Political Competition with Endogenous Party Formation and Citizen Activists

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Online Appendix

Appendix C: Extensions

C1. Higher electoral risk and convergent equilibria

Under Assumption 2, there are no robust equilibrium outcomes with platform distance below $2c$ by Proposition 1 and Corollary 1. If Assumption 2 is violated, by contrast, there may exist robust equilibria with more convergent platforms. The following proposition focuses on symmetric platforms with platform distance below $2c$. It shows that such equilibria exist if and only if the level of electoral risk as implied by the median voter distribution Φ is large enough.

For the sake of concreteness, consider a party structure such that, in each party $J \in \{L, R\}$, the median member has ideal point $m_J = 0$, the next-leftist member has an ideal point $-\hat{\omega} < 0$, and the next-rightist member has an ideal point $\hat{\omega}$. Moreover, both parties are not efficient so that they would remain active if one member leaves her party. If the electoral risk is large enough, then there is a robust political equilibrium with such a party structure and fully convergent platforms $r = \ell = 0$. In this equilibrium, any member of party L prefers to remain in her party not to avoid that the competing party R wins, but to avoid that her own party L runs with a platform that makes her (much) worse off than the platform of R. Any equilibrium of this type is fragile in the following two ways: First, any party member except the party medians and any independent agent with ideal point below $-\hat{\omega}$ or above $\hat{\omega}$ is exactly indifferent between joining a party and staying independent. Second, the equilibrium vanishes if policy preferences are (slightly) concave or (slightly) asymmetric.

PROPOSITION C.1: *If $\Gamma(\gamma^*(0), 0) \geq 2c$, there is a threshold $\hat{\beta} \in [0, 1]$ such that $(-\beta c, \beta c)$ is a robust equilibrium outcome for any $\beta \in [0, \hat{\beta}]$. If $\Gamma(\gamma^*(0), 0) < 2c$, the pair $(-\beta c, \beta c)$ is no robust equilibrium outcome for any $\beta \in [0, 1)$.*

PROOF:

Consider a potential equilibrium with platforms $\ell = m_L = -\beta c$ and $r = m_R = \beta c$, and a party structure (Ω_L, Ω_R) such that the next-midmost members of party

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L have ideal points $\ell_- < -\beta c$ and $\ell_+ > -\beta c$, respectively. A member of party L with $\omega_i \geq \ell_+$ can profitably leave L unless $\Gamma(\ell_-, r) \geq \Gamma(\ell, r) + 2c = (2+\beta)c$, and an independent agent with $\omega_i \leq \ell_-$ can profitably join party L unless $\Gamma(\ell_-, r) \leq \Gamma(\ell, r) + 2c = (2+\beta)c$. Both conditions are only compatible if the ideal point ℓ_- of the next-leftist member of L satisfies

$$\Gamma(\ell_-, \beta c) = p(\ell_-, \beta c)(\beta c - \ell_-) = (2+\beta)c\ . \tag{C.1}$$

Moreover, joining party L is profitable for some agent with ideal point $\omega_i \in (\ell_-, \ell)$ unless $\Gamma_\ell(\ell_-, r) \leq 0$. By the quasi-concavity of Γ, if $G(\beta) = \Gamma(\gamma^*(\beta c), \beta c) - (2+\beta)c \geq 0$, both conditions are satisfied for some $\omega_i \in [\gamma^*(\beta c), -\beta c)$. Otherwise, the conditions are not satisfied for any $\omega_i < -\beta c$.

By corresponding arguments, exit-stability for members with $\omega_i \leq \ell_-$ and entry-stability for members with $\omega_i \geq \ell_+$ jointly require that

$$p(\ell_+, \beta c)(\ell_+ - \beta c) = \Gamma(-\ell_+, -\beta c) = (2-\beta)c$$

and that $\Gamma_\ell(-\ell_+, -\beta c) \leq 0$. If and only if $G(-\beta) = \Gamma(\gamma^*(-\beta c), -\beta c) - (2-\beta)c \geq 0$, both conditions are satisfied for some $\ell_+ \in (\gamma^*(-\beta c), -\beta c)$.

The derivative of $G(\beta)$ is strictly negative,

$$G'(\beta) \;=\; c\ [\Gamma_r(\gamma^*(\beta c), \beta c) - 1] = c\ [2\, p(\gamma^*(\beta c), \beta c) - 1] < 0\ ,$$

because $p(\gamma^*(\beta c), \beta c) < 1/2$ due to $\gamma^*(\beta c) < -\beta c$. Hence, $G(\beta) \geq 0$ implies that $G(\beta') > 0$ for all $\beta' < \beta$. As a result, the condition on ℓ_+ is satisfied whenever the condition on ℓ_- is satisfied. Moreover, if $G(0) = \Gamma(\gamma^*(0), 0) - 2c \geq 0$, there exists a threshold $\hat{\beta} \geq 0$ such that $G(\hat{\beta}) = 0$. In this case, $G(\beta) \geq 0$ and $(-\beta c, \beta c)$ is a robust equilibrium outcome if and only if $\beta \in [0, \hat{\beta}]$. If instead $G(0) < 0$, then $(-\beta c, \beta c)$ is no robust equilibrium outcome for any $\beta \in [0, 1]$.

As a final remark, note that Assumption 2 is equivalent to $c/2 \geq \Phi(0)/\phi(0)$. By contrast, the condition $Z(0) \geq 0$ requires that $\gamma^*(0) < -4c$ or, equivalently, that

$$\Gamma_\ell(-4c, 0) = 2c\phi(-2c) - \Phi(-2c) < 0 \ \Leftrightarrow\ 2c < \frac{\Phi(-2c)}{\phi(-2c)} < \frac{\Phi(0)}{\phi(0)}\ ,$$

where the last inequality holds by the log-concavity of Φ. Hence, Assumption 2 ensures that $Z(0) < 0$, thereby ruling out the existence of robust equilibrium outcomes with platform distance below $2c$.

C2. Concave policy preferences

In the basic model, I have assumed that an agent's utility is linearly decreasing in the distance between the implemented policy x and her ideal point ω_i. In the following, I show that my main results generalize to a model with non-linear policy preferences. Specifically, I assume that policy preferences are captured by the CARA utility function

$$v(x-\omega_i)=\frac{1}{a}\left(1-e^{a|x-\omega_i|}\right) \tag{C.2}$$

with a strictly positive curvature parameter $a>0$. This implies that utility is concave and decreasing in the distance between policy x and ideal point ω_i. The limit case $a\to 0$ coincides with the linear preferences studied in the main text.

In the following, I first show that the agent's implied preferences on the platform of party L satisfy a single-crossing property. Second, I derive the equilibrium outcome at the candidate selection stage in party L given any member set Ω_L and any belief $\hat{r}$ about the platform of the competing party R. Based on these preliminary results, I then show that the platform distance in robust equilibrium outcomes is bounded both from below and from above. For the last result, I focus on symmetric outcomes such that $\ell=-r$.

LEMMA C.1: *Let the policy preferences be given by* (C.2)*. Given any $a>0$ and any platform belief $\hat{r}$, the platform preferences of party L's members satisfy a version of the single-crossing property by Gans and Smart (1996).*

PROOF:

Consider two potential platforms $\ell_1<r$ and $\ell_2\in(\ell_1,r)$, so that $0<p(\ell_1,r)<p(\ell_2,r)<1$. Agent i with ideal point ω_i prefers ℓ_1 to ℓ_2 if $F(\ell_1,\ell_2,r,\omega_i)>0$, where F is defined by (A.1). As in the benchmark case, $F(\ell_1,\ell_2,r,\ell_2)<0$. The derivative of F with respect to ω_i is given by

$$\frac{dF(\ell_1,\ell_2,r,\omega_i)}{d\omega_i}=\begin{cases}-aF(\ell_1,\ell_2,r,\omega_i) & \text{for} \quad \omega_i<\ell_1\ ,\\ -aF(\ell_1,\ell_2,r,\omega_i)-2p(\ell_1,r) & \text{for} \quad \omega_i\in(\ell_1,\ell_2)\ ,\\ aF(\ell_1,\ell_2,r,\omega_i)+2e^{a(r-\omega_i)}\left[p(\ell_2,r)-p(\ell_1,r)\right] & \text{for} \quad \omega_i\in(\ell_2,r)\ ,\\ aF(\ell_1,\ell_2,r,\omega_i) & \text{for} \quad \omega_i>r\ .\end{cases}$$

There are three possible cases. First, $F(\ell_1,\ell_2,r,\ell_1)\geq 0$ implies that

$$\begin{aligned}F(\ell_1,\ell_2,r,r) \quad = \quad & -F(\ell_1,\ell_2,r,\ell_1)\\ & +p(\ell_2,r)\left[v(r-\ell_1)+v(0)-v(r-\ell_2)-v(\ell_2-\ell_1)\right]<0\ ,\end{aligned}$$

where the term in brackets is strictly negative for any $a>0$ by Karamata's inequality. In this case, there is a unique threshold $\omega'\in(\ell_1,\ell_2)$ such that $F(\ell_1,\ell_2,r,\omega)$ is strictly positive for all $\omega<\omega'$ and strictly negative for all $\omega>\omega'$. Second, $F(\ell_1,\ell_2,r,r)\geq 0$ implies that $F(\ell_1,\ell_2,r,\ell_1)<0$. In this case, there is a unique $\omega'\in(\ell_2,r)$ such that $F(\ell_1,\ell_2,r,\omega)$ is strictly negative for all $\omega<\omega'$ and strictly positive for all $\omega>\omega'$. Finally, it is possible that $F(\ell_1,\ell_2,r,\omega)$ is weakly negative for all $\omega\in\mathbb{R}$. In all three cases, a version of the single-crossing property holds.

This implies that, for any member set Ω_L with an odd number of elements and any platform belief $\hat{r}$, the preferred platform of the party median is a Condorcet winner in the primary elections of party L. In particular, a member with ideal point ω_i prefers the platform that maximizes $E[v(x-\omega_i) \mid \ell, r] = e^{-a\omega_i}\, \Gamma^a(\ell, r) + v(r-\omega_i)$ over the set of available platforms in $[\omega_i, r]$, where

$$\Gamma^a(\ell, r) = \frac{p(\ell, r)}{a}\left[e^{ar} - e^{a\ell}\right]$$

generalizes the policy effect function $\Gamma(\ell, r)$ for non-linear preferences.

LEMMA C.2: *Let the policy preferences be given by* (C.2) *with* $a > 0$, *let* $\hat{r} > m_L$ *and* $\#\Omega_L$ *be odd. Function* $\Gamma^a(\ell, r, m_L)$ *is strictly quasi-concave in* ℓ *for* $\ell \in [m_L, r)$. *The platform of party* L *is given by the maximum of* m_L *and* $\gamma_a(\hat{r}, \Omega_L) := \arg\max_{\ell \in \Omega_L} \Gamma^a(\ell, r)$.

PROOF:

The derivative of $\Gamma^a(\ell, r)$ in ℓ is given by

$$\Gamma^a_\ell(\ell, r) = \frac{1}{2a}\,\phi\left(\frac{\ell+r}{2}\right)\,\left[e^{ar} - e^{a\ell}\right] - \Phi\left(\frac{\ell+r}{2}\right)\, e^{a\ell}\,.$$

It equals zero if

$$2\frac{\Phi\left(\frac{\ell+r}{2}\right)}{\phi\left(\frac{\ell+r}{2}\right)} = \frac{1}{a}\left[e^{a(r-\ell)} - 1\right]\,, \tag{C.3}$$

where the left-hand side is increasing in ℓ by the log-concavity of Φ, and the right-hand side is decreasing in ℓ for any $a > 0$. Hence, function Γ^a is strictly quasi-concave in ℓ. Denote by $\gamma_a(\hat{r}, \Omega_L)$ the platform that maximizes $\Gamma^a(\ell, r)$ over $\ell \in \Omega_L$. As in the linear case, an agent prefers his own ideal point ω_i to all lower platforms below ω_i or above r. With an odd number of party members, the preferred platform of the party median prevails in any pairwise vote by Lemma C.2. Hence, the platform is given by m_L if $m_L \geq \gamma_a(\hat{r}, \Omega_L)$, and by $\gamma_a(\hat{r}, \Omega_L)$ otherwise.

LEMMA C.3: *Let the policy preferences be given by* (C.2) *with* $a > 0$. *In every robust political equilibrium with two active parties and symmetric platforms, both parties are efficient.*

PROOF:

The proof follows the same steps as the proof of Lemma 3. I now provide a sketch of these steps, more details are available on request. Assume that there is a symmetric equilibrium with $\ell = -r$ in which party L is inefficient such that $\sum_{i\in\mathcal{A}} \alpha_i^L \geq C + c$.

First, in such an equilibrium, each member of L must contribute exactly c. Otherwise, she could reduce her contribution without any policy loss. Second, the party platform ℓ must equal the median member's ideal point m_L. Otherwise, a member with $\omega_i < m_L$ could leave party L without incurring a policy loss. Third, exit-robustness requires that party members with ideal points below m_L cannot profitably leave party L. Formally, we must have $F(m_L, m_{L>}, r, m_{L<}) \geq 2c$, where $m_{L>}$ and $m_{L<}$ are the party members with ideal points closest above and below, respectively, the party median m_L. Fourth, entry-robustness requires that no independent agent with ideal point $\omega_j \geq m_{L>}$ can profitably join party L. Formally, this implies that $-F(m_L, m_{L>}, r, \omega_j) \leq 2c$ must be satisfied. For an agent with $\omega_j = r + m_L - m_{L-}$, however, Karamata's inequality implies that $-F(m_L, m_{L>}, r, \omega_j) > F(m_L, m_{L>}, r, m_{L<})$. Hence, if the activist population Ω contains an agent with this ideal point and party L is inefficient, the party structure cannot be exit-robust and entry-robust at the same time. As a result, there is no robust political equilibrium with an inefficient party.

PROPOSITION C.2: *Let the policy preferences be given by* (C.2) *with some fixed parameter $a > 0$. A pair of platforms $(-\Delta/2, \Delta/2)$ is a robust equilibrium outcome if Δ is between*

$$\underline{\Delta}_a := \ln[2ac + 1]/a$$

and a threshold

$$\tilde{\Delta}_a \leq \bar{\Delta}_a := \frac{1}{a} \ln \left[\frac{e^{ax_a}\ \Phi(x_a/2) - 1/2}{\Phi(x_a/2) - 1/2} \right]$$

with $x_a := \ln \left[ac + \sqrt{a^2c^2 + 1} \right] /a$. It is no robust equilibrium outcome if $\Delta > \bar{\Delta}_a$.

PROOF:

The proof follows the same steps as the one for Proposition 1 in the main text. Consider a symmetric equilibrium with $\ell = -\Delta/2$ and $r = \Delta/2$ for some $\Delta > 0$. The platform distance is given by Δ, and the winning probability of each party is $\Phi(0) = 1/2$.

LOWER BOUND ON PLATFORM DISTANCE. — Consider a pair of platforms $\ell = -\Delta/2 < 0$ and $r = \Delta/2 > 0$. The presidential candidate of party L contributes $\alpha_i^L \geq c$. Lemma C.3 implies that, if she reduces her contribution to zero, party L becomes inactive and policy $r = \Delta/2$ is implemented for sure. The agent's utility changes by

$$\alpha_i^L - e^{a\Delta/2}\ \Gamma^a(-\Delta/2, \Delta/2) \geq c - \frac{e^{a\Delta} - 1}{2a}\ .$$

Hence, the deviation is strictly profitable if $\Delta < \underline{\Delta}^a = \ln[2ac + 1]/a$.

Upper bound on platform distance. — A symmetric pair of platforms cannot be a robust equilibrium outcome if there is an independent agent i with an ideal point $\omega_i \in (-\Delta/2, \Delta/2)$ such that

(i) $\Gamma^a(\omega_i, \Delta/2) > \Gamma^a(-\Delta/2, \Delta/2) = (e^{a\Delta/2} - e^{-a\Delta/2})/(2a)$, and

(ii) $F(\omega_i, -\Delta/2, \Delta/2, \omega_i) > c$.

Condition (i) ensures that, if i deviates by joining party L, she becomes presidential candidate. Condition (ii) implies that i profits from this deviation. The policy gain in condition (ii) equals

$$F(\omega_i, -\Delta/2, \Delta/2, \omega_i) \quad = \quad e^{-a\omega_i}\,\Gamma^a(\omega_i, \Delta/2) + \frac{1}{2a} e^{a\Delta/2}\left(e^{a\omega_i} - e^{-a\omega_i}\right) .$$

Consider an agent with ideal point $\hat{\omega}_a(\Delta) = x_a - \Delta/2$, where $x_a = \ln[ac + \sqrt{a^2c^2+1}]/a$. For this agent, both conditions (i) and (ii) are satisfied if and only if Δ exceeds $\bar{\Delta}_a = \frac{1}{a} \ln\left[\frac{e^{ax_a}\ \Phi(x_a/2)-1/2}{\Phi(x_a/2)-1/2}\right]$. This implies that the pair $(-\Delta/2, \Delta/2)$ is no robust equilibrium outcome if $\Delta > \bar{\Delta}_a$.

Existence of robust equilibrium outcomes. — In the final step, I show that there exists a threshold $\tilde{\Delta}_a \in \left(\underline{\Delta}_a, \bar{\Delta}_a\right]$ such that the pair of platforms $(-\Delta/2, \Delta/2)$ is a robust equilibrium outcome if $\Delta \in \left[\underline{\Delta}_a, \tilde{\Delta}_a\right]$. For this purpose, assume that party L is efficient and has three or more members, each of whom contributes exactly c and has ideal point $\ell = -\underline{\Delta}_a/2 = -\ln[2ac+1]/(2a)$. Correspondingly, party R is efficient with the same number of members, each of whom contributes c and has ideal point $r = \underline{\Delta}_a/2$. Then, as shown above, no member can profitably leave her party. Moreover, if an independent agent with ideal point below ℓ or above r joins a party, she cannot affect the party's platform. Finally, assume that an moderate independent agent with any ideal point $\omega_i \in (\ell, r)$ joins party L and becomes the party's candidate. Then, her net utility change is negative because

$$\begin{aligned}
F(\omega_i, \ell, r, \omega_i) \quad &= \quad \frac{p(\omega_i, r)}{a}\left[e^{a(\Delta/2-\omega_i)} - 1\right] + \frac{1}{2a}\left[e^{a(\Delta/2+\omega_i)} - e^{a(\Delta/2-\omega_i)}\right] \\
&< \quad \frac{1}{2a}\left[e^{a(\Delta/2-\omega_i)} + e^{a(\Delta/2+\omega_i)} - 1 - (2ac+1)\right] < c \\
\Leftrightarrow e^0 + a^{a\Delta} \quad &> \quad e^{a(\Delta/2-\omega_i)} + e^{a(\Delta/2+\omega_i)} ,
\end{aligned}$$

where the inequality in the last line is true by Karamata's inequality. By continuity, there is a threshold $\tilde{\Delta}_a > \underline{\Delta}_a$ such that, if $\Delta \in [\underline{\Delta}_a, \tilde{\Delta}_a]$, joining a party is not profitable for an agent with any ideal point in $(-\Delta/2, \Delta/2)$, while if Δ is slightly above $\tilde{\Delta}_a$, joining a party is profitable for some agent. As shown above, $\tilde{\Delta}_a$ must be weakly below $\bar{\Delta}_a$.

C3. Asymmetric preferences

In the basic model, I have assumed that the agent's policy preferences are symmetric so that agent i is indifferent between any pair of policies (x_1, x_2) that are equally distant from her ideal point ω_i, $x_2 - \omega_i = \omega_i - x_1 > 0$. In the following, I show that my results do not change qualitatively if policy preferences are asymmetric. Specifically, I assume that, for a leftist agent with ideal point ω_i below the expected median of 0, the policy payoff is given by

$$v_i(x - \omega_i) = \begin{cases} -(\omega_i - x) & \text{for} \quad x \leq \omega_i \ , \\ -b(x - \omega_i) & \text{for} \quad x > \omega_i \ . \end{cases} \tag{C.4}$$

For a rightist agent with ideal point $\omega_i > 0$, by contrast, the policy payoff is given by the form

$$v_i(x - \omega_i) = \begin{cases} -b(\omega_i - x) & \text{for} \quad x \leq \omega_i \ , \\ -(x - \omega_i) & \text{for} \quad x > \omega_i \ . \end{cases} \tag{C.5}$$

The basic model with symmetric preferences is nested with $b = 1$. For the case where parameter b is above 1, leftist agents are better off with a policy x_1 below their ideal point ω_i than with an equally distant policy $x_2 = 2\omega_i - x_1$. Intuitively, this implies that they are more sensitive to rightward deviations than to leftward deviations from their ideal point. The opposite is true for rightist agents.[1]

LEMMA C.4: *Let the policy preferences be given by* (C.4) *and* (C.5) *with* $b > 0$. *Given any platform belief* $\hat{r} \in \mathbb{R}$, *the platform preferences of party* L*'s members satisfy a version of the single-crossing property by Gans and Smart (1996).*

PROOF:

Consider two alternative platforms ℓ_1 and ℓ_2 such that $\ell_1 < \ell_2 < r$, and $0 < p(\ell_1, r) < p(\ell_2, r) < 1$. Agent i with ideal point ω_i prefers ℓ_1 to ℓ_2 if and only if $F(\ell_1, \ell_2, r, \omega_i) > 0$, where F is defined by (A.1). First, if an agent with ideal point $\omega_i \leq \ell_1$ strictly prefers one platform, then each agent with ideal point $\omega_i \geq r$ strictly prefers the other platform because $F(\ell_1, \ell_2, r, r) = -F(\ell_1, \ell_2, r, \ell_1)/b$. Second, the derivative of F with respect to ideal point ω_i is given by

$$\frac{dF(\ell_1, \ell_2, r, \omega_i)}{d\omega_i} = \begin{cases} 0 & \text{for} \quad \omega_i < \ell_1 \ , \\ -(1+b)p(\ell_1, r) < 0 & \text{for} \quad \omega_i \in (\ell_1, \ell_2) \ , \\ (1+b)\left[p(\ell_2, r) - p(\ell_1, r)\right] > 0 & \text{for} \quad \omega_i \in (\ell_2, r) \ , \\ 0 & \text{for} \quad \omega_i > r \ . \end{cases}$$

These properties jointly imply that, if $F(\ell_1, \ell_2, r, \ell_1) > 0$, there is a unique root

[1] For completeness, I assume that an agent with ideal point $\omega_i = 0$ has the same (symmetric) policy preferences as in the basic model.

$\omega' \in (\ell_1, \ell_2)$ such that agent i prefers ℓ_1 if and only if her ideal point satisfies $\omega_i < \omega'$. If instead $F(\ell_1, \ell_2, r, r) > 0$, there is a unique $\hat{\omega}' \in (\ell_2, r)$ such that agent i prefers ℓ_1 if and only if $\omega_i > \omega'$. Finally, it is possible that $F(\ell_1, \ell_2, r, \omega) = 0$ for all $\omega \leq \ell_1$ and all $\omega_i \geq r$. In this case, all agent with ideal points in (ℓ_1, r) strictly prefer platform ℓ_2, while the other agents are indifferent. In all three cases, a version of the single-crossing property holds.

This implies that, for any member set Ω_L and platform belief $\hat{r}$, if there is a unique party median, her preferred platform is a Condorcet winner in the primary election of party L. In particular, any member with ideal point $\omega_i < \hat{r}$ prefers the platform that maximizes $b\Gamma(\ell, r)$ over the set of available platforms in $[\omega_i, r)$. As a result, Lemma 2 continues to hold: For any $\hat{r} > m_L$ and $\#\Omega_L$ odd, the chosen platform ℓ is the maximum of m_L and $\gamma(\hat{r}, \Omega_L)$. Similarly, Lemma 3 extends to asymmetric parties: Both parties are efficient in every robust equilibrium with symmetric platforms $\ell = -\Delta/2 \leq 0$ and $r = \Delta/2 \geq 0$. Based on these intermediate results, I can now identify the set of symmetric platforms that represent robust equilibrium outcomes.

PROPOSITION C.3: *Let the policy preferences be given by* (C.4) *and* (C.5) *with* $b > 0$. *A pair of platforms* $\ell = -\Delta/2$ *and* $r = \Delta/2$ *is a robust equilibrium outcome if the platform distance* $r - \ell = \Delta$ *is between*

$$\underline{\Delta}_b \coloneqq \frac{2c}{b}$$

and a threshold

$$\tilde{\Delta}_b \leq \bar{\Delta}_b \coloneqq \tilde{c}\,\frac{\Phi(\tilde{c}/2)}{\Phi(\tilde{c}/2) - 1/2}$$

with $\tilde{c} = 2c/(1+b)$. *It is no robust equilibrium outcome if* $\Delta > \bar{\Delta}_b$. *If* $b \in (0, 1]$, *the pair* $(-\Delta/2, \Delta/2)$ *is a robust equilibrium outcome if and only if* $\Delta \in \left[\underline{\Delta}_b, \bar{\Delta}_b\right]$.

PROOF:

The proof follows the same steps as the one for Proposition 1 in the main text. Consider a pair of platforms $\ell = -\Delta/2$ and $r = \Delta/2$ for some platform distance $\Delta > 0$. The winning probability of each party is given by 1/2.

Lower bound on platform distance. — Assume that a member of party L with ideal point $\omega_i \leq \ell$ reduces her contribution to L from $\alpha_i^L \geq c$ to zero. Then, L becomes inactive and policy r is implemented for sure. For agent i, this deviation yields a policy loss of $b\Gamma(\ell, r) = b\Delta/2$. It is profitable if this policy loss is below c, i.e., if Δ is below $\underline{\Delta}_b = 2c/b$.

Upper bound on platform distance. — A symmetric pair of platforms cannot be an equilibrium outcome if there is an independent agent i with an ideal point $\omega_i \in (-\Delta/2, \Delta/2)$ that satisfies the conditions

(i) $b\Gamma(\omega_i, \Delta/2) > b\Gamma(-\Delta/2, \Delta/2) = b\Delta/2$, and

(ii) $F(\omega_i, -\Delta/2, \Delta/2, \omega_i) = b\,\Gamma(\omega_i, \Delta/2) + \Delta(1-b)/4 + \omega_i(1+b)/2 > c$.

If both conditions hold, then agent i can profitably join party L and become its presidential candidate.

Consider an agent with ideal point $\hat{\omega}_b(\Delta) = \Delta[\Phi(\tilde{c}/2) - 1]/[2\,\Phi(\tilde{c}/2)] \in (-\Delta/2, 0)$, where $\tilde{c} = 2c/(1+b)$. For this agent, both conditions (i) and (ii) are satisfied if and only if Δ exceeds $\bar{\Delta}_b = \tilde{c}\ \Phi(\tilde{c}/2)/[\Phi(\tilde{c}/2) - 1/2]$. Hence, the platforms $[-\Delta/2, \Delta/2]$ are no robust equilibrium outcome for any $\Delta > \bar{\Delta}_b$.

Existence of robust equilibrium outcomes. — In the last step, I show that there exists a threshold $\tilde{\Delta}_b \in (\underline{\Delta}_b, \bar{\Delta}_b]$ such that a pair $(-\Delta/2, \Delta/2)$ is a robust equilibrium outcome for any $\Delta \in [\underline{\Delta}_b, \tilde{\Delta}_b]$. For this purpose, assume that each party has three or more members, each of whom contribute exactly c. Moreover, each member of party L has ideal point $\ell = -\Delta/2$, and each member of party R has ideal point $r = \Delta/2$. For $\Delta \geq \underline{\Delta}_b$, both parties are exit-stable; no member can profitably leave her party. Moreover, for an independent agent with ideal point below ℓ or above r, joining a party is not profitable as she cannot affect the party platforms.

Finally, consider an independent agent with ideal point $\omega_i \in (\ell, r)$. I have to distinguish two cases. First, if $b \in (0,1)$, $F(\omega_i, -\Delta/2, \Delta/2, \omega_i)$ is strictly increasing in ω_i for all $\omega_i < 0$. For $\Delta < \bar{\Delta}_b$, both conditions (i) and (ii) are not satisfied for $\omega_i = \hat{\omega}_a(\Delta) < 0$. Hence, condition (ii) can neither be satisfied for any $\omega_i \leq \hat{\omega}_a(\Delta)$. For any $\omega_i \geq \hat{\omega}_a(\Delta)$, on the other hand, condition (i) is not satisfied by the quasi-concavity of $\Gamma(\omega_i, \Delta/2)$ in ω_i. Hence, any pair $(-\Delta/2, \Delta/2)$ with $\Delta < \tilde{\Delta}_b = \bar{\Delta}_b$ is entry-robust.

Second, if $b > 1$, $F(\omega_i, -\Delta/2, \Delta/2, \omega_i)$ may be non-monotonic in ω_i. Assume that $\Delta = \underline{\Delta}_b$ so that $\ell = -r = -c/b$. For an agent with any ideal point $\omega_i \geq 0$, on the one hand, condition (i) cannot be satisfied. For an agent with ideal point $\omega_i \in (-c/b, 0)$, on the other hand, condition (ii) cannot be satisfied because

$$\begin{aligned} F(\omega_i, -\underline{\Delta}_b/2, \underline{\Delta}_b/2, \omega_i) - c &= bp\left(\omega_i, \frac{c}{b}\right)\left(\frac{c}{b} - \omega_i\right) + \frac{1-b}{2}\,\frac{c}{b} + \frac{1+b}{2}\,\omega_i \\ &< c + \frac{1-b}{2}\left(\frac{c}{b} + \omega_i\right) < c\,. \end{aligned}$$

Hence, party L is entry-stable, and the pair $(-\underline{\Delta}_b/2, \underline{\Delta}_b/2)$ is a robust equilibrium outcome. By continuity, there is a threshold $\tilde{\Delta}_b > \underline{\Delta}_b$ such that $(-\Delta/2, \Delta/2)$ is also a robust equilibrium outcome for any $\Delta \in [\underline{\Delta}_b, \tilde{\Delta}_b]$, but not for Δ slightly above $\tilde{\Delta}_b$. As shown above, $\tilde{\Delta}_b$ is weakly below $\bar{\Delta}_b$.

C4. No exogenous costs of running

In the basic model, I assume that the campaign contributions a party collects have no effect on their winning probability, once they exceed the cost of running C. I now consider a version of the model in which each party can enter the general election whenever it has a member and a presidential candidate. Hence, there is no exogenous cost of running. Instead, I assume that, if both parties L and R compete in the general election, the winning probability of party L is increasing in their campaign expenses $C_L = \sum_{i \in \mathcal{A}} \alpha_i^L$ and decreasing in the expenses $C_R = \sum_{i \in \mathcal{A}} \alpha_i^R$ of party R. Specifically, I solve an extended model under the assumption that the winning probability of party L equals

$$\tilde{p}(\ell, r, C_L, C_R) = \tilde{\Phi}\left(\frac{\ell + r}{2} + \beta\,\frac{C_L - C_R}{C_L + C_R}\right) , \tag{C.6}$$

where $\tilde{\Phi}$ is a distribution function satisfying Assumptions 1 and 2. The fraction $(C_L - C_R)/(C_L + C_R)$ captures party L's relative campaign expenses, and β is a measure of how sensitive the electoral prospects are with respect to campaign expenses. Equation (C.6) can be micro-founded by assuming, e.g., that only a share $s \in (0, 1)$ of voters behave strategically based on policy preferences as specified in (1). The remaining share $1 - s$ of voters is *impressionable*: They cast their votes based on the relative campaign expenses $(C_L - C_R)/(C_L + C_R)$, an idiosyncratic party preference ν_i, and a common preference shock μ with distribution function $\tilde{\Phi}$, in the spirit of the probabilistic voting model by Lindbeck and Weibull (1987). To ensure the existence of political equilibria with two competing parties, I impose the following assumption on the set of parameters.

ASSUMPTION C.1: *The exogenous parameters c and β and the distribution $\tilde{\Phi}$ satisfy the condition*

$$c\,\frac{\tilde{\phi}(\beta/3)}{\tilde{\Phi}(\beta/3)} < \beta\,\tilde{\phi}(0) < 1 .$$

Otherwise, I maintain the assumptions of the basic model: All activists are policy-oriented with linear policy preferences; agent i enters party $J \in \{L, R\}$ if $\alpha_i^J \geq c$ with some $c > 0$; the members of each party nominate a presidential candidate from their ranks; and the winning candidate in the general election implements her ideal policy.

At the candidate selection stage, all insights from the basic model remain valid. Conditional on any level of the relative campaign expenses $(C_L - C_R)/(C_L + C_R)$, the agent's implied policy preferences satisfy a single-crossing condition. Each member of party L with ideal point $\omega_i < \hat{r}$ prefers the platform ℓ that maximizes $\tilde{\Gamma}(\ell, r, C_L, C_R) = \tilde{p}(\ell, r, C_L, C_R)(r - \ell)$ over the elements in Ω_L that are located in $[\omega_i, r)$. Hence, the equilibrium platform of L equals the maximum of the party median m_L and $\tilde{\gamma}(r, C_L, C_R, \Omega_L) = \arg\max_{\ell \in \Omega_L} \tilde{\Gamma}(\ell, r, C_L, C_R)$.

Henceforth, I focus on symmetric platform pairs with $\ell = -\Delta/2$ and $r = \Delta/2$. By the following proposition, the platform distance in these equilibria is bounded from above and from below as well, as in the basic model. A crucial difference is that, in the equilibria of this extended model, the parties are not efficient in the sense that any party member is pivotal for the activity of party R. However, any party member i can increase the winning probability of her party J by raising the contribution α_i^J. In an equilibrium, the sum of campaign contributions a party collects satisfies the first-order condition

$$\text{(C.7)} \qquad \frac{d\tilde{p}(\ell, r, C_L, C_R)}{dC_L}(r-\ell) - 1 = \beta\ \Delta\ \tilde{\phi}(0)\ \frac{2C_R}{(C_L + C_R)^2} - 1 \leq 0$$

for party L, and a corresponding first-order condition for party R.[2] In the following, I focus on equilibria in which condition (C.7) is satisfied with a strictly equality and both parties collect identical contributions, $C_L = C_R$.

PROPOSITION C.4: *Assume that the winning probability is given by* (C.6) *and that Assumption C.1 holds. Let $C_L = C_R$ and condition C.7 be satisfied with equality. The pair of platforms $\ell = -\Delta/2 \leq 0$ and $r = \Delta/2$ is a robust equilibrium outcome if Δ is between*

$$\tilde{\Delta}_{low}(\beta) := \frac{2c}{\beta\tilde{\phi}(0)} > 2c\ ,$$

and a threshold $\tilde{\Delta} \in \left(\tilde{\Delta}_{low}(\beta), \tilde{\Delta}_{up}(\beta)\right)$, with

$$\tilde{\Delta}_{up}(\beta) := c\ \frac{\tilde{\Phi}\left(c/2 + \beta/3\right)}{\tilde{\Phi}\left(c/2 + \beta/3\right) - \tilde{\Phi}\left(\beta/3\right)}\ .$$

It is no robust equilibrium outcome if $\Delta \geq \Delta_{up}(\beta)$.

PROOF:

Again, the proof follows the same steps as the one for Proposition 1. Fix some $\beta > 0$ and consider a symmetric pair of platforms $\ell = -\Delta/2$ and $r = \Delta/2$ with some platform distance $\Delta \geq 0$.

Lower bound on platform distance. — If $C_L = C_R$ and condition (C.7) is satisfied with equality, then this condition requires that $C_R = \beta\tilde{\phi}(0)\Delta/2$. Both parties can only run if they have at least one member, i.e., if $C_R \geq c$. This

[2] Condition (C.7) can be satisfied with a strict inequality if (i) each member i of L has an ideal point $\omega_i \leq \ell$ and contributes exactly $\alpha_i^L = c$, and (ii) no other activists makes any contribution to L. In all other cases, condition (C.7) is satisfied with a strict equality.

implies that Δ must be weakly larger than $\tilde{\Delta}_{low}(\beta) = \frac{2c}{\beta\,\tilde{\phi}(0)}$. Under Assumption C.1, $\beta\phi(0)$ is below 1, ensuring that $\tilde{\Delta}_{low}(\beta) > 2c$.

Upper bound on platform distance. — Consider an allocation such that $\ell = -\Delta/2$, $r = \Delta/2$, and $C_L = C_R = \beta\tilde{\phi}(0)\Delta/2$, see above. The relative contribution $(C_L - C_R)/(C_L + C_R)$ equals 0, and each party has a winning probability of $\tilde{\Phi}(0) = 1/2$. Assume now that an independent agent i deviates by contributing $\alpha_i^L = c$ and joining party L. This deviation raises the relative contribution $(C_L - C_R)/(C_L + C_R)$ to $c/[\beta\tilde{\phi}(0)\Delta + c] > 0$ and the winning probability of party L to $\tilde{\Phi}(\tilde{\beta}) > 1/2$, where I write $\tilde{\beta} = \beta c/[\beta\tilde{\phi}(0)\Delta + c]$ for a concise notation.

The pair $(-\Delta/2, \Delta/2)$ cannot be an equilibrium outcome if there is an agent $i \in \mathcal{A}$ with ideal point ω_i such that

(i) $\tilde{\Gamma}(\omega_i, \Delta/2, C_L + c, C_R) = \tilde{\Phi}\left(\frac{\omega_i + \Delta/2}{2} + \tilde{\beta}\right)\,(\Delta/2 - \omega_i) > \Delta\,\tilde{\Phi}(\tilde{\beta})$, and

(ii) $\Delta\,\tilde{\Phi}(\tilde{\beta}) + \omega_i > c$.

If both conditions are satisfied, then agent i can profitably join party L and become its presidential candidate.

For $\omega_i = \tilde{\omega}(\Delta, \tilde{\beta}) := \Delta/2\left[1 - 2\frac{\tilde{\Phi}(\tilde{\beta})}{\tilde{\Phi}(c/2+\tilde{\beta})}\right]$, we have

$$\tilde{\Phi}\left(\frac{\omega_i + \Delta/2}{2} + \tilde{\beta}\right)\,\left(\frac{\Delta}{2} - \omega_i\right) = \tilde{\Phi}\left(\frac{\Delta}{2}\left[1 - \frac{\tilde{\Phi}(\tilde{\beta})}{\tilde{\Phi}(c/2 + \tilde{\beta})}\right] + \tilde{\beta}\right)\,\Delta\frac{\tilde{\Phi}(\tilde{\beta})}{\tilde{\Phi}(c/2 + \tilde{\beta})}\,,$$

so that condition (i) is satisfied if and only if

$$\Delta > c\,\frac{\tilde{\Phi}(c/2 + \tilde{\beta})}{\tilde{\Phi}(c/2 + \tilde{\beta}) - \tilde{\Phi}(\tilde{\beta})}\,. \tag{C.8}$$

If (C.8) holds, condition (ii) is satisfied as well because

$$\Delta\,\tilde{\Phi}(\tilde{\beta}) + \tilde{\omega}(\Delta, \tilde{\beta}) = \Delta\,\left[\tilde{\Phi}(\tilde{\beta}) + \frac{1}{2} - \frac{\tilde{\Phi}(\tilde{\beta})}{\tilde{\Phi}(c/2 + \tilde{\beta})}\right] > c\,\frac{\tilde{\Phi}(\tilde{\beta}) + \frac{1}{2} - \frac{\tilde{\Phi}(\tilde{\beta})}{\tilde{\Phi}(c/2+\tilde{\beta})}}{1 - \frac{\tilde{\Phi}(\tilde{\beta})}{\tilde{\Phi}(c/2+\tilde{\beta})}}\,,$$

which is strictly larger than c because $\tilde{\Phi}(\tilde{\beta}) > 1/2$ for any $\beta > 0$.

To complete this step, recall that $\tilde{\beta} = \beta c/[\beta\tilde{\phi}(0)\Delta + c]$ is an endogenous object. However, for any $\Delta \geq \tilde{\Delta}_{low}(\beta)$, we have $\beta\tilde{\phi}(0)\Delta \geq 2c$ and, hence, $\tilde{\beta} \leq \beta/3$. By the log-concavity of $\tilde{\Phi}$, the right-hand side of (C.8) is strictly increasing in $\tilde{\beta}$. Thus, if the platform distance Δ is equal to or larger than the bound $\tilde{\Delta}_{up}(\beta) = c\,\tilde{\Phi}(c/2 + \beta/3)\,[\tilde{\Phi}(c/2 + \beta/3) - \tilde{\Phi}(\beta/3)]^{-1}$, there unambiguously exists an $\omega_i \in$

$(-\Delta/2, \Delta/2)$ such that both conditions (i) and (ii) are satisfied. Put differently, the pair $(-\Delta/2, \Delta/2)$ is no robust equilibrium outcome if Δ is weakly larger than the bound $\tilde{\Delta}_{up}(\beta)$, which is expressed in exogenous variables only.

Existence of robust equilibrium outcomes. — Consider a party structure such that all members of party L have ideal point $\ell = -\Delta/2$, and all members of party R have ideal point $r = \Delta/2$ with $\Delta \geq \tilde{\Delta}_{low}(\beta)$. Assume that the first-order condition (C.7) holds with equality for both parties. Hence, no member can profitably leave her party, and no agent can profitably change her party contribution.

I now show that there is a threshold $\tilde{\Delta} > \tilde{\Delta}_{low}(\beta)$ such that both parties are also entry-stable if $\Delta \in [\tilde{\Delta}_{low}(\beta), \tilde{\Delta}]$. By an adaption of the arguments in Lemma A.2, $\tilde{\Gamma}(\omega_i, \Delta/2, C_L + c, C_R)$ is strictly quasi-concave and has a unique maximizer $\tilde{\gamma}(\Delta/2) < \Delta/2$ in its first argument, where I suppress the dependence of $\tilde{\gamma}$ on C_L, C_R and c. Moreover, there is a unique $\Delta' > 0$ such that $\tilde{\gamma}(\Delta'/2) = -\Delta'/2$ or, equivalently,

$$\Delta' = 2\frac{\tilde{\Phi}\left(\frac{\beta c}{\beta\phi(0)\Delta' + c}\right)}{\tilde{\phi}\left(\frac{\beta c}{\beta\phi(0)\Delta' + c}\right)} .$$

For all $\Delta \in [0, \Delta')$, $\tilde{\gamma}(\Delta/2) < -\Delta/2$ and $\tilde{\Gamma}_\ell(-\Delta/2, \Delta/2, C_L, C_R) < 0$. This implies that, for $\Delta \in [0, \Delta')$, there is no $\omega_i \in (-\Delta/2, \Delta/2)$ such that condition (i) in the previous step is satisfied. Hence, Δ' is strictly smaller than $\tilde{\Delta}_{up}(\beta)$.

I now show that $\Delta' > \tilde{\Delta}_{low}(\beta)$ under Assumption C.1. For $\Delta = \tilde{\Delta}_{low}(\beta)$, we have $C_R = C_L = c$, $\tilde{\beta} = \beta/3$, and

$$\begin{aligned}
\tilde{\Gamma}_\ell(-\Delta/2, \Delta/2, 2c, c) &= \frac{\tilde{\Delta}_{low}(\beta)}{2}\, \tilde{\phi}(\beta/3) - \tilde{\Phi}(\beta/3) \\
&= \left[\frac{c}{\beta\, \tilde{\phi}(0)} - \frac{\tilde{\Phi}(\beta/3)}{\tilde{\phi}(\beta/3)}\right] \tilde{\phi}(\beta/3) \\
&< 0\,,
\end{aligned}$$

where the inequality follows from Assumption C.1. Thus, Δ' is strictly larger than $\tilde{\Delta}_{low}(\beta)$. As a result, the pair of platforms $(-\Delta/2, \Delta/2)$ is a robust equilibrium outcome for any $\Delta \in [\tilde{\Delta}_{low}(\beta), \Delta']$: Given any such platforms, no independent agent with ideal point $\omega_i \in (\ell, r)$ becomes presidential candidate if she joins a party.