# The Racial Wealth Gap and the Role of Firm Ownership

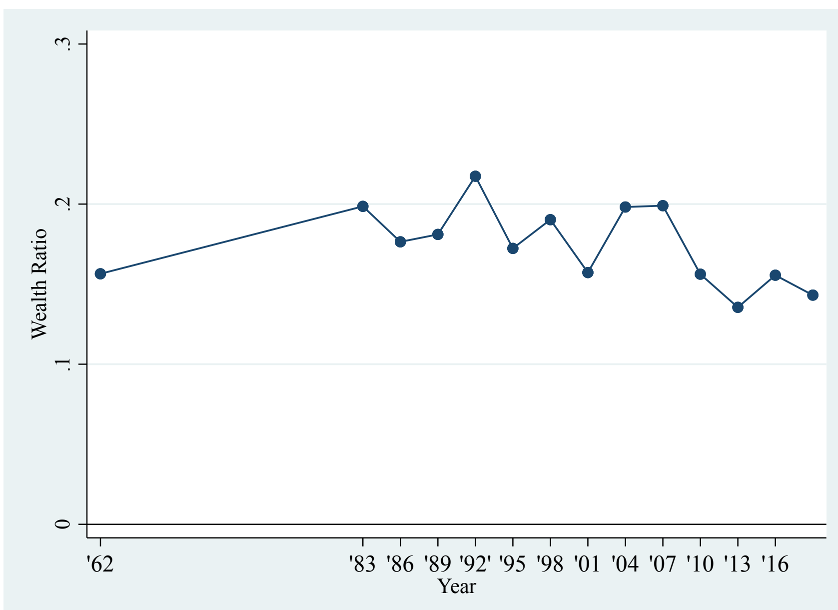
# Online Appendix

Avi Lipton

January 2022

## A Wealth Variable

Figure 1: Persistence of the Wealth Gap




Note: Each point is the estimate for the mean wealth of the black households over the mean wealth for white households in the given year.

I base my wealth variable used in figure 1 on Aliprantis et al. (2019) (ACY), who also use the Survey of Consumer Finances (Federal Reserve Board, 2019) to discuss the Racial Wealth Gap.

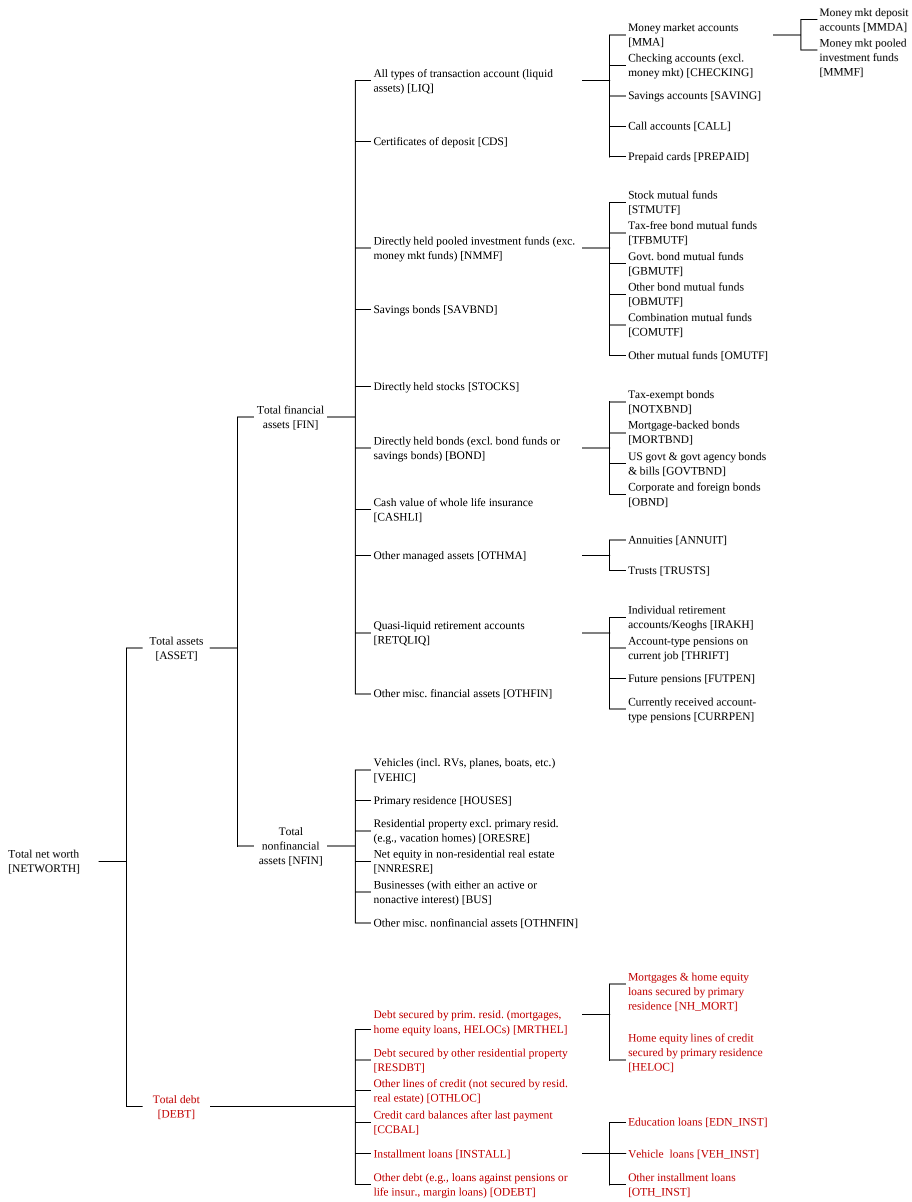
The SCF surveys a nationally representative sample of U.S. households, with an over sample of households with the highest income to better capture the upper tail of the wealth distribution, every three years from 1983 to 2019. The survey contains detailed information about household wealth and income and the variable that ACY uses to reflect wealth is household net worth, the value of all a household's assets minus the value of all its debt. The SCF provides race information about the head of households and this is how ACY delineates household race. For 1962, ACY uses a precursor to the SCF, the Survey of Financial Characteristics of Consumers (SFCC). Unfortunately, there is no data available after 1962 and before 1983. ACY limits its sample to households with heads between the ages of 20 and 100 and converts wealth data to 2019 dollars using the St. Louis Fed's GDP Implicit Price Deflator (Federal Reserve Bank of St. Louis, 2021). I follow ACY's methodology and reproduce their findings about the mean racial wealth gap from 1962 to 2019 (Figure 1). There are minor discrepancies between my results and the authors' due to slightly different variable constructions (they do not specify precisely how they construct their variables and I try to match them as closely as possible).

For the 1983 and 1986 SCF, the specific variables used for household net worth is b3324 and c1457 respectively. These two variables are equivalent and are comprised of total assets minus total debt. Total assets is the sum of paper assets — the sum of stocks and mutual funds, bonds, checking and savings accounts, IRA and Keogh accounts, money market accounts and CDs, profit sharing and thrift accounts, cash value of life insurance, and other financial assets — and real assets —- the sum of the current market value of the home, other properties, businesses, and vehicles. Total debt is the sum of total real estate debt and total consumer debt. The weight used in 1983 is the Extended Income FRB weight (b3016), which is the recommended full sample weight. The weight used in 1986 is FRB 1986 Weight #2 (c1014), which is the recommended weight for viewing the 1986 households, who were re-interviewed from 1983, as a cross section of the 1986 population. These weights are used for all calculations (SCF 1983 Code book, SCF 1986 Code book).

For 1989-2019, the data I use specifically comes from the the SCF Bulletin extract data. Figure 2 depicts the content of the net worth variable in these data sets. I modify this variable slightly by subtracting out future pensions and currently received account type pensions to bring this variable in line with the net worth variable in 1983 and 1986 that does not include this pension information. There is only one weight variable in these data sets called wgt and it is used for all calculations.

The 1962 SCFF is the most complicated data set to deal with. First, the SFCC only distinguishes race by white, non-white, and not-ascertained. As such, I follow ACY's assumption that non-white aligns with the black delineation in the SCF (the percentage of non-white households in the SFCC is close to the percentage of black Americans in the U.S. in 1962) and that not-ascertained aligns with the white delineation in the SCF (some white ethnic groups like Jews and Italians were likely delineated as not-ascertained in 1962). The next complication is that the SCFF does not provide an aggregate wealth variable. As such, I construct it from the component parts of wealth to match the net worth variable I use for the SCF. First, I add up all the assets: checking accounts (v5-v7), U.S. savings bonds (v8-v10), U.S. government bills (v11), U.S. government notes (v12), U.S. government certificates (v13), non savings U.S. government bonds (v14-v18), state and local bonds (v19), bonds, debentures, and notes for foreign corporations or governments (v20), bonds, debentures, and notes for domestic corporations (v21), mortgage assets (v22-v24), loans to business without active interest (v25), non-mortgage loan assets to individuals (v26-v28), amount paid into annuities (v29-v31), estates in probate (v32), cash surrender value for non-term life-insurance (v39-v41), balance in savings accounts (v45-v69), market value of stock (v70-v74), market value of business with active interest (v80-v84), loans to business with active interest (v85-v88), share of undistributed profits in closely held corporation (v89-v90), market value of business without active interest (v91-v95), loans to business without active interest (v96-v100), amount that can be withdraw from profit sharing plans (v101), amount that can be withdrawn from retirement plans (v102), market value of real estate (v103,v105,v107), credit in brokerage accounts (v109), market value of automobiles (v111), oil royalties, patents, and commodity contracts (v112-v114), assets held in trusts (v176). Then, from all the assets, I subtract all the debts: loans outstanding secured by life insurance polices (v42-v44), real estate debt (v104,v106,v108), debit in brokerage accounts(v110), loans secured by stock (v115), loans secured by bonds (v116), installment debt (v117-v120), non installment debt (v121-v124). Lastly, the weight variable is v4, which is used for all calculations.

Figure 2: Construction of Net Worth in SCF 1989-2019




*Names in brackets refer to variables in the SCF Bulletin extract data
For precise variable definitions, please see the documentation and programs on the SCF website.

This flow chart is provided by the 1989-2019 SCF along with the data.
See https://www.federalreserve.gov/econres/scfindex.htm

## B Formal Proofs

**Claim 1: The capital stock converges to a unique steady state $K_*$.**

*Proof.* I adapt a proof presented in Acemoglu (2009). Recall that the dynamics of the capital stock are governed by the equation:

$$K_{t+1} = \frac{\beta + \rho\beta}{\beta + \rho\beta + 1}(\frac{\rho\beta}{\beta + \rho\beta}(1 + N^{1-\alpha-\gamma}L^{\gamma}\alpha K_t^{\alpha-1} - \delta)K_t + N^{1-\alpha-\gamma}L^{\gamma}(1-\alpha)K_t^{\alpha}).$$

Setting

$$A = \frac{\beta + \rho\beta}{\beta + \rho\beta + 1},$$

$$C = N^{1-\gamma-\alpha}L^{\gamma},$$

and

$$D = \frac{\rho\beta}{\beta + \rho\beta}$$

for convenience leaves

$$\begin{aligned} K_{t+1} &= A(D(1 + C\alpha K_t^{\alpha-1} - \delta)K_t + C(1-\alpha)K_t^{\alpha}) \\ &= AD(1-\delta)K_t + ADC\alpha K_t^{\alpha} + AC(1-\alpha)K_t^{\alpha}. \end{aligned}$$

with $0 < A < 1$ and $0 < D < 1$. From the above equation it is clear that $K_{t+1} = f(K_t)$ where $f$ is a differentiable function with

$$f'(K_t) = AD(1-\delta) + ADC\alpha^2 K_t^{\alpha-1} + AC(1-\alpha)\alpha K_t^{\alpha-1}.$$

I will first prove the existence and uniqueness of the steady state. Note that $\lim_{x\to\infty} f'(x) = AD(1-\delta) < 1$ and $\lim_{x\to 0} f'(x) = \infty$. Now, note that for positive $x$

$$f(x) = x \iff \frac{f(x)}{x} = 1.$$

Using L'Hopital's rule,

$$\lim_{x \to 0} \frac{f(x)}{x} = \lim_{x \to 0} f'(x) = \infty$$

$$\lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} f'(x) < 1.$$

So, there is some value $x_1 > 0$ such that $\frac{f(x_1)}{x_1} > 1$ and a value $x_2 > 0$ such that $\frac{f(x_2)}{x_2} < 1$. Now, because $\frac{f(x)}{x}$ is continuous for all $x > 0$, the intermediate value theorem guarantees there is some $x_1 < K_* < x_2$ such that $\frac{f(K_*)}{K_*} = 1$. Regarding the uniqueness of $K_*$, note that $\frac{f(x)}{x}$ is a decreasing function. So, for all $x > K_*$, $\frac{f(x)}{x} < 1$ and for all $x < K_*$, $\frac{f(x)}{x} > 1$. Thus $K_*$ is the unique steady state of $f$ that is strictly greater than 0. To see that $K_t$ must converge to $K_*$, first note that $f'(x) > 0$ for all $x$. Then, for all $0 < K_t < K_*$,

$$K_{t+1} - K_* = f(K_t) - f(K_*) = -\int_{K_t}^{K_*} f'(x)dx < 0 \implies K_{t+1} < K_*.$$

Further,

$$\frac{K_{t+1} - K_t}{K_t} = \frac{f(K_t) - K_t}{K_t} = \frac{f(K_t)}{K_t} - 1 > 0 \implies K_{t+1} > K_t.$$

Similarly, for all $K_t > K_*$,

$$K_{t+1} - K_* = f(K_t) - f(K_*) = \int_{K_t}^{K_*} f'(x)dx > 0 \implies K_{t+1} > K_*$$

and

$$\frac{K_{t+1} - K_t}{K_t} = \frac{f(K_t) - K_t}{K_t} = \frac{f(K_t)}{K_t} - 1 < 0 \implies K_{t+1} < K_t.$$

In the model, the initial capital stock is exogenously set to some positive number so $K_1 > 0$. The previous work shows that if $K_1 < K_*$, then $K_t$ is an increasing sequence bounded above by $K_*$ and if $K_1 > K_*$, it is a decreasing sequence bounded below by $K_*$. In either case $K_t$ is a bounded monotone sequence and so must converge to some $K > 0$. Thus, applying the continuity of $f$,

$$f(K) = \lim_{t \to \infty} f(K_t) = \lim_{t \to \infty} K_{t+1} = K.$$

Thus $K$ is a steady state greater than $0$ so $K = K_*$. $\square$

.

**Claim 2: For $i \in \{R, P\}$, $b_t^i$ converges to a steady state**

$$b_*^i = \frac{\frac{\rho\beta(1+r_*)}{\beta+\rho\beta+\zeta\beta+1}(lW_* + \phi_i \frac{P_*}{100})}{1 - \frac{\rho\beta(1+r_*)}{\beta+\rho\beta+\zeta\beta+1}(1 + \frac{\zeta P_*}{100\rho v_*} + \frac{\zeta}{\rho(1+r_*)})} > 0.$$

*Proof.* Recall that the dynamics of $b_t^i$ are governed by

$$b_{t+1}^i = \frac{\rho\beta(1+r_{t+1})}{\beta+\rho\beta+\zeta\beta+1}(1 + \frac{\zeta P_t}{100\rho v_t} + \frac{v_{t+1}\zeta}{(1+r_{t+1})\rho v_t})b_t^i + \frac{\rho\beta(1+r_{t+1})}{\beta+\rho\beta+\zeta\beta+1}(lW_t + \frac{\phi_i P_t}{100}),$$

which has the form of a sequence $b_{t+1} = a_t b_t + c_t$ where $a_t$ converges to some $a > 0$ and $c_t$ converges to some $c > 0$. The proof of this claim amounts to demonstrating that $b_t$ converges to $\frac{c}{1-a} > 0$. First note that for all $n \in \mathbb{N}$, there exists some $T_n$ such that for all $t \geq T_n$, $a - \frac{1}{n} < a_t < a + \frac{1}{n}$ and $c - \frac{1}{n} < c_t < c + \frac{1}{n}$. So, for all $t \geq T_n$,

$$(a - \frac{1}{n})b_t + (c - \frac{1}{n}) < b_{t+1} < (a + \frac{1}{n})b_t + (c + \frac{1}{n}).$$

Now, let $(b_t^n)$ be the sequence $(b_t)$ after truncating off the first $T_n - 1$ terms, let $(U_t^n)$ be the sequence given by $U_0^n = b_{T_n}$ and $U_{t+1}^n = (a + \frac{1}{n})U_t^n + (c + \frac{1}{n})$, and let $(L_t^n)$ be the sequence given by $L_0^n = b_{T_n}$ and $L_{t+1}^n = (a - \frac{1}{n})L_t^n + (c - \frac{1}{n})$. It is clear from induction that $L_t^n \leq b_t^n \leq U_t^n$ for all $t$. Now, suppose for contradiction that $a > 1$. Choose $N$ such that $a - \frac{1}{N} > 1$ and $c - \frac{1}{N} > 0$. So,

$$L_{t+1}^N > (a - \frac{1}{N})L_t^N > (a - \frac{1}{N})^t L_0^N \to \infty \implies b_t^N \to \infty \implies b_t \to \infty.$$

In the specific context of the model, $b_t$ diverging to infinity means that $b_t^R$ and $b_t^P$ diverge to infinity, which implies that $B_t = q^R H b_t^R + q^P H b_t^P$ diverges to infinity. However, we know that that $B_t = \frac{\rho\beta}{\rho+\rho\beta}(1+r_t)K_t$, which converges to $\frac{\rho\beta}{\rho+\rho\beta}(1+r_*)K_*$, which is a contradiction. Next, suppose for contradiction that $a = 1$. For all $n$,

$$L_t^n = (1 - \frac{1}{n})^t L_0^n + (c - \frac{1}{n})((1 - \frac{1}{n})^{t-1} + ... + (1 - \frac{1}{n})^0).$$

The first term here will converge to $0$ and the second term is the geometric series

$$\sum_{t=1}^{\infty}(c - \frac{1}{n})(1 - \frac{1}{n})^{t-1} = \sum_{t=0}^{\infty}(c - \frac{1}{n})(1 - \frac{1}{n})^t = \frac{c - \frac{1}{n}}{\frac{1}{n}} = nc - 1.$$

So, $b_t^n \geq L_t^n$, which converges to $nc - 1$. Now, given some $m > 0$, there exists an $N$ such $Nc - 1 > m + 1$. Then, there exists a $T$ such that $t > T \implies L_t^N > m \implies b_t^N > m$. Thus, if $a = 1$, $b_t$ diverges to infinity, which as previously shown is a contradiction. So, $a < 1$. Now choose $N_0$ such that $(a + \frac{1}{N_0}) < 1$ and $(a - \frac{1}{N_0}) > 0$. For all $n > N_0$,

$$U_t^n = (a + \frac{1}{n})^t U_0^n + (c + \frac{1}{n})((a + \frac{1}{n})^{t-1} + ... + (a + \frac{1}{n}^0)) \to \frac{c + \frac{1}{n}}{1 - a - \frac{1}{n}}$$

and

$$L_t^n = (a - \frac{1}{n})^t U_0^n + (c - \frac{1}{n})((a - \frac{1}{n})^{t-1} + ... + (a - \frac{1}{n}^0)) \to \frac{c - \frac{1}{n}}{1 - a + \frac{1}{n}}.$$

Finally, take $\varepsilon > 0$. Choose $N_1 > N_0$ such that

$$\frac{c + \frac{1}{N_1}}{1 - a + \frac{1}{N_1}} < \frac{c}{1 - a} + \frac{\varepsilon}{2} \text{ and } \frac{c - \frac{1}{N_1}}{1 - a - \frac{1}{N_1}} > \frac{c}{1 - a} - \frac{\varepsilon}{2}.$$

Then, choose $T$ such that

$$t > T \implies U_t^{N_1} < \frac{c + \frac{1}{N_1}}{1 - a - \frac{1}{N_1}} + \frac{\varepsilon}{2} \text{ and } L_t^{N_1} > \frac{c - \frac{1}{N_1}}{1 - a + \frac{1}{N_1}} - \frac{\varepsilon}{2}.$$

Thus,

$$t > T \implies \frac{c}{1 - a} - \varepsilon < \frac{c - \frac{1}{N_1}}{1 - a + \frac{1}{N_1}} - \frac{\varepsilon}{2} < L_t^{N_1} < b_t^{N_1} < U_t^{N_1} < \frac{c + \frac{1}{N_1}}{1 - a - \frac{1}{N_1}} + \frac{\varepsilon}{2} < \frac{c}{1 - a} + \varepsilon.$$

So, it is clear that $b_t$ converges to $\frac{c}{1-a} > 0$ as required. $\square$

# References


**Acemoglu, Daron**, *Introduction to Modern Economic Growth*, Princeton University Press, 2009.

**Aliprantis, Dionissi, Daniel R. Carroll, and Eric R. Young**, "The Dynamics of the Racial Wealth Gap," Working Papers 201918, Federal Reserve Bank of Cleveland October 2019.

**Federal Reserve Bank of St. Louis**, "Gross Domestic Product Implicit Price Deflator," Technical Report 2021. url https://fred.stlouisfed.org/series/GDPDEF (March 2020).

**Federal Reserve Board**, "Survey of Consumer Finances 1962-2019," Technical Report 2019. url https://www.federalreserve.gov/econres/scf-previous-surveys.htm (January 2020).