Online Appendix: Not for Publication

J Numerical Examples for Section 4

J.1 Non-monotonicity of flow payoffs under a stationary privacy policy

Consider the following parametrization: $A = \{0, 1, 3\}$, $u(1) = 1$, $u(3) = 1.04$, $v = 2$, $\delta = 0.9$, $\sigma_0^2 = 0.1$, and $\gamma_t = 0$ for all $t \in \mathbb{N}$. Define $U(a) := \sum_{t=1}^{\infty} \delta^{t-1} \left[u(a) - v \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + ta} \right) \right]$. First, under the consumer's optimal policy, there is some period t^* such that $a_{t^*-1} = 1$ and $a_{t^*} = 3$ if $U(1) > U(3) > U(0)$. The reason is as follows. Proposition 1 states that the optimal policy under a stationary privacy policy is either $a_t = 0$ for all t, or a_t is positive and weakly increasing in t. $U(3) > U(0)$ implies that the consumer chooses the latter, and $U(1) > U(3)$ implies that $a_1 = 1$. Because $a_1 = 1$ and $a_t = 3$ for some finite t, there is a t^* such that $a_{t^*-1} = 1$ and $a_{t^*} = 3$. The flow payoff increases from $t^* - 1$ to t^* if $u(3) - v \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_{t^*-2}^2} + 1 + 3} \right) > u(1) - v \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_{t^*-2}^2} + 1} \right)$. The inequality holds if $u(3) - u(1) > B := \frac{3v}{\left(\frac{1}{\sigma_0^2} + 4 \right) \left(\frac{1}{\sigma_0^2} + 1 \right)}$. We can numerically show that $U(1) \approx 9.17$, $U(3) = 9.13$, $u(3) - (1) = 0.04$, and $B = 0.039$. Thus we have $U(1) > U(3) > U(0)$ and $u(3) - u(1) > B$, so the consumer receives a higher flow payoff in period t^* than in $t^* - 1$. This example shows that the consumer's flow payoffs are non-monotone, because once a_t hits a_{max}, the flow payoffs strictly decrease in t.

J.2 Non-monotonicity of a_t in equilibrium

Figure 1 depicts the equilibrium dynamics for a myopic consumer. I assume $A = \{0, 0.01, 0.02, \ldots, 2\}$ and use Claim 1 in Appendix K to compute an equilibrium. (Claim 1 also implies that long-run commitment and one-period commitment lead to the same outcome given a myopic consumer.) Figure 1(a) shows that the platform offers a decreasing privacy level, hitting zero in $t = 5$. Figure 1(b) shows that the equilibrium activity level first decreases but eventually approaches $a_{max} = 2$. The non-monotonicity of a_t^* contrasts with the case of a stationary privacy policy.

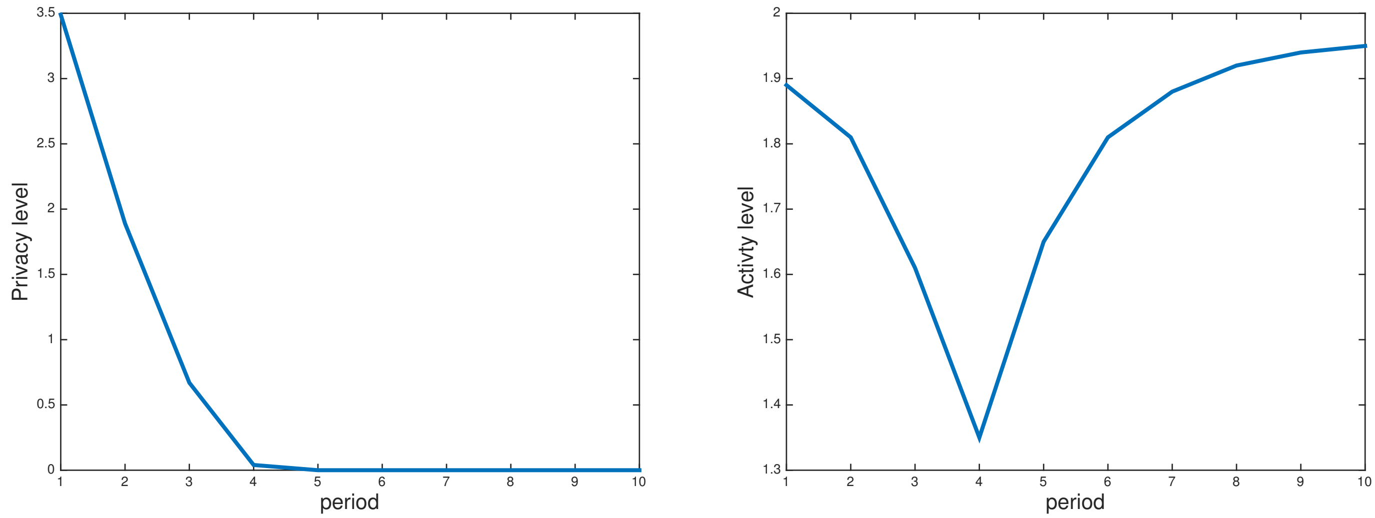

Figure 1(a): Privacy level γ_t

Figure 1(b): Activity level a_t

Figure 1: Equilibrium under $u(a) = 2a - \frac{1}{2}a^2$, $v = 10$, and $\sigma_0^2 = 1$.

J.3 Non-monotonicity of γ_t in equilibrium

Under different parameters, Figure 2 depicts another equilibrium dynamics for a myopic consumer. Figure 2(a) shows that γ_t can be non-monotone. In particular, the platform increases a privacy level from $t = 1$ to $t = 2$ because it becomes less costly to induce the highest activity level through privacy protection.

K Myopic Consumer

I characterize the equilibrium under a myopic consumer, which facilitates numerical analysis. Let $a^*(\gamma, \sigma^2) \in A$ denote the best response of a myopic consumer, given a privacy level γ in the current period and the posterior variance σ^2 from the previous period:

$$a^*(\gamma, \sigma^2) := \max \left\{ \arg\max_{a \in A} \left[u(a) - v \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma^2} + \frac{1}{\frac{1}{a} + \gamma}} \right) \right] \right\}. \tag{46}$$

The following result characterizes the equilibrium.

Claim 1. *Consider the game with long-run commitment. If the consumer is myopic, the plat-*

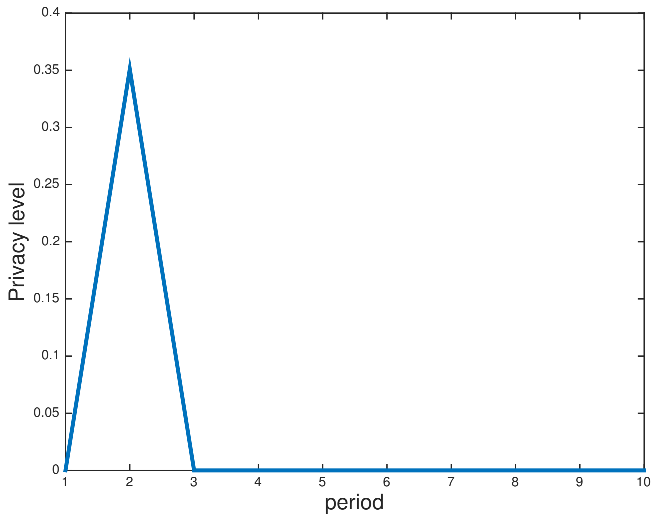

Figure 2(a): Privacy level γ_t

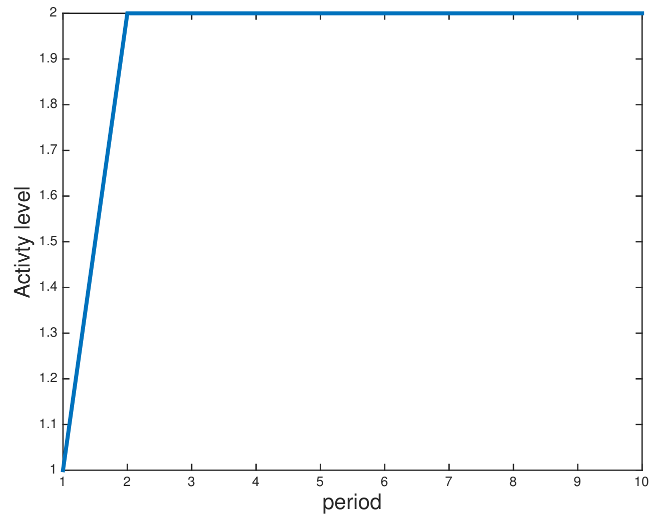

Figure 2(b): Activity level a_t

Figure 2: Equilibrium under $A = \{0, 1, 2\}$, $u(1) = 10$, $u(2) = 11$, $v = 20$, and $\sigma_0^2 = 1$.

form adopts a greedy policy that myopically maximizes the precision of the signal in each period. Formally, the equilibrium policy $(\gamma_1^*, \gamma_2^*, \dots)$ *is recursively defined as follows:*

$$\gamma_t^* \in \arg\min_{\gamma \geq 0} \frac{1}{a^*(\gamma, \hat{\sigma}_{t-1}^2)} + \gamma, \forall t \in \mathbb{N}, \tag{47}$$

$$\hat{\sigma}_0^2 = \sigma_0^2, \tag{48}$$

$$\hat{\sigma}_t^2 = \frac{1}{\frac{1}{\hat{\sigma}_{t-1}^2} + \frac{1}{\frac{1}{a^*(\gamma_t^*, \hat{\sigma}_{t-1}^2)} + \gamma_t^*}}, \forall t \in \mathbb{N}. \tag{49}$$

Proof. Lemma 1 implies $a^*(\gamma, \sigma^2)$ is increasing in γ and decreasing in σ^2. Take any privacy policy $(\gamma_t)_{t \in \mathbb{N}}$ and let $(\sigma_t^2)_{t \in \mathbb{N}}$ denote the sequence of posterior variances induced by $a^*(\cdot, \cdot)$. I show $\hat{\sigma}_t^2 \leq \sigma_t^2$ for all $t \in \mathbb{N}$. The inequality holds with equality for $t = 0$. Take any $\tau \in \mathbb{N}$. Suppose $\hat{\sigma}_t^2 \leq \sigma_t^2$ for $t = 0, \dots, \tau - 1$. It holds that

$$\sigma_\tau^2 = \frac{1}{\frac{1}{\sigma_{\tau-1}^2} + \frac{1}{\frac{1}{a^*(\gamma_\tau, \sigma_{\tau-1}^2)} + \gamma_\tau}} \geq \frac{1}{\frac{1}{\hat{\sigma}_{\tau-1}^2} + \frac{1}{\frac{1}{a^*(\gamma_\tau, \hat{\sigma}_{\tau-1}^2)} + \gamma_\tau}} \geq \frac{1}{\frac{1}{\hat{\sigma}_{\tau-1}^2} + \frac{1}{\frac{1}{a^*(\gamma_\tau^*, \hat{\sigma}_{\tau-1}^2)} + \gamma_\tau^*}} = \hat{\sigma}_\tau^2.$$

The first inequality follows from the inductive hypothesis and decreasing $a^*(\gamma, \cdot)$. The second inequality follows from (47). We now have $\hat{\sigma}_t^2 \leq \sigma_t^2$ for all t, which implies the privacy policy described by (47), (48), and (49) is optimal. □

L General Payoffs of the Platform

Most of the results continue to hold if the platform's final payoff from a sequence of posterior variances is $\Pi((\sigma_t^2)_{t\in\mathbb{N}})$, where $\Pi : \mathbb{R}_+^\infty \to \mathbb{R}$ is coordinate-wise strictly decreasing. This generalization does not change the analysis, because in equilibrium a deviation by the platform increases σ_t^2 for all $t \in \mathbb{N}$. An exception is Theorem 1, where the platform's deviation may not uniformly increase posterior variances. However, the proof of this theorem rests on the argument that if the equilibrium fails to meet certain conditions such as $\sigma_t^2 \to 0$, the platform can deviate and uniformly decrease posterior variances. Thus, Theorem 1 continues to hold with the same proof under this general $\Pi(\cdot)$.

For example, suppose the platform sells information to a sequence of short-lived data buyers. Any information sold in period t is freely replicable later and thus has a price of zero in any period $s \geq t+1$. The profit in period t equals the value of information generated in period t—i.e., the platform's ex ante payoff is $\sum_{t=1}^{\infty} \delta_P^{t-1}(\sigma_{t-1}^2 - \sigma_t^2)$, which is decreasing in each σ_t^2.

M Full Commitment

This appendix considers the platform with action-contingent commitment power: Before $t = 1$, the platform publicly commits to a mapping $\boldsymbol{\gamma}(\cdot) : \{\phi\} \cup (\cup_{s=1}^{\infty} A^s) \to \mathbb{R}_+$, which determines $\gamma_1 = \boldsymbol{\gamma}(\phi)$ and maps past actions $(a_1, \dots, a_{t-1}) \in A^{t-1}$ to the privacy level γ_t in every period $t \geq 2$.

To provide a condition under which action-contingent commitment benefits the platform, we prepare some notations. First, take any equilibrium under long-run commitment. Let $(\hat{a}_t)_t$, $(\hat{\gamma}_t)_t$, and $(\hat{\sigma}_t^2)_t$ denote the activity levels, privacy levels, and posterior variances at the equilibrium, respectively. Let $\hat{U}_2(\sigma^2)$ denote the consumer's continuation value starting from $t = 2$ when the posterior variance at the beginning of $t = 2$ is σ^2 and the consumer faces $(\hat{\gamma}_t)_{t\geq 2}$. Also, let $U^0(\sigma^2)$ denote the consumer's sum of discounted payoffs when the platform always set $\gamma_t = 0$ and the posterior variance is σ^2.

Claim 2. *Suppose $\hat{a}_1 < a_{max}$. The platform's payoff under action-contingent commitment is* *strictly greater than the one under long-run commitment if*

$$u(a_{max}) - v\left[\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \frac{1}{\frac{1}{a_{max}} + \hat{\gamma}_1}}\right] + \delta_C \hat{U}_2\left(\frac{1}{\frac{1}{\sigma_0^2} + \frac{1}{\frac{1}{a_{max}} + \hat{\gamma}_1}}\right) \tag{50}$$

$$\geq \max_{a \in A}\left\{u(a) - v\left[\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \frac{1}{\frac{1}{a} + \hat{\gamma}_1}}\right] + \delta_C U^0\left(\frac{1}{\frac{1}{\sigma_0^2} + \frac{1}{\frac{1}{a} + \hat{\gamma}_1}}\right)\right\}. \tag{51}$$

Proof. Given the deterministic policy $(\hat{\gamma}_t)_t$ under long-run commitment, we create an action-dependent policy that is strictly better for the platform. Consider the following policy $\boldsymbol{\gamma}^*(\cdot)$. If the consumer chooses $a_1 < a_{max}$ in $t = 1$, the platform sets $\gamma_t = 0$ in any period $t \geq 2$. If the consumer chooses a_{max} in $t = 1$, the platform sets $\hat{\gamma}_t$ in any period $t \geq 2$, i.e., it adopts a deterministic policy from $t = 2$ on. The left-hand side (50) is the consumer's payoff when she chooses a_{max} in $t = 1$ and behave optimally from $t = 2$ on. The right-hand side (51) is the consumer's payoff from the best possible deviation in $t = 1$. Thus the display inequality means that the consumer chooses $a_{max} > \hat{a}_1$ in $t = 1$. Note that the consumer's behavior after $t = 2$ under $\boldsymbol{\gamma}^*(\cdot)$ is different from that under long-run commitment. However, the consumer faces a lower posterior variance in $t = 2$ under the former. Proposition 4 implies that the consumer's activity level under $\boldsymbol{\gamma}^*(\cdot)$ is greater than the one under long-run commitment in any period $t \geq 2$. Thus, $\boldsymbol{\gamma}^*(\cdot)$ gives the platform a higher payoff in any period than under long-run commitment. □