ONLINE Appendix for Government Financing of R&D: A Mechanism Design Approach

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1 Online Appendices: Not for Publication

Appendix A. Calibration of parameters

In this Appendix we describe the choice of parameters used to simulate the model. We calibrate the parameters using data from the United States. The required set of parameters includes the private returns to a project, R; the distributions of success probabilities p and of externalities σ; the shadow price of public funds, λ; the project costs with partial effort c_I; the share of project cost with full effort that can be financed from the entrepreneur's own funds, $\bar{b}$; and the effectiveness of the VC in enhancing a project's success probability, β. We assume $k = c_I$, i.e., the percentage increase in cost in moving from partial to full effort is equal to the percentage increase in the associated probability of success.

To calibrate R, we begin with the equation $\bar{p}R = (1+\varphi)^{\tau}$ where φ is the internal rate of return to a VC, τ is the lag between the start-up investment at time zero and commercialisation (also assumed to be the end of VC funding), if the project succeeds, and $\bar{p}$ is the mean probability of success. Gompers et. al.(2010) estimate the average internal rate of return for VCs in the range of 14%-18%, so we use $\varphi = 0.16$. Gompers et. al. (2010) also estimate the average success rate for VC-supported start-ups – defined as a public offering, acquisition or merger for first time entrepreneurs – in the range of about 14%-27%. Thus we set the value $\bar{p} = 0.20$. We set the commercialisation lag at $\tau = 5$. These parameter values imply $R = 10.5$.

We calibrate σ so that the implied ratio of social to private rates of returns to R&D is consistent with econometric evidence from the literature. Bloom, Schankerman and Van Reenen (2013) estimate average internal social and private rates of return of 0.55 and 0.21, respectively. The ratio of social to private rates of return is slightly above 2.5 and we choose values of the externality parameter σ to match this ratio.[1] However, there is a distribution of rates of return across firms, so we also include lower and higher values of σ to check sensitivity of the optimal policy to the magnitude of the externality.[2] The chosen values of σ imply ratios of social to private internal rates of returns of $(1.1, 1.5, 2, 2.5, 3)$.

We assume that project success probabilities $\{p\}$ are drawn from the beta distribution, $B(a, b)$. To calibrate the parameters (a, b), we choose values so that the resulting distribution matches two empirical facts. The first is the fraction of projects

[1]Let φ_s and φ_p be the social and private rates of return and $\alpha = \frac{\varphi_s}{\varphi_p}$. In our model with unit costs, $(1+\varphi_s)^{\tau} = \bar{p}(R+\sigma)$ and $\left(1+\varphi_p\right)^{\tau} = \bar{p}R$ so that $\sigma = R\left[\left(\frac{1+\alpha\varphi_p}{1+\varphi_p}\right)^{\tau} - 1\right]$. Using $\varphi_p = 0.21$ and varying α around 2.5 calibrates the values of σ.

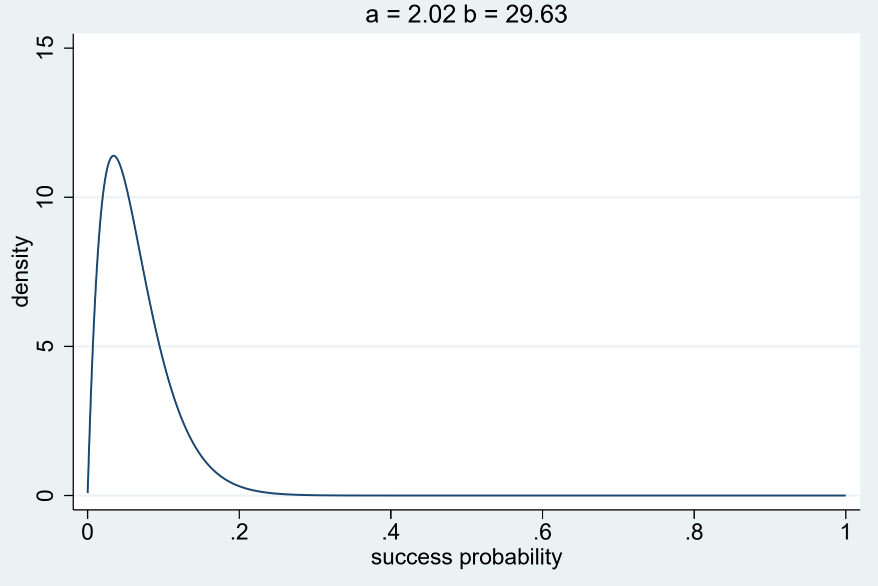
[2]In simulating welfare, we use the same distribution of success probabilities of projects for different values of σ. This procedure implicitly assumes that p and σ are independent. Since the expected private returns to R&D is $p\beta R$, this is equivalent to assuming that the private rate of return is uncorrelated with the externality σ. The correlation, computed from the estimates in Bloom, Schankerman and Van Reenen (2013), data is only -0.09. We emphasize, however, that the *model* does not impose this independence assumption.

that are financed by private VCs in the absence of a government loan program, which we set at 0.05. In the model, projects receiving funding are those with $L_1 \equiv \frac{1-k}{(\beta-k)R} + \frac{1-\bar{b}}{\beta R} \leq p \leq 1$. Thus we set the mass of such projects $1 - F(L_1; a, b) = 0.05$. Second, we require that the implied average success rate among projects that are funded by VCs, $\bar{p}$, matches the estimates from Gompers et. al. (2010). In the model, this success rate is given by the mean of the truncated distribution,

$$[1 - F(L_1; a, b)]^{-1} \left[\int_{L_1}^{1/\beta} \beta p dF(p; a, b) + 1 - F\left(\frac{1}{\beta}; a, b\right) \right],$$

which we set equal to $\bar{p} = 0.2$ for this calibration. We solve these two equations for a and b. The density function for p is shown in Figure A1. The mean of p under $F(p; a, b)$ is 0.064.

Figure A1: Probability Density Function of the Beta Distribution



To calibrate c_I, we use data on the structure of venture capital funding from Gompers (1995). He breaks funding into the early rounds (seed + startup), usually made to very young companies, middle rounds usually made to young but further developed companies, and late stage financing. We measure c_I by the ratio of the median of seed funding to total early round funding, i.e., $c_I = k = 0.21$.[3]

We calibrate λ based on estimates in the public finance literature (Dahlby, 2008). These vary depending on methodology, country coverage, and the choice of taxes used to generate the public funds. We focus on public funds raised by taxes on labor

[3]These values are similar to those computed from data on Israeli start up companies by Harel (2013). The mean (median) value of the share of seed money out of total invested funds is 0.23 (0.18) among the 1,149 firms that passed the seed stage. We thank Shai Harel for providing his data.

income; the estimated values of λ typically fall in the range of 0.25 to 1.5. We use values $\lambda = (0.25, 0.50, 0.75, 1)$. The full cost of public funds is $1 + \lambda$.

We set the fraction of project cost with full effort that can be funded by the entrepreneur at $\bar{b} = 0.25$; the qualitative results reported below are robust to using other values of $\bar{b}$.

Finally, we calibrate the effectiveness of the VC based on evidence in Hsu (2004). Using data on VC contracts, Hsu estimates that VC's with a strong reputation (above the median), based on the quality of their advice and connections, acquire start-up equity at a 10-14% discount, relative to below-median VC's. This market-based willingness to pay for more reputable VC's corresponds to our project enhancement effect, β. Thus we use the baseline value $\beta = 1.12$. This is conservative since it assumes that the below-median VC's have no positive effect on the success probability of the projects they finance. We also experiment with two alternative values, $\beta = 1.0$ (no enhancement) and $\beta = 1.24$.

Appendix B. Computing the optimal policy and welfare

In this Appendix we provide details on the computation of the optimal policy, its welfare and associated measures such as additionality and redundancy in the general case with moral hazard. We analyze two funding scenarios. In one scenario there is only the possibility of private market (VC) funding while, in the other scenario, we add the possibility of government funding. In Section B1 we first derive the entrepreneur's utility associated with the funding possibilities available in each scenario. Entrepreneurs choose the alternative giving them the highest utility. Given this behavior, in Section B2 we compute the expected social welfare associated with each funding scenario, and solve for the optimal policy. In Section B3 we present the additionality and redundancy measures, while in Section B4 we point out that the computation of welfare and costs generated by alternative – not necessarily optimal – loan policies is as in Section B2.

B1. The entrepreneur's utility

1. Private market funding only

The utility of a project of type p when funded by the private market only is given after Proposition 1, and repeated here,

$$U_P(p) = \begin{cases} 0 & \text{if } p \in \left[0, \frac{c_I}{kR}\right) & \text{no implementation} \\ kpR - c_I & \text{if } p \in \left[\frac{c_I}{kR}, \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}\right) & \text{partial implementation} \\ \min\{\beta p, 1\}R - 1 & \text{if } p \in \left[\frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}, 1\right] & \text{full implementation} \end{cases} \tag{1}$$

Note that $U_P(p) \geq 0$.[4]

2. Government funding

Given that Proposition 6 holds with our paramteres, the government offers either one of the following two contracts: a zero liability contract, or a maximum outlay contract. We analyze each case in turn.

2.1 Zero liability contract

The zero liability contract is $(b_\varepsilon, r_\varepsilon) = \left(\frac{\varepsilon c_I^2(1+\lambda)}{\sigma + c_I\varepsilon(1+\lambda)}, \frac{R}{c_I} - 1 - \varepsilon\right)$. The loan given by the government is $c_I - b_\varepsilon$. The utility of an entrepreneur of type p from taking this contract and exerting partial effort is

$$\begin{aligned} U_\varepsilon(p) &= kp\left[R - (c_I - b_\varepsilon)(1 + r_\varepsilon)\right] - b_\varepsilon \\ &= \frac{c_I\varepsilon}{\sigma + c_I\varepsilon(1+\lambda)}\left[kp\left(R(1+\lambda) + \sigma\right) - (1+\lambda)c_I\right] \end{aligned} \tag{2}$$

By design, the utility to the entrepreneur of taking this contract is close to zero (it tends to zero as $\varepsilon \downarrow 0$) when the entrepreneur exerts partial effort. The entrepreneur will not exert full effort under this contract because she may not have sufficient funds to do so and, even if she does, her expected utility from exerting full effort tends to $(c_I - 1) < 0$ as $\varepsilon \downarrow 0$.

The zero liability contract is designed in such a way that it screens out projects having negative expected welfare under partial effort. Specifically, projects with $p < \frac{c_I}{kR + \frac{k\sigma}{1+\lambda}}$ have negative expected welfare and will also not take the epsilon contract because

$$U_\varepsilon(p) \gtreqless 0 \text{ as } p \gtreqless \frac{c_I}{kR + \frac{k\sigma}{1+\lambda}} \tag{3}$$

2.2 Maximum outlay contract

The utility from taking the maximum outlay contract or, in short, the "(b, r) contract" and partially or fully implementing the project is, respectively,

$$U_{br}^P(p, b, r) = kp\left[R - (1-b)(1+r)\right] - (k - (1-b)) \tag{4}$$

$$U_{br}^F(p, b, r) = p\left[R - (1-b)(1+r)\right] - b \tag{5}$$

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[4]However, if $\frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R} < \frac{c_I}{kR}$ then

$$U_P(p) = \begin{cases} 0 & \text{if } p \in \left[0, \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}\right) \\ \min\{\beta p, 1\}R - 1 & \text{if } p \in \left[\frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}, 1\right] \end{cases}$$

Note that an entrepreneur taking the government loan (b, r) prefers to exert full to partial effort whenever

$$p > \frac{1 - c_I}{(1-k)\left(R - (1-b)(1+r)\right)}. \tag{6}$$

And will prefer to exert partial effort with the (b, r) loan than with the approximate zero liability contract whenever $U_{br}^P(p, b, r) \geq U_\varepsilon(p)$ or

$$p \geq \frac{b - b_\varepsilon - (1 - c_I)}{k\left[(c_I - b_\varepsilon)(1 + r_\varepsilon) - (1-b)(1+r)\right]}$$

which, setting $b_\varepsilon = 0$ and $(1 + r_\varepsilon) = \frac{R}{c_I}$, simplifies to

$$p \geq \frac{b - (1 - c_I)}{k\left[R - (1-b)(1+r)\right]}$$

so partial effort under the (b, r) loan is always preferred to the approximate zero liability as long as $b < 1 - c_I$ as assumed in Proposition 6 for the optimal $b = \bar{b}$. Recall that we already assume $\bar{b} \geq c_I$ so it is enough to assume, in addition, that $\bar{b} < 0.5$ for Proposition 6 to hold.

B2. Deriving the social welfare function and optimal policy

Let $\omega = \left(\sigma, \lambda, k, \beta, \bar{b}, R\right)$ denote the vector of parameters. The entrepreneurs' utility functions $U_P(p)$, $U_{br}^P(p, \bar{b}, r)$ and $U_{br}^F(p, \bar{b}, r)$ defined in Section B1 are evaluated at the optimal b, i.e., at $b = \bar{b}$. We compute social welfare in the benchmark case of private market funding only without government intervention. We then add the possibility of government intervention. Assuming $b \leq 1 - c_I$, the government offers either the zero liability contract or the maximum outlay, or (b, r), contract, whichever generates larger expected welfare.

Scenario 1: Welfare from VC funding without government intervention

Let $\tilde{W}_{prv\_only}(p; \omega)$ be the contribution to social welfare when there is only private VC funding generated by a project of type p. It equals the sum of entrepreneurs' utility (as defined in (1)), the VCs' utility (which is zero because of competition), and the expected spillover $p\sigma$ (or $kp\sigma$),

$$\tilde{W}_{prv\_only}(p; \omega) = \begin{cases} 0 & \text{if } p \in \left[0, \frac{c_I}{kR}\right) \\ kp\left(R + \sigma\right) - c_I & \text{if } p \in \left[\frac{c_I}{kR}, \frac{1-c_I}{(\beta-k)R} + \frac{1-\bar{b}}{\beta R}\right) \\ \beta p\left(R + \sigma\right) - 1 & \text{if } p \in \left[\frac{1-c_I}{(\beta-k)R} + \frac{1-\bar{b}}{\beta R}, \frac{1}{\beta}\right) \\ R + \sigma - 1 & \text{if } p \in \left[\frac{1}{\beta}, 1\right] \end{cases} \tag{7}$$

In this scenario, the entrepreneur will implement the project partially if $p \in \left[\frac{c_I}{kR}, \frac{1-c_I}{(k-\beta)R} + \frac{1-\bar{b}}{\beta R}\right)$ and fully if $p \in \left[\frac{1-c_I}{(k-\beta)R} + \frac{1-\bar{b}}{\beta R}, 1\right)$.

In practice, we write $\tilde{W}_{prv\_only}(p;\omega)$ as

$$\tilde{W}_{prv\_only}(p;\omega) = [(kp(R+\sigma) - k) \times d2] + [(\beta p(R+\sigma) - 1) \times d3] + [(R+\sigma-1) \times d4]$$

where $d2, d3$ and $d4$ are indicator functions for p falling in each of the last three intervals defined in (7) and compute the expected social welfare as

$$W_{prv\_only}(\omega) = \int_0^1 \tilde{W}_{prv\_only}(p;\omega)dF(p)$$

where $F(p)$ is the beta distribution with calibrated parameters.

Scenario 2: Welfare from government intervention

We first describe the welfare obtained by either type of contract offered and then select the contract that maximizes social welfare, given parameters ω.

Welfare when only the zero liability contract is offered We distinguish between two cases according to the project's type p: either $U_P(p) > U_\varepsilon(p)$ or $U_P(p) \le U_\varepsilon(p)$, where $U_P(p)$ is the entrepreneurs's utility from the project without government support (see (1)), and $U_\varepsilon(p)$ is the utility derived from the zero liability contract (see (2)).

1. If $U_P(p) > U_\varepsilon(p)$, which occurs when $p > \frac{c_I}{kR}$, then entrepreneurs will not accept the zero liability contract. In this case, we only observe projects funded by the private market. The contribution of such a project to welfare, denoted by $\tilde{W}_{prv,\varepsilon}(p;\omega)$, is then identical to that one under scenario 1:

$$\tilde{W}_{prv,\varepsilon}(p;\omega) = \tilde{W}_{prv\_only}(p;\omega) \tag{8}$$

2. If $U_P(p) \le U_\varepsilon(p)$ and $U_\varepsilon(p) \ge 0$, which occurs when $\frac{c_I}{kR+\frac{k\sigma}{1+\lambda}} \le p \le \frac{c_I}{kR}$ (see (1) and (3)), entrepreneurs take the zero liability contract, exert partial effort and their welfare contribution is

$$\tilde{W}_{\varepsilon,\varepsilon}(p;\omega) = kp(R+\sigma) - c_I - \lambda(c_I - b_\varepsilon)(1 - pk(1+r_\varepsilon))$$

Let $W_\varepsilon(\omega)$ be the expected social welfare generated by the ε contract. It is given by

$$\begin{aligned} W_\varepsilon(\omega) &= \int_{\frac{c_I}{kR+\frac{k\sigma}{1+\lambda}}}^{\frac{c_I}{kR}} \tilde{W}_{\varepsilon,\varepsilon}(p;\omega)dF(p) + \int_{\frac{c_I}{kR}}^1 \tilde{W}_{prv,\varepsilon}(p;\omega)dF(p) \qquad (9) \\ &= \int_{\frac{c_I}{kR+\frac{k\sigma}{1+\lambda}}}^{\frac{c_I}{kR}} \tilde{W}_{\varepsilon,\varepsilon}(p;\omega)dF(p) + W_{prv\_only}(\omega) \end{aligned}$$

The first term on the right hand side is the increment to welfare generated by the zero liability contract contract. The social cost of offering this contract is

$$C_\varepsilon(\omega) = (c_I - b_\varepsilon)(1+\lambda) \int_{\frac{c_I}{kR + \frac{k\sigma}{1+\lambda}}}^{\frac{c_I}{kR}} (1 - pk(1+r_\varepsilon))\, dF(p)$$

Welfare when the maximum outlay contract is offered The maximum outlay contract consists of a loan $1-\bar{b}$ and an interest rate r to be determined optimally. The contract is denoted by $(\bar{b}, r)$. We distinguish between three cases according to the project's type p:

1. If $U_P(p) > Max\left\{U_{br}^P(p,\bar{b},r), U_{br}^F(p,\bar{b},r)\right\}$ the entrepreneur will not accept any of the contracts offered. In this case, we only observe projects funded by the private market. The contribution of such a project to welfare, denoted by $\tilde{W}_{prv,br}(p;\omega)$, is then identical to the one under scenario 1,

$$\tilde{W}_{prv,br}(p;\omega) = \tilde{W}_{prv\_only}(p;\omega)$$

2. If $U_{br}^P(p,\bar{b},r) > Max\left\{U_{br}^F(p,\bar{b},r), U_P(p)\right\}$ the entrepreneur takes the $(\bar{b}, r)$ contract offered, exerts partial effort and contributes

$$\tilde{W}_{P,br}(p,\bar{b},r;\omega) = kp(R+\sigma) - c_I - \lambda(1-\bar{b})(1 - kp(1+r))$$

3. If $U_{br}^F(p,\bar{b},r) > Max\left\{U_{br}^P(p,\bar{b},r), U_P(p)\right\}$ the entrepreneur takes the $(\bar{b}, r)$ the contract offered, exert full effort and contributes

$$\tilde{W}_{F,br}(p,\bar{b},r;\omega) = p(R+\sigma) - 1 - \lambda(1-\bar{b})(1 - p(1+r))$$

It is convenient to define the following three indicator variables for p :

$$\Pr v\_br(p,\bar{b},r;\omega) = 1 \text{ if } U_P(p) \geq Max\left\{U_{br}^P(p,\bar{b},r), U_{br}^F(p,\bar{b},r)\right\}; = 0 \text{ else.}$$
$$Par\_br(p,\bar{b},r;\omega) = 1 \text{ if } U_{br}^P(p,\bar{b},r) \geq Max\left\{U_{br}^F(p,\bar{b},r), U_P(p)\right\}; = 0 \text{ else.}$$
$$Full\_br(p,\bar{b},r;\omega) = 1 \text{ if } U_{br}^F(p,\bar{b},r) \geq Max\left\{U_{br}^P(p,\bar{b},r), U_P(p)\right\}; = 0 \text{ else.}$$

These mutually exclusive indicators represent entrepreneurs' preferences for funding and investment of effort. These preferences depend on parameters ω as well as on p and r.

Let $W_{br}(\bar{b}, r;\omega)$ be the expected welfare from the $(\bar{b}, r)$ contract. It is given by

$$W_{br}(\bar{b},r;\omega) = \int_0^1 \left(\tilde{W}_{prv,b}(p) \times \Pr v\_br(p) + \tilde{W}_{P,br}(p) \times Par\_br(p) + \tilde{W}_{F,br}(p) \times Full\_br(p)\right) dF(p)$$

where we omit the arguments $(\bar{b}, r, \omega)$ in the functions in the integrand.

Optimal interest rate We compute $W_{br}(\bar{b}, r; \omega)$ for each value of r in the interval $\left[-1, \frac{R}{1-\bar{b}} - 1\right]$ with steps of size 0.01. The optimal value of r is obtained by selecting the r achieving the highest welfare

$$r_{opt}(\omega) = \arg \underset{r}{Max} W_{br}(\bar{b}, r; \omega)$$

giving optimal welfare of the (b, r) contract,

$$W_{br}(\omega) = W_{br}(\bar{b}, r_{opt}(\omega); \omega)$$

Cost of the program The program's cost to the government is

$$\begin{aligned} C_{br}(\bar{b}, r_{opt}; \omega) = &(1-\bar{b})(1+\lambda) \int_0^1 \left(1 - pk(1+r_{opt})\right) Par\_br(p))dF(p) \\ &+ (1-\bar{b})(1+\lambda) \int_0^1 \left(1 - p(1+r_{opt})\right) Full\_br(p)dF(p) \end{aligned}$$

Choice between the zero liability and maximum outlay contracts Finally, we compare the welfare from the offered contracts to select the optimal policy – offering either the zero liability contract or the $(\bar{b}, r_{opt})$ contract, and compute the associated welfare $W(\omega)$ derived from optimal government intervention,

$$W(\omega) = Max\left\{W_{\varepsilon}(\omega), W_{br}(\omega)\right\}$$

B3. Additionality and Redundancy

The approximate zero liability contract induces projects that would not have been executed under private funding to be partially implemented. These are the projects in the interval $\left[\frac{c_I}{kR + \frac{k\sigma}{1+\lambda}}, \frac{c_I}{kR}\right]$. Thus a measure of their 'extensive additionality' is the number of additional projects implemented,

$$Add_{\varepsilon, Ext}(\omega) = \int_{\frac{c_I}{kR + \frac{k\sigma}{1+\lambda}}}^{\frac{c_I}{kR}} dF(p)$$

This is the only effect of the zero liability contract. In particular, it does not fund projects that would have been otherwise been funded by the private market: it does not generate 'redundancy'.

The effect of offering the $(\bar{b}, r)$ contract is more complex. A project that was partially implemented under private funding, i.e., with $p \in \left[\frac{c_I}{kR}, \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}\right)$, and switches to full effort under the $(\bar{b}, r)$ contract generates 'intensive additionality'. The fraction of such projects is

$$Add_{br, Int}(\omega) = \int_{\frac{c_I}{kR}}^{\frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}} Full\_br(p, \bar{b}, r; \omega) dF(p)$$

where $Full\_br(p, \bar{b}, r; \omega)$ is the full effort indicator defined above.

The $(\bar{b}, r)$ loan may also generate extensive additionality since projects that were not implemented under private funding (i.e., with $p < \frac{c_I}{kR}$) may now become profitable to the entrepreneur, and be partially implemented.[5] The fraction of these additional projects is,

$$Add_{br,Ext}(\omega) = \int_0^{\frac{c_I}{kR}} Par\_br(p, \bar{b}, r; \omega) dF(p)$$

where $Par\_br(p, \bar{b}, r; \omega)$ is the partial effort indicator defined above.

Interestingly, the $(\bar{b}, r)$ part of the contract can also induce 'redundancy'. That is, entrepreneurs that would have implemented the project under private funding, prefer to take up the government contract. Projects that were partially implemented under private funding may find it profitable to take the government loan $(\bar{b}, r)$ but to continue exerting partial effort. The fraction of such projects is

$$\operatorname{Re} d_{br,Par}(\omega) = \int_{\frac{c_I}{kR}}^{\frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}} Par\_br(p, \bar{b}, r; \omega) dF(p)$$

Other redundant funding occurs when projects that were fully implemented under private funding (i.e., with $p \geq \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}$) find it profitable to take the government loan $(\bar{b}, r)$ and to continue exerting full effort. The fraction of such projects is

$$\operatorname{Re} d_{br,Full}(\omega) = \int_{\frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}}^{1} Full\_br(p, \bar{b}, r; \theta) dF(p)$$

The total redundancy generated by the $(\bar{b}, r)$ contract is the sum of these two components.

Tables 1 and 2 shows performance metrics when $\beta = 1.12$, while Tables B1 and B2 do so for additional values of β $(1, 1.24)$,without changing the underlying Beta distribution.

B4. Welfare from an arbitrary (b,r) contract

The welfare and the cost of an arbitrary contract (b, r) is obtained following the same steps used in the contract $(\bar{b}, r)$, provided $b \leq \bar{b}$. Table 2 shows the welfare gain, relative to the private market, divided by the cost of the program for the case of a full grant, $r = -1$, and a zero interest loan, $r = 0$.

[5]Under our calibrated parameteres, there are no projects with $p < \frac{c_I}{kR}$ that are fully implemented under the $(\bar{b}, r)$ loan because in this region of p the entrepreneur prefers to exert partial to full effort.

Table B1: Performance metrics of optimal policy and other support schemes, β = 1

		(1)	(2)	(3)	(4)	(5)	(6)	(7)
1+λ	**Optimal policy**	**Welfare gain (%)**	**% projects implemented by private market**	**% additional projects implemented by optimal policy**	**Welfare gain per dollar cost**			
	Panel A: Social/Private Rate of Returns = 1.1				**Optimal policy**	**Full grant**	**Zero interest loan**	**Constrained optimal VC**
1.25	Zero Liability	0.6	20.0	3.4	0.22	-0.23	-0.23	0.08
1.5	Zero Liability	0.5	20.0	2.8	0.22	-0.36	-0.36	-0.10
1.75	Zero Liability	0.5	20.0	2.4	0.21	-0.45	-0.45	-0.23
2	Zero Liability	0.4	20.0	2.1	0.21	-0.52	-0.52	-0.33
				Panel B: Social/Private Rate of Returns = 1.5				
1.25	Zero Liability	9.9	20.0	18.4	1.2	-0.11	-0.12	0.47
1.5	Zero Liability	8.4	20.0	15.6	1.1	-0.26	-0.27	0.23
1.75	Zero Liability	7.4	20.0	13.5	1.0	-0.36	-0.37	0.05
2	Zero Liability	6.5	20.0	11.9	0.9	-0.44	-0.45	-0.08
				Panel C: Social/Private Rate of Returns =2				
1.25	Maximum Outlay, r = -1	46.9	20.0	80.0	0.08	0.08	0.05	1.13
1.5	Zero Liability	22.2	20.0	32.3	2.09	-0.10	-0.12	0.77
1.75	Zero Liability	19.9	20.0	28.7	1.77	-0.23	-0.25	0.52
2	Zero Liability	18.0	20.0	25.8	1.57	-0.32	-0.34	0.33
				Panel D: Social/Private Rate of Returns = 2.5				
1.25	Maximum Outlay, r = -1	122.1	20.0	80.0	0.34	0.34	0.29	2.01
1.5	Maximum Outlay, r = -1	50.9	20.0	80.0	0.12	0.12	0.08	1.51
1.75	Zero Liability	32.4	20.0	43.0	2.6	-0.04	-0.08	1.15
2	Zero Liability	29.9	20.0	39.5	2.3	-0.16	-0.19	0.88
				Panel E: Social/Private Rate of Returns =3				
1.25	Maximum Outlay, r = -1	163.8	20.0	80.0	0.69	0.69	0.61	3.17
1.5	Maximum Outlay, r = -1	116.0	20.0	80.0	0.41	0.41	0.34	2.48
1.75	Maximum Outlay, r = -1	68.3	20.0	80.0	0.20	0.20	0.15	1.98
2	Zero Liability	40.7	20.0	51.3	3.1	0.05	0.01	1.6

Notes: Column 1 shows the percentage increase in welfare generated by the optimal policy relative to the welfare generated by the private market only. Column 2 shows the percentage of projects implemented by the private market. Column 3 shows the percentage of additional projects implemented by the optimal policy. Columns 4 to7 show the difference between the welfare generated by the policy and that generated by the private market divided by the cost of the policy. Self financing b in the full grant and zero interest loan is set to $\bar{b}$. In column (7), the constrained optimal subsidy rate is set to 100% because the unconstrained optimal rate is above 100% (between 130% and 186%). The unconstrained optimal subsidy rate is found by maximising expected social welfare at the value of σ implied by the mean ratio of social to private rates of return estimated by BVS (2013), for each value of λ. In this table the VC enhacement paramter β is set to 1 but the parameters of the Beta distribution are left unchanged.

Table B2: Performance metrics of optimal policy and other support schemes, β = 1.24

		(1)	(2)	(3)	(4)	(5)	(6)	(7)
1+λ	**Optimal policy**	**Welfare gain (%)**	**% projects implemented by private market**	**% additional projects implemented by optimal policy**	**Welfare gain per dollar cost**			
Panel A: Social/Private Rate of Returns = 1.1					**Optimal policy**	**Full grant**	**Zero interest loan**	**Constrained optimal VC subsidy**
1.25	Zero Liability	0.3	20.0	3.4	0.22	-0.29	-0.30	0.08
1.5	Zero Liability	0.2	20.0	2.8	0.22	-0.41	-0.42	-0.10
1.75	Zero Liability	0.2	20.0	2.4	0.21	-0.50	-0.50	-0.23
2	Zero Liability	0.2	20.0	2.1	0.21	-0.56	-0.56	-0.33
Panel B: Social/Private Rate of Returns = 1.5								
1.25	Zero Liability	4.7	20.0	18.4	1.2	-0.22	-0.23	0.47
1.5	Zero Liability	4.0	20.0	15.6	1.1	-0.35	-0.36	0.23
1.75	Zero Liability	3.5	20.0	13.5	1.0	-0.44	-0.45	0.05
2	Zero Liability	3.1	20.0	11.9	0.9	-0.51	-0.52	-0.08
Panel C: Social/Private Rate of Returns =2								
1.25	Zero Liability	12.4	20.0	36.8	2.7	-0.10	-0.13	1.14
1.5	Zero Liability	11.0	20.0	32.3	2.1	-0.25	-0.27	0.78
1.75	Zero Liability	9.8	20.0	28.7	1.8	-0.36	-0.38	0.53
2	Zero Liability	8.9	20.0	25.8	1.6	-0.44	-0.45	0.33
Panel D: Social/Private Rate of Returns = 2.5								
1.25	Zero Liability	19.5	20.0	51.7	4.5	0.07	0.02	2.02
1.5	Zero Liability	17.8	20.0	47.0	3.3	-0.11	-0.15	1.52
1.75	Zero Liability	16.3	20.0	43.0	2.6	-0.24	-0.27	1.16
2	Zero Liability	15.1	20.0	39.5	2.3	-0.33	-0.36	0.89
Panel E: Social/Private Rate of Returns =3								
1.25	Maximum Outlay, r = -1	34.7	20.0	80.0	0.3	0.28	0.21	3.20
1.5	Zero Liability	23.6	20.0	58.2	4.8	0.07	0.00	2.50
1.75	Zero Liability	22.1	20.0	54.6	3.7	-0.08	-0.14	2.00
2	Zero Liability	20.7	20.0	51.3	3.1	-0.20	-0.25	1.62

Notes: Column 1 shows the percentage increase in welfare generated by the optimal policy relative to the welfare generated by the private market only. Column 2 shows the percentage of projects implemented by the private market. Column 3 shows the percentage of additional projects implemented by the optimal policy. Columns 4 to7 show the difference between the welfare generated by the policy and that generated by the private market divided by the cost of the policy. Self financing b in the full grant and zero interest loan is set to $\bar{b}$. In column (7), the constrained optimal subsidy rate is set to 100% because the unconstrained optimal rate is above 100% (between 119% and 178%). The unconstrained optimal subsidy rate is found by maximising expected social welfare at the value of σ implied by the mean ratio of social to private rates of return estimated by BVS (2013), for each value of λ. In this table the VC enhacement paramter β is set to 1.24 but the parameters of the Beta distribution are left unchanged.

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