Debt Enforcement Procedures and the Amplification Role of Collateral Constraints*

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Abstract

This paper examine the role of credit market inefficiencies for the amplification mechanism generated by collateral constraints. To this purpose we use a model with borrowing limits *a la* Kiyotaki and Moore (1997) and inelastic capital supply. We complement previous literature by drawing theoretical considerations on the relation between the degree of debt enforcement efficiency in the credit market and the amplification of productivity shocks.

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1 Introduction

Standard Real Business Cycle theories succeed in accounting for business cycle observations of aggregate quantities relying mainly on large and persistent aggregate productivity shocks. Kiyotaki and Moore (1997) and Kiyotaki (1998) show that if debt needs to be fully secured by collateral, even small and temporary shocks can have large and persistent effects on economic activity. They documented that the credit system may act as a powerful propagation mechanism by which small shocks propagate into the economy. Kiyotaki and Moore's work has been very influential and an increasing number of papers has documented the role of collateralized debt for business fluctuations. Among others, on the role of the housing market and collateralized debt for the transmission and amplification of shocks, Iacoviello (2005) and Iacoviello and Neri (2007), the international transmission of business cycles, Iacoviello and Minetti (2007), the macroeconomic implications of mortgage market deregulation, Campell and Hercowitz (2005), and on overborrowing Uribe (2007). A common assumption in this strand of the business cycle literature is a certain degree of debt enforcement inefficiency in the credit market that limits the debt of the agents to a fraction of the value of their collateral.

Collateralized debt is becoming a popular feature of business cycle models, despite the fact that Kocherlakota (2000) and Cordoba and Ripoll (2004), demonstrated that collateral constraints *per se* are unable to propagate and amplify exogenous shocks, unless unorthodox assumptions on preferences and production technologies are assumed. Papers on the amplification role of collateral constraints neglect the role of inefficiencies in the liquidation of the collateralized asset. As documented by Djankov, Hart, McLiesh and Shleifer (2006), debt enforcement procedures around the world are significantly inefficient. They study debt enforcement with respect to an insolvent firm, documenting the time to resolve the insolvency, the cost to complete the insolvency proceeding and the computed degree of efficiency of the debt enforcement in 88 countries.[1] As a result, all procedures are extremely time consuming, costly and inefficient and the degree of inefficiency varies enormously among countries. According to their findings worldwide an average of 48% of the firm's value is lost in debt enforcement. Table 1 summarizes their results.[2]

This paper aims to reconcile the two strands of the literature by exploring the role of costly debt enforcement procedures in the amplification of productivity shocks. We limit our analysis to the class of models with borrowing limits *a la* Kiyotaki and Moore (1997) and inelastic capital supply. We document that the magnitude of amplification depends substantially on the degree of debt enforcement efficiency assumed in the credit market. As a result, for realistic degrees of efficiency in the debt enforcement procedures, collateral constraints can significantly amplify the effects of productivity shocks on output even under standard assumptions on preferences and technology.

The key insight is that the degree of inefficiency in the debt enforcement procedure affects the sensitivity of individual investment dynamics trough two major factors: the *production share* of constrained agents and the *productivity gap* between the two groups of agents. A more efficient credit market reduces the gap in terms of productivity between borrowers, that are limited in their capital holding, and lenders. On the contrary, it increases the share of total production produced by borrowers. Borrowers investment decisions and total output, are thus determined by this two opposite forces. As a result, the relation between the amplification of productivity shock to output and the degree of efficiency in the debt market displays an inverted-U shape. The model features negligible amplification in only in two particular parametrizations of the model: autarky and close to fully efficient debt enforcement procedures.

[1] Efficiency is defined as the present value of the terminal value of the firm after bankruptcy costs. It's calculated taking into account both the cost and the time to resolve insolvency.

[2] In the left column we report the measure of efficiency by income group. The right column shows data on debt enforcement for a subsample OECD countries by legal origins.

The paper is organized as follows. Section 2 presents the benchmark model and section 3 studies the model's dynamics. Section 4 discusses how the model's endogenous amplification relates to measures of efficiency of the debt enforcement procedure for the U.S. Section 5 present some extensions of the model. Section 6 draws some conclusions.

2 Benchmark Model

Model 1. Following Kiyotaki and Moore (1997) we consider a discrete time economy populated by two types of agents that trade two kinds of goods: a durable asset and a non durable commodity. The durable asset (k) does not depreciate and has a fixed supply normalized to one. The commodity good (c) is produced with the durable asset and cannot be stored. At time t there are two competitive markets in the economy: the asset market in which one unit of durable asset can be exchanged for q_t units of consumption good, and the credit market. The economy is populated by a continuum of ex-ante heterogeneous agents of unit mass: n_1 *Patient Entrepreneurs* (denoted by 1) and n_2 *Impatient Entrepreneurs* (denoted by 2). In order to impose the existence of flows of credit in this economy we assume ex-ante heterogeneity based on different subjective discount factor: $\beta_2 < \beta_1 < 1$. This assumption ensures that in equilibrium patient households lend and impatient households borrow. Both agents produce the commodity good using the same technology:

$$y_{it} = Z_t k_{it-1}^{\alpha} \tag{1}$$

where Z_t represents an aggregate technology shock. we assume that agents have access to the same concave production technology: $\alpha_1 = \alpha_2 < 1$. However, following previous literature, technology is specific to each producer and only the household that started the production has the skills necessary to conclude the production. Nevertheless, agents cannot precommit to produce. This means that if household i decides to not put his effort in the production between t and t+1 there would be no outcome of production at t+1, but only the asset k_{it}. Agent are also free to walk away from the production and the debt contracts between t and t+1. This results in a default problem that makes creditors willing to protect themselves by collateralizing the borrower's asset. Creditors know that in case the borrower runs away from production and debt obligations, they can still get his asset. However, we assume that the lenders can repossess the borrower's assets only after paying a proportional transaction cost, $[(1-\gamma)E_t q_{t+1} k_{it}]$. Thus, agents cannot borrow more than a certain amount such that what they have to reimburse in the next period cannot exceed the expected value of next period assets:

$$b_{it} \leq \gamma E_t \left[q_{t+1} k_{it}\right] \tag{2}$$

where $(1-\gamma)$ is the cost lenders must pay to repossess an asset. The lower the γ the more costly, and thus inefficient, the debt enforcement procedure. Agents face the following problem:

$$\max_{\{c_{it}, k_{it}, b_{it}\}} E_0 \sum_{t=0}^{\infty} \left(\beta_i\right)^t U\left(c_{it}\right) \qquad i = 1, 2$$

$$s.t.$$

$$c_{it} + q_t(k_{it} - k_{it-1}) = y_{it} + \tfrac{b_{it}}{R_t} - b_{it-1}$$

$$y_{it} = Z_t k_{it-1}^{\alpha}$$

$$b_{it} \leq \gamma E_t \left[q_{t+1} k_{it}\right]$$

where k_{it} is a durable asset, c_{it} a consumption good, and b_{it} the debt level. Agents' optimal choices of bonds and capital are characterized by:

$$\frac{U_{c_{i,t}}}{R_t} \geq \beta_i E_t U_{c_{i,t+1}} \tag{3}$$

and

$$q_t - \beta_i E_t \frac{U_{c_{i,t+1}}}{U_{c_{i,t}}} q_{t+1} \geq \beta_i E_t \frac{U_{c_{i,t+1}}}{U_{c_{i,t}}} \left(F_{k_i,t+1}\right) \tag{4}$$

where $U(c_{it}) = \dfrac{c_{it}^{1-\sigma}}{1-\sigma}$ and $F_{k_i,t} = \alpha Z_t k_{it-1}^{\alpha-1}$ is the marginal product of capital. The first equation relates the marginal benefit of borrowing to its marginal cost. For constrained agents the marginal benefit is always bigger than the marginal cost of borrowing. If $\mu_{i,t} \geq 0$ is the multiplier associated with the borrowing constraint, then, the euler equation becomes:

$$\frac{U_{c_{i,t}}}{R_t} - \mu_{i,t} = \beta_i E_t U_{c_{i,t+1}}$$

The second equation states that the opportunity cost of holding one unit of capital, $\left[q_t - \beta_i E_t \frac{U_{c_{i,t+1}}}{U_{c_{i,t}}} q_{t+1}\right]$, is bigger or equal to the expected discounted marginal product of capital. For constrained agents the marginal benefit of holding one unit of capital is given not only by its marginal product but also by the marginal benefit of being allowed to borrow more:

$$q_t - \beta_2 E_t \frac{U_{c2,t+1}}{U_{c2,t}} q_{t+1} = \beta_2 E_t \frac{U_{c2,t+1}}{U_{c2,t}} \left(F_{k2,t+1}\right) + \gamma E_t q_{t+1} \frac{\mu_t}{U_{c2,t}} \qquad (3.a)$$

Collateral constraints alter the future revenue from an additional unit of capital for the borrowers. Holding an extra unit of capital relaxes the credit constraint and thus, increases their shadow price of capital. This additional return encourages borrowers to accumulate capital even though they discount their revenues more heavily than lenders.

In the deterministic steady state the group of impatient households is credit constrained. Consider the euler equation of the impatient household:

$$\frac{u_{c2,t}}{R_t} - \mu_{2,t} = \beta_2 E_t u_{c2,t+1}$$

in steady state it implies:

$$\mu_2 = \left(\frac{1}{R} - \beta_2\right) u_{c_2}$$

Since the steady state interest rate is determined by the discount factor of the patient agent:

$$\mu_2 = \left(\frac{1}{R} - \beta_2\right) u_{c_2} = \left(\beta_1 - \beta_2\right) u_{c_2}$$

As long as $\beta_2 < \beta_1 < 1$, the lagrange multiplier associated with borrowing constraint for the impatient household is strictly positive in the deterministic steady state.[3] Following previous literature, we analyze the properties of the model in a neighborhood of the steady state, in which impatient households borrow up to the maximum.[4]

$$b_{2,t} = \gamma E_t \left[q_{t+1} k_{2t} \right]$$

and

$$k_{2t} = \frac{W_{2,t} - c_{2,t}}{\left[q_t - \gamma E_t \frac{q_{t+1}}{R_t} \right]}$$

where $W_{2,t} = y_{2,t} + q_t k_{2,t} - b_{2,t-1}$, is the impatient agent's wealth at the beginning of time t and $d_t = \left[q_t - \gamma E_t \frac{q_{t+1}}{R_t} \right]$, represents the difference between the price of capital and the amount he can borrow against a unit of capital, i.e. the downpayment required to buy a unit of capital. Creditors' capital decision is determined at the point in which the opportunity cost of holding capital equals its marginal product:

$$q_t - \beta_1 E_t \frac{U_{c_{1,t+1}}}{U_{c_{1,t}}} q_{t+1} = \beta_1 E_t \frac{U_{c_{1,t+1}}}{U_{c_{1,t}}} \left(F_{k_1,t+1} \right) \tag{3.b}$$

The total stock of capital k_t is given by:

$$k_t = (1-n) k_{1t} + n k_{2t} \tag{5}$$

3 Results

A look to the steady state. In what follows it's analyzed how the deterministic steady state of the model is affected by γ. As a benchmark case we

[3] In fact, given the euler equation of the patient households:

$$\frac{U_{c_{1,t}}}{R_t} = \beta_1 E_t U_{c_{1,t+1}}$$

in a deterministic steady state:

$$R = \frac{1}{\beta_1}$$

[4] In order to limit concern on the occassionally binding nature of the borrowing constraint, we base our analysis on the effects of negative productivity shocks. We condition on the initial state of the economy being the deterministic steady state and assume that the economy is hit by an unexpected shock. Thus, the lagrange multiplier associated with the collateral constraints is positive. As a result, the borrowing contraint always binds.

set patient households' discount factor equal to 0.99, and β_2 equal to $0.9 * \beta_1$. The share of capital in the production α is 0.4, the fraction of borrowing constrained population, n_2, is set to 50% and $\sigma = 2.2$. Figure 1.a shows how the marginal productivity, and thus efficiency in production, depends on γ. *Ceteris paribus* a higher γ reduces the difference between borrowers' and lenders' marginal productivity. Since in the deterministic steady state the group of impatient households is credit constrained, their capital holding is less than the level that maximizes total output.[5] Using the equations representing the households' optimal choice of capital evaluated at the steady state it is possible to show that as long as $\gamma < \frac{1}{\beta_1} = 1.0101$

$$\frac{F_{k_2}}{F_{k_1}} = \frac{\beta_1 \left[1 - \beta_2 - \gamma(\beta_1 - \beta_2)\right]}{(1 - \beta_1)\, \beta_2} > 1 \tag{6}$$

Where $F_{k_i} = \alpha \left(\frac{K_i}{n_i}\right)^{\alpha - 1}$. The steady state allocation of capital depends on the subjective discount factors, the fraction of the two groups of agents and the degree of credit market development:

$$K_2 = \frac{1}{\left\{1 + \frac{n_1}{n_2}\left[\frac{\beta_2(1-\beta_1)}{\beta_1[1-\beta_2-\gamma(\beta_1-\beta_2)]}\right]^{\frac{1}{\alpha-1}}\right\}} \tag{7}$$

Compared to the first best allocation, the allocation under credit constraints reduces the level of capital held by the borrowers and it implies a difference in the marginal productivity of the two groups. Thus, even if it is not possible

[5] The efficient allocation of capital between the two groups would be given by the equality between the marginal products of the two groups:

$$F_{k_1,t} = F_{k_2,t}$$

Thus, given the aggregate condition on capital

$$n_1 k_1 + n_2 k_2 = K_1 + K_2 = 1$$

then, since the total population is normalized to be equal to the unit interval

$$K_2^{eff} = n_2 \qquad and \qquad K_1^{eff} = 1 - n_2$$

This means that if the two groups are equally large, each group gets the same amount of capital in steady state.

to reach the efficient equilibrium ($F_{k_1,t} = F_{k_2,t}$) it is possible to reduce the efficiency loss by setting γ closer to 1. Figure 1.b shows that an increased access to the credit market implies a credit expansion and thus a rise in the level of investment by borrowers. This leads to a more efficient allocation of capital between the two groups of agents and consequently to an increase in total production. As a result, in the deterministic steady states associated to higher levels of γ, the level of total output, and thus total consumption, is higher. The price of the collateral asset is also higher.[6]

Impulse Responses. we now consider the response of the model economy to a technology shock when γ=1. Aggregate production follows an AR(1) process given by

$$\ln(Z_t) = \rho_Z \ln(Z_{t-1}) + \varepsilon_{Zt},\ \varepsilon_{Zt} \backsim^{iid} N(0, \sigma_\epsilon)$$

with $\rho_Z = 0.9$. we assume that the economy is at the steady state level at time zero and then is hit by an unexpected decrease in aggregate productivity of 1%. The results are reported in figures 2.a. An aggregate negative shock reduces production and thus the earnings of both groups of agents. Since the shock is temporary borrowers sell a part of their resources to smooth consumption. In order for the capital market to clear, lenders have to increase their demand for capital and thus the user cost of holding capital has to decrease. Movements in the relative price of capital, altering the value of the collateral asset, affect the ability to borrow and in turn investment and expenditure decisions (*collateral effect*). Thus, constrained agents are negatively affected not only by the direct impact of the shock but also by the reduced availability of credit resulted by a reduction in the price of the collateral. The reduction in borrowers' current investment expenditures propagates the effect of the shock on total production over time due to their

[6] In the steady state the asset prices depend on the marginal productivity of capital. More specifically, the households' optimal choice of capital gives

$$q = \frac{\beta_1}{1-\beta_1} F_{k_1}$$

higher marginal productivity of capital.

Figure 2.b shows the response of total output. In accordance with Cordoba and Ripoll (2004), when γ=1 the amplification generated by the model is negligible.

Amplification and Persistence. How does the efficiency in the debt enforcement procedure affect the amplification of shocks to output? Figure 3.a plots the percentage deviation of output attributed to the endogenous propagation mechanism of the model.[7] The sensitivity of output to productivity shocks varies in a non-linear way with respect to the degree of credit frictions and it is between 5% and 30% stronger than the one obtained with a fully efficient debt enforcement procedure (γ=1).

The elasticity of total output to technology shocks can be written as:[8]

$$\epsilon_{yz} = \epsilon_{yk_2}\epsilon_{k_2z} = \frac{F_{k_2} - F_{k_1}}{F_{k_2}}\alpha\frac{y_2}{y}\epsilon_{k_2z} \tag{8}$$

The first term is the productivity gap between constrained and unconstrained agents, α represents the share of collateral in production while $\frac{y_2}{y}$ is the production share of constrained agents and ϵ_{k_2z} is the elasticity of borrowers' capital to the shock (i.e. the redistribution of capital towards impatient agents). In order to explain the non linear relationship between output's amplification and the degree of credit friction let's first focus on the behavior of the redistribution of capital in the model (ϵ_{k_2z}). The right panel of figure 3.b shows the first period intensity of the reaction of investment decisions by constrained agents. The impact of the shock on capital expenditure displays an inverted U relationship with the degree of access to the credit market. This relationship is explained by two factors that work in opposite directions: *the collateral effect* and *the cost of debt*. An higher γ, induces a less remarkable reduction of borrowing and thus, asset prices. The weaker asset price reaction, in turn, decreases the sensitivity of the borrowers' capital ex-

[7]i.e. the variation that exceed the exogenous impact on output directly implied by the autocorrelation of the shock (0.9%)

[8]Since the first impact of the shock would always be equal to the shock itself, we now look at the second period effect of the shock.

penditure to shocks. On the other hand, the less remarkable decrease in debt leads to a less sizable reduction in the interest rate. This makes borrowing more costly and borrowers' investment is expected to decrease by more. As a result, the intensity of capital response depends on which of the two effects prevails.

Strictly speaking, the reaction of investment decisions and downpayment are symmetrically opposite (figure 3.c).

$$DP_t = \left[q_t - \gamma E_t \frac{q_{t+1}}{R_t} \right] \tag{9}$$

The stronger the effect on downpayment, the weaker the reaction of capital.[9] The shape of the relationship between the degree of access to credit market and the effect on downpayment can be explained by the existence of two opposite forces determining the intensity of reaction of the downpayment. Given convex marginal productivity of capital, the higher γ the weaker the reaction of q_t to the shock. If q_t falls by less also the downpayment required reduces by less. Being more expensive to buy capital, when γ is higher we expect k_{2t} to reduce by more. However, at the same time, R_t reacts by less. This increases the reduction in the downpayment. Thus, the reaction of k_{2t} is expected to be weaker. As a result, the intensity of capital response depends on which of the two opposite effect prevails. This explains the inverted U shape of the amplification on output delivered by the collateral constraint.

Despite the role of these two opposite forces on the determination of borrowers' capital decisions, it is possible to document that the sensitivity of output to z_t maintains an inverted-U shape independently of non linearities in $\epsilon_{k_2 z}$. Assume that lenders' utility function is linear in consumption, so that the interest rate is constant over the business cycle. The highest the level of γ the weaker the effect on the downpayment (since it only depends on q), and thus, the impact of the shock on capital is larger. Still the relationship between γ and the second impact of z_t on y_t has an inverted U shape (figure

[9] The difference between the price of capital and the amount agents can borrow against a unit of capital represent the amount required to buy a unit of capital.

3.d). In steady state the fraction of total output produced by constrained agents increases with γ due to the fact that more capital is held by the constrained population (*production share effect*). However, for the same reason, the *productivity gap* decreases with γ. Thus, regardless the shape of capital reaction to technology shocks, since also the second impact of the shock on total output depends on this two opposite forces it will always display a not linear shape. That is of course more pronounced when $\epsilon_{k_2 z}$ is not monotonic.

Sensitivity to Capital Share in Production. In what follows we illustrate the relation between γ and the intensity of output reaction to productivity shocks for different values of α. Figure 4.b reports that the relation between α and γ is not linear. Differently from Kocherlakota (2000), we document that lower α do not necessarily imply lower amplification of shocks. The result obtained by Kocherlakota depend on the fact that he neglects the role of inefficiency in the debt enforcement procedures and assumes $\gamma = 1$. Let's compare, α=0.4 which corresponds to the standard definition of capital, to a broader definition and include both physical and other intangible capital and set α=0.7.[10] With γ around 0.7 the amplification generated by α=0.4 is around 20% above the amplification generated by α=0.7 (figure 4.c). On the contrary, with γ about 0.95, the amplification generated by α=0.7 is about two times the one obtained with α=0.4 and around 50% stronger that the reaction induced by the variation in productivity itself (figure 4.d). In the model presented here, Kocherlakota's results only hold for values of γ close to unity.

4 Quantitative Results

Results presented above show that for values of γ below unity the model with collateral constraints can generate amplification and persistence of productivity shocks of non-negligible magnitude. However, the relation being hump-shaped the magnitude of amplification varies significantly. In what

[10] See among others Angeletos and Calvet (2006).

follows we investigate the quantitative relevance of the amplification generated by the model when the efficiency in the debt market is set to be equal to the one reported for the US. To conduct a quantitative experiment we set patient households' discount factor equal to 0.99, such that the average annual rate of return is about 4%. The baseline choice for the fraction of borrowing constrained population, n_2, is set to 50% and $\sigma = 2.2$. Regarding the discount factor of borrowers, Lawrance (1991) estimates that the discount factors of poor households are in the 0.95 to 0.98 range, while according to Carroll and Samwick (1997), the empirical distribution of discount factors lies in the 0.91 to 0.99 interval. Thus we compare the results for three different values of β_2 : 0.91, 0.95 and 0.97. The share of capital in the production α is more difficult to pin down. Thus, we report the amplification for two different values of this parameters: α=0.4 which corresponds to the standard definition of capital, and α=0.7 that reflects a broader definition and includes both physical and other intangible capital.[11]

Table 3.a shows that the amplification endogenously generated by the model with a degree of efficiency in the debt market equal to the one reported for the US is quantitatively significant. Not surprising the higher the discount fact of impatient agents the lower the endogenous amplification generated by the model. A lower β_2 means a higher degree of heterogeneity in the model, this implies a wider productivity gap between borrowers and lenders (see eq.6) and thus greater amplification. The role of capital intensity in production in generating amplification is such that for β_2 =0.91 a α equal to 0.4 amplifies the effect of the shock by more. However, when the gap in discount factors reduces, stronger amplification in given by α=0.7.

Depending on the choice of parameters' value, the degree of endogenous amplification generated by the model can be as low as19% (β_2 =0.97, α=0.4) and as high as 39% (β_2 =0.95, α=0.7). In any case, the magnitude of amplification is sizable and significantly higher than the one generated by the version of the model in which inefficiencies in the liquidation of the collateral-

[11] See Angeletos and Calvet (2006).

ized asset are neglected ($\gamma = 1$). As a result, for realistic degrees of efficiency in the debt enforcement procedures, collateral constraints can significantly amplify the effects of productivity shocks on output even under standard assumptions on preferences and technology.

5 Introducing Labor Supply

Model 2. To explore the robustness of the results presented above, we now consider the case in which household work is also an input of production. we assume that each household works in his own firm and gets utility from leisure.[12] Following Greenwood et al. (1988) we assume that the utility function is

$$U(c_{it}, L_{it}) = \frac{1}{1-\sigma}\left(c_{it} - \chi\frac{L_{it}^{\eta}}{\eta}\right)^{1-\sigma} \tag{10}$$

and

$$y_{it} = Z_t k_{it-1}^{\alpha} L_{it}^{1-\alpha} \tag{11}$$

As figure 5.a shows including household work in the model increase both amplification and persistence of productivity shocks to output. Moreover, endogenous amplification of the shocks is already present in the first period. To a 1% decrease in productivity, total output decreases by 1.47%. However, it is possible to show that the first period amplification is independent of γ. Given the household's labor supply

$$\chi L_{it}^{\eta-1} = (1-\alpha) Z_t k_{it}^{\alpha} L_{it}^{-\alpha} \tag{12}$$

it is possible to write each individual productions only in terms of the capital input

$$y_{it} = Z_t^{\frac{\eta}{\alpha+\eta-1}} k_{it-1}^{\alpha + \frac{(1-\alpha)\alpha}{\alpha+\eta-1}} \frac{1-\alpha}{\chi}$$

When productivity decreases by 1%, output decreases by $\frac{\eta}{\alpha+\eta-1}\% = 1.47\%$. As figure 5.b shows, different values of γ imply different magnitude of the

[12] I set the labor supply elasticity to 0.5 (η=2)and the weight on leisure is chosen so that hours worked in in the initial steady state is around 1/3 depending on the given γ (χ=1).

endogenous amplification of output to the same shock. The second impact still varies with the degree of credit friction. The elasticity of total output to technology shocks can be written as in the previous setup but multiplied by $\frac{\eta}{\alpha+\eta-1}$

$$\epsilon_{yz} = \frac{\eta}{\alpha+\eta-1}\epsilon_{yk_2}\epsilon_{k_2z} \tag{13}$$

In what follows we compare the response of output to a productivity shock in the model with collateral constraints with the response obtained in the standard representative agent model. In this last framework the economy is populated only by patient agents and there are no limits to credit. The output response delivered by the model with collateral constraints can be much stronger and persistent than the response generated by the representative agent model. The reaction of output to a productivity shock is between 50% and 130% higher than the variation directly induced by the shock. So, despite the non linearity featured by the model, a degree of amplification significantly higher than the one generated by the representative agent model is displayed by the model.

Model 3. In order to take into account the implications for amplification of the wealth effects on labor supply the following utility function is assumed[13]:

$$U(c_{it}, L_{it}) = \frac{1}{1-\sigma}\left(c_{it}^{(1-\varphi)}(1-L_{it})^{\varphi}\right)^{1-\sigma} \tag{14}$$

As in Cordoba and Ripoll (2004) introducing labor supply according to a standard utility function of this type, is detrimental for the amplification of shocks. However the result holds for a very restricted range of the debt enforcement procedure parameter. Figure 5.c shows that for any value of $\gamma < 0.98$ the magnitude of the second period amplification is bigger than the one reproduced by the equivalent representative agent model. While, incorporating collateral constraints generates amplification and persistence of shocks for a wide range of γ. Overall, output responds between 5% and 25% more than the reaction directly induced by the variation in productivity.

[13] I set $\varphi = 0.6$ so that hours worked in in the initial steady state are around 1/3.

Quantitative Results. Table 3 presents the results. First we document that, if the model doesn't take into account differences in terms of efficiency of the debt enforcement procedures among countries (or group of countries) it predicts either negligible amplification of productivity shocks to output (model 1) or even a detrimental effects of collateral constraints on output reaction (model 3). Only exception the case in which wealth effects of labor supply are ignored (model2). Then we set the efficiency in the debt market to be equal to the one reported for the US. The three models display a significant amplification of the shock on output. Model 1 report an amplification of 33%, model 3 of about 30% and an exceptional high amplification is generated by model 2. Similar results are obtained if we use the degree of inefficiency reported for OECD economies by legal origins. Setting γ equal to the average level of efficiency of each group implies a reaction of output of between 15% and 48% higher than the one obtained by the representative agent model. Thus, if realistic values of the liquidation cost are assumed ($\gamma < 1$) the model significantly improves the role of collateral constraints in terms of amplification of productivity shocks.

6 Conclusion

The aim of this paper is to quantify the amplification generated by collateral constraints in relation to the degree of frictions in the credit market. To this purpose we analyze a stylized business cycle version of Kyiotaki and Moore (1997) model. We document that taking into account the existence of costly debt enforcement procedures – and thus calibrating the degree of debt enforcement efficiency as in the data– makes the model with collateral constraints generate significantly amplification of productivity shocks to output. Previous literature ignoring the possibility of inefficiencies in the debt enforcement procedures, neglects a relevant source of amplification for this class of models and thus minimize the role of collateral constraints in the amplification of productivity shocks to output.

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Appendix .1 Benchmark Model: Equilibrium Conditions

The system of non-linear equations is given by 4 first order conditions

$$\frac{U_{c_{1,t}}}{R_t} = \beta_1 E_t U_{c_{1,t+1}} \tag{E.1}$$

$$\frac{U_{c_{2,t}}}{R_t} - \mu_{2,t} = \beta_2 E_t U_{c_{2,t+1}} \tag{E.2}$$

$$q_t - \beta_1 E_t \frac{U_{c_{1,t+1}}}{U_{c_{1,t}}} q_{t+1} = \beta_1 E_t \frac{U_{c_{1,t+1}}}{U_{c_{1,t}}} F_{k_1,t+1} \tag{E.3}$$

$$q_t - \beta_2 E_t \frac{U_{c_{2,t+1}}}{U_{c_{2,t}}} q_{t+1} = \beta_2 E_t \frac{U_{c_{2,t+1}}}{U_{c_{2,t}}} F_{k_2,t+1} + \gamma E_t q_{t+1} \frac{\mu_{2t}}{U_{c_{2,t}}} \tag{E.4}$$

4 aggregate conditions

$$n_1 k_{1t} + n_2 k_{2t} = K_{1t} + K_{2t} = 1 \tag{E.5}$$

$$y_t = n_1 y_{1t} + n_2 y_{2t} \tag{E.6}$$

$$n_1 b_{1t} + n_2 b_{2t} = 0 \tag{E.7}$$

1 budget constraint[14]

$$c_{2t} + q_t(k_{2t} - k_{2t-1}) = y_{2t} + \frac{b_{2t}}{R_t} - b_{2t-1} \tag{E.8}$$

1 borrowing constraint

$$b_{2,t} = \gamma E_t \left[q_{t+1} k_{2t} \right] \tag{E.9}$$

the resource constraint

$$y_t = n_1 c_{1t} + n_2 c_{2t} \tag{E.10}$$

the two technologies:

$$y_{1t} = Z_t k_{1t-1}^{\alpha} \qquad y_{2t} = Z_t k_{2t-1}^{\alpha} \tag{E.11}$$

12 equations and 12 unknowns:$\{\mu_{2t}, q_t, R_t, y_t\}$ and $\{c_{it}, k_{it}, b_{it}, y_{it}\}_{t=0}^{\infty}$ for i=1,2.

[14]Using the Walras' Law we can drop at each t one of the two budget constraints.

Appendix .2 Benchmark Model: Steady State

From E.1 I find the steady state interest rate:

$$\frac{1}{R} = \beta_1 \tag{ss.1}$$

from E.2 the lagrange multiplier:

$$\mu_2 = (\beta_1 - \beta_2)\, u_{c_2} \tag{ss.2}$$

Using E.3 and E.4:

$$q = \frac{\beta_1}{1-\beta_1} F_{k_1} = \frac{\beta_2}{1-\beta_2-\gamma(\beta_1-\beta_2)} F_{k_2} \tag{ss.3}$$

and substituting for K_1 using the aggregate condition on capital: $K_1 = 1 - K_2$ I find the steady state allocation of capital to the group of borrowers: K_2

$$\frac{\beta_1}{1-\beta_1}\left(\frac{1-K_2}{n_1}\right)^{\alpha-1} = \frac{\beta_2}{1-\beta_2-\gamma(\beta_1-\beta_2)}\left(\frac{K_2}{n_2}\right)^{\alpha-1}$$

Thus:

$$K_2 = \frac{1}{\left\{1 + \frac{n_1}{n_2}\left[\frac{\beta_2(1-\beta_1)}{\beta_1[1-\beta_2-\gamma(\beta_1-\beta_2)]}\right]^{\frac{1}{\alpha-1}}\right\}}$$

Thus I find the steady state borrowing level:

$$b_2 = \gamma\,[qk_2] = -b_1 \tag{ss.4}$$

and the total production:

$$y = n_1 y_1 + n_2 y_2 \tag{ss.5}$$

where

$$y_1 = k_1^{\alpha} \quad y_2 = k_2^{\alpha} \tag{ss.6}$$

From E.8 I find the consumption of the borrowers

$$c_2 = y_2 - b_2\left(1 - \frac{1}{R}\right) \tag{ss.7}$$

and from the resource constraint the consumption of the group of lenders

$$n_1 c_1 = y - n_2 c_2 \tag{ss.8}$$

Table 1: Debt Enforcement around the World			
Around the World		OECD	
income level	efficiency	legal origins	efficiency
high	77.35	*english*	77.0
upper middle	46.11	*french*	69.7
lower middle	35.03	*german*	72.2
Total	51.97	*nordic*	84.9
Source: Djankov,Hart, McLiesh and Shleifer (2006)			

Table 2: Parameter Values			
preferences		**shock process**	
discount rate	$\beta_1 = 0.99$	autocorrelation	$\rho_z = 0.9$
	$\beta_2 = 0.9 * \beta_1$		
	$\sigma = 2.2$	**population**	$n = 0.5$
technology	$\alpha = 0.4$	**borrowing limit**	$\gamma \in [0, 1]$

Table 3.a: Model Results			
γ=85.8			
	β_2=0.91	β_2=0.95	β_2=0.97
α=0.4	0.3046	0.2437	0.1716
α=0.7	0.3033	0.3506	0.3070
γ=1			
	β_2=0.91	β_2=0.95	β_2=0.97
α=0.4	0.0523	0.0263	0.0132
α=0.7	0.1889	0.1018	0.0529
Other parameter values' as in table 2.			

Table 3.b: Model Results			
	output amplification		
	model 1	model 2	model 3
representative agent model	0	0.54	0
γ=1	0.0645	0.8196	-0.0834
γ=85.8	0.3046	1.0584	0.2167
OECD,efficiency by legal origins			
english 77.0	0.2504	0.8348	0.1827
french 69.7	0.2027	0.7192	0.1461
german 72.2	0.2186	0.7525	0.1584
nordic 84.9	0.2980	1.0201	0.2141

model 1: no labor supply
model 2: household labor,utility eq.9
model 3: household labor, utility eq.14
Parameter values' as in table 2.

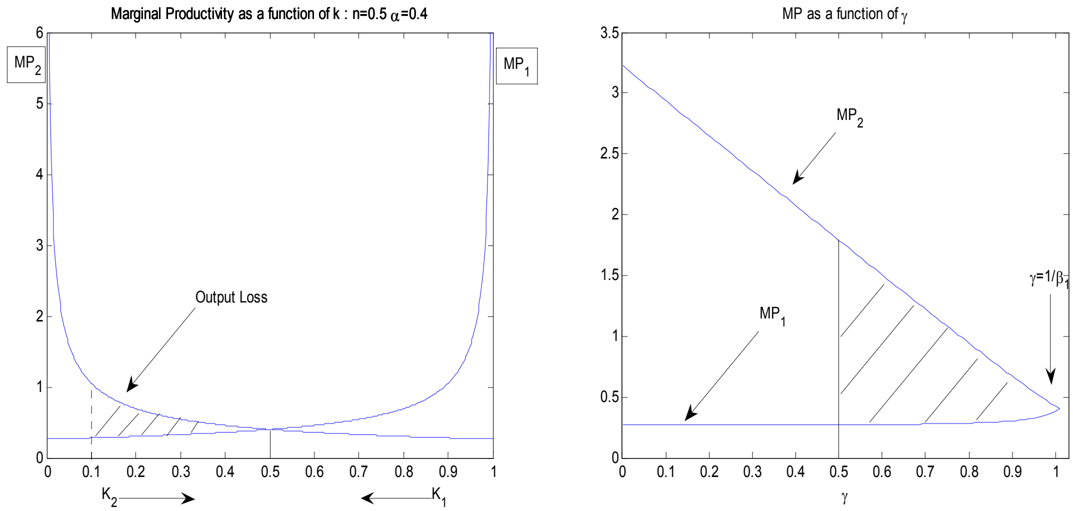

Figure 1.a shows how the steady state productivity gap production between the two groups of agents varies with respect to γ.

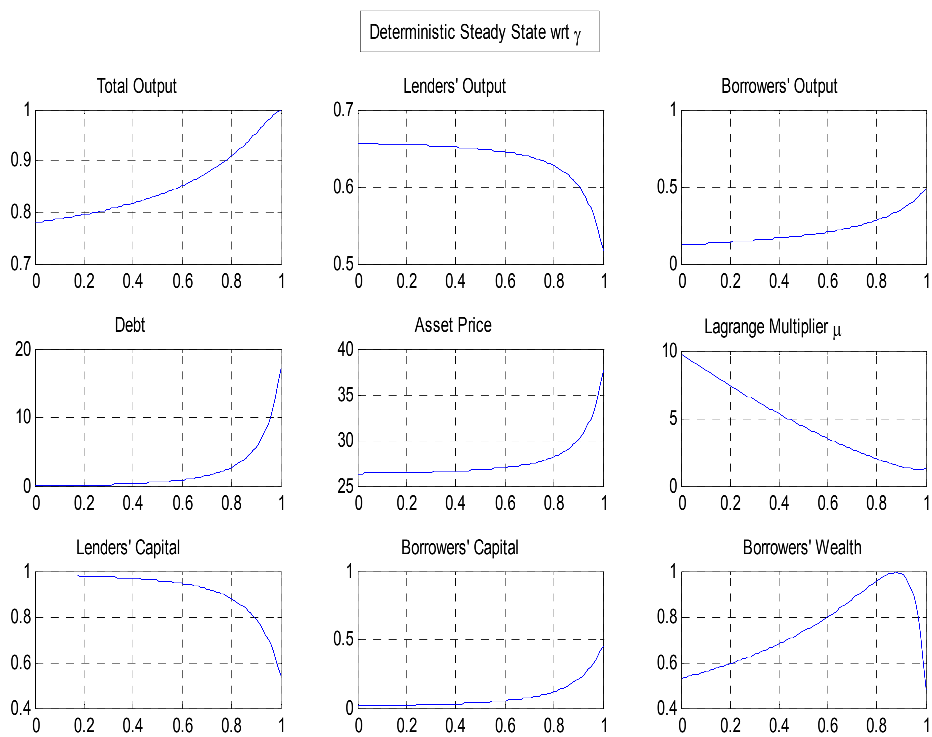

Figure 1.b show how the steady state values of the model's variables change with respect to the degree of credit market development γ.

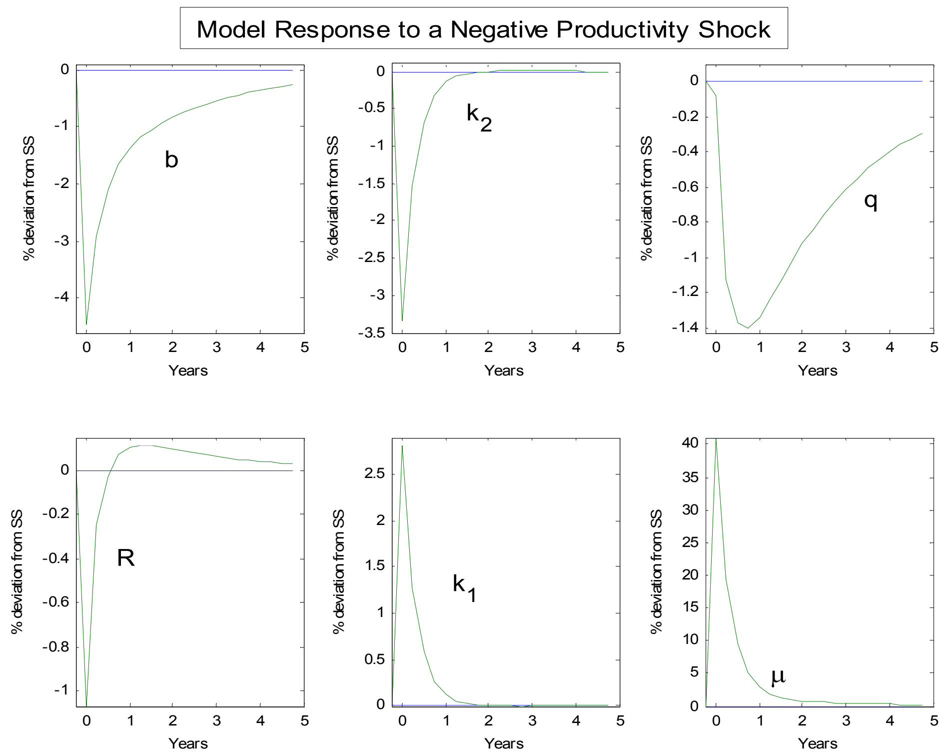

Figure 2.a shows the responses to a 1% decrease in productivity. The units on the vertical axes are percentage deviations from the steady state, while on the horizontal axes are years.

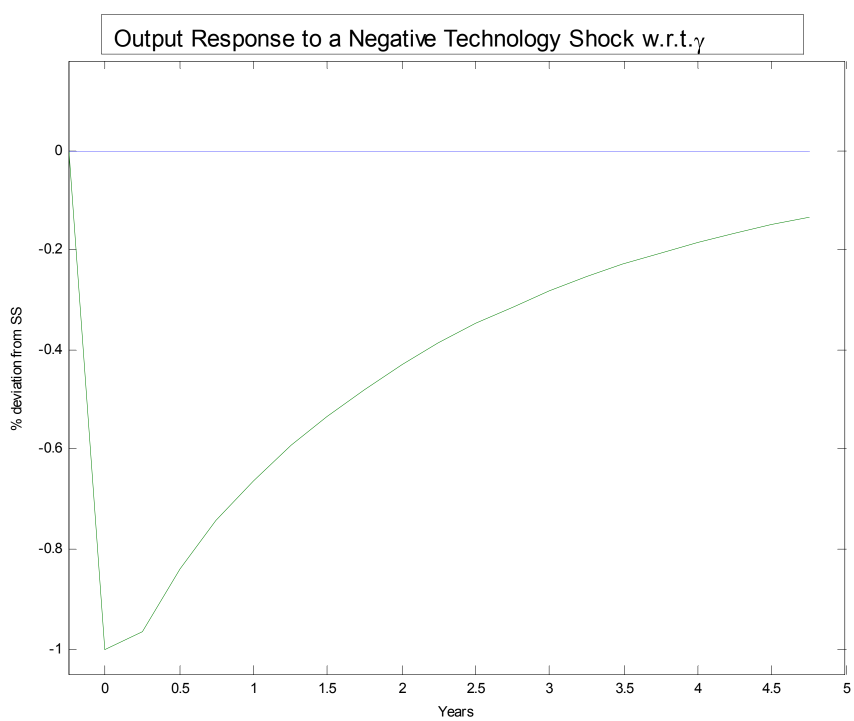

Figure 2.b shows the response of total aggregate output to a 1% decrease in productivity

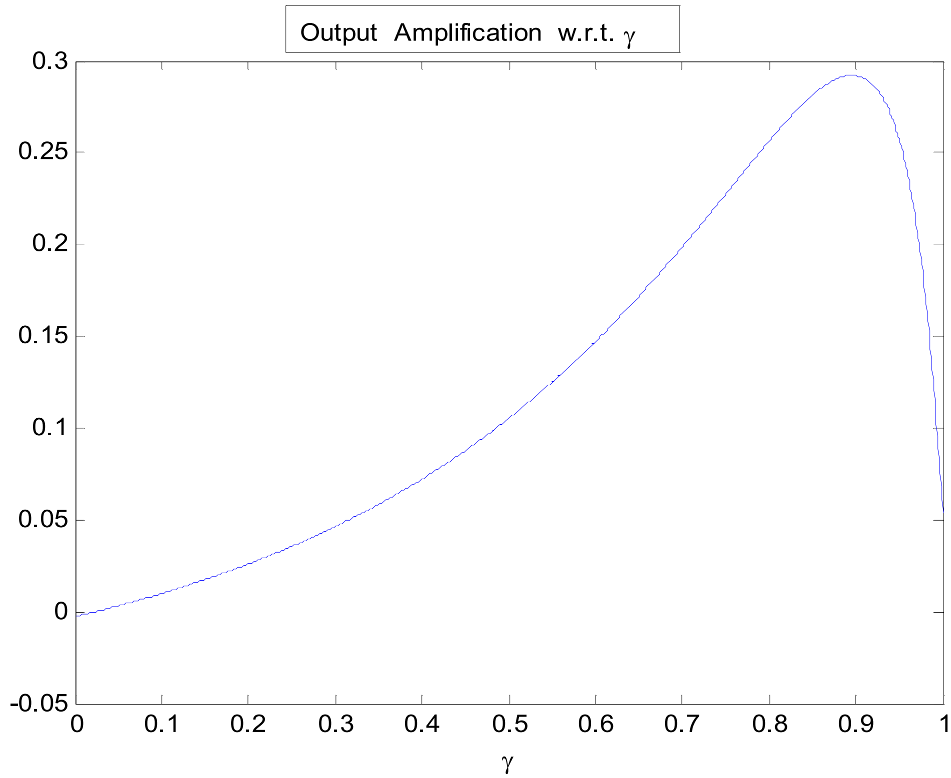

Figure 3.a second period amplification of the shock on production— endogenous reaction to shocks

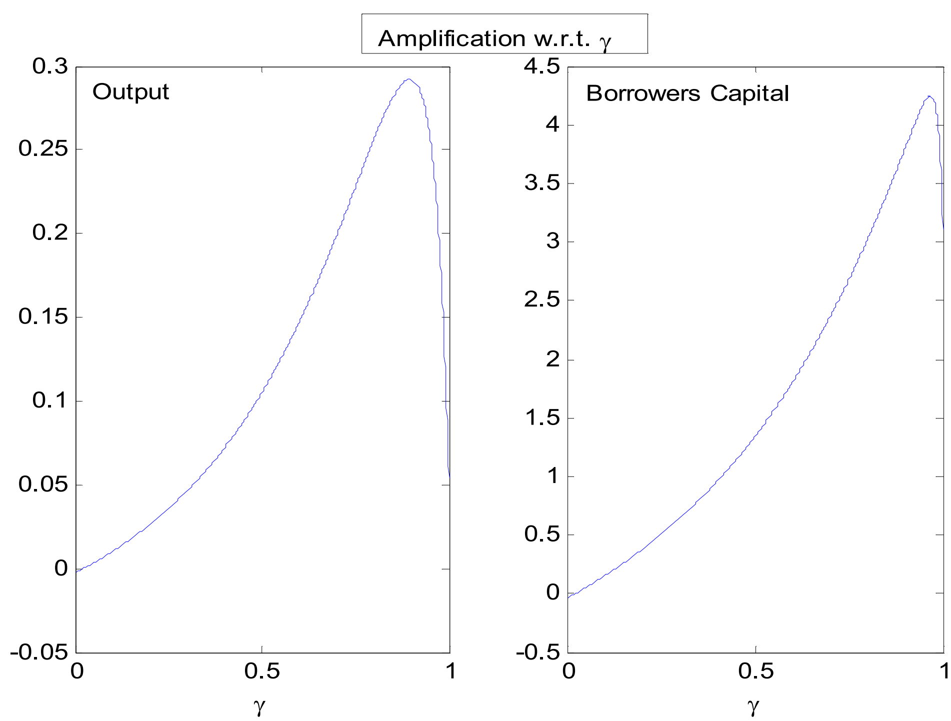

Figure 3.b second period amplification of the shock on production— endogenous reaction to shocks

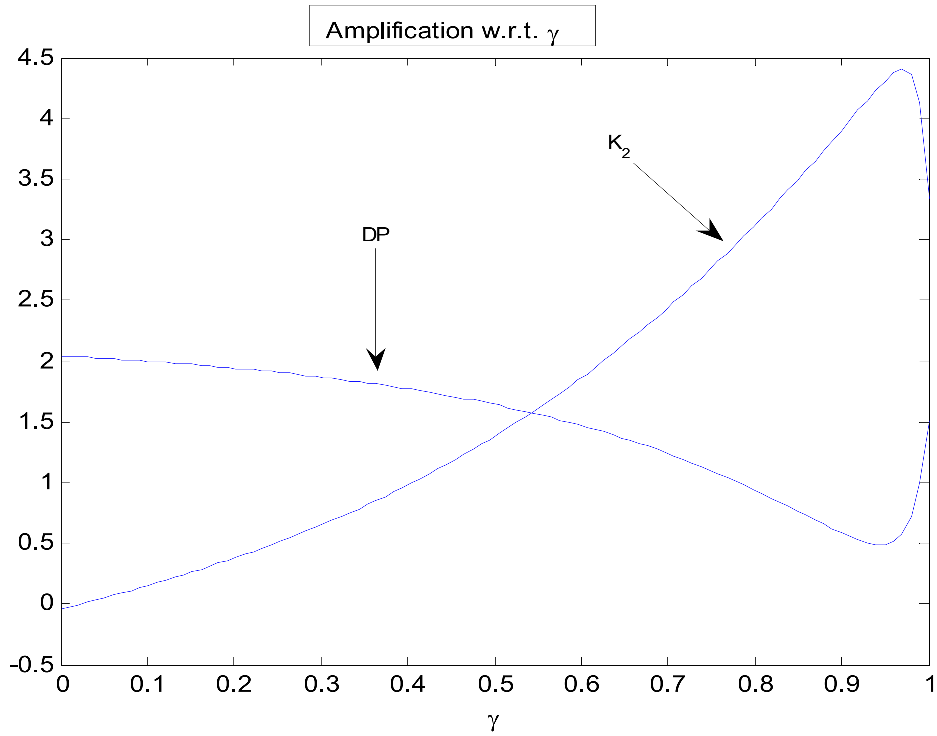

Figure 3.c second period amplification of the shock on production— endogenous reaction to shocks

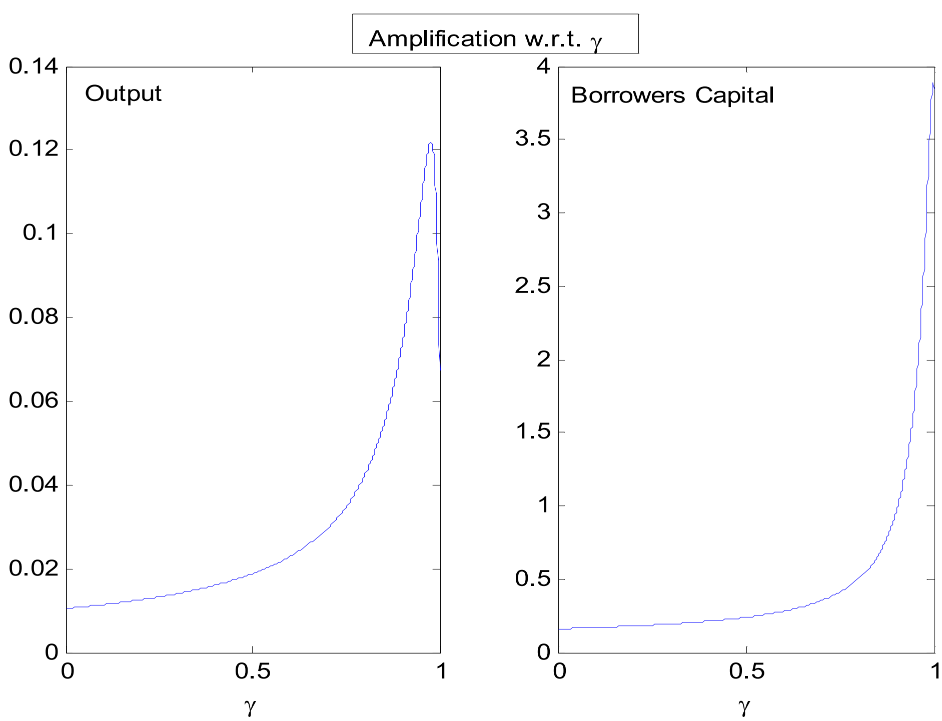

Figure 3.d second period amplification of the shock on production— endogenous reaction to shocks

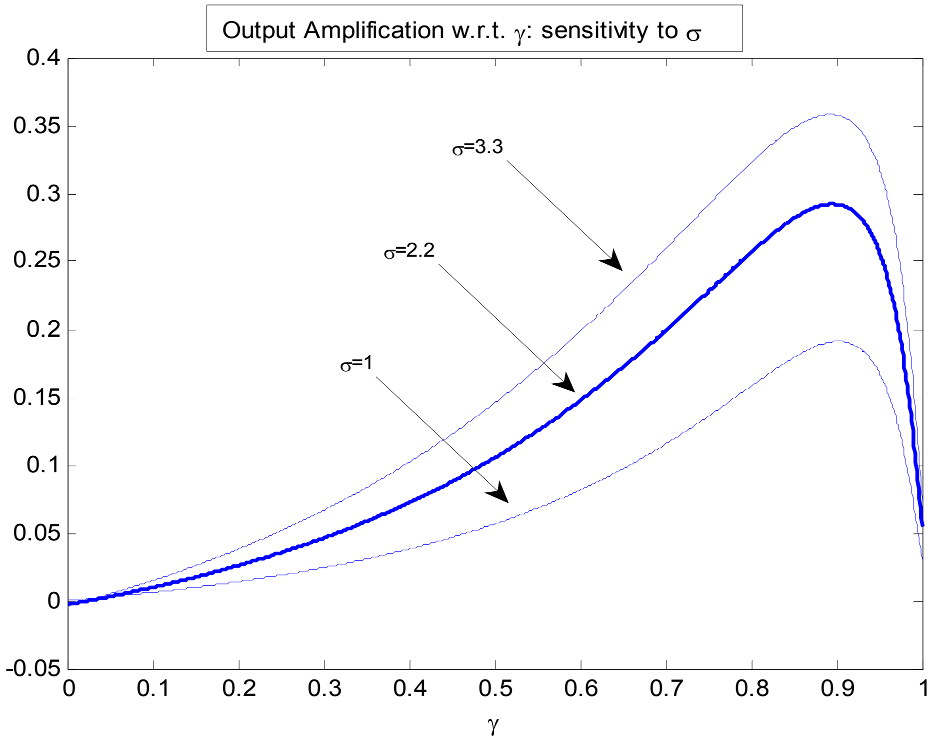

Figure 4.a second period amplification of the shock on production— endogenous reaction to shocks

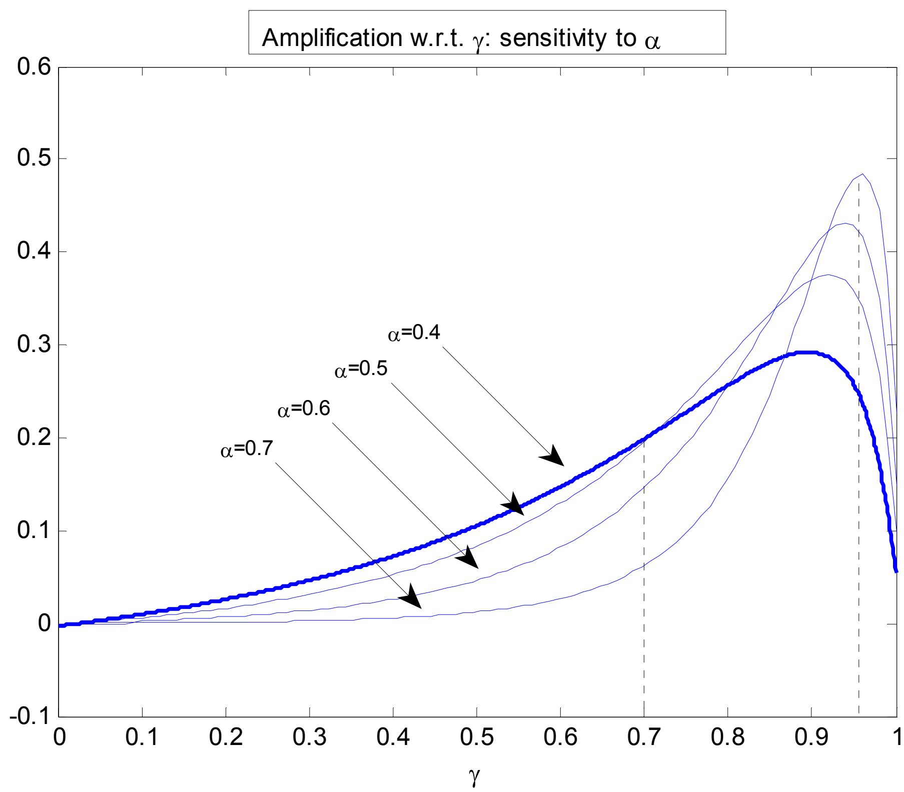

Figure 4.b second period amplification of the shock on production— endogenous reaction to shocks

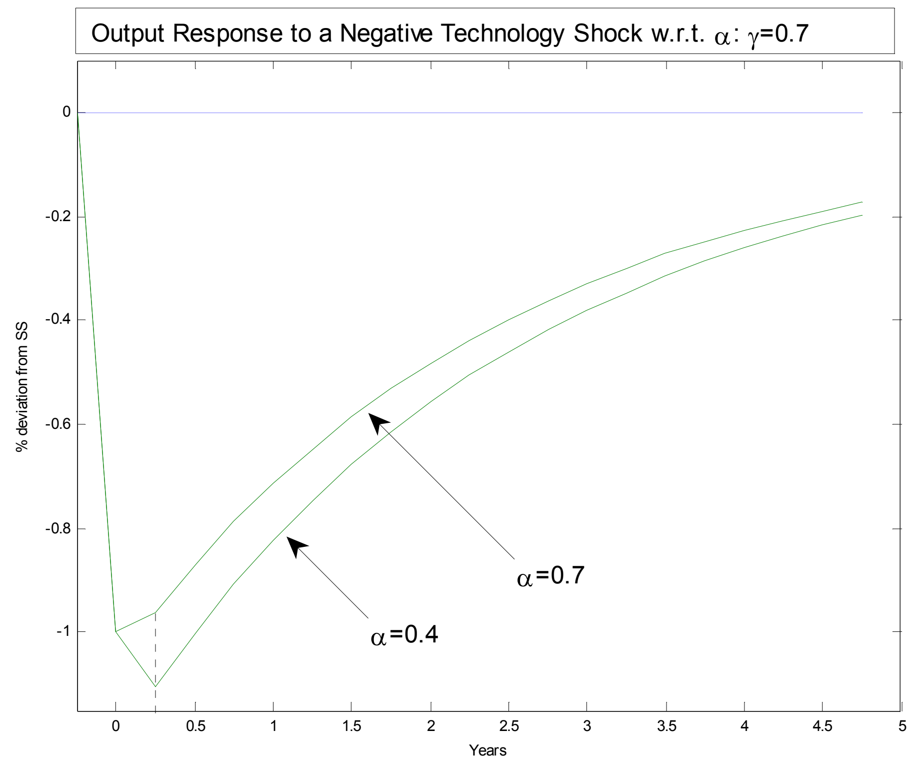

Figure 4.c shows the response of total aggregate output to a 1% decrease in productivity

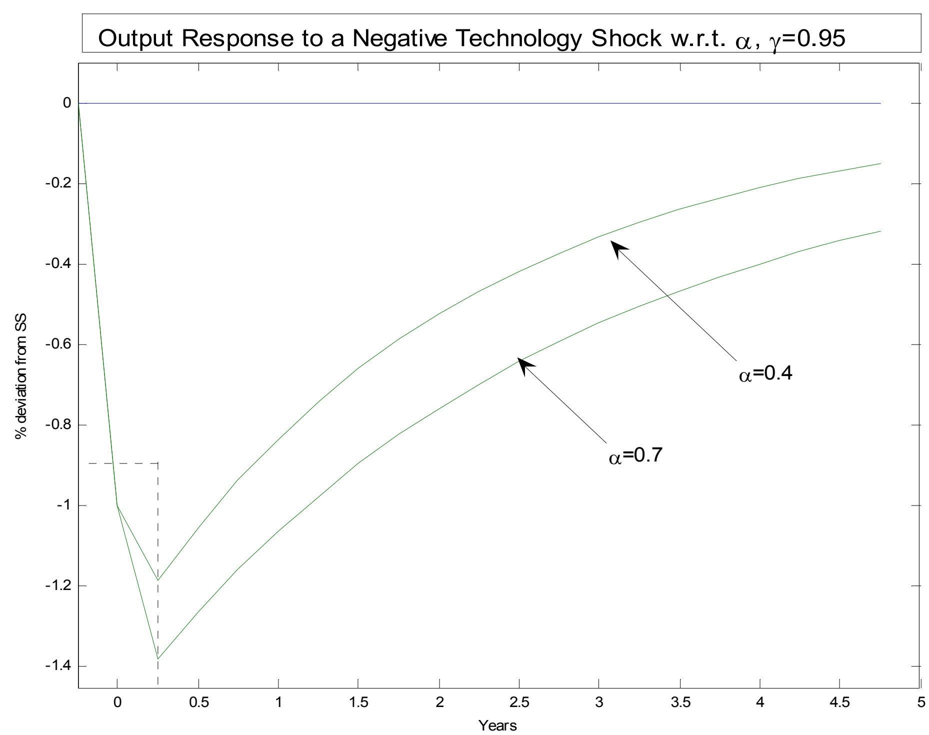

Figure 4.d shows the response of total aggregate output to a 1% decrease in productivity

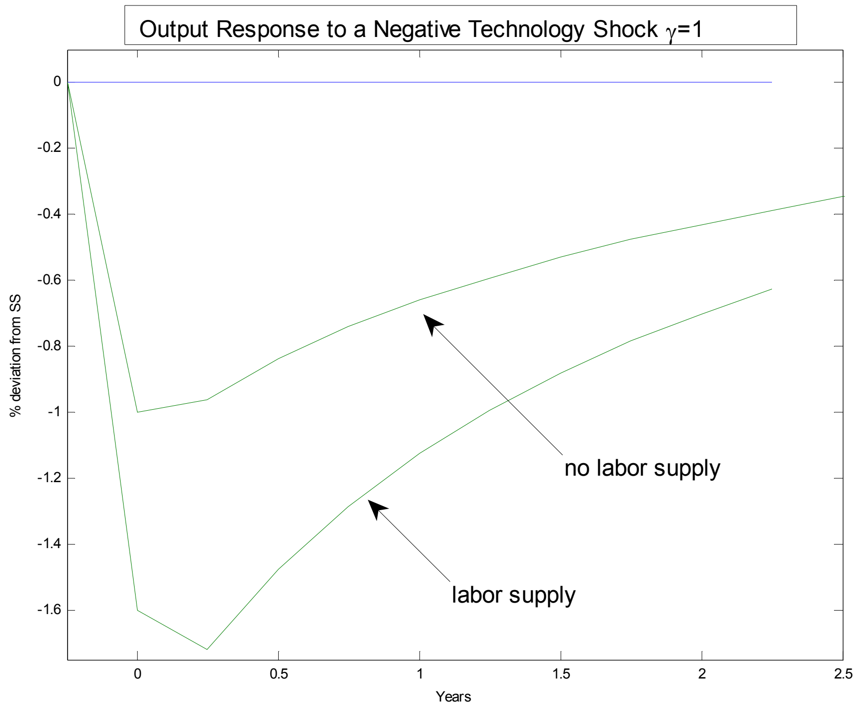

Figure 5.a shows the response of total aggregate output to a 1% decrease in productivity

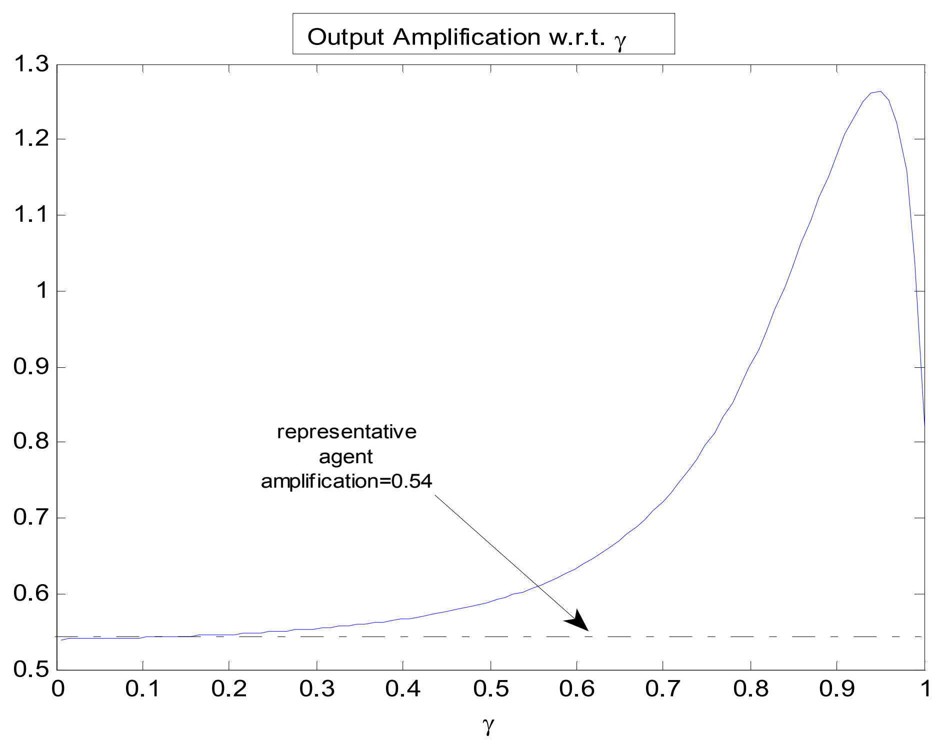

Figure 5.b second period amplification of the shock on production— endogenous reaction to shocks

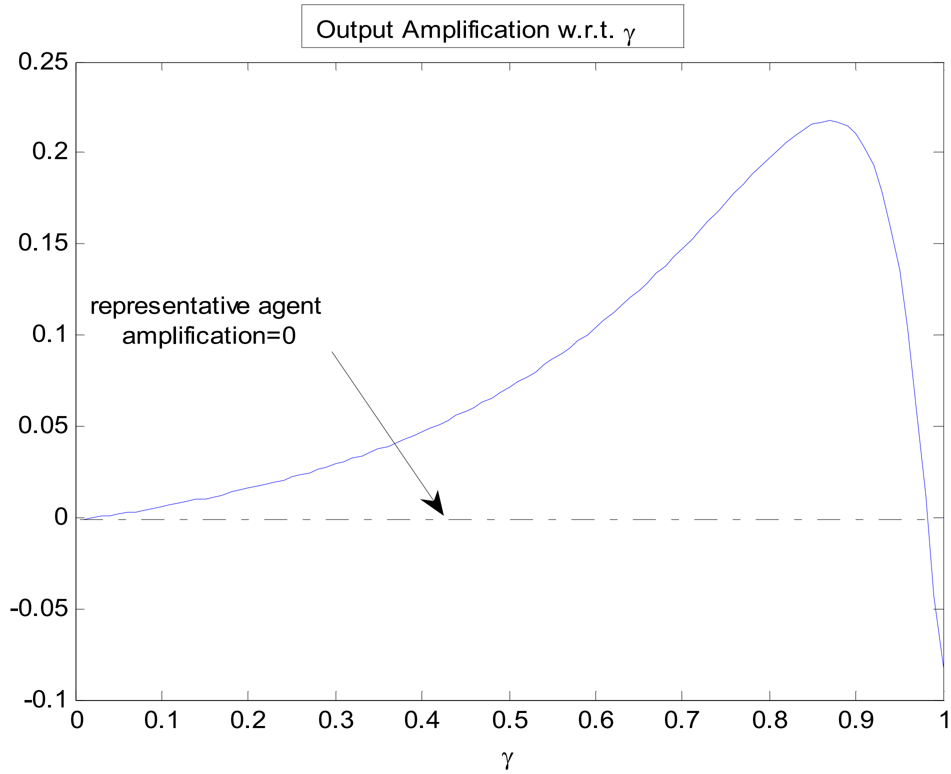

Figure 5.c second period amplification of the shock on production— endogenous reaction to shocks