The Impact of Payroll Taxes upon Employment, Wages, the Unemployment Rate and Informality - Theory and Simulations for Colombia

by

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Abstract

Recent models of the Informal Sector emphasize the free choice of sectors by firms or workers and define informal firms as those that do not pay taxes. One dominant group of models examines identical firms producing a homogeneous final good, where legal firms pay taxes and receive a public good, while non-legal firms do not pay taxes and receive a smaller amount of the public good. These models may be consistent with a stable equilibrium where there are both Formal (legal) and Informal (non-legal) firms, as observed in developing countries.

This paper first presents a generic and somewhat general version model with public goods and taxation ("g-t model"), finding that most equilibria are unstable, so that all firms choose to be either legal or non-legal, which is inconsistent with the observed facts for developing countries. However, when the government finances detection of non-legal firms and fines those firms, stable 'internal' solutions may dominate. This requires a probability of detection that falls as the number of legal firms increases.

A general model of detection, penalties and their public finance is presented and analyzed, finding that the probability of detection often, though not always, falls with the number of legal firms. The role of exogenous financing of detection is emphasized, where greater exogenous financing reduces informality and if high enough constitutes complete prevention, inducing all firms to be formal. This is both a *positive* and *normative* finding, potentially explaining the absence of informal firms in some countries, and the presence in others, while also constituting a policy instrument for eradicating informality.

As in the g-t model, the Loayza model of informality requires a probability of detection that falls as the number of legal firms rises for a stable internal solution. Thus, this general model of detection partially validates the Loayza model. It is also observed that in the Loayza model, governments may eliminate informality without penalties, by lowering the tax rate sufficiently. This also has *positive* and *normative* implications.

The final section of the paper applies the foregoing results to model firms that face the choice between paying payroll taxes (legality) and evading payroll taxes (non-legality). Non-legal firms face an expected penalty. Because endogenous financing of detection, with revenues from detected and fined non-legal firms, generally leads to a probability of detection that rises with the share of non-legal ("informal") firms, the model is consistent with a Stable Internal Solution, where legal and non-legal firms coexist, as observed in most developing countries, and to a lesser degree in some developed countries. We show that, as is consistent with intuition, rising payroll taxes lead to greater non-compliance, as in the g-t-p and L96 models. And, as in the earlier models, given tax rates, higher exogenous taxation or greater monitoring efficiency leads to greater compliance.

We then couple this structure with a model of aggregate employment, wages and unemployment, in the context of the two-sector framework. We find that higher taxes lead to higher equilibrium unemployment rate. Higher taxes increase "informality" and the equilibrium unemployment rate. In the final section, we present simulations of the actual increases in Colombian payroll tax rates upon the equilibrium unemployment rate. We find that the increases in payroll rates over the 1990-2008 (planned) period may have contributed to an increase of approximately three percent in the equilibrium unemployment rate, along with having raises non-compliance, or "informality".

I. INTRODUCTION

Many recent theories of the Informal Sector have turned from non-competitive segmentation theories, such as Todaro (1969), to competitive models emphasizing the payment of taxes and adherence to norms dictated by the State, where workers and firms freely choose to be legal or non-legal. Here the "Informality Sector" is understood to refer to non-legal firms that do not pay taxes or adhere to State norms. This focus on competitive models reflects growing evidence against segmentation theories [e.g. Magnac (1991), MacIsaac and Rama (1997), Heckman and Hotz (1986)].

Recent competitive theories of the Informal Sector emphasize the role of public goods, often in the absence of any penalties for non-legal behavior. One important competitive model is that of Marcouiller and Young (1995). That model assumes that legal firms produce one good and non-legal firms produce a different good. They seek to show how the State may be empowered to impose high taxes and extract high levels of rents, without resorting to persecuting and penalizing non-legal firms. While rising tax rates induce firms to move to the non-legal sector, this is limited because this leads to high relative supplies of the non-legal good. And this rising relative supply of the non-legal good depresses the prices of that good, vis-à-vis the legal good. This constitutes a self-regulating mechanism that determines the size of the non-legal sector and limits its growth in the face of a "predatory" State.

Another group of models assumes a homogenous final good. Grossman and Yoshiaki (2003), for example, present a model of firms producing a homogeneous final good, requiring a public good for production. Non-legal firms produce a substitute public good. This model, assumes that firm's sectoral choices are governed by differences in exogenous endowments of capital and the structure of taxation. Another often cited model is Loayza (1996). That model is similar to the Grossman-Yoshiaki model, but differs in that it assumes that firms are identical and that non-legal firms have access to some part of the public good, because the public good is not entirely "excludable".

The Loayza model seeks to explain the simultaneous presence of both legal and non-legal firms in a stable equilibrium. To achieve this result, the model assumes that non-legal firms face penalties and that the State dedicates resources towards the detection of non-legal firms, avoiding tax payments. The model crucially assumes a structure of the probability of detection that is neither convincingly motivated nor modeled. Moreover, the model also assumes that the public good is strictly rival.

This paper seeks to explain the coexistence of legal and non-legal firms, where legality consists in the payment of taxes. In the vernacular of the paper, this stable coexistence is a stable internal solution (SIS), where in equilibrium the fraction of legal (non-legal) firms lies between zero and one and is stable. The initial framework employed is similar to that in Loayza(1996), "L96", where identical firms choose between legality and non-legality. This paper focuses upon competitive models of the Informal Sector, where firms are identical and produce a homogeneous final good.

We first develop a generic model of tax payment and public goods, deriving the conditions for a SIS and finding that this model does not generally satisfy those conditions. Thus, in Section II develops a simple generic model of public goods (g) and taxation (t), the g-t model. It models the public finance of public good production and offers a general formulation that allows for variations in three key parameters: the returns to scale in the production of the public good; the degree of rivalry of the public good; and the degree of excludability of the public good. Analysis of different parameter structures reveals some stable internal equilibria, but a preponderance of unstable equilibria. As such, the g-t model cannot adequately explain the dominant facts for developing countries, where both legal and non-legal firms coexist (a stable internal solution, or SIS). These results point towards the need for endogenous detection and penalties as a key to understanding SIS in some countries. Section II ends by showing how the existence of endogenous penalties, where the probability of detection varies with the size of the non-legal sector, as assumed in L96, can explain the coexistence of legal and non-legal firms (the g-t-p model).

Because L96 does not provide a conceptual basis or model of this assumption regarding the probability of detection, we next generate a simple, general model of monitoring and the detection probability. Section III develops a model of detection and penalties, and their public finance. Detection is assumed to always be partly self-financed, from fines on non-legal firms. In addition, detection may be financed by general tax revenues. When detection is partially self-financed, the condition for a SIS, that the detection probability rises with the size of the non-legal sector, is generally satisfied. This provides a conceptual foundation for the determinants of size of the non-legal sector, and provides insights into potential normative policy tools to influence the size of that sector, as well as positive analysis of data on the size of the "informal sector", within countries over time and over countries. Increases in general tax funds dedicated to taxation shifts the probability of detection function upwards and can reduce or even eliminate non-legal firms. Increased monitoring efficiency or fines have similar impacts.

Section IV synthesizes another model of taxation, public goods and detection, the Loayza(1996) model, "L96", and applies the foregoing detection model L96. The L96 model, though similar in spirit to the g-t model, is entirely different in its specification of the role of the public good and the tax structure. Despite this difference, without detection, this model collapses and all firms choose to either legal or non-legal. A SIS would not exist, thus validating the qualitative results of the prior, simpler models. Then we show how the foregoing detection model can be applied to the L96 model, to obtain essentially the same results as in L96, underpinned by a simple, rigorous theory of detection.

In Section V the foregoing conceptual framework and probability model is applied to modeling the impact of payroll taxes on the size of the non-legal sector and the unemployment rate. There firms that face the choice between paying payroll taxes (legality) and evading payroll taxes (non-legality). Non-legal firms face an expected penalty. Because endogenous financing of detection, with revenues from detected and fined non-legal firms, generally leads to a probability of detection that rises with the share of non-legal ("informal") firms, the model is consistent with a Stable Internal Solution, where legal and non-legal firms coexist, as observed in most developing countries, and to a lesser degree in some developed countries. We show that, as is consistent with intuition, rising payroll taxes lead to greater non-compliance, as in the g-t-p and L96 models. And, as in the earlier models, given tax rates, higher exogenous taxation or greater monitoring efficiency leads to greater compliance.

We then apply this structure to a model of aggregate employment, wages and unemployment, in the context of the two-sector framework. We find that higher taxes lead to higher equilibrium unemployment rate. Higher taxes increase "informality" and the equilibrium unemployment rate. In the final section, we present simulations of the actual increases in Colombian payroll tax rates upon the equilibrium unemployment rate. We find that the increases in payroll rates over the 1990-2008(planned) period may have contributed to an increase of approximately three percent in the equilibrium unemployment rate, along with having raises non-compliance, or "informality".

II. THE GENERIC g – t MODEL OF PUBLIC GOODS, TAXATION AND INFORMALITY (NON-LEGALITY)

Some models attempt to explain "informality" assuming that legal firms pay taxes and received public goods, in the absence of detection and penalties for non-legal firms. This section presents a simple, generic model of the finance of public good production, and examines whether such a model can typically explain the coexistence of legal and non-legal firms in equilibrium, or a SIS. This model allows for variations in three key parameters: the returns to scale in the production of the public good, α (as alpha rises, returns to scale increase); the degree of rivalry of the public good, *m* (as *m* falls from 1, rivalry falls); and the degree of excludability of the public good, γ (as gamma rises from 0, excludability falls). As with other models discussed in this paper, we assume one final good and identical technologies across firms.

Legal firms pay a lump sum tax and receive a share of total public good production, G, equal to "g". Non-legal firms do not pay taxes, but typically receive a smaller share of the public good, where this is referred to as partial-excludibility of the public good.

Total profits V_L and V_N in legal and non-legal sectors, respectively, are given by:

$$V_L = (pQ - wE - rK) + (g - t)\,, \tag{2.1}$$

and

$$V_N = (pQ - wE - rK) + \gamma g\,. \tag{2.2}$$

Where '*g*' is the public good per firm in the legal sector. γg is the per firm public good in the non-legal sector. If gamma is zero, non-legal firms receive no public goods.

$$g = \frac{G}{\left[L + \gamma N\right]^m}\,. \tag{2.3}$$

Here,

L = number of legal firms;
N = number of non-legal firms;
γ = coefficient that indicates the degree of exclusion of the non-legal firms[1], $0 \le \gamma \le 1$;
m = coefficient that indicates the degree of rivalry of the public good[2], $0 \le m \le 1$.

Production of the aggregate public good, *G*, is a function of available funding, which equals the lump sum per firm tax, *t*, times the number of legal firms. Thus,

$$G = \left(Lt\right)^{\alpha}, \tag{2.4}$$

with $\alpha > 0$ the coefficient that indicates the returns to scale and where *t* is a lump-sum tax per legal firm, $t \ge 1$. The per firm public good for legal firms is then:

$$g = \frac{\left(Lt\right)^{\alpha}}{\left[L + \gamma N\right]^m}\,. \tag{2.5}$$

Assuming that the total number of firms is constant and normalizing such that $L, N \in \left[0,1\right]$ and $L + N = 1$, we can rewrite the (2.5) as:

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[1] A value of $\gamma = 0$ indicates that the public good is completely excludable; $\gamma = 1$ implies that the public good is completely non-excludable.

[2] When $m = 0$ the public good is non-rival; when $m = 1$ the public good is rival.

$$g = \frac{(Lt)^{\alpha}}{\left[L(1-\gamma)+\gamma\right]^{m}}\ ;\ g = g(L) \tag{2.6}$$

Firms freely choose to be legal or non-legal. The condition of an internal equilibrium (where some firms are legal and some firms are non-legal) is one of profits equalization between legal and non-legal sectors. From 2.1 and 2.2, this implies that:

$$g - t = \gamma g \text{. Equivalently, } g = \frac{t}{1-\gamma}. \tag{2.7}$$

We will denote the right-hand side expression, $\frac{t}{1-\gamma}$, by $\tilde{t}$.

If $g > \tilde{t}$ then profits in the legal sector are higher than profits in the non-legal sector and vice versa. In equilibrium, the fraction of legal (and non-legal) firms depends on the function *g(L).* If the slope of *g(L)* is negative, then there may exist an internal solution, *L**, that is stable, or "SIS" (see Figure 1, column 1). At any initial distribution of firms below (above) *L**, profits in the legal sector are higher (lower) than profits in the non-legal sector, because $g > \tilde{t}$ ($g < \tilde{t}$). Therefore more firms will choose to be legal (non-legal). This continues until reaching the point *L**, where firms are indifferent between being legal or being non-legal.

On the other hand, if the function *g(L)* has a positive slope, while there could be an internal equilibrium, it will be unstable (see Figure 1, column 2). Then, if L falls below *L**, $\tilde{t}$ will exceed the per firm public good, *g*, so that *L* will fall to zero. And if *L* were to rise above *L**, *g* will exceed $\tilde{t}$, so that *L* increases to one. Thus, when *g'(L)*>0, while there may exist an internal solution, it will be unstable. It will collapse to either of two cases, where *L*=0 or *L*=1.

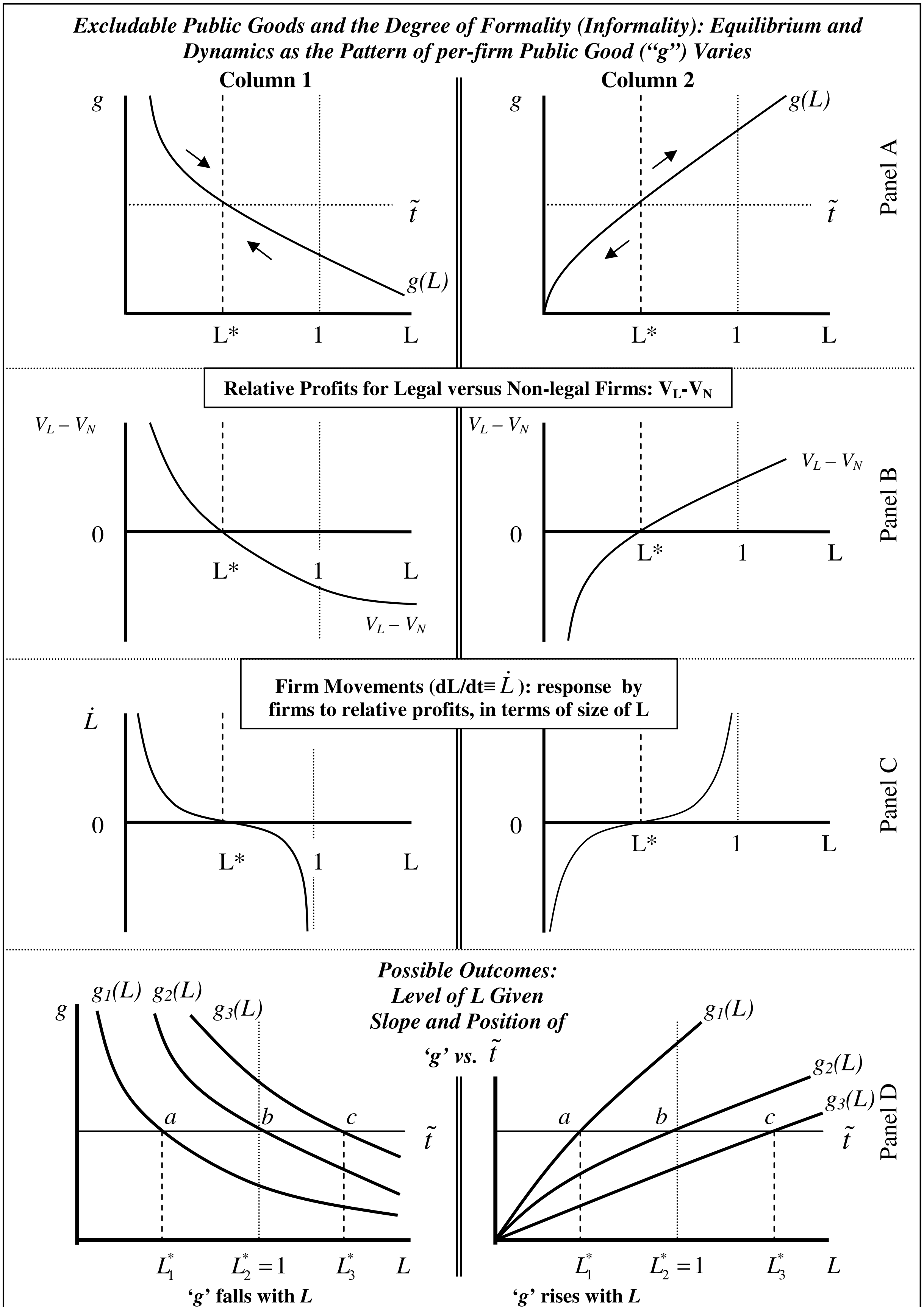

Figure 1. Graphic analysis of the fraction of firms that choose to be legal in equilibrium.

The slope of *g(L)* is given by:

$$\frac{\partial g}{\partial L}=\frac{\alpha L^{\alpha-1}t^{\alpha}[L(1-\gamma)+\gamma]^{m}-L^{\alpha}t^{\alpha}m[L(1-\gamma)+\gamma]^{m-1}(1-\gamma)}{[L(1-\gamma)+\gamma]^{2m}}\,. \qquad (2.8)$$

Because the denominator is typically positive[3], the sign of this derivative depends on the sign of the numerator:

$$\frac{\partial g}{\partial L}\gtreqless 0 \text{ as } \alpha L^{\alpha-1}t^{\alpha}[L(1-\gamma)+\gamma]^{m}\gtreqless L^{\alpha}t^{\alpha}m[L(1-\gamma)+\gamma]^{m-1}(1-\gamma)\,. \qquad (2.9)$$

This expression simplifies to:

$$\alpha\gamma\gtreqless L(1-\gamma)(m-\alpha)\,.^{4} \qquad (2.10)$$

The synthesis for the sign of the derivative *g'(L)* is reported in Table 1.

Table 1. Sign of the derivative *g'(L)*

α versus m	**Values of γ**	
	$\gamma=0$ (1)	$\gamma\in(0,1)$ (2)
$\alpha>m$ (includes m=0)	(+)	(+)
$\alpha=m$	0	(+)
$\alpha<m$	(-)	$g'(L)\gtreqless 0$ as $\alpha\gamma\gtreqless L(1-\gamma)(m-\alpha)$

Thus, in general, in the g-t model, the conditions for a SIS do not generally exist. There is only one case where a SIS may exist.

Simulations show that the possible patterns for *g(L)* when we have partial excludability and where alfa is less then m. These simulations are summarized in Figure 2[5], where we see that in this case there is no SIS.

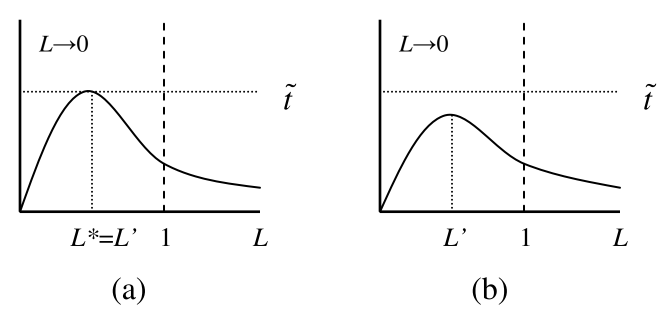

Figure 2. Possible patterns for the public good per firm (*g*) when the public good is partially excludable ($\gamma\in(0,1)$) and $\alpha<m$. No Stable Internal Solution results.

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[3] It is zero if $\gamma=L=0$.

[4] If we do not normalize L and N, the restriction for the derivative being positive or negative is $\alpha\gamma\gtreqless\frac{L}{\overline{L}}(1-\gamma)(m-\alpha)$, where $\overline{L}=L+N$. This restriction is equal to that when *L* and *N* are normalized.

[5] Note that in simulations, the case where *g(L)* is non-monotonic and $\tilde{t}$ crosses below its maximum (twice) is not observed.

Thus, there is only one possible case where a SIS could exist, and this case is unlikely in practice, because it assumes that the public good is entirely excludable, and in practice is it likely that some "leakage" of the public good to non-tax paying firms.

Summary and Interpretation

The foregoing g-t model is one where legal firms pay taxes, receive some of the public good and where non-legal firms do not pay taxes and receive less of the public good (none or a smaller share than legal firms). The model is somewhat general in that it contemplates differing returns to scale in the production of the public good, differing levels of rivalry of the public good and differing degrees of excludability of the public good. In theory, no penalties are required for a stable internal solution, with both legal and non-legal firms, to exist.

However, analysis of this model yields few cases where a stable internal solution exists. Complex interactions between returns to scale, rivalry, and excludability exist. In the empirically important case of a partially excludable public good, in general there is no stable internal solution. These results are at odds with the reality of most developing countries, where legal and non-legal firms coexist.

However, if we add detection and penalties for non-legal firms to this model, the modified model can lead to stable internal solutions. Moreover, while many authors (notably Grossman and Yoshiaki, 2003) have focused on models with public goods without penalties, in practice most governments do dedicate resources to detecting and penalizing non-legal firms.

The g-t Model with Detection and Penalties: g-t-p Model

Here we modify the basic g-t model by introducing detection and penalties. If the State dedicates resources to detection, so that there is a positive probability of detecting non-legal firms, ρ, and fines detected non-legal firms in the amount *M*, then the equalization of profits between legal and non-legal sectors implies:

$$g - t = \gamma \cdot g - \rho M \ , \tag{2.11}$$

or

$$\tilde{g} \equiv \left(g + \frac{\rho M}{(1-\gamma)} \right) = \left(\frac{t}{(1-\gamma)} \right) \equiv \tilde{t} \ . \tag{2.12}$$

This is the same condition for equilibrium as before, except for that instead of '*g*' on the left hand side we have $\tilde{g}$, which is equal to '*g*' plus the term $\frac{\rho M}{(1-\gamma)}$. It follows directly that if the derivative of the probability of detection with respect to the fraction of legal firms, or $\frac{\partial \rho}{\partial L}$, is negative and large relative to the derivative of '*g*' with respect to *L*, then the derivative of $\tilde{g}$ with respect to *L* can be negative. Then if there is an internal solution it will be stable.

$$\text{For } \frac{\partial \tilde{g}}{\partial L} < 0 \text{, requires } \frac{\partial \rho}{\partial L} < -\frac{\partial g}{\partial L}\left(\frac{1-\delta}{M} \right) < 0 \, . \tag{2.13}$$

Thus, for $\left| \frac{\partial \rho}{\partial L} \right|$ sufficiently large, the introduction of detection and penalties will lead to a stable internal solution. The endogenous probability of detection will serve as a brake upon the expansion of the non-legal sector. This will constitute a self-regulating mechanism that can lead to a SIS.

In the following section we develop a simple, general model of detection and penalties. For a number of important parameter values, the penalty falls with the fraction of non-legal firms, as required in the g-t-p model, above.

Solving for the threshold probability that equalizes profits over sectors, we have: $\rho^{**} = \left(\hat{t} - g\right)\frac{\left(1-\gamma\right)}{M}$. Thus, a rise in the tax rate increases the threshold. This implies that firms will shift from the legal to non-legal sector. In turn this progressively increases the probability of detection, so that N* grows, but does not N* may stop before reaching one, where all firms are non-legal. Higher taxes increase "informality", but are still consistent with a Stable Internal Solution where legal and non-legal firms coexist.

III. A GENERAL MODEL OF THE PROBABILITY OF DETECTION AND ITS PUBLIC FINANCE

This section develops a model of endogenous detection with penalties. Above we saw that a probability of detection that falls with the share of legal firms (rises with the share of non-legal firms) may lead to stable internal solutions with both legal and non-legal firms, in the g-t-p model. And in the subsequent section, we will examine the Loayza (1996) model which assumes, but does not prove, this to be the case.

We begin by assuming that there are *H* firms, *N* non-legal and *L* legal, or $L+N=H$. The State dedicates resources to generating a random sample of all firms, S. This random sample is a function of resources, *Y*. Because the sampling is random and once a firm is sampled its nature is revealed (to be an *L* or *N* firm), the probability of detection of a non-legal firm is given by the ratio of the sample to the overall population. The Baysian framework provides us with:

$$p(D \mid N) = \frac{p\left(D\right)p\left(N\right)}{p\left(N\right)},$$

Where P(D|N) is the condicional probabilty of detection if a firm is non-legal, P(D) is the unconditional probability of detection, and P(N) is the unconditional probability of being a non-legal firm. Because of the random sample and the random distribution of firms in the overall population, we have:

$$p(D \mid N) = p\left(D\right)$$

and, thus:

$$p(D) = \frac{S}{H}. \qquad (3.1)$$

Thus, the conditional probability of detection simplifies to the unconditional probability of detection, which is the ratio of the sample to the overall population.

The sample, S, is a function of the resources, *R*, available to produce that sample. We assume a general (AK type) production function for the generation of the sample:

$$S = \beta R^{\lambda}, \qquad (3.2)$$

The parameter β reflects the efficiency in the production of the sample. As β goes to infinity, information becomes perfect and the State knows the actions of all firms without cost or resort to sampling. Returns to scale in the production of the sample are reflected by λ.

Government resources for detecting firms that do not pay taxes may derive from general tax funds, *T*, and from funds collected from non-complying firms which are caught and penalized. Thus, the probability of detection is endogenous and will depend upon the number of non-legal firms, *N*.

The expected amount of penalties collected is equal to the probability of detection, ρ, times the fine, *M*, times the number of non-legal firms, *N*. We also include a corruption term via ε, where 0<ε<1. Then,

$$R = T + \varepsilon\rho NM \text{ .}^{6} \tag{3.3}$$

Thus:

$$\rho = \frac{\beta\left[T + \varepsilon\rho NM\right]^{\lambda}}{H} \text{ .}^{7} \tag{3.4}$$

Anticipating results below, note briefly that as T or β, or both, rise, the intercept of the probability of detection rises. Thus, for high values of T or β the expected penalty could be high enough to prevent firms from ever contemplating non-legality. In a *normative* sense, this offers the State policy instruments for preventing *informality,* and in a positive sense may help explain variations in the share of firms that are non-legal, or informal.

Case A: No Autonomous Financing of Detection (T=0)
When there is no autonomous financing, the closed form solution for ρ is as follows:

$$\rho^* = \left[\frac{\beta}{H}\left(\varepsilon NM\right)^{\lambda}\right]^{\frac{1}{1-\lambda}}. \tag{3.5}$$

The derivative of ρ^* with respect to *N* is given by:

$$\frac{\partial\rho^*}{\partial N} = \frac{\lambda}{1-\lambda}\left(\frac{\beta}{H}(\varepsilon M)^{\lambda}\right)^{\left(\frac{1}{1-\lambda}\right)} N^{\left(\frac{\lambda}{1-\lambda}-1\right)}. \tag{3.6}$$

Thus,

$$\frac{\partial\rho^*}{\partial N} \gtrless 0 \quad \text{as } \lambda \lesseqgtr 1^{8}. \tag{3.7}$$

This case, however, is less likely than that where there is some degree of exogeneous financing of detection.

Case B: Autonomous Financing of Detection (T>0)

When detection is financed by funds from general taxes, so that $T > 0$, there is no explicit solution for the probability of detection, ρ^*. Therefore, we analyze the slope of $\rho^*(N)$ by calculating the implicit derivative of ρ^* with respect to *N*. Define the implicit function, *F*, as follows:

$$F = \rho - \frac{\beta}{H}\left[T + \varepsilon\rho NM\right]^{\lambda}. \tag{3.8}$$

Then,

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[6] Or $Y = (T - FC) + \varepsilon\rho NM$, where FC are fixed costs.

[7] Technically, $\rho = \min\left[\frac{\beta[T+\varepsilon\rho NM]^{\lambda}}{H}, 1\right]$.

[8] $\frac{\partial\rho^*}{\partial N}$ is not defined for λ=1.

$$\frac{d\rho^*}{dN} = -\frac{F_N}{F_\rho}, \tag{3.9}$$

where F_N and F_ρ are the partial derivatives of F with respect to N and ρ, respectively. These partial derivatives are:

$$F_N = -\frac{\lambda\beta\varepsilon\rho M}{H}\left[T + \varepsilon\rho NM\right]^{\lambda-1}, \tag{3.10}$$

and:

$$F_\rho = 1 - \frac{\lambda\beta\varepsilon NM}{H}\left[T + \varepsilon\rho NM\right]^{\lambda-1}. \tag{3.11}$$

Because the numerator of the implicit derivative of ρ with respect to N, $(-F_N)$, is positive, the sign of $\frac{\partial\rho^*}{\partial N}$ is equal to the sign of the denominator F_ρ:

$$\frac{\partial\rho^*}{\partial N} \gtrless 0 \text{ as } 1 - \frac{\lambda\beta\varepsilon NM}{H}\left[T + \varepsilon\rho NM\right]^{\lambda-1} \gtrless 0. \tag{3.12}$$

Multiplying (3.12) through by $\frac{H}{\beta\left[T + \varepsilon\rho NM\right]^{\lambda}}$, we obtain:

$$\frac{\partial\rho^*}{\partial N} \gtrless 0 \text{ as } \frac{H}{\beta\left[T + \varepsilon\rho NM\right]^{\lambda}} - \frac{\lambda\varepsilon NM}{T + \varepsilon\rho NM} \gtrless 0. \tag{3.13}$$

Noting that the term $\frac{H}{\beta\left[T + \varepsilon\rho NM\right]^{\lambda}}$ is equal to ρ^{-1}, we may rewrite (3.13) as follows:

$$\frac{\partial\rho^*}{\partial N} \gtrless 0 \text{ as } \frac{(T/\varepsilon NM) + \rho}{\rho} \gtrless \lambda, \tag{3.14}$$

or, equivalently:

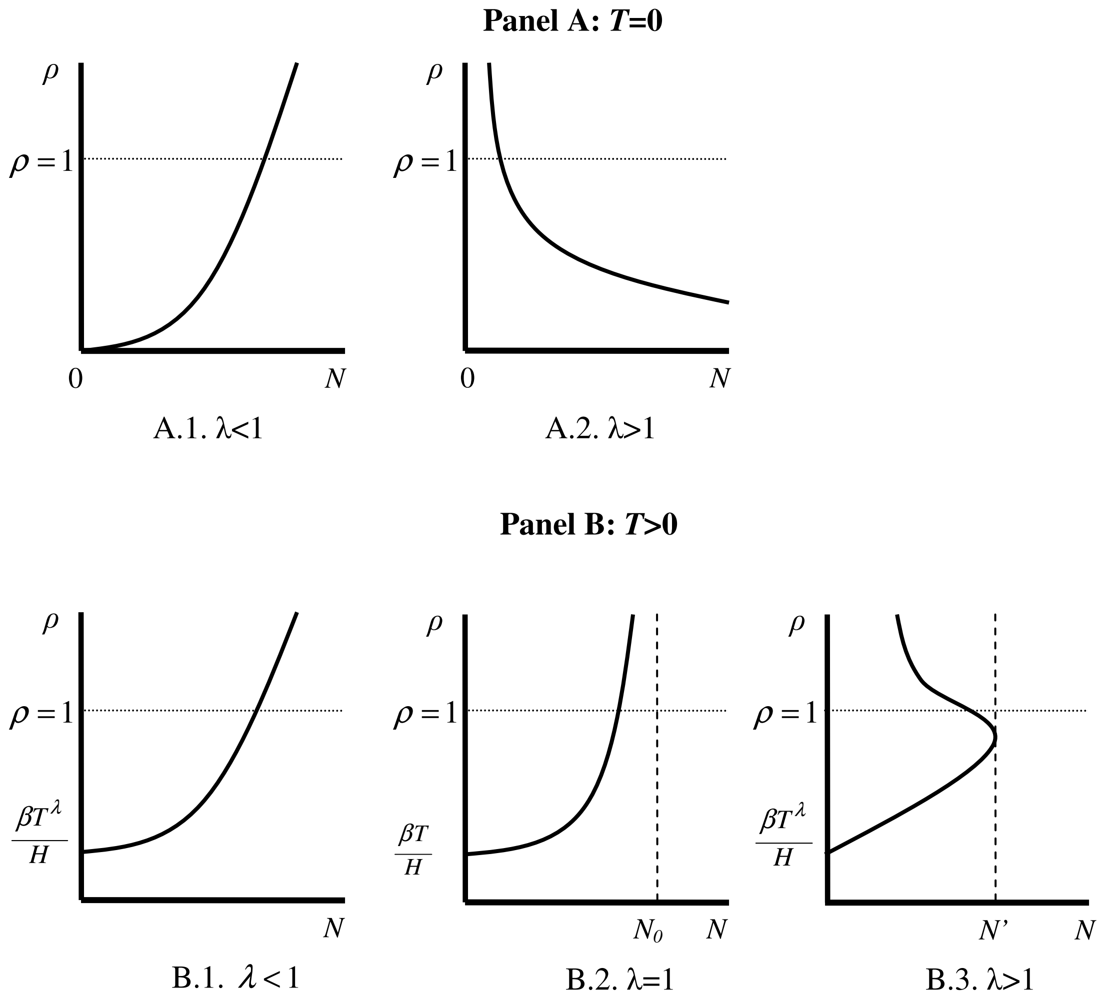
$$\frac{\partial\rho^*}{\partial N} \gtrless 0 \text{ as } \frac{T}{\varepsilon\rho NM} \gtrless \lambda - 1. \tag{3.15}$$

Noting that in the empirically likely case, where detection receives some exogenous financing, or $T > 0$, the term $\frac{(T/\varepsilon NM) + \rho}{\rho}$ exceeds one. Thus, when $\lambda \le 1$, the derivative of the probability of detection with respect to N will be positive.

These results are summarized below in Table 3. The dynamics of the model is illustrated in figure 3.

Table 3. Derivative of the Probability of Detection with Respect to the Size of the Non-legal Sector

Quasi exogenous financing of sample production T	Returns to scale in production of sample λ	Sign of $\frac{d\rho}{dN}$
$T = 0$	> 1	-
	= 1	?
	< 1	+
$T > 0$	> 1	$\frac{\partial \rho^*}{\partial N} \gtrless 0$ as $\frac{T}{\varepsilon \rho NM} \gtrless \lambda - 1$
	= 1	+
	< 1	+



Notation:

N: number of non-legal firms;
ρ: probability of detection;
T: autonomous financing;
λ: returns to scale in the production of the sample;
β: efficiency in the production of the sample; and
H: total number of firms.

Figure 3. The probability of detection.

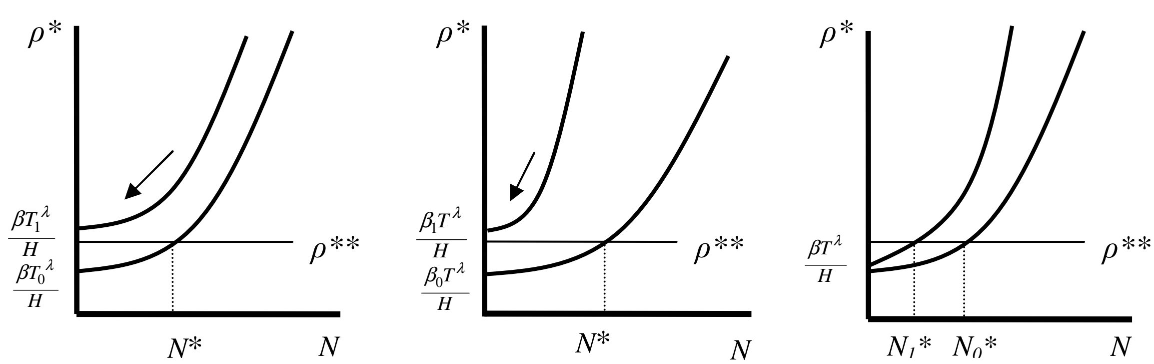
Summary

When the probability of detection is partially self-financed by fines collected, then the probability becomes a function of the share of non-legal firms, N. It is likely that detection financing is often a combination of both exogenous and endogenous financing. Though not universally true, in most cases the probability of detection is an increasing function of the share of non-legal firms.

As seen early, such a structure can produce a self-regulating mechanism which would lead to the stable coexistence of both legal and non-legal firms, or SIS.

We can integrate this detection probability model into different models, such as g-t Model and Loayza's (1996). Assume that in the absence of detection and fines, legal firms have higher profits than non-legal firms. Then the addition of detection and penalties for non-legal firms will lower profits for non-legal firms, while not affecting profits of legal firms. Thus, it will often be the case that there exists a threshold level of the probability of detection at which profits in both sectors are equalized. If the probability of detection varies with the share of legal (or non-legal) firms, as in the model above, the intersection of the probability of detection curve, $\rho^*(N)$, and the threshold level of the probability, ρ^{**}, determines the equilibrium level of non-legal (legal) firms, *N* (L*)*. And if *N** is less than the total number of firms, *H*, this constitutes an internal solution. Finally, if, as we have shown is often the case, the probability of detection rises with *N* (falls with *L*), this internal solution will be stable.

This logic is illustrated in figure 4, below. V_L and V_N are the net profits of a firm choosing to be in the legal (*L*) or non-legal (*N*) sector. Assume there is a ρ^{**} such that $V_L = V_N$. Figure 4 shows a possible equilibrium and the dynamics when *T*, *β* or *M* vary.



a) Increases in T: $T_1 > T_0$ b) Increases in β: $\beta_1 > \beta_0$ c) Increases in M

Figure 4. Probability of detection and possible equilibriums varying parameters: *T*, *β* and *M*.
When *T* or *β* increases, the intercept of the function *ρ*(N)* shifts upward. Additionally, when *β* increases, there is a large increase in the slope of the curve. When *M* increases, the slope increases, but the intercept remains unchanged.

Normative and Positive Implications of the Detection Model – Early Deterrence

It is important to note that if the intercept of the probability function is sufficiently high, there will never be any non-legal firms. This has both *positive* and *normative* implications. Countries without "informal" firms may be those which attain high probabilities of detection even when there are no non-legal firms. The intercept of the probability function rises with β or *T*. Thus, the autonomous detection financing, *T*, constitutes a central policy instrument for the State, to achieve normative goals. By increasing this autonomous financing high enough, the State can eliminate "informality", defined here as non-legal firms. Increases in the degree of perfection of information, β, also shift the intercept of the probability function upwards. However, the State's ability to affect the degree of information perfection would arise from technological innovation, modernization, or structural reforms affecting detection, which are more difficult to effect than changes in exogenous financing of detection, *T*. Elimination of 'informality', should that be a desired social goal, could be achieved initially by designating high levels of general tax funds to detection, while more gradually working to raise the efficiency of detection (increasing β), through technological innovation and modernization.

Note that increasing the level of the fine charged to the non-legal firm that is detected, *M*, will increase the slope of the probability of detection, and shift that curve leftwards. This will lower the level of non-legal firms in equilibrium. However, this will not by itself lead to the complete elimination of non-legal firms. Increases in β or T are required to eliminate entirely non-legal firms, or to entirely eradicate 'informality'.

Summary

The foregoing model may provide insights into how different structures of public goods, taxes and penalties lead to differing shares of non-legal firms across countries and within countries over time. In general, high levels of exogenous funding for detection will lower or even eliminate the percentage of firms which are non-legal. This may be used to explain differences over countries and over time, or as policy instruments to change outcomes.

In the next section we synthesize Loayza (1996), which presents a model of public goods, taxation and detection. That model assumes a probability of detection but neither explains nor models it. The above model of detection, therefore, is a crucial complement to the Loayza (1996) model, lending it greater clarity and legitimacy. At the same time, because we have found that the probability of detection is not always an increasing function of the share of non-legal firms, we find that the conclusions of Loayza (1996) are not completely robust to different assumptions regarding the detection process.

IV. LOAYZA-1996(L96): MODELING PENALTIES WITH PROBABILITY MODEL

This section synthesizes a model that, while similar in spirit to the g-t and g-t-p models, is entirely different in its specification of the roles of the public good and the tax structure. Before, taxes were lump sum. Now taxes are proportional to sales. Before the public good entered into profits additively and was not necessary for production in the non-legal sector. Here, the public good enters into the production function as an input and is a necessary input in both sectors. Despite these differences, the general results of the g-t and g-t-p models obtain. In the absence of penalties, a SIS will not exist. This section also serves to illustrate the use of the foregoing detection model, and how we may provide more rigor to the L96 model, supporting its basic conclusions.

One much noted article which models firm sector choice with public goods, taxation and penalties for firms that do not pay taxes is Loayza (1996), or L96. L96 assumes a homogeneous final good and identical firms. L96 assumes an AK production function where the public good is necessary for production, regardless of the firm's sector choice, and that the public good is partially non-excludable, $\gamma \in (0,1)$. Unlike Grossman and Yoshiaki (2003), non-legal firms do not produce a substitute public good, so that given the assumption that the public good is a necessary input to production, it follows that the partial non-excludability of the public good is necessary if non-legal firms are to exist.

The profits of legal and non-legal firms given by V_L and V_N are the net profits for firms if they choose to be legal or non-legal:

$$V_L = (1-\tau)Ag^b K - rK \text{ (legal)} \tag{4.1}$$

and:

$$V_N = (1-\pi)A(g\gamma)^b K - rK \text{ (non-legal).} \tag{4.2}$$

Here g is the per firm public good and A is an exogenous productivity parameter. Gross output in the legal sector is $Ag^b K$ and $A(g\gamma)^b K$ in the non-legal sector, where the non-legal firm receives $(g\gamma)$ units of the public good, and where Loayza assumes that gamma is positive but less than one. τ is the tax rate and π is the expected penalty rate. This expected penalty consists of the probability of detection, ρ, times the penalty rate, μ (the total per firm penalty for firms caught and fined, *M*, is equal to the penalty rate times the non-legal firms production: $\mu \cdot (g\gamma)^{\lambda} K$).

Note that, in addition to Loayza's formulation, it is possible here to bring to bear the formulation for the public good and the per firm public good presented in the g-t model, above:

$$G = \left[c\tau g^b K - T \right]^{\alpha}. \text{ Then } g = \frac{[c\tau g^b K - T]^{\alpha}}{[L(1-\gamma)+\gamma]^m}. \qquad (4.3)$$

Note that 'c' here is the share of revenues not siphoned off by corruption.

In contrast to L96, this formulation models more explicitly the public finances of the production of the public good, and introduces a net deduction of total tax funds generated to be applied to the detection of non-tax paying firms, *T*.

The equilibrium where firms are indifferent between being legal or non-legal is where net profits equalize: $V_L = V_N$. Thus:

$$(1-\tau)g^b K - rK \ = \ (1-\pi)(g\gamma)^b K - rK. \qquad (4.4)$$

Noting that $\pi = \rho\mu$, we have:

$$1-\tau = (1-\rho\mu)\gamma^b. \qquad (4.5)$$

This equality defines a threshold level of the probability of detection, ρ^{**}, which will equalize profits in both sectors:

$$\rho^{**} = \frac{1}{\mu}\left(1 - \frac{(1-\tau)}{\gamma^b}\right). \qquad (4.6)$$

Note that for the threshold probability to be positive, $\rho^{**}>0$, this requires $\gamma^b > (1-\tau)$. If the public good is "fully non-excludable" ($\gamma = 1$), the equilibrium probability becomes $\rho^{**} = \frac{\tau}{\mu}$. If the public good is completely excludable ($\gamma = 0$), the non-legal sector disappears, because they have no access to the necessary public good.

Loayza focuses upon a potential stable internal equilibrium, where both legal and non-legal firms coexist, consistent with the observed reality in developing countries. However, it is useful to note that this model points to a normative result. If the government lowers the tax rate enough, it can induce all firms to choose to be legal, without the introduction of penalties. If there are no penalties, equation (4.5) simplifies to $(1-\tau) = \gamma^b$, which defines a threshold level of taxation, τ^*:

$$\tau^* = 1 - \gamma^b. \qquad (4.7)$$

If the government sets the tax rate below τ^* ($\tau = \tau^* - \varepsilon$), then all firms will choose to be legal.

Absent penalties, there would be no self-regulating mechanism leading to an SIS. In general, the model would collapse into corner solutions, where all firms were either legal or non-legal, in contrast to the empirical regularity.

Thus, for Loayza's model to be relevant, in a *positive* sense, there must be reasons why the government would not lower taxes enough to induce full compliance with the law. And in a *normative* sense, it is important to keep this potential policy avenue in mind: non-legal firms, or "Informality", could be potentially eliminated by lowering the tax rate sufficiently. If the tax rate is rigid and/or the public good is almost non-excludable (γ is near to one), penalties would be necessary. There is an internal solution of the model with penalties, if the following condition is satisfied:

$$\rho^{**} > 0 \text{ as } \gamma^{b} + \tau > 1. \tag{4.8}$$

The equilibrium and dynamics that L96 emphasizes are summarized in the following figure:

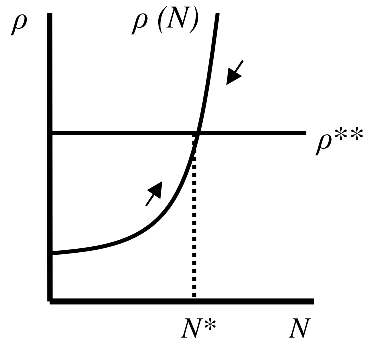


Figure 5. Probability of detection and equilibrium in the Loayza's model**.**

Equilibrium occurs where the probability $\rho(N)$ crosses with the threshold value of the probability, consistent with the equalization of profits across sectors, ρ^{**}. This equilibrium is stable, because for $N<N^*$, profits in the non-legal sector exceed those of firms in the legal sector, leading to an increase in N and convergence to N^*. For $N>N^*$ the opposite occurs, with N falling to N^*.

If taxes rise, the threshold probability rises, and N must rise to compensate. And if the fine rises, the threshold probability falls, so that N must fall.

The Probability of Detection in Loayza

L96 *assumes* that the probability of detection rises with the percent of all firms that are non-legal firms (or falls as the percent of legal firms rises). This assumption is key to that model. If the probability of detection falls with the number of non-legal firms, there may be an internal solution where the equilibrium fraction of legal firms is between zero and one, but this equilibrium will not be stable. The fraction of legal firms would converge to zero or one, and the model would not lead to the observed reality of developing countries.

However, L96 neither provides a clear intuitive justification for this assumption, not makes an attempt at modeling it.[9] In the prior section we have developed a simple, general model of detection of non-legal firms, and the underlying public finances.

Implications of the General Model of Detection for the Loayza Model, L96

As shown in the probability model (previous section), the probability of detection often, though not always, has a positive slope. Thus, combining the general model of detection with the L96 model leads to a more complete model, where as in L96, stable internal solutions may dominate. Below we extend the L96 model in combination with the probability model for the case of constant returns to scale in the production of the sample in the presence of both exogenous (T>0) and endogenous financing.

[9] Some mention is made of the expected tax rate rising as the size of the non-legal sector increases, as a reaction by legal firms. No probability model is provided and not discussion of the financing of detection is developed.

Applying the Generic Probability Model to Loayza (1996)

In this section, we apply the generic probability model developed in Section III to the model of Loayza (1996).

Given that M can be expressed as

$$M = \mu \cdot y_I \tag{4.11}$$

where y_I is output per firm in the informal economy, we can rewrite (4.6) as:

$$\rho^{**} = \frac{y_I}{M}\left(1 - \frac{(1-\tau)}{\gamma^b}\right). \tag{4.12}$$

To solve for N^*, we equalize ρ from (3.4) and ρ^{**} from (4.12), and obtain:

$$N^* = \frac{1}{\varepsilon \cdot y_I \cdot F}\left[\left(F \frac{y_I}{M}\frac{H}{\beta}\right)^{\frac{1}{\lambda}} - T\right], \tag{4.13}$$

where $F \equiv \left(1 - \frac{(1-\tau)}{\gamma^b}\right)$.

The sign of the partial derivatives of N^* are:

$$\frac{\partial N}{\partial M} < 0\,;\ \frac{\partial N}{\partial \beta} < 0\,;\ \frac{\partial N}{\partial T} < 0\,;\text{ and }\ \frac{\partial N}{\partial \varepsilon} < 0\,.$$

$\frac{\partial N}{\partial \tau}$ is positive if $\lambda<1$, and it is negative if $\lambda>1$ and simultaneously $T=0$. The sign depends on parameter values in any other case.

Thus, we have seen that L96 requires penalties, without with the model will collapse into corner solutions, as in the g-t model. We also say that non-legality can be eliminated by lowering the tax rate sufficiently, so that an implicit necessary assumption for the L96 model is downward rigidity of the tax rate. We then incorporated the formal detection model into the L96 framework, and derived the implied comparative statics, consistent with the original L96 model.

V. WAGE CURVE WITH PAYROLL TAXES, UNEMPLOYMENT AND TWO SECTORS (LEGAL, NON-LEGAL)

Building upon the prior models with endogenous monitoring, we now turn to the modeling of payroll taxes. The model parallels the g-t-p model structure and that of L96 in important ways, applies the probability model developed above, generates a two-sector aggregate labor model with informality, and then couples this structure with the Wage-Curve paradigm of equilibrium unemployment. Finally, we simulate changes in the equilibrium unemployment level in Colombia, using the historic changes in payroll taxes in Colombia and calibrating the model using parameter estimates for the wage curve from the international literature (see Robbins, Ruiz and Salinas (2007) for a synthesis of this literature), and performing sensitivity analysis.

As in the prior models, developed above, we assume identical firms, where legal firms pay taxes, here payroll taxes. In contrast to models with public goods, payroll taxes do not produce clear benefits to the firm. Thus, for a stable internal solution where legal and non-legal firms coexist, there must exist a positive expected fine for firms evading payroll taxes. In the absence of such expected penalties, all firms would find it optimal to evade payroll taxes.

The initial logic of the model follows closely upon that presented in the g-t-p and L96 models, where we incorporate the probability model presented in section III. An internal solution requires the equality of expected profits of legal and non-legal firms, defining a threshold level of the probability of detection. This threshold probability is an increasing function of the payroll tax rate. Because the probability of detection, as shown above, tends to be a decreasing function of the share of legal firms (or an increasing function of the share of non-legal, or "informal" firms), a higher tax which increases the threshold requires a smaller share of legal firms (higher degree of "informality") to restore equilibrium. Higher taxes increase "informality".

As in the prior models with detection, shifts in the detection function alter the equilibrium level of "informality". In particular, higher exogenous financing of detection or greater efficiency in detection reduces the equilibrium level of informality, for a given level of payroll taxes.

The model incorporates a Wage Curve quasi-supply curve (e.g. Blanchflower and Oswald(1994); Card(1995); Blanchard and Katz(1997); Blanchard(1999); Robbins(2007)) to also incorporate equilibrium unemployment in a 'simplified general equilibrium model' approach (Stigliz and Shapiro(1984)). We first model aggregate employment demand as the sum of legal and non-legal sectoral demands, where firms face sector specific wages, and where legal firms wage costs reflect the legal sector wage and payroll taxes upon those wages. Non-legal firms net wage costs simply reflect wage costs in that sector. Competition among workers leads to the equality of net worker wages, which permits us to collapse the dimensionality of the model in terms of one wage. The dimensionality of labor taxes is also reduced by noting that employer and employee payroll taxes typically maintain a relation of proportionality, so that one tax is a sufficient statistic (this assumption is not necessary, but reflects the dominant institutional reality in many countries).

Total employment demand is therefore able to be expressed as a function of one sectoral wage and one tax rate. This employment demand is then expressed as an employment rate, dividing it by the aggregate nominal labor supply, or the total potential labor force.

In the Wage Curve paradigm, aggregate effective labor supply differs from aggregate nominal labor supply. For example, in the shirking model of Efficiency Wages, the effective labor supply, referred to as the Non-Shirking-Condition (NSC), is a convex curve in the wage-employment-rate space, with a finite intercept and asymptotically infinite as the employment rate tends towards one. Equilibrium occurs at the intersection of this quasi-labor supply curve and the standard demand curve, expressed in the wage-employment-rate space. In equilibrium the employment rate is less than one, so that there is equilibrium involuntary unemployment.

In the current model, the aggregate quasi-supply curve is expressed in terms of the expected net wage received by workers participating and randomly allocated across the legal and non-legal sectors. This relation may also be expressed in terms of a single sectoral wage.

The macro-economic equilibrium level of employment is then directly solved for. Comparative statics show that rising payroll taxes, in addition to leading to growth in the size of the "informal" sector, also lead to a lower equilibrium employment rate, except when valuation of payroll taxes by workers is complete. In the latter case we have a variant of the classic "Full Pass-Through" results seen in models of one sector without unemployment. Here, however, we have a substantially more complete model that embraces key components of the labor market in LDCs, in particular: equilibrium informality and equilibrium unemployment.

The remainder of this part of the paper is organized as follows: first, we model payroll taxes, penalties and detection, where the detection model was presented above, in section III. We show that higher payroll taxes lead to a higher threshold probability of detection, which, in turn, leads to a higher share of non-legal firms, or higher "informality". Second, we construct a model of aggregate labor demand and aggregate quasi-labor supply in terms of one, common, wage. Third, we solve for the equilibrium employment rate, and, forth, we examine comparative static properties, emphasizing the effects of changes in the payroll tax rate. In the fifth section, we present simulations of the effects of changes in payroll taxes in Colombia since 1990 upon the equilibrium employment rate.

These simulations suggest that the historical increases in the payroll tax rate in Colombia may have decreased the equilibrium employment rate by 2 to 4 percent, depending largely upon the degree that workers value payroll tax contributions. Finally, we conclude discussing directions for future research.

A Two Sector Model with Payroll Taxes, Informality and Unemployment

As in the g-t, g-t-p, Loayza(1996) models, we assume that firms, technology and workers are identical (the results generalize when relaxing the technology assumption). Firms may choose between two states or sectors, legal and non-legal. Firms choosing to be legal pay payroll taxes and firms choosing to be non-legal shirk the payment of payroll taxes, but face an expected penalty, where the penalty if caught is proportional to the wage bill.

And, as in the prior models with detection, firms will change states, or sectors, if profits are higher in the other state. In a stable internal solution, this movement will continue until expected profits between the two states equalize. Unlike the models of public goods, firms that pay payroll taxes do not receive direct benefits in return. Therefore, in the absence of an expected penalty all firms will choose to be non-legal.

In the model of detection, presented above, we found that when part of the detection process is self-financed that the probability of detection tends to rise with the size of the non-legal sector, for most parameter values. This serves as an endogenous mechanism that may lead to a stable internal solution. As pointed out above, L96 assumes this structure, but did not provide a clear conceptual basis for it.

In the current model, we incorporate the results of the detection model, and assume, as in L96, that the probability of detection rises with the size of the non-legal sector. The fraction of legal or non-legal firms in equilibrium is determined as follows. Assume initially that all firms are legal and that when all firms are legal that the probability of detection is low, because the financing from detected non-legal firms is zero. Because they pay payroll taxes, but in other respects are identical to non-legal firms, their profits will be lower than legal firms. Thus, legal firms begin to change states, from legal to non-legal, or "migrate" to the non-legal sector. The increase in the size of the non-legal sector increases revenues from detection and penalties, and therefore increases the probability of detection and the expected penalty. A threshold level of the probability of detection will exist, where expected profits in both sectors are equalized. This threshold will be a decreasing function of the level of taxes in the legal sector. If the threshold is sufficiently low, then the intersection of this threshold with the probability of detection function will occur before all firms become non-legal, constituting an internal solution. The increase of the fraction of non-legal firms and consequent higher expected penalties in the non-legal sector will lower expected profits of non-legal firms. Firms will continue to move to the non-legal sector until either all firms are non-legal, or until the increase in expected penalties equalizes expected profits in both sectors.

An internal solution here will be stable, as seen in the g-t-p and L96 models. Stability is immediate in this case. Assume that the intersection of the probability of detection function, p(N) with the threshold probability of detection, p** , occurs at the fraction N* of all firms. Then if the initial fraction of non-legal firms, N_0, is below N*, N<N*, then the expected penalty for a non-legal firm is low and expected profits for a firm choosing to be non-legal exceed expected profits if choosing to be legal. Thus, legal firms begin to change status, and N rises. Similarly, if No exceeds N*, then N will fall. Thus, N converges to N*

Equilibrium consistent with an internal solution requires, as in prior models, the equality of expected profits:

$$V_L = V_N \quad . \tag{5.1}$$

Legal firms pay a payroll tax, δ_1, and non-legal firms face an expected penalty, π, which is the product of the probability of detection times 'm', the penalty rate, which is proportional to the wage bill, wE:

$$\underline{\text{Legal}} \qquad\qquad\qquad\qquad \underline{\text{Non-legal}}$$

$$pQ - Ew_L(1+\delta_1) - rK = pQ - Ew_N(1+\pi) - rK \tag{5.2}$$

where : $\pi = \rho(L)m$; ρ is the detection probability and m is the penalty rate.

This condition implicitly defines a threshold probability of detection, ρ^{**},at which profits equalize. Before solving explicitly for ρ^{**}, it is evident that given that the determination of the equilibrium degree of legality ("formality"), L*, or non-legality, N* ("informality") will be essentially identical to that of the g-t-p or L96 models. A given tax level will determine a given threshold probability, ρ^{**}, where higher taxes will lower the threshold. If N<N*, then the probability of detection is below ρ^{**}, so that N will rise until $\rho(N) = \rho^{**}$, and N=N*.

The Condition for an Internal Equilibrium and the Threshold Probability

Simplifying the expression for the equality of profits across sectors, (5.2), we see that equilibrium with an internal solution requires the equality of the payroll tax rate and the expected penalty:

$$W_L(1+\delta_1) = W_N(1+\pi) \,, \tag{5.3a}$$

or

$$W_L D_1 = W_N(1+\pi) \text{, where } D_1 \equiv (1+\delta_1) \,. \tag{5.3b}$$

Note that in equilibrium the cost structures of legal and non-legal firms equalize so that the level of employment per firms will be equal across sectors (again, homogeneity of technology may be relaxed leading to different levels of employment per firm).

To evaluate the expression in (5.3b), we derive expressions relating wages in both sectors and collapsing the dimensionality of the tax structure in terms of the employer tax rate alone (this is not necessary, but reflects the usual institutional structure). Given free worker mobility over sectors, net expected workers' wages will equalize across sectors. Net wages of workers in the non-legal sector are simply their base wage, W_N. Net wages for workers in the legal sector, however, must subtract payroll tax payments and add their valuation of total payroll tax contributions. If we assume that workers' valuations of payroll tax contributions are equal with respect to employer and employee contributions and equal to "τ", the net wage of a worker in the legal sector will be equal to:

$$W_L^{net} \cdot e = W_L \cdot D2 \cdot e \qquad , D2 \equiv (1-\delta_2 + \tau(\delta_1 + \delta_2)) \,, \tag{5.4a}$$

or:

$$W_L^{net} = W_L \cdot D2 \tag{5.4b}$$

where δ_1 is the payroll tax rate paid by the employer and δ_2 is the payroll tax rate paid by the employee, D2 captures the net effects of the tax paid by the worker and the benefits they receive, and 'e' is the employment rate.[10]

In many countries, the payroll tax rates of employers and employees are in constant proportion, and we may reduce the dimensionality of the taxes:

[10] Later we will assume a wage-curve theory of unemployment, so that 'e'<1 in equilibrium, where workers' employment probabilities are identical over sectors and equal to the aggregate employment rate.

$$\Rightarrow \delta_2 = c\delta_1 , \tag{5.5a}$$

where 'c' < 1, because worker contributions are almost universally lower than employer contributions. Thus, we may rewrite D2 in (5.4) , where we define 'g':

$$g \equiv \tau(1+c) - c : \tag{5.5b}$$

$$D_2 = (1 + g\delta_1) . \tag{5.6}$$

Substituting (5.6) into (5.4b) and this into (5.3b) the equilibrium condition becomes:

$$W_L D1 = W_L (1 + g\delta_1)(1 + \pi) , \tag{5.7a}$$

recalling that $\pi = \rho \cdot m$:

$$W_L D1 = W_L (1 + g\delta_1)(1 + \rho m) . \tag{5.7b}$$

Solving for the threshold probability that equates profits, we have:

$$\begin{aligned} \rho^{**} &= \left(\frac{D_1}{D_2} - 1 \right) m^{-1} \\ &= \left(\tilde{d} - 1 \right) m^{-1}, \qquad \tilde{d} \equiv D1 / D2 \end{aligned} \tag{5.8a}$$

or

$$\rho^{**} = \left(\frac{1 + \delta_1}{1 + g\delta_1} - 1 \right) m^{-1} \tag{5.8b}$$

And the derivative of the threshold with respect to the tax rate is:

$$\frac{\partial \rho^{**}}{\partial \delta_1} \gtrless 0 \qquad as \quad 1 \gtrless g \ . \tag{5.9}$$

Because g= τ (1+c)-c, where $c \in (0,1)$, the threshold is an increasing function of the tax rate, when, as is empirically likely[11], valuation is incomplete ($\tau < 1$), and the derivative converges to zero, when valuation is complete[12]:

$$\frac{\partial \rho^{**}}{\partial \delta_1} > 0, \qquad where \quad \tau < 1 . \tag{5.10a}$$

and,

$$\lim_{\tau \to 1} \left(\frac{\partial \rho^{**}}{\partial \delta_1} \right) = 0 . \tag{5.10b}$$

It follows directly then that, with less than complete valuation by workers of total payroll tax contributions, an increase in the payroll tax rate, which increases the threshold probability, requires an increase in the size of the non-legal sector to restore equilibrium. In other words: a higher payroll tax rate increases "informality", or reduces the size of the legal sector:

$$\partial L^* \big/ \partial \delta_1 < 0 \equiv \partial N^* \big/ \partial \delta_1 > 0 \ . \tag{5.11}$$

This result is illustrated in the figure below.

[11] If there is inefficiency or corruption in the provision of benefits financed by payroll taxes, or if workers do not value the uses of these monies completely, then valuation will be incomplete, or less than 1.

[12] The sign of g depends on the valuation parameter, $\tau : g \gtrless 0$ as $\tau \gtrless \frac{c}{1+c}$.

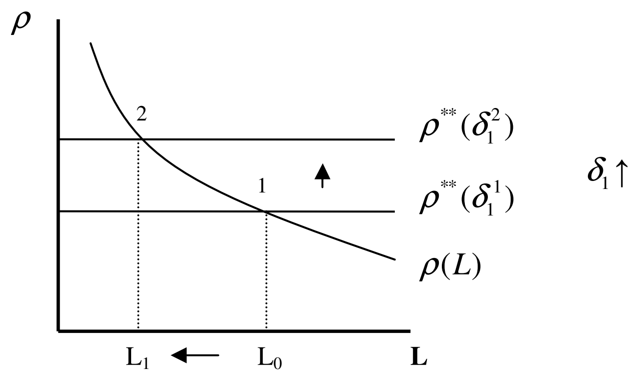


Figure. **Impact of Higher Payroll Taxes on the Degree of Formality(Informality)**.
Higher payroll taxes ($\delta_1^{\ 2} > \delta_1^{\ 1}$) increases the threshold probability, leads to a smaller fraction of legal firms, or rises "informality".

It is important to recall from our early discussion that, as before, for a given tax rate, increases in the exogenous component of detection will decrease the degree of informality, as will increase in efficiency in the detection or monitoring function. Thus, the government can reduce or eliminate informality through these channels. This is important for both their normative policy implications, and for their positive implications, potentially providing insight into why the degree of informality varies over time within countries and across countries.

Next we extend the model to address the employment rate and the impact of payroll taxes upon the employment rate.

Modeling Aggregate Employment, Wages and the Employment Rate in the Two-Sector model with Payroll Taxes and Detection

In the previous sub-section, we presented the basis for a two sector model with payroll taxes in the legal sector and expected penalties in the non-legal sector, which incorporate the prior model of endogenous detection revenues and probabilities. We derived the threshold probability consistent with a stable internal solution, where legal and non-legal firms coexist, and showed that rising payroll tax rates will increase the size of the non-legal sector.

In this section we extend the above two-sector model to the aggregate labor market, where we focus on the equilibrium employment rate, in a Wage-Curve framework, and the impact of rising payroll taxes upon the equilibrium employment rate. We find that rising payroll taxes lower the equilibrium employment rate, or increase "informality".

We first formulate total employment demand in terms of one sectoral wage. Then we normalize total employment demand as an employment rate, and solve for the employment rate demand in terms of the sectoral wage. Next we model aggregate labor supply, employing the Wage-Curve paradigm. We then equalize wage rates in terms of employment rates, solve for the equilibrium employment level and examine the comparative static properties of the model, focusing upon the elasticity of the employment rate to the payroll tax rate.

Total Employment Demand

Total employment demand is the sum of sectoral employment demands. We may express total employment demand as the employment demand per form in each sector, L and N, weighted by the share of total firms, F = L+N, in each sector, times the total number of firms:

$$E^d(total) = F\left(E_L^d\left(\frac{L}{F}\right) + E_N^d\left(\frac{N}{F}\right)\right). \qquad (5.12)$$

In equilibrium, per firm employment demand will be the same in both sectors, as the technology and cost structures are the same, since the share of legal (non-legal) firms adjusts until labor costs equalize, or $E_L^d = E_N^d$.[13] Thus we may rewrite total labor demand where firm mobility between N and L has equalized cost structures as follows:

$$E^d(total) = F\left(E_L^d\left(\frac{L}{F}\right) + E_N^d\left(\frac{N}{F}\right)\right) \qquad (5.13a)$$

or:

$$E^d(total) = E_L^d\left[F\left(\left(\frac{L}{F}\right) + \left(\frac{N}{F}\right)\right)\right]. \qquad (5.13b)$$

And this simplifies further to:

$$E^d(total) = E_L^d \cdot F \quad , \qquad (5.13c)$$

[13] Profit equalization implies that $W_L D1 = W_N(1+\pi)$, and intersectoral labor mobility implies that net wages equalize, or that $W_N = W_L D2$, so that profit equalization becomes: $W_L \cdot D1 = W_L \cdot D2 \cdot (1+\pi)$, where $\pi = \rho(N)\cdot m$. In equilibrium adjusts to equalize net labor costs, so that the cost structures of firms in both sectors equalizes, and their optimal employment levels will be the same as well. As mentioned, we may relax the identical technology assumptions and will generate, grounded upon the same logic, different optimal levels of employment for legal and non-legal firms in equilibrium.

so that total employment demand, where the firm mobility has adjusted to equalize cost structures across sectors, is simply the employment demand for the representative firm times the number of firms, F. For simplicity of exposition, we assume that the number of firms is fixed. [14]

Firm Employment Demand

We next make explicit firm labor demand, which follows standard results, to derive an expression for aggregate labor demand. We assume Cobb-Douglas production, where output, Q is, $Q = AE^{a}K^{(1-a)}$. Thus, employment demand in the legal sector is:

$$E_L^d = \left(\frac{w_L D_1}{\Omega}\right)^{\lambda}, \tag{5.14}$$

where $\lambda \equiv \dfrac{1}{a-1}$ and $\Omega \equiv aAK^{(1-a)}$.

Multiplying by F and normalizing by the population size, P, we have an expression for aggregate employment demand as an employment rate:

$$e^d = \frac{E_L^d \cdot F}{P}. \tag{5.15}$$

Substituting in from (5.14) and solving for the wage level we have:

$$w_L = \left(\frac{\Omega}{D_1}\right)\left(\frac{P}{F}\right)^{1/\lambda} e^{1/\lambda} \tag{5.16}$$

Next we model the aggregate labor supply, based upon the Wage-Curve paradigm.

Aggregate Effective Labor Supply

As stated earlier, the Wage Curve, a theoretic proposition with extensive empirical support (see Stiglitz and Shapiro(1984); Blanchflower and Oswald(1994,2005); Card(1995); Blanchard and Katz(1997); Blanchard(1999); Robbins, Ruiz, Salinas(2007) and various articles by Akerlof on Efficiency Wages) posits that the aggregate supply of effective labor services differs from aggregate nominal labor supply, and is a function of the aggregate employment rate, not the aggregate employment level. In equilibrium, aggregate labor demand, normalized in terms of the employment rate, intersects with the wage-curve at a point where the equilibrium employment rate, or the national rate of employment, is less than one. In equilibrium there is involuntary unemployment.

The Aggregate Wage Curve in a Two Sector Model with Payroll Taxes and Detection

We construct the aggregate wage curve for our model with payroll taxes, evasion and detection of evaders. The aggregate quasi-labor supply may be expressed in terms of the expected real wage if employed, 'w^e'. This reduces to simply he net wage in the legal sector, 'w_L':

$$W^e = \varphi W_L D_2 + (1-\varphi) W_N. \tag{5.17a}$$

Given that intersectoral labor mobility equalizes net wages in both sectors, we have, as seen above, $w_L D_2 = w_N$. Substituting this in (5.17), we obtain:

[14] In this model, higher labor costs in the legal sector are transmitted to higher labor costs in the non-legal sector, thereby raising overall costs of production for all firms. This will, as modeled explicitly here, decrease the optimal employment level per firm. In addition, if we include 'scale effects", such higher costs would potentially decrease total final demand, and reduce the optimal number of firms. However, there is implicitly another sector, the benefits provider sector, which would expand with higher payroll taxes, thus mitigating this effect on the number of firms.

$$W^e \quad = \quad \varphi\, w_L D_2 + (1-\varphi)\, w_L D_2 \,, \tag{5.17b}$$

or simply:

$$W^e \quad = \quad w_L D_2 \,. \tag{5.17c}$$

In a one sector model without payroll taxes (e.g. Blanchard (1999)) the wage curve may be modeled as:

$$w = \theta\left(e^s\right)^{\sigma}, \tag{5.18a}$$

where 'e' is the employment rate and sigma the elasticity of the aggregate employment rate to the wage.[15] In the current two-sector model, we model the aggregate wage curve in terms of the expected wage if employed, which is equal to $w_L D_2$. Thus we have:

$$w_L^s = \frac{\theta\left(e^s\right)^{\sigma}}{D_2} \tag{5.18b}$$

Equilibrium in the Aggregate Labor Market

Equalizing supply and demand, we have:

$$w_L^d = w_L^s \,, \tag{5.19}$$

or:

$$\left(\frac{\Omega}{D_1}\right)\left(\frac{P}{F}\right)^{1/\lambda} e^{\frac{1}{\lambda}} = \frac{\theta\left(e^s\right)^{\sigma}}{D_2} \tag{5.20}$$

Solving for the equilibrium employment rate we have:

$$e^* = \left[\frac{\theta}{\Omega}\tilde{d}\left(\frac{F}{P}\right)^{\frac{1}{\lambda}}\right]^{R}, \tag{5.21a}$$

where $R \equiv \dfrac{1}{\frac{1}{\lambda} - \sigma}$ and $\tilde{d} = \dfrac{D_1}{D_2}$, or, in logs:

$$\ln e^* = R \ln\left[\frac{\theta}{\Omega}\tilde{d}\left(\frac{F}{P}\right)^{\frac{1}{\lambda}}\right]. \tag{5.21b}$$

Comparative Statics

We are interested principally in the impact of changes in the payroll tax rate upon the equilibrium employment level. Thus, examining the derivative of e* with respect to the payroll tax we have:

$$\frac{\partial \ln e^*}{\partial \delta_1} = R\,\frac{\partial \ln \tilde{d}}{\partial \delta_1}, \tag{5.22a}$$

---

[15] In our simulations we use both this formulation as in Blanchard(1999) and an alternative formulation which better reflects the empirical specification of the wage-curve, but is analytically slightly more complex.

or:

$$\frac{\partial \ln e^*}{\partial \delta_1} = R\left[\frac{\partial \ln D_1}{\partial \delta_1} - \frac{\partial \ln D_2}{\partial \delta_1}\right], \tag{5.22b}$$

and:

$$\frac{\partial \ln e^*}{\partial \delta_1} = R\left[\frac{1}{1+\delta_1} - \frac{g}{1+g\delta_1}\right] \tag{5.22c}$$

This derivative is of the form:

$$\frac{\partial \ln e^*}{\partial \delta_1} = R\left[H\right]. \tag{5.22d}$$

Given that $R < 0$ and $H \gtreqless 0$, as $\tau \gtreqless 1$, we have:

R.1 $\dfrac{\partial \ln e^*}{\partial \delta_1} \gtreqless 0$ as $\tau \gtreqless 1$. (5.22e)

An increase in the payroll tax rate lower or increase the employment rate depending on the degree of workers' valuations of benefits, τ. These results parallel the standard results for the payroll taxes in the one-sector neoclassical model, without unemployment or tax evasion. In that model, full valuation of benefits leads to what is called "Full Pass-through", where an increase in the payroll tax rate does not alter the equilibrium employment rate, and is entirely absorbed in the form of lower wages. Given full valuation of benefits, workers are, in theory, indifferent to increases in the payroll tax rate, which while lowering the equilibrium base wage, raise their net benefits, shifting out the supply curve to fully compensate the inward shift in aggregate employment demand

In the current context we also obtain full pass through, but in terms no only of the level of employment, as in the standard neoclassical model, but in terms of the employment rate.

R.1.a $\dfrac{\partial \ln e^*}{\partial \delta_1} = 0$ for $\tau = 1$.

However, in general it would seem more reasonable that workers value contributions only partially. This is plausible even more likely in developing countries with substantial inefficiencies in service provision and, often, endemic corruption. Under this assumption, higher payroll taxes lower the equilibrium employment rate.

R.1.b. $\dfrac{\partial \ln e^*}{\partial \delta_1} < 0$ for $\tau < 1$.

Summary

In the foregoing model identical firms face the choice between paying payroll taxes (legality) and evading payroll taxes (non-legality). Non-legal firms face an expected penalty. Because endogenous financing of detection, with revenues from detected and fined non-legal firms, generally leads to a probability of detection that rises with the share of non-legal ("informal") firms, the model is consistent with a Stable Internal Solution, where in equilibrium, legal and non-legal firms coexist, as observed in most developing countries, and to a lesser degree in some developed countries. We showed that, as is consistent with intuition, rising payroll taxes lead to greater non-compliance, as in the g-t-p and L96 models. And, as in the earlier models, given tax rates, higher exogenous taxation or greater monitoring efficiency leads to greater compliance.

We then coupled this structure with a model of aggregate employment, wages and unemployment, in the context of the two-sector framework. We found that higher taxes lead to higher equilibrium unemployment rate.Thus, higher taxes increase "informality" and the equilibrium unemployment rate. In the next section, we present simulations of the actual increases in Colombian payroll tax rates upon the equilibrium unemployment rate.

Simulations

We employ the historic changes in Colombian payroll taxes since 1990 through 2008 (planned) in combination with the derivative of the natural log of the equilibrium employment rate with respect to the payroll tax, and do sensitivity testing of the predicted changes, varying parameters of the production function, a, the elasticity of the wage-curve, σ, and the degree or worker valuation of benefits, τ. We vary these parameters around consensus international estimates, and calibrate the model from the employment level in 1990. The table below summarizes these simulations.

The rise in Colombian payroll tax rates over this period would have generated an increase in the unemployment rate ranging from 0 ($\tau = 1$, or Full-Pass-Through) to a maximum of 9.2 percent. This maximum would occur if workers did not value benefits and if the wage curve where significantly more inelastic than international evidence suggests.

Simulations For the Wage Curve Model with Two Sectors

(Assumptions: c, the ratio of employee to employer payroll taxes, =0.33 (relation between delta 1 y delta 2); change in delta1, $d\delta_1 = 0.1$; initial employment rate of 85%)

$\sigma = 0.7$						$\sigma = 0.8$						$\sigma = 0.9$					
σ	a	τ	δ_1 (initial)	$\frac{\partial \ln e^*}{\partial \delta_1}$	**Change in Employment Rate**	σ	a	τ	δ_1 (initial)	$\frac{\partial \ln e^*}{\partial \delta_1}$	**Change in Employment Rate**	σ	a	τ	δ_1 (initial)	$\frac{\partial \ln e^*}{\partial \delta_1}$	**Change in Employment Rate**
0.7	0.5	0.0	0.3	-0.950	-7.7%	0.8	0.5	0.0	0.3	-0.877	-7.1%	0.9	0.5	0.0	0.3	-0.814	-6.6%
		0.0	0.5	-0.889	-7.2%			0.0	0.5	-0.821	-6.7%			0.0	0.5	-0.762	-6.2%
		0.1	0.3	-0.818	-6.7%			0.1	0.3	-0.755	-6.2%			0.1	0.3	-0.701	-5.8%
		0.1	0.5	-0.741	-6.1%			0.1	0.5	-0.684	-5.6%			0.1	0.5	-0.635	-5.2%
		0.3	0.3	-0.587	-4.8%			0.3	0.3	-0.541	-4.5%			0.3	0.3	-0.503	-4.2%
		0.3	0.5	-0.502	-4.2%			0.3	0.5	-0.463	-3.8%			0.3	0.5	-0.430	-3.6%
		0.5	0.3	-0.389	-3.2%			0.5	0.3	-0.359	-3.0%			0.5	0.3	-0.333	-2.8%
		0.5	0.5	-0.317	-2.7%			0.5	0.5	-0.293	-2.5%			0.5	0.5	-0.272	-2.3%
		0.7	0.3	-0.217	-1.8%			0.7	0.3	-0.201	-1.7%			0.7	0.3	-0.186	-1.6%
		0.7	0.5	-0.171	-1.4%			0.7	0.5	-0.158	-1.3%			0.7	0.5	-0.147	-1.2%
		0.9	0.3	-0.068	-0.6%			0.9	0.3	-0.063	-0.5%			0.9	0.3	-0.058	-0.5%
		0.9	0.5	-0.052	-0.4%			0.9	0.5	-0.048	-0.4%			0.9	0.5	-0.044	-0.4%
		1.0	0.3	0.000	0.0%			1.0	0.3	0.000	0.0%			1.0	0.3	0.000	0.0%
		1.0	0.5	0.000	0.0%			1.0	0.5	0.000	0.0%			1.0	0.5	0.000	0.0%
	0.7	0.0	0.3	-1.140	-9.2%		0.7	0.0	0.3	-1.036	-8.4%		0.7	0.0	0.3	-0.950	-7.7%
		0.0	0.5	-1.067	-8.6%			0.0	0.5	-0.970	-7.9%			0.0	0.5	-0.889	-7.2%
		0.1	0.3	-0.982	-8.0%			0.1	0.3	-0.893	-7.3%			0.1	0.3	-0.818	-6.7%
		0.1	0.5	-0.889	-7.2%			0.1	0.5	-0.808	-6.6%			0.1	0.5	-0.741	-6.1%
		0.3	0.3	-0.704	-5.8%			0.3	0.3	-0.640	-5.3%			0.3	0.3	-0.587	-4.8%
		0.3	0.5	-0.602	-5.0%			0.3	0.5	-0.547	-4.5%			0.3	0.5	-0.502	-4.2%
		0.5	0.3	-0.466	-3.9%			0.5	0.3	-0.424	-3.5%			0.5	0.3	-0.389	-3.2%
		0.5	0.5	-0.381	-3.2%			0.5	0.5	-0.346	-2.9%			0.5	0.5	-0.317	-2.7%
		0.7	0.3	-0.261	-2.2%			0.7	0.3	-0.237	-2.0%			0.7	0.3	-0.217	-1.8%
		0.7	0.5	-0.205	-1.7%			0.7	0.5	-0.186	-1.6%			0.7	0.5	-0.171	-1.4%
		0.9	0.3	-0.081	-0.7%			0.9	0.3	-0.074	-0.6%			0.9	0.3	-0.068	-0.6%
		0.9	0.5	-0.062	-0.5%			0.9	0.5	-0.056	-0.5%			0.9	0.5	-0.052	-0.4%
		1.0	0.3	0.000	0.0%			1.0	0.3	0.000	0.0%			1.0	0.3	0.000	0.0%
		1.0	0.5	0.000	0.0%			1.0	0.5	0.000	0.0%			1.0	0.5	0.000	0.0%

The central role of valuation is evident in the prior table. It is methodologically difficult, with available data, to obtain reliable estimates of τ. And estimates of τ do not exist for Colombia, presenting an important challenge for future research. In our opinion, for a country such as Colombia, where there is widespread evidence that inefficiency and corruption is common, generally, we believe that a more reasonable assumption for the value of τ is one half.

The next table simulates effects assuming $\tau = .5$. The table is divided into two parts. The first part simulates the effect of the increases in payroll taxes over 1990-2008(planned), while the second part simulates the effects from the reforms of 2003(lay 797) through 2008, taking as the initial employment level that which obtained in 2003. [16]

Simulations for the Wage Curve Model with Two Sectors for Colombia							
$\tau = 0.5$							
Wage Curve: σ	Production Function: *a*	**1990-2008** (Employment rate in 1990=89.8%) (Unemployment rate in 1990=10.2%)			**2003-2008** (Employment rate in 2003=83%) (Unemployment rate in 2003=17%)		
		Change in Employment Rate	**Predicted employment rate for 2008**	**Predicted unemployment rate for 2008**	**Change in Employment Rate**	**Predicted employment rate for 2008**	**Predicted unemployment rate for 2008**
0.8	0.5	*-2.7%*	87.1%	12.9%	*-0.6%*	82.4%	17.6%
	0.7	*-3.2%*	86.6%	13.4%	*-0.8%*	82.2%	17.8%
0.9	0.5	*-2.5%*	87.3%	12.7%	*-0.6%*	82.4%	17.6%
	0.7	*-3.0%*	86.8%	13.2%	*-0.7%*	82.3%	17.7%
1.2	0.5	*-2.1%*	87.7%	12.3%	*-0.5%*	82.5%	17.5%
	0.7	*-2.4%*	87.4%	12.6%	*-0.6%*	82.4%	17.6%
Assuming a 1% rise in pension contribution in 2008.							

Based upon these assumptions, the simulations suggest that the overall increase in the equilibrium unemployment rate over the 1990-2008 period may have ranged from 2.1 to 3.2 percent. These are substantial risings in permanent unemployment. However, they are far short of the cyclic increases in the unemployment rate experienced over the 1995-2000 period. For the 2003-2008 period, the predicted increase in the equilibrium unemployment rate from the 2003 reform, ranges from .5 to .8 percent.

Summary

In the foregoing model identical firms face the choice between paying payroll taxes (legality) and evading payroll taxes (non-legality). Non-legal firms face an expected penalty. Endogenous financing and detection constitutes a self-regulating mechanism and is consistent with a Stable Internal Solution, where legal and non-legal firms coexist, as observed in developing countries. Rising payroll taxes lead to greater non-compliance, as in the g-t-p and L96 models. And, higher exogenous taxation or greater monitoring efficiency leads to greater compliance.

Combining this structure with a model of aggregate employment, wages and unemployment, in the context of the two-sector framework, we found that higher taxes lead to higher equilibrium unemployment rate. Higher taxes increase "informality" and the equilibrium unemployment rate.

Simulations suggest that the secular upward trend in Colombian payroll taxes is likely to have led to substantial increases in the equilibrium unemployment rate. In addition, this upward trend almost certainly would have increased the degree of non-compliance with payroll taxes, swelling the size of the "informal" sector.

[16] That employment level is, however, unlikely to have been at the long-run equilibrium level, given that the effects of the major recession in Colombia that began in 1996, were still being felt.

Section VI. Conclusion

Recent models of the Informal Sector emphasize the free choice of sectors by firms or workers and define informality firms as those that do not pay taxes. One dominant group of models examines identical firms producing a homogeneous final good, where legal firms pay taxes and receive a public good, while non-legal firms do not pay taxes and receive a smaller amount of the public good. These models may be consistent with a stable equilibrium where there are both Formal (legal) and Informal (non-legal) firms, as observed in developing countries.

This paper first presents a generic and somewhat general version model with public goods and taxation model ("g-t model"), finding that most equilibria are unstable, so that all firms choose to be either legal or non-legal, which is inconsistent with the observed facts for developing countries. However, when the government finances detection of non-legal firms and fines those firms, stable 'internal' solutions may dominate. This requires a probability of detection that falls as the number of legal firms increases.

A general model of detection, penalties and their public finance is presented and analyzed, finding that the probability of detection often, though not always, falls with the number of legal firms. The role of exogenous financing of detection is emphasized, where greater exogenous financing reduces informality and if high enough constitutes complete prevention, inducing all firms to be formal. This is both a *positive* and *normative* finding, potentially explaining the absence of informal firms in some countries, and the presence in others, while also constituting a policy instrument for eradicating informality.

As in the g-t model, the Loayza model of informality requires a probability of detection that falls as the number of legal firms rises for a stable internal solution. Thus, this general model of detection partially validates the Loayza model. It is also observed that in the Loayza model, governments may eliminate informality without penalties, by lowering the tax rate sufficiently. This also has *positive* and *normative* implications.

In Section V we presented a model of payroll taxes with detection, based on the model of detection in Section III and the structure of taxation and detection in Loayza (1996). This provides the basis of a consistent model of payroll taxes and the size of the non-legal or 'informal' sector.

The final section of the paper, Section V, applied the foregoing results to model firms that face the choice between paying payroll taxes (legality) and evading payroll taxes (non-legality). Non-legal firms face an expected penalty. Because endogenous financing of detection, with revenues from detected and fined non-legal firms, generally leads to a probability of detection that rises with the share of non-legal ("informal") firms, the model is consistent with a Stable Internal Solution, where legal and non-legal firms coexist, as observed in most developing countries, and to a lesser degree in some developed countries. We showed that, as is consistent with intuition, rising payroll taxes lead to greater non-compliance, as in the g-t-p and L96 models. And, as in the earlier models, given tax rates, higher exogenous taxation or greater monitoring efficiency leads to greater compliance.

We then coupled this structure with a model of aggregate employment, wages and unemployment, in the context of the two-sector framework. We found that higher taxes lead to higher equilibrium unemployment rate. Higher taxes increase "informality" and the equilibrium unemployment rate. In the final section, we presented simulations of the actual increases in Colombian payroll tax rates upon the equilibrium unemployment rate. We found that the increases in payroll rates over the 1990-2008(planned) period may have contributed to an increase of approximately three percent in the equilibrium unemployment rate, along with having raises non-compliance, or "informality".

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