenous shocks.

Labor unions

As in the representative agent model, households supply labor to unions, who set wages. Saver and debtor households are members of the same unions, and labor supply from debtors is a perfect substitute for labor supply from savers. This means that the wage and labor supply from each union is the same for both household types.

Unions set wages to maximize the average expected discounted utility of their members. The problem of labor unions is therefore:

$$\max_{W_t(i)} E_t \sum_{s=0}^{\infty} \Bigg\{ \tilde{\Lambda}_{t+s} W_{t+s}(i) h_{t+s}(i) - \tilde{\beta}^s (\kappa^h)^{-\frac{1}{\sigma^h}} e^{\zeta^{\kappa h}_{t+s}} \frac{1}{1+\frac{1}{\sigma^h}} (h_{t+s}(i))^{1+\frac{1}{\sigma^h}} \\ - \tilde{\Lambda}_{t+s} \frac{\chi^w}{2} \left(\frac{W_{t+s}(i)}{W_{t+s-1}(i)\Xi^w_{t+s}} - 1 \right)^2 W_{t+s} \Bigg\} \tag{O.171}$$

subject to labor demand (equation O.23) and the definition of wage indexation Ξ^w_t (equation O.26). This is exactly as in the representative-agent model, except that wages and wage adjustment costs are discounted using average preferences across both household types:

$$\tilde{\beta} \equiv (1-q_d)\beta + q_d \beta^d \tag{O.172}$$

$$\tilde{\Lambda}_{t+s} \equiv (1-q_d)\beta^s \Lambda_{t+s} + q_d (\beta^d)^s \Lambda^d_{t+s} \tag{O.173}$$

Following the steps as in the representative-agent model, union wage setting generates a wage Phillips curve given by:

$$(1-\sigma^w) h_t = -(\kappa^h)^{-\frac{1}{\sigma^h}} e^{\zeta^{\kappa h}_t} \sigma^w \frac{h_t^{1+\frac{1}{\sigma^h}}}{w_t \tilde{u}_{ct}} + \frac{\chi^w w_t \pi_t}{w_{t-1}\Xi^w_t} \left(\frac{w_t}{w_{t-1}\Xi^w_t}\pi_t - 1 \right) \\ - E_t \frac{(\beta(1-q_d)u_{ct+1} + \beta^d q_d u^d_{ct+1})\chi^w w^2_{t+1}\pi_{t+1}}{w_t^2 \Xi^w_{t+1} \tilde{u}_{ct}} \left(\frac{w_{t+1}}{w_t \Xi^w_{t+1}} \pi_{t+1} - 1 \right) \tag{O.174}$$

where $\tilde{u}_{ct} = \tilde{\Lambda}_t P C_t$ is the average marginal utility of consumption across all households.

Firms and price setting

Firms are unchanged from the representative-agent model. In particular, firms are owned by savers only, so continue to discount future profits based on saver preferences alone. The price-setting problem is therefore unchanged from the representative-agent model.

Monetary and fiscal policy

Monetary policy is as in the representative-agent model. The government budget constraint is however different in two respects. First, I change the supply of real bonds to $(1-q_d)b$, so that the equilibrium quantity of bonds held by savers remains at b as in the representative-agent model. Second, the government is the source of funds for the lending banks. This ensures that saving and borrowing are treated symmetrically from the bank side. This means that the government lends out $q_d d$ real bonds in period t, and is repaid $q_d d(1+i_t^{CB})$ in period $t+1$. The government budget constraint in real terms therefore becomes

$$\tau_t = p_t^h g_t + ((1-q_d)b - q_d d)i_{t-1}^{CB} \tag{O.175}$$

Market clearing

Domestic goods market clearing is

$$y_t^h = \tilde{c}_t^h + inv_t + g_t \tag{O.176}$$

Export goods market clearing is as in the representative-agent model (equation O.68). Total domestic output is as in equation O.69.

Foreign variables, exchange rates, and the balance of payments

These are all as in the representative-agent model.

Steady state

As in Galí et al. (2007), I set the steady-state inter-household transfer τ_0 such that steady state consumption is identical across saver and debtor households. As a result, the steady states of all variables that appear in the representative-agent model are unchanged by the introduction of debtors, with the exception of steady state taxes $\bar{\tau}$, which become

$$\bar{\tau} = \bar{g} + ((1-q_d)b - q_d d)\bar{i}^{CB} \tag{O.177}$$

The steady states of the new variables $c_t^d, \tilde{c}_t, i_t^{ed}, p_t^{gd}, \lambda_t^d, i_t^{gd}, i_t^{bd}$ are given by the following equations, in which $\bar{x}$ refers to the steady state of the corresponding variable x_t.

$$\bar{c}^d = \bar{\tilde{c}} = \bar{c} \tag{O.178}$$

where $\bar{c}$ is unchanged from the representative-agent model.

As in the attention to saving block of the representative-agent model, I now define two new steady state objects, which are calibration targets. First, $\overline{mn}^d$ is the spread between the policy rate and the effective interest rate on debt:

$$\overline{mn}^d \equiv \bar{i}^{ed} - \bar{i}^{CB} \tag{O.179}$$

Second, $\overline{sd}^d$ as the standard deviation of available interest rates on debt:

$$\overline{sd}^d \equiv \frac{\bar{i}^{bd} - \bar{i}^{gd}}{2} \tag{O.180}$$

Both of these are calibrated to moments from the data, as described in Appendix E.3.

Since $\bar{i}^{CB}$ is already pinned down by the deposit bank block (as in the representative-agent model), we can use equation O.179 to obtain $\bar{i}^{ed} = \bar{i}^{CB} + \overline{mn}^d$.

From the attention first order condition (combining equations O.156 and O.157), we have

$$\bar{\lambda}^d = \frac{\mu}{\beta^d d\bar{u}_c^d} \tag{O.181}$$

where the marginal utility in steady state is $\bar{u}_c^d = \bar{u}_c$, because consumption is the same in steady state across households.

Rearranging equation O.163 yields

$$\bar{p}^{gd} = \frac{\exp\left(\frac{2\overline{sd}^d}{\bar{\lambda}^d}\right)}{\exp\left(\frac{2\overline{sd}^d}{\bar{\lambda}^d}\right) + 1} \tag{O.182}$$

Using equation O.164 and O.180, we obtain expressions for each of the borrowing interest rates available in steady state

$$\bar{i}^{gd} = \bar{i}^{ed} - 2(1 - \bar{p}^{gd})\overline{sd}^d \tag{O.183}$$

$$\bar{i}^{bd} = \bar{i}^{ed} + 2\bar{p}^{gd}\overline{sd}^d \tag{O.184}$$

Finally, the lending bank first order conditions (O.167 and O.168) imply

$$\bar{i}^{gd} = \bar{i}^{CB} + \chi_0^{gd} + \frac{\bar{\lambda}^d}{1 - \bar{p}^{gd}} \tag{O.185}$$

$$\bar{i}^{bd} = \bar{i}^{CB} + \chi_0^{bd} + \frac{\bar{\lambda}^d}{\bar{p}^{gd}} \tag{O.186}$$

Substituting these into the definitions of $\overline{mn}^d$ and $\overline{sd}^d$ (O.179 and O.180) gives two conditions pinning down the steady state bank costs required to meet those calibration targets. Solving those conditions yields

$$\chi_0^{gd} = \overline{mn}^d - 2(1 - \bar{p}^{gd})\overline{sd}^d - \frac{\bar{\lambda}^d}{1 - \bar{p}^{gd}} \tag{O.187}$$

$$\chi_0^{bd} = \overline{mn}^d + 2\bar{p}^{gd}\overline{sd}^d - \frac{\bar{\lambda}^d}{\bar{p}^{gd}} \tag{O.188}$$

References

Adolfson, M., Laséen, S., Lindé, J., and Villani, M. (2007). Bayesian estimation of an open economy DSGE model with incomplete pass-through. *Journal of International Economics*, 72(2):481–511.

Born, B. and Pfeifer, J. (2020). THE NEW KEYNESIAN WAGE PHILLIPS CURVE: CALVO VS. ROTEMBERG. *Macroeconomic Dynamics*, 24(5):1017–1041.

Galí, J., López-Salido, J. D., and Vallés, J. (2007). Understanding the Effects of Government Spending on Consumption. *Journal of the European Economic Association*, 5(1):227–270.

Ghironi, F. and Melitz, M. J. (2005). International Trade and Macroeconomic Dynamics with Heterogeneous Firms*. *The Quarterly Journal of Economics*, 120(3):865–915.

Harrison, R. and Oomen, O. (2010). Evaluating and Estimating a DSGE Model for the United Kingdom. *Bank of England working papers*, 380.

Smets, F. and Wouters, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach. *American Economic Review*, 97(3):586–606.