Online Appendix to
“On the Alignment of
Consumer Surplus and Total Surplus
Under Competitive Price Discrimination”

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This document contains the online appendices to “On the Alignment of Consumer Surplus and Total Surplus Under Competitive Price Discrimination.” Appendix B studies the known-cost and known-value model when producers may price below cost. Appendix C studies the case when producers may not know their own costs.

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B Pricing Below Cost

In the known-values and known-costs model of Section 3, the lower bound $\underline{PS}_i$ on producer i's surplus relies on the hypothesis that producers do not price below cost. If we allow producers to price below cost, then some rather extreme welfare outcomes can be supported in equilibrium, as the following result shows:

Theorem 4 (Pricing below cost)**.**

If producers can price below cost, consumer surplus and total surplus can be aligned, and consumer surplus and producer surplus are opposed. Moreover, for every $\varepsilon > 0$, there exists an information structure (S, ϕ) and equilibrium strategies ρ so that $PS \leq \varepsilon$ and $CS \geq \overline{TS} - \varepsilon$.

Proof. We take $S_i = \mathbb{R}^{2N}$, and $\phi\,(ds, dv, dc)$ puts probability one on $s_i = (v, c)$ for all i, that is, the information structure publicly reveals all of the values and costs. If a sale is inefficient, or if there is more than one efficient producer, then all producers simply price at c. If there is only one efficient producer, who we take to be producer i, then producer i sets a price

$$p_i = \min\left\{c_i + \varepsilon, \left(c_i + r_i\right)/2\right\},$$

where

$$r_i = v_i - \max_{j \neq i} v_j + c_j$$

The inefficient producers then randomize on the interval $[p_i, (p_i + r_i)/2]$, according to the distribution

$$\rho_j\left([p_i, x]\,|s_j\right) = \begin{cases} 0 & \text{if } x < p_i; \\ 1 - \frac{p_i - c_i}{x - c_i} & p_i \leq x < (p_i + r_i)/2; \\ 1 & \text{if } x \geq (p_i + r_i)/2. \end{cases}$$

By construction, $p_i < r_i \leq v_i - v_j + c_j$ for all $j \neq i$, so the only way for a producer $j \neq i$ to make a sale is by setting a price below cost, which would give non-negative profit. Hence, inefficient producers have no profitable deviations. On the other hand, if the efficient producer prices at

$x > (p_i + r_i)/2$, they make zero profit, at any price $x \leq p_i$ they make a sale with probability one and hence profit is weakly lower than at $x = p_i$, and for $x \in [p_i, (p_i + r_i)/2]$, expected profit is

$$\begin{aligned}(x - c_i) \prod_{j \neq i} \rho_j \left([x, (p_i + r_i)/2] \,|s_j \right) &= (x - c_i) \left(\frac{p_i - c_i}{x - c_i} \right)^{N-1} \\ &\leq (x - c_i) \left(\frac{p_i - c_i}{x - c_i} \right) \\ &= p_i - c_i.\end{aligned}$$

Hence, the efficient producer does not have a profitable deviation either. Since the efficient producer always makes a sale, $TS = \overline{TS}$. But the efficient producer's price is always less than $c_i + \varepsilon$, so $PS \leq \varepsilon$, and therefore $CS \geq \overline{TS} - \varepsilon$, as desired. □

The proof idea is quite simple. These extreme outcomes can be sustained when the producers have complete information about (v, c). The efficient producer prices at the minimum of $c_i + \varepsilon$ and whatever price would tie with the runner-up producer. The runner-up producer either prices at cost (when there is a tie) or randomizes over prices (below their own cost) so that the residual willingness to pay is distributed on $[c_i + \varepsilon, c_i + 2\varepsilon]$. Moreover, we can pick the shape of this distribution so that pricing at $c_i + \varepsilon$ is a best response for the efficient producer.

Thus, with unrestricted prices, it is possible to sustain hypercompetitive outcomes in equilibrium, where producers know that they are pricing well below cost, but they are willing to do so because they expect to not make a sale. Restricting attention to equilibria in which producers price above cost is a straightforward and intuitive way to rule out such implausible scenarios.

C Unknown Costs

In this appendix, we explore what happens when the consumer knows their values but producers may not know their own costs. Operationally, what this means is that each producer i's strategy can only depend on their signal s_i, and cannot depend directly on their cost, i.e., a strategy ρ_i associates to each s_i a distribution over prices. We will continue to require that producers not set prices that are certainly below cost. In particular, under the information structure (S, π), a strategy is a measurable function $f : S_i \to \mathbb{R}$ such that $\pi\left(\left\{(s, v, c)\left|c_i \geq f\left(s_i\right)\right.\right\}\right) = 1$, we have that

$$\int_{(v,c)} \rho_i\left(\left[f\left(s_i\right), \infty\right)|s_i\right) \pi\left(ds, dv, dc\right) = 1.$$

In other words, we restrict attention to strategies for which there is probability zero that producers price strictly below a lower bound on their cost, where the lower bound depends only on their own signal.

Obviously, in the special case where producers' costs are certain, this assumption on strategies reduces to the requirement that producers price above cost, and our existing results would go through without modification. However, we will argue that with even a small amount of uncertainty, this restriction loses much of its bite. In fact, Theorem 5 shows that there are cases in which it is possible to approximate]the same hypercompetitive outcomes as those obtained in Theorem 4, where we dropped the restriction on pricing altogether. The critical issue is that producers may be frequently pricing below cost, but they cannot distinguish those situations from when they would also be setting similarly low prices as the efficient producer.

We say that the prior μ is *weakly competitive* if whenever there is positive probability that producer i is *uniquely efficient*—meaning that they are the only efficient producer—and has cost $c_i = x$, then there exists a producer $j \neq i$ such that there is positive probability that producer j is uniquely efficient and has cost $c_j = x$. The substantive implication of weak competitiveness is that a producer cannot infer the identity of the efficient producer just from knowing the efficient producer's cost: For any given efficient cost, there are always at least two producers who could be uniquely efficient with that cost. This condition would be trivially satisfied if the prior distribution of (v, c) is exchangeable.

Theorem 5 (Alignment with Unknown Costs)**.** *Suppose that $N \geq 2$, costs are unknown, values are homogeneous, and the prior is weakly competitive. Then consumer surplus and total surplus can be aligned, and consumer surplus and producer surplus are opposed. In particular, for any $\varepsilon > 0$, there exists an information structure and equilibrium in which $TS = \overline{TS}$, $PS < \varepsilon$ and $CS \geq \overline{TS} - \varepsilon$.*

In the proof of the theorem we construct an information structure and equilibrium of the following form: Each producer's signal is a "recommended" price, and in equilibrium, producers set prices equal to their signals. Because values are homogeneous, the efficient producer is simply the producer with the lowest cost. The low cost producer i is recommended a random price $p_i \in [c_i, c_i + \varepsilon]$, where $c_i + \varepsilon < \min_{j \neq i} c_j$. By weak competitiveness, there is a producer $j \neq i$ who also is sometimes uniquely efficient with the cost c_i. That producer is recommended a random price in $[p_i, c_i + \varepsilon]$, according to a distribution that makes producer i prefer p_i to prices in $[p_i, c_i + \varepsilon]$. This incentivizes producer i to price close to c_i, and moreover, the strategy of following the recommendation is not dominated, since producer j cannot tell whether they are recommended such a price because they are efficient, or because they are inefficient and being used to pressure the efficient producer to price close to cost.[1]

We now present the formal proof:

Proof of Theorem 5. Fix $\varepsilon > 0$. Because the support of costs is finite, we may assume that ε is small enough so for any c and c' that are in the support of μ, if $c_i \neq c'_j$, then $\left|c_i - c'_j\right| > \varepsilon$.

[1]Theorem 5 generalizes Theorem 2 of Bergemann, Brooks, and Morris (2017), presented in the setting of a private-value first-price auction. The result in that paper corresponds to the special case in which values are homogeneous and certain, i.e., there is a commonly known v which is the value for every producer's cost. Moreover, Bergemann, Brooks, and Morris (2017) assumed that the prior distribution is exchangeable, which implies weak competitiveness. The structure of the proofs is largely the same.

Consider the information structure where each producer is recommended a price. If trade is inefficient, or if trade is efficient but there is more than one efficient producer, then all producers are recommended to price at cost. Otherwise, there is a unique efficient producer, and since values are homogeneous, the efficient producer is the one who has the lowest cost. We recommend a price p_i to the efficient producer that is drawn from any full support, non-atomic distribution (say uniform) on $[c_i, c_i + \varepsilon]$. As a result, the price set by the efficient producer is necessarily low enough that other producers would have to price weakly below cost in order to make a sale. By the richness assumption, there is a producer $j \neq i$ who with positive probability is efficient with the same cost. We draw a price p_j for that producer on the interval $[p_i, (p_i + c_i + \varepsilon)/2]$, according to the distribution

$$Prob\,(p_j \leq x) = \begin{cases} 0 & \text{if } x < p_i; \\ 1 - \frac{p_i - c_i}{x - c_i} & p_i \leq x < \frac{p_i + c_i + \varepsilon}{2}; \\ 1 & \text{if } x \geq \frac{p_i + c_i + \varepsilon}{2}. \end{cases}$$

All other producers $k \neq i, j$ are recommended prices $p_k = c_k$.

We claim that under this information structure, it is an equilibrium for each producer to set a price equal to their signal, i.e., to obey the recommendation. To see why, suppose that producer i is recommended to price at p_i. We will consider three events: (i) $p_i = c_i$, (ii) producer i is inefficient and $p_i < c_i$, or (iii) producer i is efficient and $p_i \geq c_i$. In fact, we will argue that a producer would not have a profitable deviation, even if they knew which case (i)–(iii) had obtained. In case (i), then either trade is inefficient, there is more than one efficient producer and all producers are pricing at cost, or there is another producer that is efficient and is setting a price below $p^*(v, c)$. In any of these cases, the only way for producer i to make a sale with positive probability would be to lower their price, which would be to a value less than their cost. Hence, a producer cannot make positive profit on this event by deviating. Case (ii) is similar: By setting the recommended price, producer i will not make a sale. The only way to make a sale is by lowering their price, which is already below cost, so the producer would make negative profit. Finally, in case (iii), producer i is making a sale with probability one by obeying the recommendation. Deviating to a lower price will only result in lower profit, and deviating to a higher price x will result in a sale with probability zero if $x > (p_i + c_i + \varepsilon)/2$, a profit of

$$\frac{p_i - c_i + \varepsilon}{2} \frac{1}{2} \frac{p_i - c_i}{(p_i - c_i + \varepsilon)/2} = \frac{p_i - c_i}{2}$$

if $x = (p_i + c_i + \varepsilon)/2$ (because of the mass point on $(p_i + c_i + \varepsilon)/2$), and otherwise results in profit

$$(x - c_i)(1 - Prob(p_j \leq x)) = p_i - c_i,$$

the same as that obtained by following the recommendation. Thus, there is also no profitable deviation in case (iii).

Finally, we verify that the proposed strategies have the property that no producer is setting a price that is strictly below cost with probability one. Signals take the form of recommended prices. This will be achieved by demonstrating that any lower bound $f : \mathbb{R} \to \mathbb{R}$ such that $\pi(\{(s, v, c) | c_i \geq f(s_i)\}) = 1$ must satisfy $f(p_i) \leq p_i$ with probability one. Suppose not. Because there are finitely many costs, then there must be some cost x so that the prices for which $f(p_i) > p_i$ occurs with positive probability when the efficient cost is x, meaning that the prices are in the interval $[x, x + \varepsilon]$. Let us compute the conditional distribution of producer i's cost, given a recommendation p_i in this interval. Let γ be the probability that they are recommended such a price when $c_i > x$ (case (ii)), and let γ' be the likelihood of being recommended the price when $c_i = x$ (case (iii)). The conditional probability of the cost being x is therefore

$$\frac{\gamma'/\varepsilon}{\gamma'/\varepsilon + \gamma \int_{y=x}^{p_i} \frac{y-x}{(p_i-x)^2} dy/\varepsilon} = \frac{\gamma'}{\gamma' + \gamma/2} > 0.$$

(It is also possible that in the event that $c_i > x$, the efficient producer was told to set a price y so that $p_i = (y + x + \varepsilon)/2$, in which case there is a conditional mass point on the recommendation of p_i of size $(y - c_i)/(p_i - c_i)$, but since this occurs with probability zero conditional on $c_i > x$, omitting it does not affect the interim belief conditional on the recommendation p_i.) Thus, conditional on a recommendation of $p_i \in [x, x + \varepsilon]$, a producer assigns positive probability to the event that $c_i = x$, and hence $f(p_i) \leq x \leq p_i$, as desired. $\square$

Note that the outcome described in Theorem 5 simultaneously maximizes consumer surplus and total surplus, which shows that consumer surplus and total surplus can be aligned. However, the theorem also shows that unknown costs are consistent with some rather extreme and hypercompetitive outcomes in which producer surplus is driven down to zero.

A critical assumption of Theorem 5 is that there are at least two producers. The case of a single producer has been studied by KZ and looks quite different. They showed that as long as there is common knowledge of gains from trade, there is an information structure and and optimal strategy for the producer which results in an efficient outcome, but where the producer does not benefit from the information at all. Hence, with monopoly producer, an analogue of the main result of BBM obtains, and consumer surplus and total surplus can be aligned. But when there is a single producer and there is not common knowledge from gains from trade, then consumer surplus and total surplus may not be aligned, as the following example shows:

Example 5 (Monopoly without alignment)**.** The value cost profile (v, c) is either $(3, 3 + \varepsilon)$ or $(2, 0)$, both equally likely, and where ε is close to zero. In an efficient outcome, it would have to be that the producer always sets a price above 3 when the value is 3, and sets a price below 2 when the value is 2. Clearly, this would require the producer to learn the consumer's value exactly, in which case the producer will set a price equal to 2 when $v = 2$, so that consumer surplus is zero. On the other hand, under no information, the producer will optimally price at 2 and earn a producer surplus of $2 - (3 + \varepsilon)/2 > 0$, and the resulting consumer surplus is 5/2. In effect, by pooling efficient and inefficient outcomes, the producer is forced to sometimes sell at a loss, in a manner that benefits the consumer.

The issue of whether or not there is common knowledge of gains from trade becomes moot when there are at least two producers and if the prior is weakly competitive, because the producers drive one another's prices down to cost. By focusing on the case of homogeneous values in Theorem 5, we have opted for simplicity of exposition rather than providing the most general conditions under which this kind of hypercompetitive outcome can be supported.