The Long Run Determinants of the US External Imbalances*

Andrea Ferrero†
Federal Reserve Bank of New York

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Abstract

This paper develops a tractable two-country model with life-cycle structure to investigate the interaction and the relative importance of productivity growth, demographic factors and fiscal policy in accounting for the evolution of the US external imbalances during the last three decades. The results suggest that (i) productivity growth differentials drive the directionality of capital flows in the medium run, (ii) the different evolution of demographic factors across countries explain a large portion of the long run trend, and (iii) fiscal policy plays, at best, a minor role. The main prediction of the analysis is that among industrialized countries capital should generally be expected to flow toward relatively young and rapidly growing economies. The paper also shows that international demographic trends might be an important force behind the recent declining pattern of the world real interest rate and hence sheds some new light on the real side of the interest rate conundrum.

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†Macroeconomic and Monetary Studies Function – 33 Liberty Street, 3rd floor – New York, NY 10045.
E-mail: andrea.ferrero@ny.frb.org – URL: http://nyfedeconomists.org/ferrero/

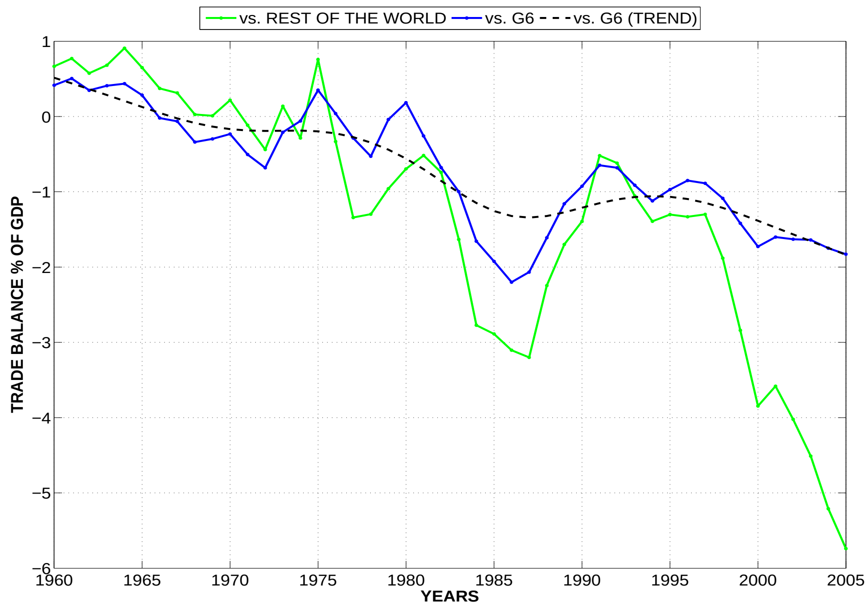

Figure 1: US TRADE BALANCE

1 Introduction and Related Literature

The dynamics of the US external imbalances have recently received a great deal of attention in the economic community. In 2006 the US trade balance reached a record deficit of 758.5 billions of dollars, close to 6% of GDP.

Figure 1 shows that the current levels of external imbalances are not a recent phenomenon but the result of a long run trend that started roughly three decades ago. The objective of this paper is to investigate the role of productivity growth, demographic trends and fiscal policy in explaining this long run trend.

The analysis yields two main results. First, productivity growth differentials are mainly responsible for medium run variations in the US external imbalances. Second, demographic differentials explain a substantial fraction of the low frequency component in the US external position. These two findings are more tightly connected than what it might appear at a first glance. If productivity growth differentials are temporary, persistent demographic differentials are crucial to obtain widening external imbalances.

Despite recent commentaries that government deficits in the US after 2000 might have substantially contributed to the widening of the external deficits, this paper only finds a marginal role for fiscal policy to determine the trade deficit. The main reason is that fiscal policy outside the US, at least for the countries considered in this study, has been expansionary too, possibly at a similar rate.

This paper centers in particular on the bilateral trade balance between the United States and the other six major industrialized countries of the world economy (Canada, France, Germany, Italy, Japan and the United Kingdom - henceforth, the G6). As figure 1 shows, the US trade balance vis-a-vis the G6 accounted for the bulk (about 80% on average) of the total US external imbalances until the late 1990s.[1] Therefore, this bilateral trade flow captures well the idea of permanent imbalances that represents the object of interest for this paper. Given the fact that the analysis considers a subset of the rest of the world as the US international trade partner, it is then more appropriate to discuss the dynamics of the external position in terms of trade rather than current account imbalances.[2]

The reason for narrowing the focus on this subset of the US trade partners is twofold. First, the G6 features a degree of goods and factor markets development that is highly comparable to the US, at least at a first pass. Second, international financial markets between these two regions have become increasingly deep and well integrated over the last three decades. Despite these similarities, the G6 and the US display notable differences in terms of productivity growth, demographic trends and fiscal policy, as the remainder of this section will illustrate more thoroughly. These three factors constitute the driving forces of the external imbalances in this paper.

Lane and Milesi-Ferretti [2001] provide reduced-form evidence that the level of external debt of one country can be parsimoniously explained by the level of output per-capita, the level of government debt and a set of demographic indicators. Hence, variations in these factors should account for the evolution of external imbalances. This paper starts from this premise to systematically investigate the relative contribution of each of those factors in the context of a simple open economy life-cycle model.

The recent work by Engel and Rogers [2006] has emphasized the role of GDP growth differentials (see figure 2) in explaining the US external deficits.[3] While this simple story could

[1] The recent departure of the US trade balance series vis-a-vis the rest of the world from the series vis-a-vis the G6 depends upon the capital flows towards the US from oil producers, newly developed and emerging market economies (most notably China). This aspect of the US external imbalances (the "global saving glut", in the words of Bernanke [2005]) is not the focus of this paper. See Caballero, Fahri and Gourinchas [2007] and Mendoza, Quadrini and Rios-Rull [2007] for explanations of this phenomenon based capital markets imperfections.

[2] For the US, the trade and current account balance almost perfectly align. This anomaly (the "exorbitant privilege" pointed out by Gourinchas and Rey [2007]) is due to the fact that, despite being a net debtor, the US earns on its gross asset position a much higher return than what it pays off on its stock of gross liabilities.

[3] This is also the mechanism considered in an earlier version of the paper by Caballero, Fahri and Gourinchas

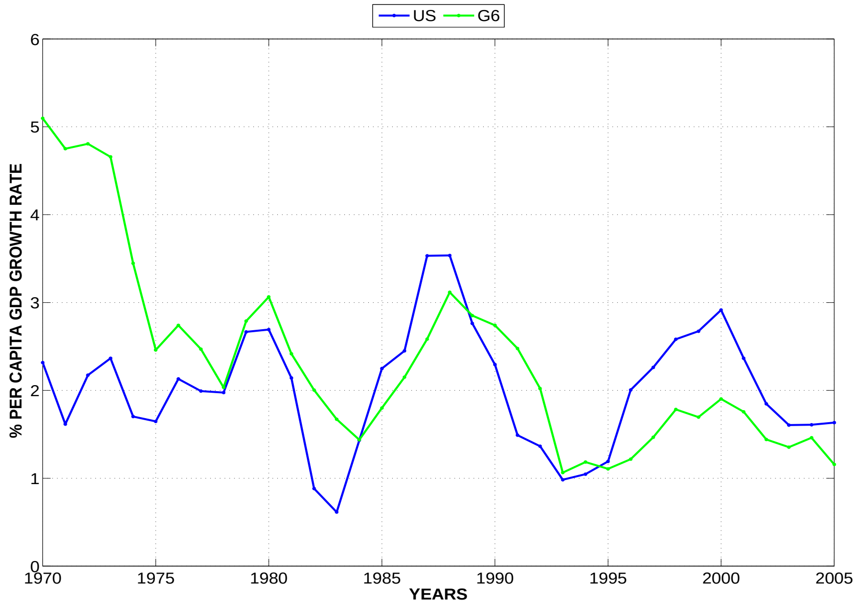

Figure 2: GDP GROWTH RATE

easily account for the level of the US external position, it would need to rely on expectations of positive and increasing GDP growth differentials between the US and the rest of the world in order to capture the dynamic evolution of the external imbalances. Engel and Rogers [2006] report evidence from the Survey of Professional Forecasters on the expected US share of world GDP in the next decade that delivers a dynamic pattern of US external imbalances consistent with the data. This paper sticks to the standard assumption of stationarity for expected productivity growth. Nevertheless, in the model, observed productivity growth differentials between the US and the G6, once coupled with persistent demographic differentials that generate a negative long run trend, are able to generate realistic external imbalances.

The paper develops a simple open economy model with life-cycle structure. The setup is tractable enough to illustrate analytically the channels through which the exogenous factors affect the trade balance and, at the same time, delivers reasonable quantitative implications when confronted with the data. Basically, the framework extends the neoclassical growth model to allow for life-cycle behavior, along the lines of Blanchard [1985] and Weil [1989].

[2007] to explain the imbalances between the U-block (US, UK and Austrialia) and the E-block (Europe and Japan) of their multi-country model.

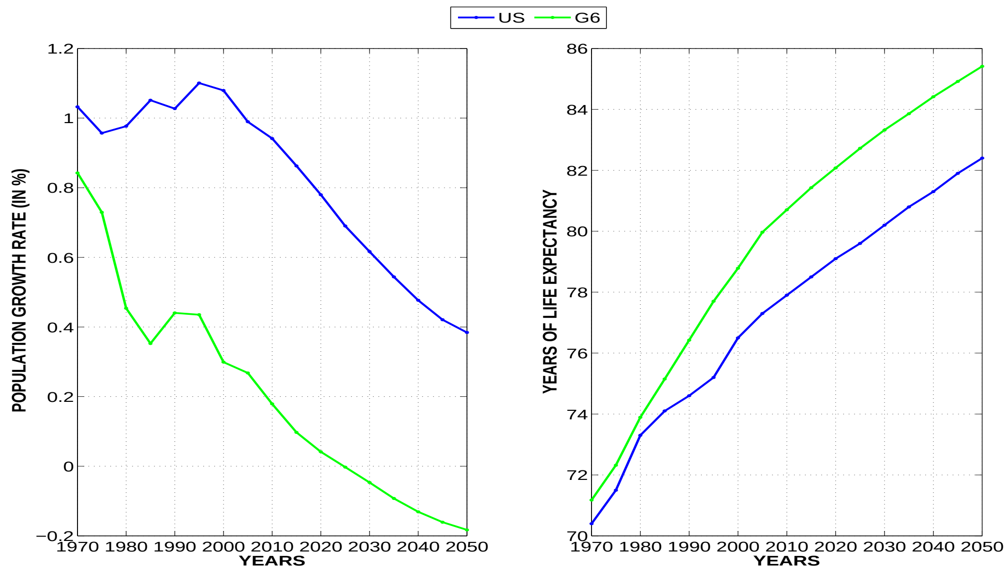

Figure 3: DEMOGRAPHIC INDICATORS

The key difference is the addition of a (random) transition probability from employment into retirement that constitutes the essence of the life-cycle aspect of the model. One important consequence is that, in this paper, the probability of surviving is decoupled from the population growth rate. Since the probability of surviving is directly related to life expectancy, the historical and expected trends of this variable can be analyzed separately from those of population growth (see figure 3). Previous models that failed to disentangle these two aspects of the demographic transition counterfactually predicted, as far as industrialized countries are concerned, that economies with higher population growth rates should be net lenders in international capital markets.[4] This paper emphasizes the importance of variable life expectancy in shaping the individual consumption-savings decisions. Countries where individuals live on average longer (the G6) are associated with higher savings rates. Since those countries typically also feature lower population growth rates, the results in this paper are largely consistent with the data and reverse the argument stemming from the traditional life-cycle literature.

The quantitative importance of cross-country demographic differentials on the trade balance has recently been stressed by Henriksen [2002], Feroli [2003] and Domeij and Flóden

[4]See, for instance, the survey chapter in Obstfeld and Rogoff [1996]. The result can be overturned by the presence of investment. However, the data for the US suggest that the fundamental reason behind the external imbalances is the action along the consumption-saving margin rather than movements in investment.

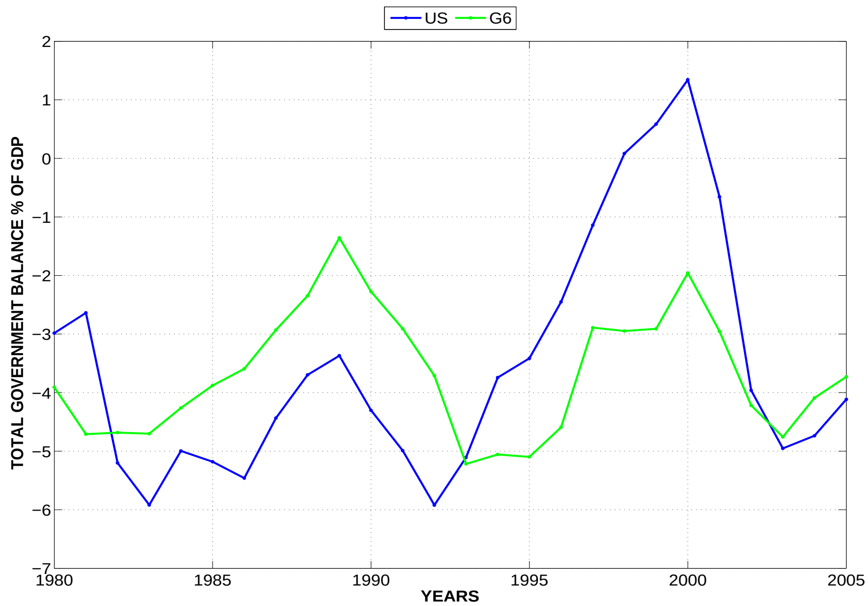

Figure 4: FISCAL BALANCE

[2006] in the context of open economy computable overlapping generation (OLG) models.[5] All three studies find a quantitatively significant effect of demographic variables on external imbalances. The contribution of this paper is to highlight the key margin (low frequency trend) along which demographic differentials operate and to demonstrate that combining the demographic trends with productivity growth rate differentials can explain most of the US trade deficits vis-a-vis the G6 in the last three decades.

Without voluntary bequests, the presence of life-cycle structure breaks Ricardian equivalence in the model. Changes in fiscal policy within each country affect private consumption-saving decisions. Hence, fiscal policy differentials across countries potentially influence the trade balance. This paper finds that observed fiscal policy differentials alone lead to modest effects on external imbalances, as also recently suggested by Erceg, Guerrieri and Gust [2005]. Despite the departure from Ricardian equivalence and the life-cycle effects, the differentials in government balances between the US and the G6 (see figure 4) are simply not large enough to generate sizeable external imbalances. Interestingly, however, fiscal policy

[5]These papers extend the seminal work of Auerbach and Kotlikoff [1987] to a two-country setup. Brooks [2003] and Attanasio, Kitao and Violante [2006] study the implications of demographic trends for capital flows between developed and developing countries in a similar context.

differentials explain in part the fluctuations in the long run trend of the US external imbalances generated by the demographic transition. In particular, even in the absence of a boom in relative productivity, the international demographic transition reconciles the coexistence of non-negligible trade deficits with periods, like the 1990s, of relatively contractionary US fiscal policies, often cited as evidence in favor of Ricardian equivalence.

Finally, the model naturally generates a decreasing pattern for the world real interest rate, which is broadly consistent with the evidence in the data. The decline of the real rate is the direct consequence of the global excess of savings associated with the demographic transition. This finding carries important implications for the persistence and sustainability of fiscal and external deficits.[6] Low interest rates increase the attractiveness of debt and lower the burden of outstanding liabilities. In this respect, the model improves upon large strands of the existing literature which tends to ignore the dynamics of the real interest rate.[7]

The next two sections present the model and the equilibrium for the two-country world economy. The central section of the paper illustrates analytically and quantitatively the results. The description of the data and the details of the derivations are confined in the appendix.

2 A Life-Cycle Model of the US and the G6

The world consists of two countries, Home (US) and Foreign (G6), initially identical in all respects. This section describes in details the structure of the Home economy. If necessary, Foreign variables are denoted by an asterisk "*".

At time t, the Home country is populated by a continuum of agents of mass N_t who can be either workers (w) or retirees (r). The commodity space is constituted by a single consumption good which can be traded internationally at no shipping cost and serves as numeraire. There is no aggregate uncertainty. All agents have perfect foresight but can be surprised by unexpected exogenous shocks.

The life-cycle structure is very simple. Between period t and $t+1$, a worker remains in the labor force with probability $\omega_{t,t+1}$ and retires with the complementary probability. If retired, an individual survives from period t to period $t+1$ with probability $\gamma_{t,t+1}$. The total number of workers in period t is N_t^w. In period $t+1$, $(1-\omega_{t,t+1}+n_{t,t+1})\,N_t^w$ new individuals

[6] Gourinchas and Rey [2006] study the sustainability of the US trade balance in light of the intertemporal approach to the current account and emphasize valuation effects associated to the rate of return. Bohn [2004] finds evidence in favor of sustainability of the US fiscal policy based on a test of the intertemporal government budget constraint. The level of the interest rate plays a crucial role for both results.

[7] One exception is Caballero, Fahri and Gourinchas [2007], who also obtain a decreasing path for the real interest rate in their model.

(workers) are born. Consequently, the law of motion for the aggregate labor force is

$$N^w_{t+1} = (1 - \omega_{t,t+1} + n_{t,t+1})\, N^w_t + \omega_{t,t+1} N^w_t = (1 + n_{t,t+1})\, N^w_t, \tag{1}$$

so that $n_{t,t+1}$ represents the growth rate of the labor force between period t and $t+1$. The number of retirees is denoted by N^r_t and evolves over time according to

$$N^r_{t+1} = (1 - \omega_{t,t+1})\, N^w_t + \gamma_{t,t+1} N^r_t. \tag{2}$$

The ratio between the number of retirees and the number of workers (the "dependency ratio", defined as $\psi_t \equiv N^r_t / N^w_t$) summarizes the heterogeneity in the population. The law of motion of the dependency ratio follows from combining expressions [1] and [2]

$$(1 + n_{t,t+1})\, \psi_{t+1} = (1 - \omega_{t,t+1}) + \gamma_{t,t+1} \psi_t. \tag{3}$$

If population growth rates and transition probabilities were constant, so would be the dependency ratio ($\psi = (1-\omega)\,/\,(1+n-\gamma)$). In this case, the number of retirees and the total population would also grow at rate n as the number of workers.

Workers supply inelastically one unit of labor and retirees do not work.[8] Individual preferences belong to the recursive non-expected utility family, originally introduced by Kreps and Porteus [1978] and extended by Epstein and Zin [1989] to an infinite horizon framework. The general formulation for an individual of cohort $z = \{w, r\}$ reads as

$$V^z_t = \left\{ (C^z_t)^\rho + \beta^z_{t,t+1} \left[E_t \left(V^\theta_{t+1} \mid z \right) \right]^{\frac{\rho}{\theta}} \right\}^{\frac{1}{\rho}}, \tag{4}$$

where C^z_t denotes consumption and V^z_t stands for the value of utility in period t. The discount factor of a retiree differs from that of a worker in order to account for the probability of death

$$\beta^z_{t,t+1} = \begin{cases} \beta\gamma_{t,t+1} & \text{if } z = r \\ \beta & \text{if } z = w \end{cases}$$

The preferences in [4] feature the well-known property of separating the elasticity of intertemporal substitution, $\sigma \equiv 1/\,(1-\rho)$, from the coefficient of relative risk-aversion, equal

[8] Gertler [1999] shows how to introduce variable labor supply in this framework without sacrificing the overall analytical tractability of the model. The assumption of inelastic labor supply constitutes the natural benchmark for the purpose of this paper. The long run trends in demographics would induce individuals to supply more hours. This result would stand in sharp contrast with the data (see figure 13 in the appendix).

to $1-\theta$.

The expected continuation value in [4] differs across cohorts because the future value of utility depends on the current employment status

$$E_t\left\{V_{t+1}^{\theta} \mid z\right\} = \begin{cases} \left(V_{t+1}^{r}\right)^{\theta} & \text{if } z = r \\ \omega_{t,t+1}\left(V_{t+1}^{w}\right)^{\theta} + \left(1-\omega_{t,t+1}\right)\left(V_{t+1}^{r}\right)^{\theta} & \text{if } z = w \end{cases}$$

Two assumptions make the model analytically tractable: (i) the independence of the transition probabilities ω and γ from age and retirement period, and (ii) the existence of a perfect annuity market in which retirees can fully insure against the risk of death. These two assumptions allow to study the decision problem of a representative agent within each cohort without explicit reference to individual characteristics.

The first assumption, however, comes at a cost. The independence of the probability of retirement from age implies a strong precautionary saving motive for young agents. The problem is that this effect is at odds with the data. To prevent this counterfactual excess of savings by young workers, I impose that households are risk-neutral with respect to variations in income and set $\theta = 1$. For a given level of the interest rate, only average consumption tomorrow (appropriately weighted) matters for current consumption decisions. Despite this restriction, the separation of intertemporal substitution from risk aversion can still guarantee a reasonable behavior of consumption-saving decisions in response to interest rate variations.

The assumption of perfect insurance markets against the risk of death, instead, is quite common in the literature (see, for instance, Yaari [1965] and Blanchard [1985]). Notice that a similar mechanism to insure against labor income fluctuations would smooth consumption perfectly across work and retirement, hence shutting off the life-cycle channel which is at the very heart of the analysis. The assumption of risk neutrality, on the other hand, mitigates the artificially high degree of labor income fluctuations induced by the constant transition probability into retirement, while still allowing for an arbitrary elasticity of substitution σ.

I now turn to the characterization of the consumption-savings problem for each cohort. I present the problem of a generic retiree first and then the problem of a generic worker.

Retirees. A retiree i chooses consumption $C_t^r(i)$ and assets $A_t^r(i)$ to solve

$$V_t^r(i) = \max \ \left\{\left(C_t^r(i)\right)^{\rho} + \beta\gamma_{t,t+1}\left[V_{t+1}^r(i)\right]^{\rho}\right\}^{\frac{1}{\rho}}, \tag{5}$$

subject to

$$A_{t+1}^{r}(i) = \frac{R_{W,t} A_t^r(i)}{\gamma_{t-1,t}} - C_t^r(i). \tag{6}$$

Retirees turn their wealth over to a perfectly competitive mutual fund industry which invests the proceeds and pays back a premium over the market return to compensate for the probability of death.[9] For a retiree who survives between period $t-1$ and t, the return on a one dollar investment is $R_{W,t}/\gamma_{t-1,t}$, where $R_{W,t}$ is the world interest rate that clears the international capital markets. The extra-return over the market rate compensates for the probability of death.

Workers. Individuals are born workers and start their life with zero assets. A generic worker j chooses consumption $C_t^w(j)$ and assets $A_{t+1}^w(j)$ to solve

$$V_t^w(j) = \max \ \left\{ \left(C_t^w(j)\right)^{\rho} + \beta \left[\omega_{t,t+1} V_{t+1}^w(j) + \left(1 - \omega_{t,t+1}\right) V_{t+1}^r(j) \right]^{\rho} \right\}^{\frac{1}{\rho}}, \tag{7}$$

subject to

$$A_{t+1}^w(j) = R_{W,t} A_t^w(j) + W_t - T_t^w - C_t^w(j), \tag{8}$$

where W_t represents the market wage and T_t^w is the total amount of lump-sum taxes paid by each worker. The value function $V_{t+1}^r(j)$ is the solution of the problem [5]–[6] above and enters the continuation value of a worker to discount the possibility that retirement occurs between time t and $t+1$.

Firms. Firms operate in perfect competition using a constant returns to scale technology. The aggregate production function is Cobb-Douglas

$$Y_t = \left(X_t N_t^w\right)^{\alpha} K_t^{1-\alpha}, \tag{9}$$

where $\alpha \in (0,1)$ is the labor share and X_t is the level of exogenous labor-augmenting productivity at time t. Productivity grows over time according to

$$X_{t+1} = \left(1 + x_{t,t+1}\right) X_t.$$

[9] It is convenient to assume that in each country the mutual fund operates at the national level only. If the opposite were true, the presence of cross-country differences in aging profiles would imply equalization of returns in the insurance market (and thus not in the capital market) via a no arbitrage argument. Since the mutual fund industry is a only device to keep the model tractable, the restriction on insurance markets appears to be appropriate.

The law of motion of capital is given by

$$K_{t+1} = (1-\delta) K_t + I_t, \tag{10}$$

where δ is the depreciation rate (constant and equal across countries).

Government. The government levies lump-sum taxes and issues one-period debt B_{t+1} to finance a given amount of wasteful spending G_t according to the flow budget constraint

$$B_{t+1} = R_{W,t} B_t + G_t - T_t, \tag{11}$$

where $T_t = N_t^w T_t^w$ represents the total tax revenues.

3 Equilibrium in the World Economy

All markets are competitive and all agents take prices as given. Formally, a competitive equilibrium is a sequence of quantities and prices such that in each country (i) households maximize utility subject to their budget constraint, (ii) firms maximize profits subject to their technology constraints, (iii) the government chooses a path for taxes and debt, compatible with intertemporal solvency, to finance an exogenous level of total spending, (iv) all markets clear. This section describes in details the equations that constitute the competitive equilibrium. Again, the focus is on the Home country.

Individual Decision Rules. As demonstrated by Farmer [1990], under the assumption of risk neutrality in preferences [4], the solution of the individual problem consists of an explicit decision rule for consumption, given prices and policy variables.[10] A retiree consumes optimally a fraction out of total wealth according to

$$C_t^r(i) = \xi_t \frac{R_{W,t} A_t^r(i)}{\gamma_{t-1,t}}. \tag{12}$$

From the Euler equation, it is possible to show that ξ_t, the marginal propensity to consume for a retiree, evolves according to

$$\xi_t = 1 - \gamma_{t,t+1} \beta^\sigma R_{W,t+1}^{\sigma-1} \frac{\xi_t}{\xi_{t+1}}, \tag{13}$$

[10] The details of the derivation are provided in the appendix. See also Backus, Routledge and Zin [2004] for a discussion of other examples in the literature.

The crucial point of equation [13] is that the evolution of ξ_t is independent of individual characteristics.

The solution for the problem of a worker can be characterized along similar lines. Workers consume out of financial and human wealth according to

$$C_t^w(j) = \pi_t \left[R_{W,t} A_t^w(j) + H_t^w(j) \right], \tag{14}$$

where π_t is the marginal propensity to consume for a worker. As for retirees, from the Euler equation, it is possible to retrieve the law of motion of the workers' marginal propensity to consume

$$\pi_t = 1 - \beta^\sigma \left(\Omega_{t+1} R_{W,t+1} \right)^{\sigma-1} \frac{\pi_t}{\pi_{t+1}}, \tag{15}$$

where $\Omega_t \equiv \omega_{t-1,t} + (1 - \omega_{t-1,t}) \epsilon_t^{\frac{1}{1-\sigma}}$ and $\epsilon_t \equiv \xi_t / \pi_t$ are two additional variables that link the marginal propensities to consume of the two cohorts.[11] Also for workers, the marginal propensity to consume only depends on aggregate or cohort-specific variables but not on individual characteristics. The variable $H_t^w(j)$ in [14] represents the present discounted value of current and future human wealth net of taxation, that is

$$H_t^w(j) \equiv \sum_{v=0}^{\infty} \frac{W_{t+v} - T_{t+v}^w}{\prod_{s=1}^{v} \left(\Omega_{t+s} R_{W,t+s} / \omega_{t+s-1,t+s} \right)} = W_t - T_t^w + \frac{\omega_{t,t+1} H_{t+1}^w(j)}{\Omega_{t+1} R_{W,t+1}}. \tag{16}$$

The term $\Omega_{t+1} R_{W,t+1} / \omega_{t,t+1}$ constitutes the effective discount rate for a worker. The first component of the higher discounting captures the effect of the finite lifetime horizon (less value attached to the future). The actual discount factor is also augmented by the term $\omega_{t,t+1}$ which induces higher savings to finance consumption during the retirement period (positive probability of retiring).

Aggregation. In order to derive the aggregate consumption function, the first step consists of aggregating consumption by cohort, which is possible because individuals within the same cohort share the same marginal propensity to consume.[12] Aggregate consumption for retirees

[11] In the appendix, I formally show that ϵ and Ω are both larger than one in steady state. For plausible parameter choices, including the calibration adopted in the numerical experiment section, this result holds also outside the steady state. The fact that $\epsilon_t > 1$ implies that the marginal propensity to consume is higher for retirees than for workers. As the next section clarifies, this result constitutes a key element to understand the response of consumption and savings to changes in demographic factors.

[12] Aggregate variables take the form $Z_t^r \equiv \int_0^{N_t^r} Z_t^r(i)\, di$ for retirees and $Z_t^w \equiv \int_0^{N_t^w} Z_t^w(j)\, dj$ for workers.

is given by

$$C_t^r = \xi_t R_{W,t} A_t^r, \tag{17}$$

where A_t^r is total wealth that retirees carry from period $t-1$ into period t, with aggregate return given by $R_{W,t}$.[13] Individual consumption for workers can be aggregated along the same lines to yield

$$C_t^w = \pi_t \left(R_{W,t} A_t^w + H_t \right), \tag{18}$$

where A_t^w is total aggregate financial wealth held by workers. The aggregate value of human wealth evolves according to

$$H_t = N_t^w W_t - T_t + \frac{\omega_{t,t+1} H_{t+1}}{\left(1 + n_{t,t+1}\right) \Omega_{t+1} R_{W,t+1}}. \tag{19}$$

The second step consists of writing the aggregate consumption function C_t as the sum of [17] and [18]. I denote by $\lambda_t \equiv A_t^r / A_t$ the share of total assets A_t held by retirees. It follows that the aggregate consumption function

$$C_t = \pi_t \left[\left(1 - \lambda_t\right) R_{W,t} A_t + H_t + \epsilon_t \lambda_t R_{W,t} A_t \right]. \tag{20}$$

Finally, I need to characterize the evolution of the distribution of assets, as captured by λ_t. This additional state variable keeps track of the heterogeneity in wealth accumulation introduced by the life-cycle hypothesis. Aggregate assets for retirees depend on the total savings of those who are retired in period t but also on the total savings of the fraction of workers who retire between t and $t+1$

$$A_{t+1}^r = R_{W,t} A_t^r - C_t^r + \left(1 - \omega_{t,t+1}\right) \left(R_{W,t} A_t^w + N_t^w W_t - T_t - C_t^w \right). \tag{21}$$

Aggregate assets for workers depend only the savings of the fraction of workers who remain in the labor force

$$A_{t+1}^w = \omega_{t,t+1} \left(R_{W,t} A_t^w + N_t^w W_t - T_t - C_t^w \right). \tag{22}$$

After substituting expressions [17] and [22] into [21], the law of motion for the distribution of wealth across cohorts can be written as

$$\lambda_{t+1} A_{t+1} = \left(1 - \omega_{t,t+1}\right) A_{t+1} + \omega_{t,t+1} \left(1 - \xi_t\right) R_{W,t} \lambda_t A_t. \tag{23}$$

[13] Each individual retiree earns a return given by $R_{W,t}/\gamma_{t-1,t}$ but only a fraction $\gamma_{t-1,t}$ of retirees survive between two periods.

Expression [23] relates the evolution of the distribution of wealth λ_t to the aggregate asset position A_t.

Closing the Model. The problem of the firm is standard. Firms hire labor to the point that the real wage equals the marginal product of labor

$$W_t = \alpha \frac{Y_t}{N_t^w}. \tag{24}$$

The investment decisions are consistent with the equality between the marginal product of capital net of depreciation and the real return that prevails in the international asset markets

$$R_{W,t} = (1-\alpha)\frac{Y_t}{K_t} + (1-\delta). \tag{25}$$

Finally, in equilibrium, the government is restricted to choose combinations of taxes and debt that respect the intertemporal solvency condition

$$R_{W,t}B_t = \sum_{v=0}^{\infty} \frac{(T_{t+v} - G_{t+v})}{\prod_{s=1}^{v} R_{W,t+s}}. \tag{26}$$

The current value of debt (principal and interests) must be equal to the present discounted value of current and future primary surpluses. For later purposes, it is also convenient to express fiscal policy variables as fractions of GDP. Given the path for G_t, I assume that the government fixes the ratio $g_t \equiv G_t/Y_t$ and chooses combinations of tax rates $\tau_t \equiv T_t/Y_t$ and debt-to-GDP ratios $b_t \equiv B_t/Y_t$ that satisfy [26].

The market clearing conditions for goods and assets complete the characterization of the equilibrium. The portfolio of assets held by the private sector is composed by capital and by government and international bonds (net foreign assets)

$$A_t = K_t + B_t + F_t. \tag{27}$$

The evolution of net foreign assets links the goods and the asset markets together. Net foreign assets represent the payment received from the rest of the world in exchange for exporting goods[14]

$$F_{t+1} = R_{W,t} F_t + NX_t, \tag{28}$$

where the trade balance NX_t is defined as the difference between domestic production and absorption

$$NX_t = Y_t - (C_t + I_t + G_t). \tag{29}$$

International capital flows equalize the return $R_{W,t}$ across countries. In equilibrium, Home and Foreign holdings of net foreign assets must add up to zero

$$F_t + F_t^* = 0.$$

Despite the tractability of the model, a complete closed form solution cannot be derived. In the appendix, I characterize a symmetric steady state of the model in which exogenous variables are constant and equal across countries. Given the presence of exogenous growth, quantities are not stationary. A steady state with balanced growth exists for detrended variables, that is, for variable expressed in terms of efficiency units $X_t N_t^w$. In such a steady state, quantities grow at the constant rate $(1+x)(1+n) \cong 1 + x + n$. In what follows, I denote a generic variable Z_t in terms of efficiency units by lower case letters $z_t \equiv Z_t / (X_t N_t^w)$. The solution for the steady state and for the transition dynamics is obtained by employing a non-linear Newton method.[15] In order to have a determinate steady state, I calibrate from the data an initial level of steady state government debt relative to GDP as the "fiscal rule" which closes the model.

As a final remark, it is interesting to note that in this model the steady state is determinate.[16] The life-cycle structure pins down endogenously the steady state value of net foreign asset. Ghironi [2006] shows a similar result in a framework with overlapping families of infinitely lived agents based on Weil [1989].

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[14] Following Obstfeld and Rogoff [1996], the current account balance can be defined as the one-period variation in the net foreign asset position

$$CA_t \equiv F_{t+1} - F_t = (R_{W,t} - 1) F_t + NX_t.$$

For the US, the current account and trade balance almost coincide, which essentially means that the net interest rate payment on the outstanding stock of net foreign assets must be very small. See Tille [2005], and the references therein, for a discussion of the valuation effects associated with the maturity and denomination of the US stock of foreign assets and liabilities.

[15] See Juillard [1996] for details about the implementation of the specific Newton-Raphson algorithm used to solve this model.

[16] It is well known that open economy models with incomplete markets generally feature steady state indeterminacy and non-stationary dynamics of net foreign assets. See Schmitt-Grohé and Uribe [2003] for the discussion of a number of alternative mechanisms to circumvent this problem.

Section 4 below discusses the calibration and results of the quantitative experiments. In order to build some intuition, however, it is useful to note that, in the model, the assumption of perfectly integrated international financial market implies that in each period capital in efficiency units must be equal across countries. The result follows from the no arbitrage relation

$$R_{W,t} - 1 + \delta = (1-\alpha)(k_t)^{-\alpha} = (1-\alpha)(k_t^*)^{-\alpha}, \tag{30}$$

which entails $k_t = k_t^*$, $\forall t$. Since both countries have access to the same technology, also output per efficiency unit is equalized across borders ($y_t = y_t^*$). Under the assumption that $g_t = g_t^*$, the national account identity [29] gives an expression for the trade balance as a function of consumption and investment differentials[17]

$$nx_t = -\frac{1}{2}(c_{R,t} + i_{R,t}), \tag{31}$$

where, for any z_t and z_t^*, $z_{R,t} \equiv z_t - z_t^*$.

The objective of the rest of this paper is to understand how productivity, demographic and fiscal policy differentials can determine persistent external imbalances by affecting relative consumption and investment.

4 A Quantitative Investigation of the US Trade Deficit

This section evaluates the quantitative importance of differentials in productivity, demographics and fiscal policy to account for the bilateral trade balance between the US and the G6 during the period 1970-2005. The first step is to show that the model predictions are consistently in line with the data over the entire sample period. The second step consists of assessing the relative contribution of each factor towards explaining the external imbalances. Three main results stand out:

1. Productivity differentials are mainly responsible for the medium frequency variations in the trade balance of the US vis-a-vis the G6.

2. Demographic differentials are crucial to explain the declining long run trend in the US trade deficit.

3. Fiscal policy differentials generate the "swings" along the long run trend in the US external imbalances.

[17] In what follows, I refer to $c_{R,t}$ also as (the negative of) relative savings. The result is a consequence of the definition of aggregate savings as investment plus the trade rather than the current account balance.

4.1 Description of the Experiment

The time period is one year. The central numerical experiment of the paper consists of simulating an unexpected series of shocks to productivity growth rates, demographic variables and fiscal stances in 1970 and to compute the transition path to the final steady state, which is reached sometime after 2030 (the year after which the exogenous variables become constant). Under perfect foresight, agents are surprised by the shock in the initial period but perfectly anticipate the evolution of the exogenous variables thereafter.[18]

The choice of 1970 as the base year is mainly motivated by the consistency between the model assumption of perfectly integrated international capital markets and the collapse of the Bretton Woods system in the early 1970s. The choice of 2030 as the final year reflects the need to average out some of the uncertainty in the demographic projections and to prevent an excessive drop in the real interest rate as a consequence of the demographic transition. Appendix *B* contains the details on the data used for the calibration.

The high frequency variations in productivity growth (possibly due to variations in capacity utilization) are smoothed out by taking five years averages. I use the realized values of net debt as a percentage of GDP in the two countries as the exogenous fiscal stances in the simulation. Given the constant level of government spending as a fraction of output, the tax rate and the government surplus are an outcome of the model which depends on the equilibrium output and interest rate.[19] The probabilities of surviving during the transition result from linear interpolation of the values for life expectancy in the initial and final steady states, under the assumption that the average permanence in the labor force is constant at 45 years.[20] Finally, I infer the growth rate of the labor force from the data on population growth rates, adjusting for convergence in 2030 at 0.2%. Figure 14 in the appendix reports the exogenous processes fed into the model.

Fixed Parameters The choice of the values for preference and production parameters is mostly in line with the existing literature. The elasticity of intertemporal substitution σ is

[18] While typical of large scale OLG models, the assumption of perfect foresight is clearly extreme. Even in this simple framework, however, abstracting from aggregate uncertainty helps to preserve the tractability of the model and the computations. Given that the agents in the economy are surprised by the initial shock but then anticipate the entire path of the transition, it seems safe to speculate that the explicit consideration of aggregate uncertainty could lead to higher volatility and lower persistence in the trade balance. This missing component is at least in part compensated by the absence of frictions in international financial markets that allows capital to move freely across countries at no cost.

[19] The use of the debt-to-GDP ratio as the exogenous fiscal variable simplifies the computations. The results are nevertheless unchanged if the fiscal instrument is assumed to be the tax rate.

[20] Linear interpolation partly compensates for using data on life expectancy at birth, rather than at 20, by understating the surviving probabilities during the entire transition relative to the data.

assumed to be equal to 0.5, which represents a compromise between the low values usually adopted in the public finance literature (e.g. 0.25 as in Auerbach and Kotlikoff [1987]) and in the real business cycle literature (e.g. 1 as in Cooley [1995]).[21] The discount factor β is set equal to 1. The resulting real interest rate in 1970 is 5.4%. Productivity is computed as the Solow residual of the production function [9] using data on GDP and investment from the World Bank (World Development Indicators) and on employment from GGDC. The capital stock is inferred from investment using the perpetual inventory method. The steady state value of productivity growth equals 1.3%, in line with the average of the estimates for the G7 during the last three decades. The production parameters are standard in the real business cycle literature. The labor share α is assumed to be 2/3 and the depreciation rate δ is set to 6%. The analysis of fiscal policy focuses on the government financing decisions given a constant level of public expenditure, equal to 20% of GDP. This value corresponds approximately to the average US value during the post-war period, as reported by the Bureau of Economic Analysis.[22]

Initial Steady State. The initial steady state is symmetric.[23] The "United Nations World Population Prospects: The 2004 Revision" is the main source for the calibration of the demographic variables. Population grows at a rate of 1%, which is in line with the observed value for both US and G6 in 1970. In the model economy, individuals are born workers at the age of 20. Since the average permanence in the labor force is $(1-\omega)^{-1}$ years, I set the probability of retiring for a worker (ω) equal to 0.9778, as to match the average retirement age of 65 used by Auerbach and Kotlikoff [1987] in their quantitative experiments. Moreover, since the average retirement period is $(1-\gamma)^{-1}$, I further choose the probability of surviving for a retiree (γ) equal to 0.8, as to match the average expected lifetime horizon of an agent in the model $(65+(1-\gamma)^{-1})$ to the empirical counterpart in 1970 (roughly 70 years both for

[21] See Attanasio [1999] for a comprehensive survey. Gertler [1999] analyzes the sensitivity of the steady state interest rate and capital stock to alternative values for the elasticity of intertemporal substitution in this model and finds a negative relationship between such parameter and the steady state interest rate. A low value of this elasticity also reinforces the crowding out effect of fiscal policy on the capital stock.

[22] This value obviously underestimates the role of government spending in European countries and Japan. However, the model excludes features, such as a public pension system or a public health system, which account for large parts of government spending in those countries and are likely to provide utility to households. Hence, the value of 20% seems a reasonable approximation in a context in which all government spending is considered pure waste.

[23] In a symmetric steady state, the external position is balanced so that the impact of the exogenous transition can be evaluated without the adjustment effects induced by an existing stock of either assets or liabilities. This assumption squares well with the state of the demographic indicators in 1970. Moreover, productivity growth was higher in the G6 but fiscal policy was more expansionary in the US. Therefore, a symmetric steady state appears to be a reasonable benchmark scenario.

the US and G6).[24]

The ratio of government debt to GDP is calibrated using data on net debt from the International Monetary Fund (World Economic Outlook Database), which are available only starting in 1980. I set the ratio of debt to GDP in the initial symmetric steady state to 26%, which represents the average between US and G6 in 1980. For the US, this value represents a reasonable approximation also for the 1970s, a decade during which the ratio of net debt to GDP was roughly constant. In the experiment presented here, I assume that the ratio of debt to GDP is constant at 26% also in the G6 throughout the 1970s.[25]

Given the benchmark calibration, the model delivers a capital-income ratio equal to 2.92 in the initial steady state. Investment constitutes 24.3% of GDP and the tax rate is equal to 20.8%.

Final Steady State. The transition from the initial to the final steady state is governed by the evolution of productivity (TFP), demographic factors (expected lifetime horizons and population growth rates) and fiscal stances (debt-to-GDP ratios) in both countries. Productivity reverts back to its initial steady state value of 1.3% in 2030. Convergence for population growth rates occurs also in 2030 at a value equal to 0.2%, which coincides with the average of the UN projections for the US and the G6 for the period $2030 - 2050$.[26] The UN projections anchor also life expectancy in both regions during the same period. The implied values of the average retirement period are 15 and 18 years for the US and G6 respectively. For debt-to-GDP ratios, I assume slow convergence in 2030 at 60% from the levels of 2006 reported in the IMF World Economic Outlook Database (approximately 50% in the US and 70% in the G6).

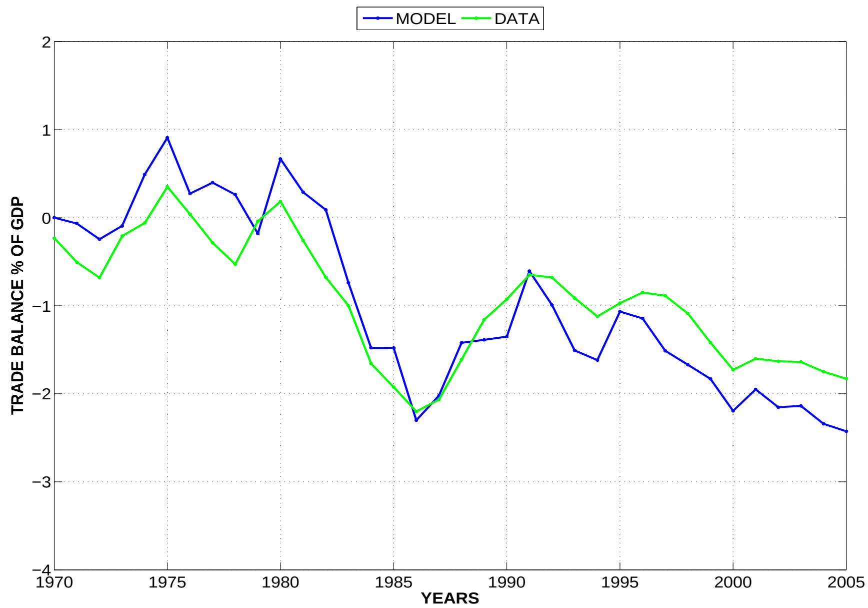

Figure 5: THE US TRADE BALANCE: MODEL VS DATA

4.2 Results

Figure 5 compares the time series of the US trade balance (as a percentage of GDP) vis-a-vis the G6 with the correspondent variable generated by the model for the period $1970 - 2005$.

Overall, the model fits the data quite well. The simulated series for the US trade balance predicts slightly excessive surpluses during the 1970s whereas the actual external position was closer to balance. The first half of the 1980s corresponds to the first episode of large imbalances followed by a partial correction during the second part of the same decade. With

[24] A decision maker in the model starts making consumption-savings decisions at the age of 20. Unfortunately, data on life expectancy at 20 are not available on a consistent basis. Therefore, I use life expectancy at birth to calibrate the model. Table 3 shows life expectancy at 20 for a number of survey years (source: United Nations Demographic Yearbook). The differentials between the US and the G6 in life expectancy at birth are generally preserved at 20.

[25] Data on net debt for the US are available also from the Congressional Budget Office for the entire post war period. While the pattern is essentially the same, there exist some differences in terms of absolute magnitudes, although the order never exceeds 5%. For international comparisons, the World Economic Outlook Database appears to be the most reliable source for the time series of net debt. Results (largely unchanged) based on an alternative debt series extrapolated from gross government debt (source OECD Economic Outlook Database) starting in 1970 are available upon request.

[26] The convergence of productivity and population growth rates ensures that, in the limit, no country ends up representing the entire world economy.

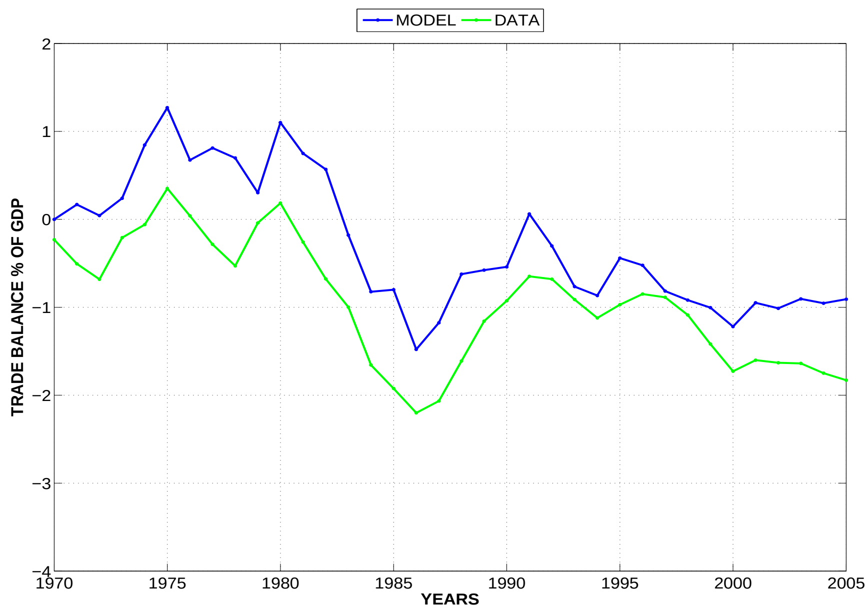

Figure 6: THE US TRADE BALANCE: MODEL (TFP ONLY) VS DATA

the 1990s, the US trade balance starts deteriorating and that trend currently persists. The model partly overpredicts also in this last part of the sample. These extra capital flows might be due to unmodeled frictions in international financial markets, such as capital controls still in place after the breakdown of Bretton Woods or transaction costs.

In any event, the objective of this paper is not to provide a forecasting framework for the US trade balance. Rather, the main contribution is to illustrate the role of productivity, demographics and fiscal policy in accounting for the evolution of the US trade balance during the last 35 years. The fact that the predictions of the model for the last three decades are largely consistent with the data, both qualitatively and quantitatively, is reassuring for the relevance of the decomposition that follows. In order to evaluate the relative importance of each factor, I simulate a number of counterfactual experiments that shut off one or more exogenous variables and compare the time series of the trade balance so obtained with the baseline case reported in figure 5 or with other transformations of the empirical and artificial data.

Figure 6 plots the data against a counterfactual scenario in which only productivity growth rates differ across countries.

Result #1.

Productivity differentials account for the medium frequency variations of the US trade balance.

The correlation coefficient between the simulated series and the data is higher than 0.9. However, it is clear that the model systematically underpredicts the US external imbalances.

Result #1 is in line with the findings of Engel and Rogers [2006] that emphasize the role of faster growth in the US relative to other industrialized economies to explain the US external imbalances. Nevertheless, faster growth needs to be complemented by a mechanism that imposes substantial persistence in productivity growth differentials.

The intuition for the result is typical of a neoclassical growth model. Along the investment channel, a difference in productivity growth rates contemporaneously influences relative investment

$$i_{R,t} = \left(x_{t,t+1} - x^*_{t,t+1}\right) k_t, \tag{32}$$

where, for simplicity, I have abstracted from population growth differentials. In the absence of any frictions in international financial markets, capital flows to the country that can employ it more efficiently.

On the consumption-savings margin, expectations of higher productivity growth in the future lead to higher current consumption. Together with the investment channel, this typical consumption smoothing effect explains both why the trade deficit widened during the second part of the 1990s, when the US was experiencing a series of positive productivity shocks, and why, under the hypothesis of convergence, the model predicts a rebalancing of the US external position.

In order to explain this last part of the sample, Engel and Rogers [2006] appeal to survey evidence to argue that the US share of world output will increase in the next decade. This possibility essentially corresponds in this model to an increasing path of productivity growth in the US relative to the G6 for the next several years, similar to the productivity boom of 1990s. In this paper, instead, I assume that productivity growth slowly reverts back to trend after 2005. Nevertheless, the complete model is capable to generate a widening trade deficit after 2000. The demographic differentials between the US and the G6 are the key mechanism that generates a downward trend in the US trade balance.

Figure 7 compares the low frequency component of the actual data with the simulated time series of the US trade balance in the absence of productivity differentials. Figure 8 further decomposes the relative contribution of demographics and fiscal policy to account for the movements in the long run trend of the simulated US trade balance.

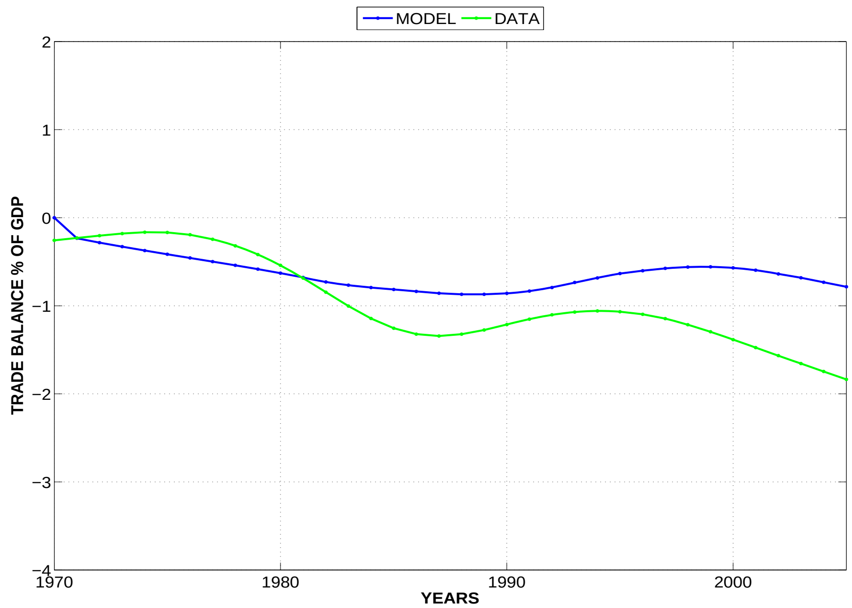

Figure 7: THE US TRADE BALANCE: MODEL (NO TFP) VS DATA (HP FILTER)

Result #2.

Demographic differentials drive the low frequency component of the US trade balance.

The demographic transition alone explains roughly two thirds of the negative trend in the trade balance observed in the data. An important result of this paper is to show (see figure 9) that the differentials in life expectancy are indeed responsible for most of the variation. This finding is interesting because it highlights a mechanism neglected in most of the earlier work on demographics and external imbalances (see Obstfeld and Rogoff [1996] for a survey).

Life expectancy differentials affect the trade balance only via relative consumption-savings decisions. Relative consumption is given by

$$c_{R,t} = R_{W,t} \left\{ \left[\pi_t \left(1 - \lambda_t\right) a_t - \pi_t^* \left(1 - \lambda_t^*\right) a_t^* \right] + \left(\xi_t \lambda_t a_t - \xi_t^* \lambda_t^* a_t^* \right) \right\} + \left(\pi_t h_t - \pi_t^* h_t^* \right). \qquad (33)$$

The first two components of [33] represent the differentials in the propensity to consume out of financial wealth by workers and retirees respectively. The third term captures the differentials in the propensity to consume out of human wealth.

The adjustment of the trade balance to a change in surviving probabilities depends pri-

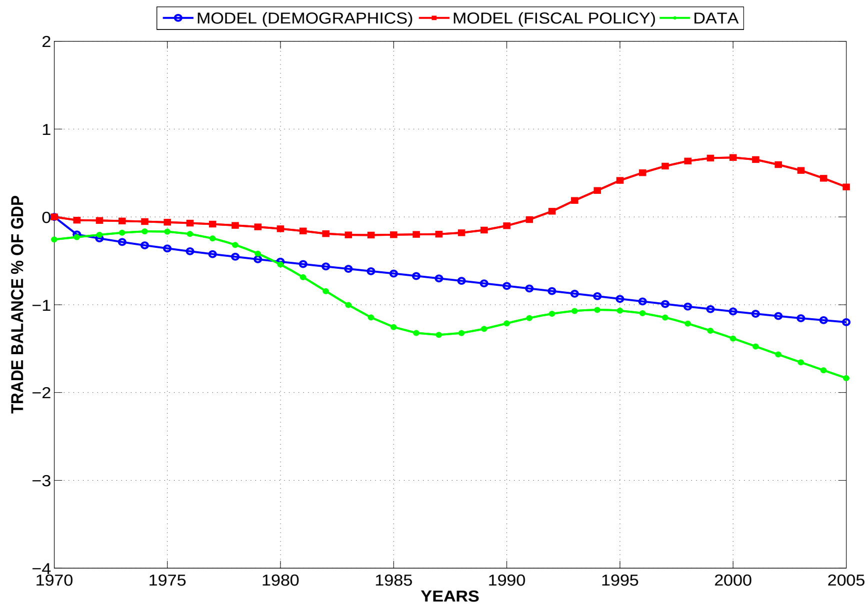

Figure 8: DECOMPOSITION OF LOW FREQUENCY COMPONENT

marily on the response of the marginal propensities to consume.

In steady state, the relative marginal propensity to consume for retirees is

$$\xi_R = -\beta^{\sigma} R_W^{\sigma-1} \left(\gamma - \gamma^*\right).$$

Hence, retirees in the relatively older country experience a lower marginal propensity to consume.

Life expectancy differentials affect the relative marginal propensity to consume for workers through the terms Ω and Ω^*

$$\pi_R = -\beta^{\sigma} R_W^{\sigma-1} \left[(\Omega)^{\sigma-1} - (\Omega^*)^{\sigma-1} \right].$$

For a given interest rate, a higher surviving probability induces a reduction in the marginal propensity to consume for workers too.[27]

At the aggregate level, an increase in the expected lifetime horizon increases the saving rate. On the one hand, workers save more in order to finance a longer retirement period. On

[27] See section $A.2$ of the appendix for details.

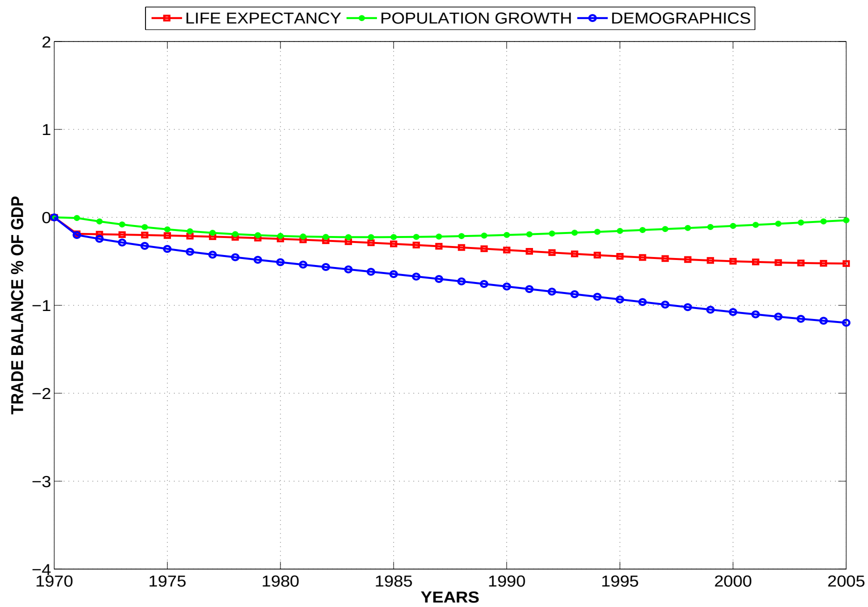

Figure 9: DECOMPOSITION OF THE CONTRIBUTION OF DEMOGRAPHICS

the other hand, for a given level of wealth, retirees spread their consumption over a longer retirement period. The crucial implication for the trade balance is that, ceteris paribus, the level of savings is higher in the country with higher life expectancy. The relative excess of savings generates a trade surplus associated with the accumulation of a positive stock of net foreign assets. The life-cycle structure of the population is at the core of this mechanism.

Alongside the effects on the marginal propensity to consume, a higher probability of survival also increases the dependency ratio. From [3], in steady state, the cross-country differential is

$$\psi_R \mid_{\gamma\neq\gamma^*} = \frac{(1-\omega)(\gamma-\gamma^*)}{(1+n-\gamma)(1+n-\gamma^*)}.$$

Quite obviously, the dependency ratio is higher in the country with longer life expectancy. Since retirees have a higher marginal propensity to consume relative to workers, over time this effect partly offsets the initial fall of aggregate consumption. Nonetheless, for plausible parameterizations, the reduction of the marginal propensity to consume among retirees always dominates the cross-sectional adjustment within each country.

Population growth rate differentials influence the trade balance along two margins.

The direct channel operates through the investment sector, similarly to productivity. If the two countries differ only in population growth rates, relative investments is

$$i_{R,t} = \left(n_{t,t+1} - n^*_{t,t+1}\right) k_t. \tag{34}$$

As for productivity, capital flows to the country that can employ it more efficiently. The only difference is that the efficiency argument now depends on the growth rate of the labor force as opposed to the growth rate of technology.

The indirect effect of population growth rate differentials depends on the life-cycle structure of the model. For two countries initially identical in all respects, the relative dependency ratio is inversely proportional to the relative population growth rate after the shock

$$\frac{\psi_{t+1}}{\psi^*_{t+1}} = \frac{1 + n^*_{t,t+1}}{1 + n_{t,t+1}}.$$

Since retirees consume relatively more than workers, this channel generates an increase of aggregate consumption in the country with the higher dependency ratio.

The sign of the net adjustment (investment versus consumption-saving decisions) of the trade balance following a shock to the relative population growth rate is ambiguous. Figure 9 suggests a mild predominance of the investment channel in the experiment considered here, although the overall effect appears to be rather small.

Figure 9 also shows that the two margins along which demographic differentials operate display in fact some interesting interaction. The overall effect of the demographic transition on the trade balance is not simply the sum of a relatively lower population growth rate and a relatively longer life expectancy in the G6. The combination of these two forces exerts a more than proportional pressure on the dependency ratio. The number of retirees grows relatively to the number of workers because (i) the average lifetime horizon is longer (aging) and (ii) less people are born in each period (growth). The pool of retirees in the G6 is more numerous than in the US but those individuals live on average longer and hence need to save relatively more than their US counterparts.

The quantitative effects of the demographic transition crucially depends on the accurateness of the UN projections used to anchor the final steady state. In particular, the differentials in life expectancy, much more than differentials in population growth rates, play a crucial role for determining the underlying long run trend of the US external imbalances. As it should be clear from the previous discussion, the larger are the differentials in life-expectancy, the more pronounced is the downward slope in the trend component of the external imbalances.[28]

[28] Robustness analysis based on the alternative scenarios (high and low) prepared by the UN Population Database are available upon request.

The previous discussion has emphasized that a substantial part of the negative trend in the US trade balance is the result of demographic differentials persisting over time. Figure 8 shows that fiscal policy differentials generate movements in the long run trend while having almost no medium frequency effect on external imbalances.

Result #3.
Fiscal policy amplifies the trend induced by demographic differentials.

Fiscal policy differentials cause the three major swings in the simulated trade balance which replicate quite closely the fluctuations in the observed trend, especially during the last twenty five years.

Figure 8 also implies that it is possible to reconcile the persistent US trade deficits with the conduct of fiscal policy in the last three decades once demographic dynamics are properly taken into account. Even in the absence of productivity growth differentials, the coexistence of fiscal surpluses and trade deficits in the US during the 1990s is not necessarily evidence in favor of Ricardian equivalence.

On the other hand, fiscal policy differentials alone play a limited role in explaining the US external imbalances, especially at higher frequency. This result is not built into the model. In principle, the deviations from Ricardian equivalence in the model are substantial.[29] The reason why fiscal deficits only account for a small fraction of the US external imbalance vis-a-vis the G6 is that the observed relative fiscal stance is not large and persistent enough to support sizeable trade deficits.

Fiscal policy differentials affect the trade balance only through relative consumption. For simplicity, assume that productivity growth rates and surviving probabilities are constant and equal across countries and that population growth rates are zero.[30] The absence of life expectancy differentials implies that the marginal propensity to consume for each cohort is identical across countries. Expression [33] can be rewritten as

$$c_{R,t} = \pi_t \left[h_{R,t} + R_{W,t} a_{R,t} + (\epsilon_t - 1) R_{W,t} \left(\lambda_t a_t - \lambda_t^* a_t^* \right) \right]. \tag{35}$$

The first component of [35] is relative human wealth, which depends negatively on the relative

[29] The effects of fiscal deficits on external imbalances are almost four times larger than the estimates in Erceg, Guerrieri and Gust [2005]. Under the baseline calibration, a permanent rise of one dollar in the fiscal deficit generates more than a 0.75 dollars deterioration of the trade balance.

[30] This assumption permits to illustrate more starkly the non-Ricardian effects associated to fiscal policy in this framework. Weil [1989] shows that a positive population growth rate (overlapping families of infinitely lived agents) is a sufficient condition for Ricardian equivalence to fail in an otherwise neoclassical world.

tax rate

$$h_{R,t} = -\sum_{v=0}^{\infty} \frac{\tau_{R,t+v} y_{t+v}}{\prod_{s=1}^{v} \left[\Omega_{t+s} R_{W,t+s} / (1+x)\, \omega\right]}. \tag{36}$$

The second term of [35] is the gross return on the relative aggregate asset position which can be rewritten, using expressions [26] and [27], as

$$R_{W,t} a_{R,t} = 2 R_{W,t} f_t + \sum_{v=0}^{\infty} \frac{\tau_{R,t+v} y_{t+v}}{\prod_{s=1}^{v} \left[R_{W,t+s} / (1+x)\right]}. \tag{37}$$

Finally, the third element of [35] captures relative asset holdings by retirees (remember that $\lambda_t = A_t^r / A_t$). From expression [23], this difference evolves according to

$$\lambda_t a_t - \lambda_t^* a_t^* = (1-\omega)\, a_{R,t} + \omega \left(\frac{1-\xi_t}{1+x}\right) R_{W,t-1} \left(\lambda_{t-1} a_{t-1} - \lambda_{t-1}^* a_{t-1}^*\right). \tag{38}$$

In general, relative asset holdings by retirees can be expressed as a function of the cross-country difference in aggregate assets.[31]

Starting from a symmetric steady state, a relative fiscal deficit in the US rises the debt-to-GDP ratio b_t and reduces the tax rate τ_t. On impact, the trade balance depends only on future relative tax rates. Since the Home country is running a fiscal deficit, the relative tax rate is negative, boosting relative human wealth and turning the trade balance into a deficit. Along the transition, the tax cut stimulates private consumption and reduces private savings. As time goes by, however, the tax rate in the Home country increases in order to smooth the tax hike that takes place at the end of the fiscal expansion. Moreover, in general equilibrium, the drop of the savings rate increases the real interest rate. This effect partially limits the benefits of the fiscal expansion on consumption.

Fiscal deficits entail non-Ricardian effects because of the life-cycle structure of the economy. In expressions [36] and [37], relative taxes are discounted at different rates. In other words, a fraction of government debt constitutes net wealth for the private sector. Because of the transition into retirement, a worker who benefits from a tax cut today, with some probability, will not be subject to the future higher tax rate, which is necessary, ceteris paribus, to satisfy the government intertemporal solvency condition. Hence, the capitalization of future

[31] Whether expression [38] admits a backward or forward solution depends on the autoregressive coefficient being larger or smaller than one. Fiscal and demographic forces will typically exert opposite effects on this term. For high enough values of the equilibrium interest rate, possibly due to expansionary fiscal policies, the autoregressive coefficient will be larger than one so that only current and future fiscal decisions matter for the equilibrium trade balance.

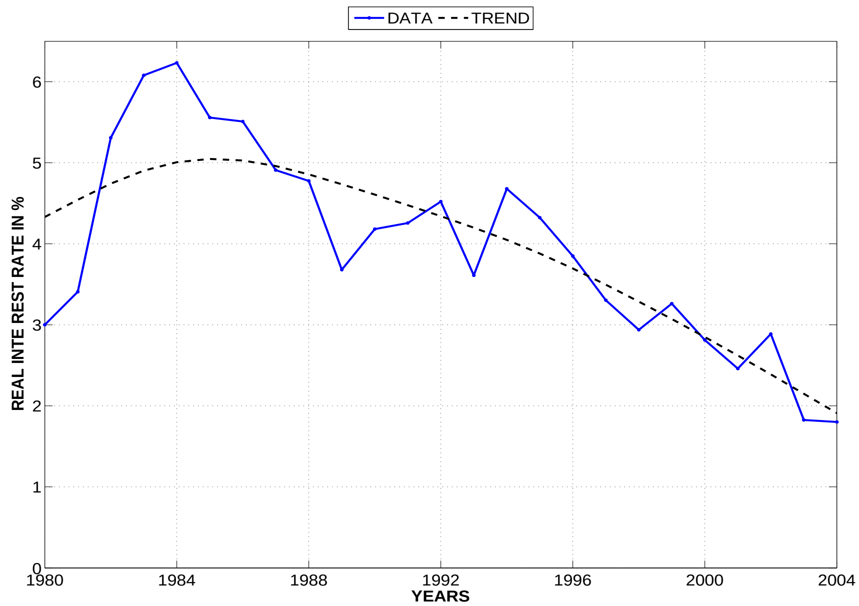

Figure 10: THE WORLD REAL INTEREST RATE

tax rates by workers is less than complete (as measured by the factor Ω_{t+v}/ω). In this case, fiscal and trade deficits are "twins", although the synchronization is less than perfect. The persistence in the trade deficit is augmented by the term in [38], which measures the variation in the distribution of wealth across cohorts following an expansion.

The bottom line of this analysis is that the relative fiscal stance, not its absolute magnitude, determines the equilibrium effects of fiscal deficits on the trade balance. A fiscal deficit in the US is neither necessary nor sufficient to generate a US trade deficit vis-a-vis the G6. As figure 4 shows, during the last three decades, the relative government balance has not been large and persistent enough to drive the transition of external imbalances.

4.3 The World Real Interest Rate

This section discusses the implications of the driving forces behind the US external imbalances for the equilibrium real interest rate. Figure 10 reports the average long run ex-post real interest rate for the G7 (source: IMF International Financial Statistics Database). In the data, long term real interest rates have been declining for the last couple of decades. In a recent speech, Bernanke [2005] has proposed a joint explanation for the US external

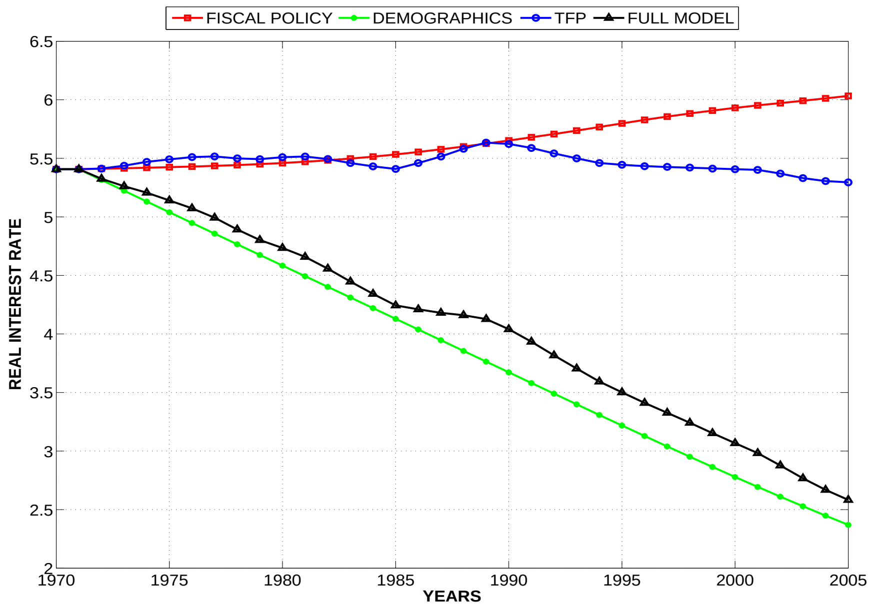

Figure 11: DECOMPOSITION OF THE WORLD REAL INTEREST RATE IN THE MODEL

imbalances and low real interest rates based on the significant increase in the world supply of savings (the "saving glut" hypothesis). Caballero, Fahri and Gourinchas [2007] and Mendoza, Quadrini and Rios-Rull [2007] have elaborated on this idea by linking the global saving glut to capital markets imperfections in emerging economies. This thesis is obviously appealing given the prominent role of emerging market economies in accounting for the growing US external imbalances after the Asian crises of the late 1990s. Nevertheless, two questions remain unanswered. First, why did real rates start falling in the late 1980s, well before the emergence of the global saving glut? Second, why did real rates decreased not only in the US but also in the other industrialized countries during the same period?

This paper too suggests that the global imbalances and decreasing real rates are indeed related. The demographic transition is the key factor. Figure 11 displays the equilibrium world real interest rate generated by the model along the transition described in the previous section. The graph decomposes the path of the real rate from the complete experiment in the marginal contributions of each exogenous force.

While crucial to capture the medium frequency variation in the trade balance, productivity growth alone cannot explain the decrease in the real return. A productivity boom raises savings, and hence reduces the real interest rate, by stimulating investment. However, as argued in the previous section, high productivity growth is also associated with high consumption, and hence low saving rates, because of the consumption smoothing motive. The overall effect is a fairly flat pattern of the world real interest rate.

The effect of demographics instead is perfectly consistent with a persistent drop in the equilibrium world real interest rate. In particular, the increase in life expectancy plays a crucial role to generate a reduction in the real return. As agents expect to live longer, the saving rate of both workers and retirees increases. Even though lower population growth rates would tend to reduce the long run world saving rate by increasing the dependency ratio, the interaction with longer life expectancy more than compensates this effect. The larger pool of retirees actually experiences on average a longer lifetime relative to before the transition. Therefore, the overall saving rate increases.[32]

Finally, from a global perspective, fiscal policy has been mildly expansionary during the last three decades. As any non-Ricardian model would suggest, the world real interest rate should have increased as a consequence of the fiscal expansion, due to the crowding out of private capital. This prediction is clearly at odds with the data. This result is important because it also sheds new light on the empirical (lack of) relationship between fiscal deficits and interest rates. Evans [1987] suggests that the absence of high interest rates in periods of substantial fiscal deficits, both for the US and at the international level, supports the hypothesis of Ricardian equivalence.[33] The analysis in this paper implies that, even if fiscal deficits do trigger a positive response of the real interest rate, the failure to control for demographic trends might substantially bias the results.

Importantly, low (and decreasing) real interest rates generally favor deficits by reducing the cost of borrowing and maintaining a low burden of outstanding debt. This mechanism constitutes an intertemporal valuation effect for both net foreign debt and government debt which increases the persistence of existing liabilities.

Figure 12 illustrates the valuation mechanism associated with low interest rates with two simple examples.

The left panel of figure 12 reports the response of the trade balance to an increase in life expectancy in the case of a two-country world and in the case of a small open economy which

[32] This result might seem at odds with the evidence on the negative saving rate in the US. Two observations however are in order. First, while the households' saving rate in the US has been historically declining for the last two decades and turned negative in the last couple of years, the private sector saving rate (households and firms) is much higher, around 15% of GDP. Second, part of the increase in consumption in the second part of the 1990s as accounted by official statistics might actually reflect unmeasured investment (McGrattan and Prescott [2006]).

[33] See also Elmendorf and Mankiw [1999] for a critical review of the literature on this theme.

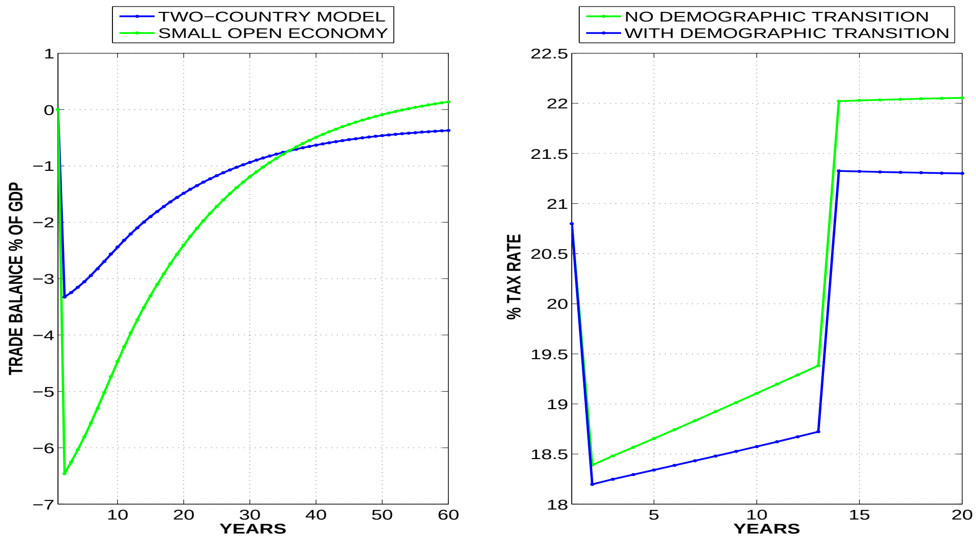

Figure 12: GENERAL EQUILIBRIUM EFFECTS

faces a constant interest rate. In the two-country model, the demographic shock hits the Foreign economy (γ^* increases from 0.8 to 0.85 over 10 years) and generates a trade deficit in the Home country according to the mechanism discussed in the previous section. For a small open economy, the same demographic transition generates a trade deficit in the rest of the world which, in this case, represents the Home country. The simulation shows that the general equilibrium effects associated with the interest rate absorb on impact half of the adjustment in the trade balance. Moreover, a decreasing interest rate implies that the trade balance is more persistent and that the rebalancing towards the new steady state occurs more gradually given the reduced burden of foreign debt.[34]

The right panel of figure 12 displays the adjustment of the domestic tax rate after a permanent fiscal expansion in the Home country (b increases from 26% to 60% over 12 years) with and without demographic transition. The demographic transition consists of an increase in the probability of surviving γ from 0.8 to 0.85 over 10 years. The low interest rate associated with the demographic transition reduces the burden of outstanding public debt. Therefore, during the expansion, the government of the Home country can afford a larger reduction of the tax rate. In the new steady state, the tax rate for an economy that experiences a

[34]In this experiment, the unilateral demographic shock leads the Home country to run a long run trade surplus that compensates the accumulation of net foreign debt experienced along the transition.

demographic transition is 1% lower than in the case of stationary demographics. This effect is indeed quantitatively relevant if one considers that the demographic transition occurs over a ten-year horizon and only leads to an increase of 1.5 years in life expectancy.

5 Conclusions

This paper has studied the contribution of productivity growth, demographic factors and fiscal policy to account for the evolution of the US external imbalances in the last three decades. The main results are that productivity growth accounts for most of the variation at medium frequency whereas demographic factors explain a substantial part of the low frequency component. Fiscal policy only plays a minor role because the differential fiscal stance of the US against the rest of the industrialized countries is minimal. International capital flows are generally directed towards relatively young and fast-growing countries.

The explicit consideration of demographic differentials turns out to be crucial. In particular, differentials in life expectancy help to explain the recent widening of the US trade deficit even under the assumption that productivity growth in the US and in the G6 will revert to trend in the next few years. Moreover, even in an environment in which fiscal policy entails non-Ricardian effects, the demographic transition limits the rebalancing impact of fiscal consolidations, like the one in the US during the 1990s, and amplifies the expansionary consequences of fiscal deficits, like the ones of the early 1980s and 2000s. Interestingly, fiscal deficits appear to be associated with the major swings in the long run trend of the US trade balance.

Finally, the interest rate generated by the model mimics well the declining trend observed in the data. The global excess of savings due to longer retirement periods leads to decreasing returns in financial markets which contribute to increase the persistence of deficits by reducing the burden of debt and lowering the cost of borrowing.

The general lesson that emerges from the analysis is that among industrialized countries capital should generally be expected to flow toward relatively young and rapidly growing economies. It should not be a surprise then that countries with these characteristics experience large and possibly persistent external imbalances.[35]

One important limitation of the model is the absence of an explicit role for social security systems. This aspect currently represents an important difference between the US and several

[35] Another example besides the US is Australia, with an average current account deficit at 4.3% of GDP over the last decade accompanied by GDP growth rates higher than 3%, population growth rate around 1% and life expectancy only slightly higher than the US. See also Backus, Henriksen, Lambert and Telmer [2006] for a comprehensive survey.

other countries, including the majority of the G6. In principle, it would be easy to include a simple mechanism of transfers to retirees that could be calibrated to the different sizes of the social security systems in the US and in the G6. On the other hand, it becomes crucial to make assumptions about the sustainability of such systems, and, most importantly, about the agents' expectations of possible reforms. In fact, one possible response to a longer expected lifetime horizon would be the increase of the working age. While this topic is present in the economic and political debate, most countries have yet to take explicit measures in this direction. Nor the data suggest any evidence of increased participation by the fraction of 65+ years old in the population. Nonetheless, this question deserves further research.

The analytical tractability of the model, on the other hand, should prove useful to investigate a number of additional problems. For example, the heterogeneity of the population might deliver interesting insights for the study of optimal fiscal policy. One specific application could be to analyze the welfare costs of fiscal rules, with particular reference to the current debate in the European Monetary Union. This question too is left for future research.

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A Analytical Appendix

The first appendix describes the solution of the individual maximization problem and the characterization of the steady state.

A.1 The Individual Problem

The first order condition with respect to asset accumulation for a retiree born in period j and retired in period k yields

$$\left(C_t^{jkr}\right)^{\rho-1} = \beta\gamma_{t,t+1}\left(V_{t+1}^{jkr}\right)^{\rho-1}\frac{\partial V_{t+1}^{jkr}}{\partial A_{t+1}^{jkr}}.$$

The Envelope condition is

$$\frac{\partial V_t^{jkr}}{\partial A_t^{jkr}} = \left(V_t^{jkr}\right)^{1-\rho}\left(C_t^{jkr}\right)^{\rho-1}\frac{R_{W,t}}{\gamma_{t-1,t}}.$$

The resulting Euler equation takes the standard form

$$C_{t+1}^{jkr} = \left(\beta R_{W,t+1}\right)^{\sigma} C_t^{jkr}, \tag{39}$$

where $\sigma \equiv (1-\rho)^{-1}$. I can solve for the policy function of a retiree by guessing that consumption is a fraction of total wealth

$$C_t^{jkr} = \xi_t\left(\frac{R_{W,t}A_t^{jkr}}{\gamma_{t-1,t}}\right), \tag{40}$$

where ξ_t is the marginal propensity to consume for a retiree. If I substitute [40] into [39], I obtain

$$\xi_{t+1}\left(\frac{R_{W,t+1}A_{t+1}^{jkr}}{\gamma_{t,t+1}}\right) = \left(\beta R_{W,t+1}\right)^{\sigma}\xi_t\left(\frac{R_{W,t}A_t^{jkr}}{\gamma_{t-1,t}}\right).$$

On the other hand, from the budget constraint of a retiree, one can substitute for consumption from [40] and express assets as

$$A_{t+1}^{jkr} = (1-\xi_t)\frac{R_{W,t}A_t^{jkr}}{\gamma_{t-1,t}}.$$

I can then rewrite the last equation as

$$\frac{R_{W,t+1}A_{t+1}^{jkr}}{\gamma_{t,t+1}} = (1-\xi_t)\,\frac{R_{W,t+1}}{\gamma_{t,t+1}}\left(\frac{R_{W,t}A_{t}^{jkr}}{\gamma_{t-1,t}}\right)$$

If I plug back the result into the Euler equation, it follows that the marginal propensity to consume follows a non-linear first order difference equation of the form

$$\xi_t = 1-\gamma_{t,t+1}\beta^{\sigma}R_{W,t+1}^{\sigma-1}\frac{\xi_t}{\xi_{t+1}}. \tag{41}$$

I can also conjecture that the value function is linear in consumption according to

$$V_t^{jkr} = \Delta_t^r C_t^{jkr}. \tag{42}$$

Then, it must be the case that

$$\left(\Delta_t^r C_t^{jkr}\right)^{\rho} = \left(C_t^{jkr}\right)^{\rho} + \beta\gamma_{t,t+1}\left(\Delta_{t+1}^r C_{t+1}^{jkr}\right)^{\rho}.$$

I substitute for consumption at $t+1$ from [39] and simplify the consumption term. The result is

$$\left(\Delta_t^r\right)^{\rho} = 1+\gamma_{t,t+1}\beta^{\sigma}R_{W,t+1}^{\sigma-1}\left(\Delta_{t+1}^r\right)^{\rho}.$$

From [41], it then follows that the proportionality term is

$$\Delta_t^r = \xi_t^{-\frac{1}{\rho}}.$$

The problem of a worker born in period j who is employed in period t is slightly more complicated but can be solved along the same lines. The first order condition for asset holding is

$$\left(C_t^{jw}\right)^{\rho-1} = \beta\left[\omega_{t,t+1}V_{t+1}^{jw} + (1-\omega_{t,t+1})\,V_{t+1}^{jr}\right]^{\rho-1}\left[\omega_{t,t+1}\frac{\partial V_{t+1}^{jw}}{\partial A_{t+1}^{jw}} + (1-\omega_{t,t+1})\,\frac{\partial V_{t+1}^{jr}}{\partial A_{t+1}^{jw}}\right].$$

The envelope conditions are

$$\frac{\partial V_t^{jw}}{\partial A_t^{jw}} = \left(V_t^{jw}\right)^{1-\rho}\left(C_t^{jw}\right)^{\rho-1}R_{W,t}$$

and

$$\frac{\partial V_t^{jr}}{\partial A_t^{jw}} = \frac{\partial V_t^{jr}}{\partial A_t^{jr}} \frac{\partial A_t^{jr}}{\partial A_t^{jw}} = \frac{\partial V_t^{jr}}{\partial A_t^{jr}},$$

where the last result is driven by the fact that individuals are risk neutral with respect to labor income fluctuations and hence choose the same asset profile independently of the employment status. I can then use the Envelope condition for retirees with the further adjustment that workers do not turn their wealth over to the mutual fund and hence do not receive the additional return that compensates for the probability of death. The Euler equation becomes

$$\begin{aligned} \left(C_t^{jw}\right)^{\rho-1} &= \beta R_{W,t+1} \left[\omega_{t,t+1} V_{t+1}^{jw} + (1-\omega_{t,t+1}) V_{t+1}^{jr}\right]^{\rho-1} \\ & \left[\omega_{t,t+1} \left(V_{t+1}^{jw}\right)^{1-\rho} \left(C_{t+1}^{jw}\right)^{\rho-1} + (1-\omega_{t,t+1}) \left(V_{t+1}^{jr}\right)^{1-\rho} \left(C_{t+1}^{jr}\right)^{\rho-1}\right] \end{aligned} \tag{43}$$

I conjecture that the value function for a worker has the same form of [42]

$$V_t^{jw} = \Delta_t^w C_t^{jw}. \tag{44}$$

I substitute the guess back into the Euler equation [43] together with [42]

$$\begin{aligned} \left(C_t^{jw}\right)^{\rho-1} = \beta R_{W,t+1} & \left[\omega_{t,t+1} \Delta_{t+1}^w C_{t+1}^{jw} + (1-\omega_{t,t+1}) \Delta_{t+1}^r C_{t+1}^{jr}\right]^{\rho-1} \\ & \left[\omega_{t,t+1} \left(\Delta_{t+1}^w\right)^{1-\rho} + (1-\omega_{t,t+1}) \left(\Delta_{t+1}^r\right)^{1-\rho}\right]. \end{aligned}$$

I further define the ratio of the quantity

$$\Omega_t \equiv \omega_{t-1,t} + (1-\omega_{t-1,t}) \left(\frac{\Delta_t^r}{\Delta_t^w}\right)^{1-\rho}.$$

The Euler equation can then be rearranged to give

$$\omega_{t,t+1} C_{t+1}^{jw} + (1-\omega_{t,t+1}) \left(\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w}\right) C_{t+1}^{jr} = \left(\beta \Omega_{t+1} R_{W,t+1}\right)^{\sigma} C_t^{jw}. \tag{45}$$

I guess that the decision rule for a worker is

$$C_t^{jw} = \pi_t \left(R_{W,t} A_t^{jw} + H_t^{jw}\right). \tag{46}$$

From [40], I can also notice that the decision rule for a retiree who just abandoned the labor force is

$$C_t^{jr} = \xi_t R_{W,t} A_t^{jw}.$$

The variable H_t^{jw} represents the present discounted value of current and future human wealth and is defined in section 3. By substituting the guess into the Euler equation, I obtain

$$\omega_{t,t+1}\pi_{t+1}\left(R_{W,t+1}A_{t+1}^{jw} + H_{t+1}^{jw}\right) + (1-\omega_{t,t+1})\left(\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w}\right)\xi_{t+1}R_{W,t+1}A_{t+1}^{jw}$$
$$= \left(\beta\Omega_{t+1}R_{W,t+1}\right)^{\sigma}\pi_t\left(R_{W,t}A_t^{jw} + H_t^{jw}\right)$$

I can rearrange the last expression to yield

$$\omega_{t,t+1}\left(A_{t+1}^{jw} + \frac{H_{t+1}^{jw}}{R_{W,t+1}}\right) + (1-\omega_{t,t+1})\left(\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w}\right)\epsilon_{t+1}A_{t+1}^{jw}$$
$$= \left(\beta\Omega_{t+1}\right)^{\sigma}R_{W,t+1}^{\sigma-1}\frac{\pi_t}{\pi_{t+1}}\left(R_{W,t}A_t^{jw} + H_t^{jw}\right),$$

where $\epsilon_t \equiv \xi_t/\pi_t$. I use the definition of Ω_t to rewrite

$$A_{t+1}^{jw} + \frac{\omega_{t,t+1}H_{t+1}^{jw}}{\Omega_{t+1}R_{W,t+1}} = \beta^{\sigma}\left(\Omega_{t+1}R_{W,t+1}\right)^{\sigma-1}\frac{\pi_t}{\pi_{t+1}}\left(R_{W,t}A_t^{jw} + H_t^{jw}\right)$$

From the budget constraint of a worker and the guess [46], it is possible to see that

$$A_{t+1}^{jw} + \frac{\omega_{t,t+1}H_{t+1}^{jw}}{\Omega_{t+1}R_{W,t+1}} = (1-\pi_t)\left(R_{W,t}A_t^{jw} + H_t^{jw}\right).$$

If I substitute this result back into the Euler equation, I can show that the marginal propensity to consume for a worker evolves according to

$$\pi_t = 1 - \beta^{\sigma}\left(\Omega_{t+1}R_{W,t+1}\right)^{\sigma-1}\frac{\pi_t}{\pi_{t+1}} \tag{47}$$

Finally, I can verify the validity of the original guess for the value function [44]. The following expression must be satisfied

$$\left(\Delta_t^w C_t^{jw}\right)^{\rho} = \left(C_t^{jw}\right)^{\rho} + \beta\left[\omega_{t,t+1}\Delta_{t+1}^w C_{t+1}^{jw} + (1-\omega_{t,t+1})\Delta_{t+1}^r C_{t+1}^{jr}\right]^{\rho}.$$

I can use expression [45] to obtain

$$(\Delta_t^w)^\rho = 1 + \beta^\sigma \left(\Omega_{t+1} R_{W,t+1}\right)^{\sigma-1} \left(\Delta_{t+1}^w\right)^\rho .$$

Expression [47] then implies that

$$\Delta_t^w = \pi_t^{-\frac{1}{\rho}}.$$

I can also observe that $\Delta_{t+1}^r / \Delta_{t+1}^w = \epsilon_t^{-(1/\rho)}$ and that

$$\Omega_t \equiv \omega_{t-1,t} + \left(1 - \omega_{t-1,t}\right) \epsilon_t^{\frac{1}{1-\sigma}}.$$

This concludes the characterization of the individual problem in the model.

A.2 The Steady State

In this section, I characterize the stationary symmetric steady state of the model. Symmetry implies that the stock of net foreign assets and the trade balance are zero.

For retirees and workers, it suffices to describe the steady state value of their respective marginal propensities to consume. For a given level of the world interest rate, the marginal propensity to consume of each cohort is given by

$$\xi = 1 - \gamma \beta^\sigma R_W^{\sigma-1} \tag{48}$$

and

$$\pi = 1 - \beta^\sigma \left(\Omega R_W\right)^{\sigma-1}, \tag{49}$$

where

$$\Omega = \omega + (1 - \omega)\, \epsilon^{\frac{1}{1-\sigma}} \tag{50}$$

and

$$\epsilon = \xi / \pi. \tag{51}$$

At this point, I prove two results related to the individual marginal propensities to consume.

First, I want to show that in steady state, $\epsilon > 1$. To see this, suppose the contrary is true ($\epsilon \leq 1$). Taking the ratio of [48] and [49] gives

$$\gamma \geq \Omega^{\sigma-1} = \left[\omega + (1 - \omega)\, \epsilon^{\frac{1}{1-\sigma}}\right]^{\sigma-1}.$$

Since $\gamma < 1$, the last condition is satisfied only if $\epsilon > 1$ which constitutes a contradiction.

For a given level of the world interest rate, the system of equations [48]-[51] determine the solution of the individual marginal propensities to consume and for the variables Ω and ϵ.

Second, I show that in steady state, for a given interest rate, the probability of surviving has a negative effect on the marginal propensity to consume for workers. To see this, I can rewrite expression [49] making use of [48], [50] and [51] as

$$\pi = 1 - \beta^{\sigma} \left\{ \left[\omega + (1-\omega) \left(\frac{1 - \gamma \beta^{\sigma} R_W^{\sigma-1}}{\pi} \right)^{\frac{1}{1-\sigma}} \right] R_W \right\}^{\sigma-1}.$$

In order to sign the effect of γ on π (for a given R_W), I can just apply the implicit function theorem to the function

$$F(\gamma, \pi) = \pi - 1 + \beta^{\sigma} \left[\Omega(\gamma, \pi) R_W \right]^{\sigma-1}.$$

Some algebra allows to obtain the following partial derivatives

$$\frac{\partial F}{\partial \gamma} = (1-\omega) \beta^{2\sigma} \Omega^{\sigma-2} R_W^{\sigma-1} \epsilon^{\frac{\sigma}{1-\sigma}} / \pi > 0$$

and

$$\frac{\partial F}{\partial \pi} = 1 + (1-\omega) \beta^{\sigma} \Omega^{\sigma-2} R_W^{\sigma-1} \epsilon^{\frac{1}{1-\sigma}} / \pi > 0.$$

It then follows that

$$\frac{\partial \pi}{\partial \gamma} = -\frac{\partial F(\gamma, \pi) / \partial \gamma}{\partial F(\gamma, \pi) / \partial \pi} < 0,$$

which implies a negative effect of population aging on the marginal propensity to consume for workers, holding fixed the world interest rate. I now return to the description of the steady state equations.

From the production function it is easy to see that

$$y = k^{1-\alpha} \tag{52}$$

The firm first order condition for capital pins down the capital-ouput ratio for a given level of the world return

$$R_W = (1-\alpha) \frac{y}{k} + (1-\delta).$$

The last two expressions combined deliver a solution for capital per unit of efficient labor

$$k = \left[\frac{1-\alpha}{R_W - (1-\delta)}\right]^{\frac{1}{\alpha}}. \tag{53}$$

Investment is a constant share of the capital stock to compensate for depreciation and population growth

$$i = (x + n + \delta)\, k. \tag{54}$$

Again, for a given level of the world interest rate, the solution for capital, output and investment can be retrieved from the system of equations [52]-[54].

As a second step, I consider aggregate consumption and the distribution of assets. The equation for aggregate consumption [20] yields

$$c = \pi \left\{ \left[1 + (\epsilon - 1)\,\lambda\right] R_W a + h \right\}. \tag{55}$$

In steady state, the distribution of wealth across cohorts is determined by

$$\lambda = \frac{(1-\omega)}{(1+x+n) - \omega\,(1-\xi)\,R_W}.$$

The expression for λ can also be readjusted as to make explicit the dependence on the ratio of retirees to workers in the population

$$\lambda = \frac{(1+n-\gamma)\,\psi}{(1+x+n) - \gamma\omega\,(\beta R_W)^{\sigma}}, \tag{56}$$

where

$$\psi = \frac{1-\omega}{1+n-\gamma}. \tag{57}$$

Given that the dependency ratio is only a function of exogenous parameters, the steady state distribution of wealth only depends on the level of the interest rate. The capitalized value of human wealth that enter the steady state consumption function is

$$h = \frac{(\alpha - \tau)\,y}{1 - \omega\,(1+x)\,/\Omega R_W}. \tag{58}$$

In a symmetric steady state, net foreign assets are equal to zero so that total assets are given by

$$a = k + by. \tag{59}$$

From the government budget constraint, taxes equal expenditure plus the net interest rate payment on the stock of steady state debt

$$\tau = g + [R_W - (1 + x + n)]\, b. \tag{60}$$

The system of equations [55]-[60] determines the demand side of the model in steady state for a given interest rate, which can be derived from the resource constraint

$$(1 - g)\, y = c + i, \tag{61}$$

given the steady state value of the stock of government debt b and of government spending g.

The algorithm of solution for the steady state corresponds to the one described in the text and the practical implementation is carried out using DYNARE.

B Data Appendix

In this appendix, I provide the details about the data used in this paper and the methodology to construct aggregates for the G6.

The series for the US trade balance (versus the rest of the world and versus the G6) are from the Bureau of Economic Analysis (International Transactions) and are expressed in percentage of GDP (same source, NIPA Tables). Since bilateral trade data for France, Germany and Italy are available only starting in 1986, I proxy the G6 series using Western Europe plus Canada and Japan for the period $1960 - 1966$ and the EU 6 (Belgium, France, Germany, Italy, Luxembourg and Netherlands) plus Canada, Japan and United Kingdom for the period $1966 - 1985$. All series are expressed in current US dollars. The trend for the US trade balance vis-a-vis the G6 in percentage of GDP is the long run component of the Hodrick-Prescott filtered series.

The data used to construct real GDP growth rates per capita are from the Groningen Growth and Development Centre (GGCD). All values are expressed in PPP at 2002 US dollars. Since total GDP is expressed in the same unit for all countries, I create a series for the G6 GDP by simply summing GDP across countries. The data on average annual hours are also from GGDC.

The data on output and investment used to construct TFP are from the World Bank (World Development Indicators). The data for labor (civilian employment) are from GGDC. The series for capital stock is derived from investment using the perpetual inventory method, given the steady state depreciation and the average GDP growth rate over the sample period.

The data on historical and projected demographic trends are from United Nations World Population Prospects: The 2004 Revision. Although availability dates back to 1950, I use data and projections for the period $1970-2050$. Population growth rates for the G6 are computed after aggregating total population across countries. The years of life expectancy are a population-weighted average of the national counterparts. The data for life expectancy by age are available for selected years from the United Nations Demographic Yearbook.

The data for fiscal policy are from the International Monetary Fund (World Economic Outlook Database) and cover the period $1980-2004$. I use PPP conversion factors from the same source to express the levels of net government debt and total government deficit in each G6 member in 2000 US dollars. I then aggregate across countries and divide by the series of GDP constructed along the same lines.

Finally, the data used to construct the world interest rate are from the International Monetary Fund (International Financial Statistics) for the period $1980-2004$. For each country, I compute the ex-post real interest rate by subtracting the realized CPI inflation rate from the available long term (generally 10-year) government bond yield. The "world" interest rate is computed using population weights. Aggregation using GDP weights delivers almost identical results.

C Additional Graphs and Tables

Table 1. INITIAL SYMMETRIC STEADY STATE

n	1%	POPULATION GROWTH RATE
$(1-\omega)^{-1}$	45	AVG. WORKING PERIOD (YEARS)
$(1-\gamma)^{-1}$	5	AVG. RETIREMENT PERIOD (YEARS)
β	1	SUBJECTIVE DISCOUNT FACTOR
σ	0.5	ELASTICITY OF INTERTEMPORAL SUBSTITUTION
α	2/3	LABOR SHARE
δ	6%	DEPRECIATION RATE
x	1.3%	GROWTH RATE OF TECHNOLOGY
g	20%	GOVERNMENT SPENDING (% OF GDP)
b	26%	GOVERNMENT DEBT (% OF GDP)

Table 2. FINAL STEADY STATE

	US	G6	
n	0.2%	0.2%	POPULATION GROWTH RATE
$(1-\gamma)^{-1}$	15	18	AVG. RETIREMENT PERIOD (YEARS)
b	60%	60%	GOVERNMENT DEBT (% OF GDP)

Table 3. LIFE EXPECTANCY

COUNTRY	YEAR	AT BIRTH		AT 20	
		MALE	FEMALE	MALE	FEMALE
UNITED STATES	1975	68.7	76.5	50.8	58.1
	1983	71	78.3	52.6	59.5
	1998	73.8	79.5	55	60.3
	2003	74.8	80.1	55.8	60.9
CANADA	1975-1977	70.19	77.48	52.09	58.95
	1985	73.04	79.73	54.27	60.65
	1992	74.6	80.9	55.6	61.7
	2003	77.4	82.4	58.2	63
FRANCE	1974	69	76.9	50.8	58.3
	1988	72.33	80.46	53.53	61.36
	1993-1995	73.64	81.7	54.57	62.39
	2003	75.9	82.9	56.6	63.4
GERMANY	1973-1975	68.04	74.54	50.59	56.49
	1999	74.7	80.7	55.5	61.3
	2002-2004	75.9	81.5	56.6	62.1
ITALY	1974-1977	69.69	75.91	51.99	57.75
	1995	74.6	81	55.6	61.7
	2002	77.1	83	57.8	63.5
JAPAN	1975	71.76	76.95	53.31	58.11
	1985	74.84	80.46	55.8	61.18
	1995	76.36	82.86	57.15	63.46
	2004	78.6	85.6	59.2	66
UK	1987-1989	72.42	78.03	53.63	58.94
	2000	75.3	80.1	56.1	60.8

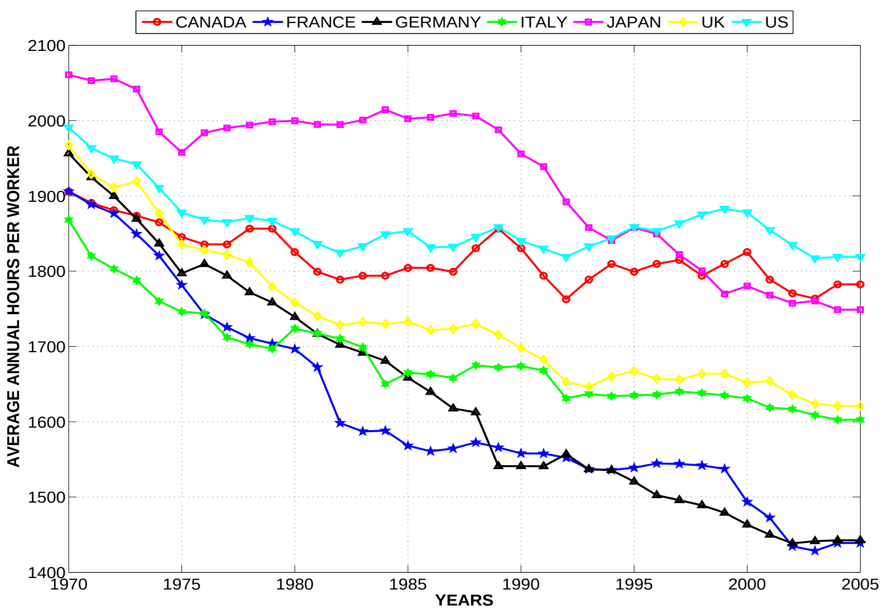

Figure 13: HOURS IN G7 COUNTRIES

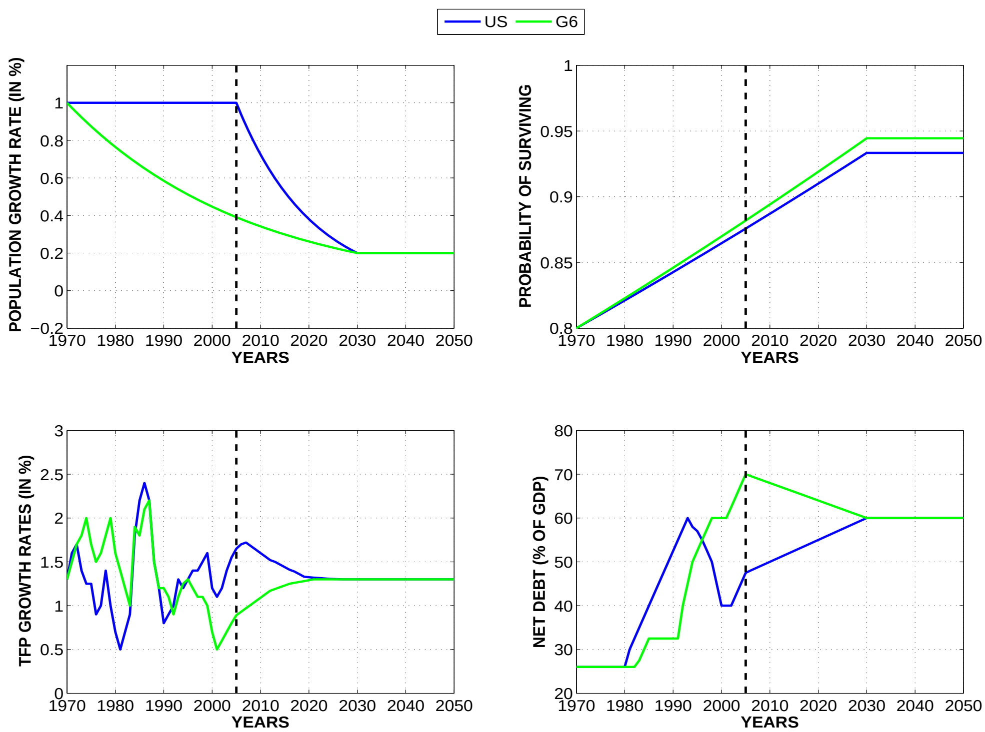

Figure 14: THE EXOGENOUS TRANSITION